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QUANTUM ELECTRODYNAMICS WITH ZERO BARE FERMION MASS

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It is the purpose of this Letter to show that the logarithmic divergence of the fermion self-mass in lowest order perturbation theory of quantum electrodynamics disappears if the approximation is sufficiently improved and the bare fermion mass is assumed to be zero.

We start from the Dyson equation for the fermion without the bare mass term; as in lowest order perturbation theory we neglect the corrections in the photon propagator and in the vertex and put $Z_2=1$:

$$S_F'^{-1}(p)=i\not{p}-\frac{i\alpha}{4\pi^3}\int\gamma_\mu S_F'(k)\gamma_\nu \times\left[g^{\mu\nu}-\frac{(p-k)^\mu(p-k)^\nu}{(p-k)^2}\right]\frac{1}{(p-k)^2-i\epsilon}d^4k. \quad (1)$$

The use of the Landau gauge[1] has the result that the $\not{p}$ part of the self-energy integral vanishes.[2] Defining the function $m(p^2)$ by[1]

$$S_F'^{-1}(p)=i\not{p}+m(p^2), \quad (2)$$

one finds the following nonlinear integral equation:

$$m(p^2)=-\frac{i3\alpha}{4\pi^3}\int\frac{m(k^2)}{m^2(k^2)+k^2-i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}d^4k. \quad (3)$$

This equation can be reduced to a nonlinear differential equation in momentum space[3] which, however, is difficult to solve.

Because the theory does not contain a constant with the dimension of a length, Eq. (3) is dilatationally invariant[3,5]: If $m(p^2)$ is a solution, the same is true of $\lambda^{-1}m(\lambda^2p^2)$. As we will see, the solutions of Eq. (3) are, for not too high momenta, closely approximated by the usual second-order perturbation result:

$$m(k^2)=m+\alpha f(k^2) \quad (4)$$

with $f(-m^2)=0$. The constant m corresponds to the mass of the particle and may be used as a label to characterize a certain member of the set of solutions. Concentrating on one of the solutions of Eq. (3), defined by $m=m_1$, we make for nearly all momenta only a small error if we replace in the denominator of the right-hand side of Eq. (3) $m(k^2)$ by m_1. For not too high values of k^2 this is true because of Eq. (4) and for ultra-high momenta $m^2(k^2)$ is negligible compared with

k^2. The only considerable change we may have made in the equation is in the immediate vicinity of the pole, but this is an infrared problem which is at present not of interest. The usual lowest order perturbation approximation is equivalent to also replacing the numerator in the integrand by a constant; we will show this to be a very poor approximation at ultrahigh values of k^2.

We now have

$$m(p^2) = -\frac{i3\alpha}{4\pi^3}\int\frac{m(k^2)}{m_1^2+k^2-i\epsilon}\frac{1}{(p-k)^2-i\epsilon}d^4k. \quad (5)$$

This is a linear equation, but our approximation, together with Eq. (2), implies the normalization

$$m(-m_1^2) = m_1. \quad (6)$$

We will show that Eq. (5), together with condition (6), allows an exact and physically reasonable solution for <u>all</u> values of the coupling constant satisfying the condition $0<\alpha<\pi/3$. Putting

$$A(k^2) = \frac{m(k^2)}{m_1^2+k^2-i\epsilon} \quad (7)$$

and going over to configuration space, we get for $x \neq 0$

$$(\Box - m_1^2)A(x^2) = -(3\alpha/\pi)(1/x^2)A(x^2),$$

or

$$\left[4u\frac{d^2}{du^2}+8\frac{d}{du}+\left(\frac{3\alpha}{\pi}\frac{1}{u}-m_1^2\right)\right]A(u)=0,\ u\equiv x^2\neq 0. \quad (8)$$

This equation has a solution with only positive-frequency components

$${}_\nu\Delta^+(x,m_1) = i\frac{m_1^2}{8\pi}\lim_{\eta\to 0}\left[\frac{i^{\nu+1}H_\nu^{(1)}(im_1[(x-i\eta)^2]^{1/2})}{m_1[(x-i\eta)^2]^{1/2}}\right], \quad (9)$$

where η is a positive-timelike vector and the index ν of the Hankel function is $\nu=(1-3\alpha/\pi)^{1/2}$. A solution with only negative frequencies is given by

$${}_\nu\Delta^-(x,m_1) = -{}_\nu\Delta^+(-x,m_1). \quad (10)$$

From Eq. (7) it follows that the solution $A(x)$ has the form of a Feynman amplitude:

$$A(x) = C[\theta(t)\,{}_\nu\Delta^+(x,m_1) - \theta(-t)\,{}_\nu\Delta^-(x,m_1)] = C\,{}_\nu\Delta_F(x,m_1), \quad (11)$$

C being a normalization constant. For spacelike x one has

$${}_\nu\Delta^-(x,m_1) = -{}_\nu\Delta^+(x,m_1)$$

as is necessary for the Lorentz invariance of Eq. (11).

