

INVESTIGATION OF MEDIUM AND HEAVY NUCLEI BY
QUASI-FREE PROTON-PROTON SCATTERING*

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(Received August 16, 1960)

Recently, experimental groups in Uppsala,¹ Harwell,² and Chicago³ have directly demonstrated the shell structure of several light nuclei by measuring the energy and angular correlations of proton pairs resulting from high-energy (150-440 Mev) quasi-free knockout processes. The binding energies and interesting features of the momentum distributions of the protons in the $1s$ and $1p$ shells and the spin-orbit splitting of the $1p$ shell have been determined in this way.

Up to now these measurements have been limited to light nuclei, mainly because the cross section for knockout processes in the inner shells and the level distances in the upper shell decrease rapidly with increasing nuclear radius.⁴ It is the purpose of this note to point out that high-energy quasi-free $p-p$ scattering is not only able to supply new information on the light nuclei, but is also a promising tool for the investigation of medium and heavy nuclei. It is expected that the limitation in size of the effective interaction zone, resulting from the absorption of the protons by the nucleus, should give rise to a typical angular correlation pattern from which several important quantities may be read off directly.

Throughout this paper the indices 0, 3, 1, 2, i , and f refer, respectively, to the incoming and the nuclear proton before the collision, to the two outgoing protons, and to the initial and final nucleus. The symbol \vec{k} refers to momentum; α_i is the angle which \vec{k}_i makes with the projection of \vec{k}_0 on the plane through \vec{k}_1 and \vec{k}_2 (see Fig. 1); m denotes a magnetic quantum number and \bar{P} the parity operator.

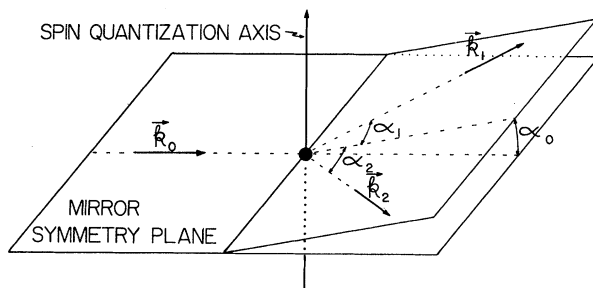


FIG. 1. Definition of angles and plane of mirror symmetry.

Consider as an example the coplanar ($\alpha_0 = 0$) quasi-free scattering of 150-Mev protons by medium and heavy nuclei and choose α_1 and α_2 not too different from 45° . Comparing the mean free paths in nuclear matter of the involved protons with the total length of their trajectories in the nucleus, one sees that the bulk of the protons which suffer only one collision originates from the two pole caps at the ends of the nuclear symmetry axis orthogonal to the scattering plane.^{5,6} The contribution of the multiply scattered proton pairs to the correlation cross section is smeared out over the whole available energy and angular range. It results in a smooth background in the measured energy spectrum on which the peaks caused by the relevant clean knockout collisions are superimposed. It is to be expected that the height of the aforementioned pole caps is small compared with their mutual distance D . The contributions of the two pole caps will, because of symmetry reasons, be equal in magnitude, though they have in general a phase difference η . If one changes α_0 to small nonzero values (keeping α_1 , α_2 , k_1 , k_2 constant), the magnitudes of the pole cap contributions remain approximately equal, but their phase difference becomes $\eta + \alpha_0 k_0 D$. For the transition to a definite final state of the nucleus, the contributions of the two pole caps will add coherently and the correlation cross section will show an interference pattern of the form

$$|G(\alpha_0, \alpha_1, \alpha_2, k_1, k_2)|^2 \cos^2[\frac{1}{2}(\alpha_0 k_0 D + \eta)]. \quad (1)$$

As a function of α_0 , $|G|^2$ varies slowly as compared to the \cos^2 term (which in our example has a period of about 10°).

