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Analysis of low-x gluon density from the F_2 scaling violations

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Abstract

We present an analysis of the current methods of the approximate determination of the gluon density $G(x,Q^2)$ at low x from the scaling violations of the proton structure function $F_2(x,Q^2)$. The gluon density is obtained from the GLAP equation by expansion at distinct points of expansion. We show that the different results given by the proposed methods can be obtained from one new expression considering the adequate points of expansion. It is shown in which case the approximate determination of $G(x,Q^2)$ is reasonable.

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1. Introduction

The standard perturbative QCD framework predicts that at a regime of low values of the Bjorken variable x ($x \approx 10^{-4}$) and large values of Q^2, a nucleon consists predominantly of sea quarks and gluons. The gluons couple only through the strong interaction, consequently the gluons are not directly probed in the DIS, only contributing indirectly via the $g \to q\bar{q}$ transition. Therefore, its distribution is not so well determined as those of the quarks.

Recently, a direct determination of the gluon density was performed in the kinematical region $x > 10^{-3}$ [1]. A leading order determination of the gluon density in the proton was performed by measuring multijet events from boson-gluon fusion in deep-inelastic scattering with the detector H1 at the electron-proton collider HERA. The data agree with the dramatic rise of the gluon density with decreasing x predicted by the traditional analysis of $F_2(x,Q^2)$ [2]. These analysis have performed next-to-leading order (NLO) QCD fits based on the Altarelli-Parisi (GLAP) [3] evolution equations of the parton densities. The initial densities should be determined by the experiment at some scale $Q^2 = Q_0^2$.

The parton densities are well determined for $x > 0.02$, where there are many different types of high-precision constraints on the individual parton distributions. The exception is the gluon density which is mainly constrained by (i) the momentum sum rule, (ii) prompt photon production and (iii) the scaling violations of F_2. On the other hand at small x ($x \leq 10^{-3}$) we have only the measurement of the structure function $F_2(x,Q^2)$, and many different partonic descriptions at small x, for example, GLAP [3] and BFKL [4]. Therefore it is important to obtain more information in this region, mainly of the gluon den-

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sity, which is the dominant parton at small x.

At low x the GLAP equation [3] for F_2 becomes, for four flavors

$$\frac{dF_2(x,Q^2)}{d\log Q^2} = \frac{10\alpha_s}{9\pi}\int_0^{1-x} dz P_{qg}(z) \times G\left(\frac{x}{1-z}, Q^2\right). \qquad (1)$$

where $G(x,Q^2) = xg(x,Q^2)$ is the gluon density momentum with $g(x,Q^2)$ being the gluon density, $\alpha_s = \alpha_s(Q^2)$ is the strong coupling constant and the splitting function $P_{qg}(x')$ gives the probability of finding a quark inside a gluon with momentum x' of the gluon.

This equation describes the evolution of the proton structure function $F_2(x,Q^2)$ in LO at small x. From experiments one can measure the structure function and then determine its derivative $\frac{dF_2(x,Q^2)}{d\log Q^2}$. Some methods [5,6] were proposed in the literature to isolate the gluon density by its expansion inside Eq. (1), but have different results for $G(x,Q^2)$. We demonstrate here that one of the methods is not totally correct and that the results of both methods can be obtained by using different points of expansion from one general expression. This letter is organized as follows. In Section 2 the methods proposed in the literature are presented. We show that one of these methods is not totally correct and requires a modification. In Section 3 a general expression for the approximate determination of the gluon density is proposed. We determine the appropriate points of expansion using data for $\frac{dF_2(x,Q^2)}{d\log Q^2}$, obtained by HERA [7] and also for a screening model [8] which has recently appeared in the literature. Finally, in Section 4, we present our conclusions.

2. Analysis of approximative methods

The methods of the approximate determination of the gluon density are based on a simplification of the convolution $P_{qg} \otimes g$ (Eq. (1)) by the expansion of the gluon density. The result is the gluon density $G(k.x)$ proportional to the derivative of $F_2(x,Q^2)$, where the constant k is associated with the choice of a point of expansion and $x = x_B$. In DIS we do not probe the gluon directly, but instead via the process $g \rightarrow q\bar{q}$. The longitudinal fraction x_g of the proton's momentum carried by the gluon is therefore sampled over an interval bounded below by the Bjorken x_B variable $x_B \equiv \frac{Q^2}{2p\cdot q}$, where as usual p and q are the four-momenta of the incoming proton and virtual photon respectively, and $Q^2 \equiv -q^2$. Therefore, the choice of the point of expansion, and consequently of k, is associated with the relation between x_g and x_B. We discuss below the methods of approximate determination of gluon density proposed in the literature.