The ${}_\nu\Delta$ functions defined in Eqs. (9), (10), and (11) are generalizations of the corresponding free invariant functions ($\nu=1$), which they approach asymptotically for large x^2.

From Eqs. (5), (6), and (7), A should be normalized to give

$$m(p^2) = -\frac{i3\alpha}{4\pi^3}\int A(p-k)\frac{1}{k^2-i\epsilon}d^4k = m_1 \quad \text{for } p^2=-m_1^2. \quad (12)$$

From a physical point of view this equation reflects the condition that, for zero bare mass, the electromagnetic self-mass should be the total one. The integral in Eq. (12) can be written as a four-dimensional integral in Euclidean momentum space.[3] Going over to Euclidean configuration space and integrating over angles, we finally obtain

$$m(p^2) = \frac{3\alpha}{\pi}m_1C\int_0^\infty\frac{K_\nu(m_1x)J_1(px)}{px}dx, \quad (13)$$

using the standard notation for Bessel functions.

In contradistinction to the free case ($\nu=1$), the integral with $\nu=(1-3\alpha/\pi)^{1/2}$ has no divergence at $x=0$ [corresponding to infinite k^2 in Eq. (12)] and consequently the <u>self-mass integral is finite.</u> Evaluating the integral (13), one obtains for $|p^2| \leq m_1^2$

$$m(p^2) = C\frac{3\alpha}{4\pi}\Gamma\left(\frac{1+\nu}{2}\right)\Gamma\left(\frac{1-\nu}{2}\right) \times {}_2F_1\left(\frac{1+\nu}{2},\frac{1-\nu}{2},2;-\frac{p^2}{m_1^2}\right), \quad (14)$$

from which follows, for $p^2=-m_1^2$,

$$C = m_1. \quad (14')$$

From Eq. (2) the Fermion propagator is

$$S_F'(p) = [m(p^2)-i\not{p}]/[m^2(p^2)+p^2-i\epsilon]. \quad (15)$$

We could now proceed to solve Eqs. (7) and (11) for $m(k^2)$, or, equivalently, to analytically continue expression (14), and insert the result in Eq. (15). At present, however, we are only interested in a <u>zero-order</u> propagator which is also suitable for ultrahigh momenta. For this purpose one may approximate expression (15) by

$$S_F^0(p) = \frac{m(p^2) - i\not{p}}{m_1^2 + p^2 - i\epsilon} = A(p^2) - \frac{i\not{p}}{m_1^2 + p^2 - i\epsilon}. \quad (16)$$

Inserting $A(x)$ from Eqs. (11) and (14′) one obtains

$$S_F^0(x, m) = m\,{}_\nu\Delta_F(x, m) - \not{\partial}\,{}_1\Delta_F(x, m). \quad (17)$$

For $\nu = 1$ this is the usual free propagator. As expected on general grounds[4] the total propagator is not less singular at $x^2 = 0$ than the free one, but, as we are working in the Landau gauge, only the first term on the right-hand side contributes to the self-energy integral.

Because ${}_\nu\Delta_F(x, m)$ and ${}_1\Delta_F(x, m)$ have the same asymptotic behavior for large x^2 (small momenta), one may in this region use the free propagator as a zero-order approximation. For small values of x^2, however, ${}_\nu\Delta_F$ goes like $x^{-1-\nu}$; expanding $x^{-1-\nu}$ in a power series of α, one obtains

$$x^{-1-\nu} = x^{-2}[1 + (3\alpha/2\pi)\log x + \alpha^2(\cdots) + \cdots].$$

The convergence of this expression becomes poor for values of x of the order of $\exp(-2\pi/3\alpha)$ crudely corresponding to momenta of the order of $m\exp(2\pi/3\alpha)$. Although the left-hand side gives a finite result if inserted in the self-mass integral (12), this is not true of any of the individual terms on the right-hand side. In particular the first term reproduces the well-known logarithmic divergence of the lowest order perturbation theory.

In order to compensate for this divergence it is usual to introduce either an infinite bare fermion mass or a cutoff into the theory. As we saw, the assumption of a zero bare fermion mass results in the usual renormalized quantum electrodynamics. The present approach has several attractive features. The basic equations of the theory are better defined than in the case of an infinite bare mass, the mass renormalization is finite without the use of a cutoff, and a connection between the massive charged leptons and the massless neutral ones appears. Furthermore, the dilatational invariance of the theory is appealing from a general point of view[5] and could justify some hope for a calculation of the e-μ mass ratio and of the ratio between the mass scale of the lepton system and that of the group of strongly interacting particles.[6,3]

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The figure numbers and captions should be interchanged for Figs. 2 and 3. Figure 2, the resonance with and without stress, is at the upper left; Figure 3, the resonance after preferential bleaching, is at the lower right.