XXXVI Ibero-Latin American Congress on Computational Methods in Engineering

CILAMCE

Rio de Janeiro, 22-25 Nov 2015

BENDING ANALYSIS OF A WIRE STRAND ANALYTIC MODEL

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Abstract. *An analytical approach, using the spatial beam theory, to determine the mechanical response of cables is proposed in this work. Experimental results are limited and expensive to be obtained and there is a lack of good benchmarks solutions to check the FEM models before starting complex analysis. Therefore, in order to obtain the results the parametric spatial description were derived for one strand, considering one wire surrounding a core, since the response of all wires are similar. Solving the differential equilibrium equations of the spatial beam theory and applying bending loads it was possible to obtain the mechanical response of the wire. Six loading cases were analyzed for the wire helix analytically and numerically. Good results were obtained when the both methodologies were compared for the cable bending.*

Keywords: *Wire, Spatial Beam Theory, Differential Geometry, Parameterization, Bending.*

1 INTRODUCTION

Cables are structures that can be used in many applications such as cable-stayed bridges, offshore platforms and prestressed concrete structures. Its geometry is defined as a straight core surrounded by layers of helical strands. The simplest, and most used, configuration is the so called 1 + 6, where six helical wires are laid around the core forming just one layer. The main reason to be used so diversely is the high strength-to-weight ratio and the capacity to support large axial loads in comparison to bending and torsion loads.

Several authors have tried to develop theories and models to study cables. Many numerical analyses, specially using Finite Element Method (FEM), are available and can be used to study these structures. Although is a very powerful tool this approach demands time and must be carefully employed. Stanova *et al.* (2011), Nawrocki and Labrosse (2000), and Jiang (1999) are examples of works where this analysis was carried out using finite elements.

Analytical studies have been produced based on assumptions as curved beams theory, purely tension wires and friction effects (Ghoreishi *et al*. (2007)). Some of the most know theory are presented by Østergaard *et al.* (2011), Argatov (2011), Labrosse and Nawrocki (2000) and Costelo (1990).

In order to validate the FEM or even the methodology used to represent the cable behavior analytically, some classical experimental work are cited. The most known procedures were performed by Machida. and Durelli (1973) and Utting and Jones (1987)

2 STRAND MODELING

In order to simulate the cable behavior the structure was modeled according to the space curved beam theory. To do so, only one wire was considered, since the results can be used to simulate the whole cable, this condition is valid for a simplified case where a non contact behavior is assumed for the structure. The cable geometry is set as a helix and characterized by its center line.

Parameterizing the curve, as Labrouse and Nawrocki (2000), to obtain the mathematical representation of the wire as

$$\begin{aligned} x_1 &= Rcos\left(\frac{Scos(\alpha)}{R}\right), \\ x_2 &= Rsin\left(\frac{Scos(\alpha)}{R}\right), \\ x_3 &= Ssin(\alpha), \end{aligned} \tag{1}$$

where R is the rod radius, α is the helix angle and S is the position on the helix length. With this parameterization it is possible to determine any point in the curve that represents the strand laid helically around a straight core.

It is also important to define the Frenet-Serret triad that defines the kinematic properties of a particle moving along the curve. Three components can be obtained from this theory considering only the dependence on the arc length: the tangent, normal and binormal vectors. The first one indicates the tangent direction of the point in the curve, the second points at the direction of the curvature radius and the latter is a cross product of the other two. Consequently these are orthogonal to each other (Kraus, 1967).

Using the equation found in Feyrer (2007) and the description of a generic space curve (**r**), where the position vector depends only on the length along the wire (S), to determine the tangent vector and the parameterization in Eq. (1) it is possible to find

$$\boldsymbol{t}(S) = \frac{d\boldsymbol{r}}{dS} = \begin{bmatrix} -sin\left(\frac{Scos(\alpha)}{R}\right)cos(\alpha) \\ cos\left(\frac{Scos(\alpha)}{R}\right)cos(\alpha) \\ sin(\alpha) \end{bmatrix}, \tag{2}$$

and the derivative of the triad unit vectors leads to (Ramsey, 1988)

$$\frac{d}{dS}\begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n} \\ \boldsymbol{b} \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n} \\ \boldsymbol{b} \end{bmatrix}, \tag{3}$$

where **n** and **b** denotes the normal and binormal vectors, respectively, and κ and τ represents the curvature and torsion of the space curve. Hence, the normal vector can be determined by solving the first equation of the system in Eq. (3) and as the triad is an orthogonal set of vector the binormal direction can be obtained by the cross product of **t** and **n**. Therefore, this triad represents a local reference system and can be used to study any point of the wire individually.

Using the parameterization it is possible to obtain κ, which express the tangent vector change rate of a point moving along the curve, and τ, that indicate the twist of the same point. Using the Eq. 1 and the relations in Eq. 2 it is possible to define both parameters as

$$\begin{aligned} \kappa &= \frac{-cos^2(\alpha)}{R}, \\ \tau &= \frac{sin(\alpha)cos(\alpha)}{R}. \end{aligned} \tag{4}$$

So, the triad can be expressed now as a function depending only on the length of the wire, as the normal and binormal vectors written as

$$\boldsymbol{n}(S) = \frac{d\boldsymbol{t}}{dS}\Big/\kappa = \begin{bmatrix} cos\left(\frac{Scos(\alpha)}{R}\right) \\ sin\left(\frac{Scos(\alpha)}{R}\right) \\ 0 \end{bmatrix}, \tag{5}$$

$$\boldsymbol{b}(S) = \boldsymbol{t} \times \boldsymbol{n} = \begin{bmatrix} -sin\left(\frac{Scos(\alpha)}{R}\right) sin(\alpha) \\ cos\left(\frac{Scos(\alpha)}{R}\right) sin(\alpha) \\ cos(\alpha) \end{bmatrix}.$$