2.1. Prytz's approach

Expanding the gluon distribution around $z = \frac{1}{2}$ and approximating the upper integration limit $1 - x \approx 1$ in Eq. (1) we can get [5]

$$G(2x) \approx \frac{27\pi}{10\alpha_s}\frac{dF_2(x,Q^2)}{d\log Q^2}. \qquad (2)$$

Estimates of gluon density [1] have demonstrated that this approximation agrees with the gluon density obtained with other indirect determinations made by H1 and Zeus Collaborations.

2.2. Bora et al.'s approach

This method is based in expanding the gluon distribution around $z = 0$. Substituting the result in Eq. (1) one gets [6]

$$\frac{dF_2(x,Q^2)}{d\log Q^2} \approx \frac{5\alpha_s}{9\pi}A(x) \times \left\{G(x) + \frac{B(x)}{A(x)}x\frac{dG}{dx}\right\} \qquad (3)$$

where

$$A(x) = \frac{2(1-x)^3}{3} - (1-x)^2 + (1-x)$$

$$B(x) = \frac{(1-x)^2}{2} - \frac{2(1-x)^3}{3} + \frac{(1-x)^4}{2}.$$

Bora et al. [6] claim that this expression in the limit $x \rightarrow 0$ can be approximated by

$$G(\tfrac{4}{3}x) \approx \frac{9\pi}{5\alpha_s 3}\frac{dF_2(x,Q^2)}{d\log Q^2}. \qquad (4)$$

In Fig. 1 we compare the results of $\frac{dF_2(x,Q^2)}{d\log Q^2}$ for the approximate relations of Prytz, Eq. (2), Bora et

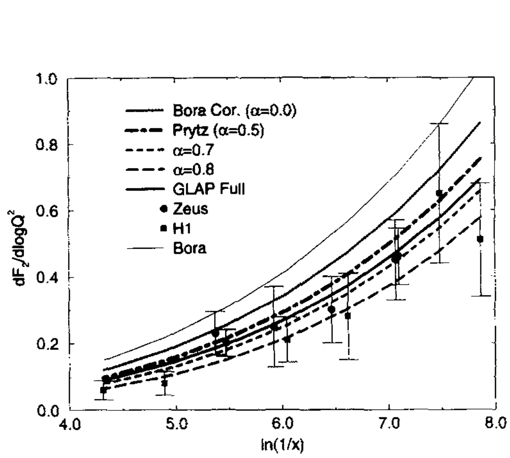

Fig. 1. Results of the $\frac{dF_2(x,Q^2)}{d\log Q^2}$ obtained from relations of Prytz (2), Bora (3), (1) and of Eq. (7), using GRV94(LO) [2]. Also plotted the behaviour obtained at several points of expansion α. Dat from H1 and ZEUS [7] at $Q^2 = 20$ GeV^2.

al., Eq. (3), and the complete expression, Eq. (1), using the gluon distribution GRV94(LO) [2]. In our calculations the value of α_s is obtained at $\Lambda^{(4)}_{\overline{MS}} = 0.232$ GeV^2. The figure shows that Prytz approximation is more accurate.

The result (4) differs from Prytz's result (2). Bora et al. explain that the difference arises because they have retained the x dependence in the upper limit of integration in Eq. (1) after the expansion of the gluon density. Calculating Eq. (1) without approximation in the upper integration limit, expanding the gluon distribution around $z = 0$, we find that this is not the case. Also, Bora et al.'s method requires that

$$\frac{[B(x)]^2}{A(x)[A(x)+2B(x)]} \approx 0$$
$$\frac{A(x)+B(x)}{A(x)+2B(x)} \approx 1\,. \tag{5}$$

At small values of x the behaviour of these expressions disagrees with the approximations. Consequently, the Bora approximation is not valid in this x range.

A more adequate approximation of Eq. (3) is

$$\frac{dF_2(x,Q^2)}{d\log Q^2} \approx \frac{5\alpha_s}{9\pi} A(x) G\left(x + \frac{B(x)}{A(x)} x\right). \tag{6}$$

In the limit $x \to 0$, this equation becomes

$$\frac{dF_2(x,Q^2)}{d\log Q^2} \approx \frac{5\alpha_s}{9\pi} \tfrac{2}{3} G(\tfrac{3}{2}x)\,. \tag{7}$$

In Fig. 1 we presented the behaviour of the Prytz, Eq. (2), and Bora, Eq. (3), results and the one obtained from Eq. (7), called Bora corrected. Eq. (7) also differs from the Prytz expression (2). This is due to the election of a different point of expansion of the gluon density in both cases.