The value of η can be found by a symmetry argument. Consider the operator Q_z which mirrors the wave function through the plane $\alpha_0 = 0$. We quantize the spins in the direction orthogonal to this plane and note that Q_z equals the parity operator times a rotation by an angle π around the quantization axis. The initial state is an eigenstate of Q_z with eigenvalue $P_i \exp[i(m_i + m_0)\pi]$. The mirror parity operator of the final state is $P_f \exp[i(m_f + m_1 + m_2)\pi] Q_{\alpha_0}$, where Q_{α_0} is the mirror parity of the interference pattern. Parity

conservation results in the condition:

$$Q_{\alpha_0} = P_i P_f \exp[i\Delta m\pi], \quad (2)$$

with

$$\Delta m = (m_i + m_0) - (m_f + m_1 + m_2).$$

Because of fermion conservation, Δm is an integer. Thus, only the cases $Q_{\alpha_0} = \pm 1$ occur.

Using the single-particle model and the impulse approximation (for which, by the same symmetry arguments, $m_0 + m_3 - m_1 - m_2 = 0$), one obtains $Q_{\alpha_0} = (-1)^{l+m}$, where l and m refer to the Y_l^m component of the state from which the proton has been knocked out.

From (1) and (2) it follows, independent of the experimental energy resolution, that the correlation cross section will have the form

$$a(\alpha_0, \alpha_1, \alpha_2, k_1, k_2) \cos^2(\frac{1}{2}\alpha_0 k_0 D) + b(\alpha_0, \alpha_1, \alpha_2, k_1, k_2) \sin^2(\frac{1}{2}\alpha_0 k_0 D), \quad (3)$$

where a and b are slowly varying functions of α_0 .

If a measurement is made in such a way that many final states of different character contribute statistically to the cross section, a and b will have similar magnitudes and the α_0 -dependent interference pattern will fade out. On the basis of the shell model and of the nature of the reaction, one expects to be able to resolve groups of states which correspond to the shell-model state from which the proton has been ejected. But, because the m degeneracy cannot be resolved, one in general will still have a mixture of \cos^2 and \sin^2 contributions.

For this last case, detailed calculations⁶ have been made with a single-particle model, the impulse approximation, and a simplified distortion for the incoming and outgoing waves. After

summing over the magnetic quantum numbers, the coefficients a and b are shown to be strongly different for small momentum transfers (nonrelativistically: $\alpha_1 \approx \alpha_2 \approx 45^\circ$; $k_1 \approx k_2 \approx \frac{1}{2}\sqrt{2} k_0$). The expected α_0 -dependent interference pattern is therefore pronounced and gives by its period the distance between the pole caps, by its modulation the height of the pole caps, and by its phase the parity of the shell-model state from which the proton has been knocked out. Furthermore, it turns out that the orbital angular momentum of this state can be found from the (α_1, α_2) dependence of the correlation cross section.

Finally, we want to point out two necessary conditions for the applicability of our general treatment. At a given bombarding energy the nucleus to be investigated must be at least so heavy that the heights of the contributing pole caps are rather small compared to the nuclear radius. On the other hand, these heights should not be too small, because otherwise the absolute magnitudes of the pole cap contributions become already very different for small values of α_0 and the correlation pattern fades out. A more quantitative prediction requires distorted wave calculation for each case.

*Work partially supported by Comissão Supervisora dos Institutos and Conselho Nacional de Pesquisas, Brasil and by the U. S. National Science Foundation.

¹H. Tyrén, Peter Hillman, and Th. A. J. Maris, *Nuclear Phys.* **7**, 10 (1958). Peter Hillman, H. Tyrén, and Th. A. J. Maris (to be published).

²T. J. Gooding and H. J. Pugh (to be published). H. G. Pugh and K. F. Riley (private communication).

³H. Tyrén and P. Isacson (private communication).

⁴Th. A. J. Maris, Peter Hillman, and H. Tyrén, *Nuclear Phys.* **7**, 1 (1958).

⁵A. J. Kromminga and I. E. McCarthy, *Phys. Rev. Letters* **4**, 288 (1960).

⁶Gerhard Jacob and Th. A. J. Maris (to be published).