3. Expansion at an arbitrary point of expansion

Using the expansion of the gluon distribution $G(\frac{x}{1-z})$ at an arbitrary $z = \alpha$ and retaining terms only up to the first derivative in the expansion, we get in the limit $x \to 0$ that the gluon distribution for a point of expansion $\alpha < 1$ can be expressed by

$$G\left[\frac{x}{1-\alpha}(\tfrac{3}{2} - \alpha)\right] \approx \frac{9\pi}{5\alpha_s} \frac{3}{2} \frac{dF_2(x,Q^2)}{d\log Q^2}\,. \tag{8}$$

Eq. (8) can be reduced to Prytz (2) and to the gluon distribution (7), in their respective points of expansion $\alpha = \frac{1}{2}$ and $\alpha = 0$.

In Fig. 1 we present the results of $\frac{dF_2(x,Q^2)}{d\log Q^2}$ at some points of expansion α using the gluon distribution GRV94(LO). We can see that the election of expansion point is very important to obtain one reliable gluon distribution in this approach. For current data of $\frac{dF_2(x,Q^2)}{d\log Q^2}$ the points $\alpha \geq 0.5$ are favoured. This means that in this kinematical region the longitudinal momentum of the gluon x_g is more than twice the value of the longitudinal momentum of the probed quark (or antiquark) in DIS.

One can ask if the approximative determination of the gluon density from $F_2(x,Q^2)$ scaling violations can indicate the presence of new effects at low x, for example, recombination. In Fig. 2 we present the behaviour obtained using (8) for distinct points of expansion and $\frac{dF_2(x,Q^2)}{d\log Q^2}$ data calculated using the Glauber (Mueller) approach [8]. This approach includes shadowing corrections and $\frac{dF_2(x,Q^2)}{d\log Q^2}$ can be calculated using the total cross section of the gluon pair with the nucleon in the eikonal approach.

We can conclude that, when compared with the behaviour of GRV94(LO), the screening may cancel for some values of α, demonstrating the importance of the choice of the expansion point, and that the better ones (with greater sensitivity to screening) are in the range $0.5 \leq \alpha < 0.8$. This conclusion agrees with that

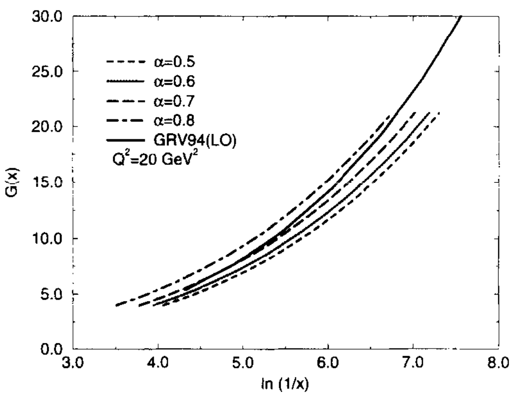

Fig. 2. Gluon distributions obtained of (8) using $\frac{dF_2(x,Q^2)}{d\log Q^2}$ with screening [8]. Also shown the gluon distribution of GRV94(LO).

of Ryskin et al. [9] who estimate that the value of the longitudinal gluon momenta x_g is approximately 3 times larger than the Bjorken x_B, that corresponds to the expansion at $\alpha = 0.75$.

4. Conclusions

The gluon is by far the dominant parton in the small x regime. Its distribution is not as well determined as those of the quarks, and there are several methods proposed in the literature in order to determine it in different regions of x. More recently, new methods [5,6,10] based on the dominant behaviour of the gluon distribution at small x were proposed.

In this work we discussed the approximative determination of the gluon density at low x from $F_2(x,Q^2)$ scaling violations [5,6]. We show that one of these methods is not correct. We proposed one general expression for an approximative determination of the gluon density at an arbitrary point of expansion of $G(\frac{x}{1-z})$. Using data of H1 and Zeus Collaborations [7] and results obtained from the Glauber (Mueller) approach [8] that includes shadowing corrections, we conclude that the more suitable points of expansion are in the range $0.5 \leq \alpha < 0.8$. This conclusion agrees with recent results obtained in [9].

The approximative determination of the gluon density obtained in this work was developed in the GLAP framework. Consequently the approximation is valid in the GLAP limit, which should break down at sufficiently small x. When $\alpha_s \ln\frac{1}{x} \approx 1$ we should resum also the $\alpha_s \ln\frac{1}{x}$ contributions. In LO, $(\alpha_s \ln\frac{1}{x})^n$ resummation is accomplished by the BFKL equation. Therefore, a rigorous determination of the transition region to small x dynamics is very important in order to isolate correctly the gluon distribution.

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