Using the spatial beam theory, developed by Love (1944), to represent the wire by its center line, where the equilibrium equations are written as

$$\frac{dN_1}{dS} - \kappa N_2 + q_1 = 0,$$

$$\frac{dN_2}{dS} + \kappa N_1 - \tau N_3 + q_2 = 0, \tag{6}$$

$$\frac{dN_3}{dS} - \tau N_2 + q_3 = 0,$$

where N_i and q_i are the load and distributed load, respectively, in each direction. For the equilibrium of moments, the system is

$$\frac{dM_1}{dS} - \kappa M_2 + m_1 = 0,$$
$$\frac{dM_2}{dS} + \kappa M_1 - \tau M_3 - N_3 + m_2 = 0, \tag{7}$$
$$\frac{dM_3}{dS} + \tau M_2 + N_2 + m_3 = 0.$$

where M_i and m_i are the moment and distributed moment, respectively, in each direction. Therefore, the response of the cable due to the loads applied can be calculated.

Solving both systems (Eq. 6 and 7) it is possible to obtain the cable mechanical behavior. Considering that the wire is clamped at one end, the boundary conditions, in order to determine the 6 integration constants, can be used in the solutions. Since the 5 parameters are know ($\boldsymbol{t}$, $\boldsymbol{n}$, $\boldsymbol{b}$, κ and τ) one can describe the wire geometrically at any point.

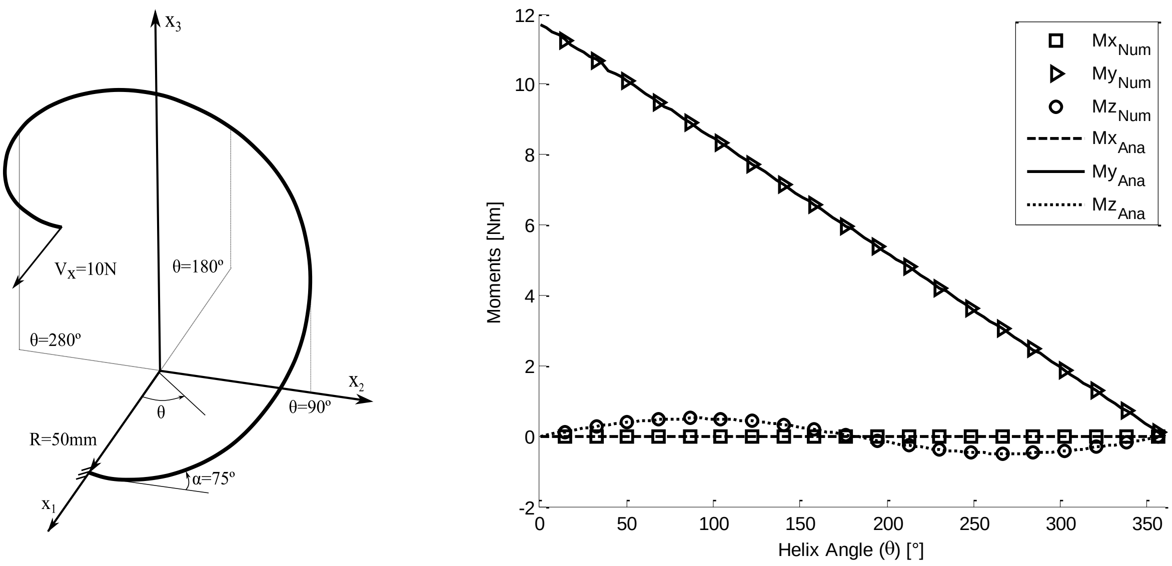
To do so, the loads applied in the structure are decomposed into the local system, using the triad as the transformation matrix written as

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{n} \\ \boldsymbol{b} \end{bmatrix}, \tag{8}$$

so, each component of the force is evaluated in any point of the helix. This procedure is reversible, that means, the Frenet-Serret formulas makes it possible to transform any information, regarding the local system, back to the global system, and vice-versa. All the study developed here is focused on the global analysis, hence the diagrams are presented regarding the fixed system in the center of the cable.

The triad is used to transform not only the force vector but also the moments. So the position vector (***r***) determines the distance for the moments and the cross product aligns the boundary condition to the triad.

3 RESULTS



(a) **(b)**

Figure 1 – (a) Case 1 studied, V_x parallel to the x_1 direction and (b) moments diagram.

Using the formulation proposed, one turn and one wire of the cable was simulated. This methodology provided results that can be used to characterize the role structure. Setting the helix angle as 75º and the radius 50mm the maximum value of S is calculated, due to the parameterization (Eq. 1), as $S = 1.172m$. With the geometry defined, the loads were changed and the moments diagram, due to bending, analyzed. Six cases of bending were studied. The first case is a force applied in one end of the wire, the other is clamped in all the cases, in the $\boldsymbol{x_1}$ direction as shown in Fig. 1(a).

The moments diagram (Fig. 1(b)) presents the numerical solution ($M_{x_{num}}$, $M_{y_{num}}$ and $M_{z_{num}}$) and the model results ($M_{x_{ana}}$, $M_{y_{ana}}$ and $M_{z_{ana}}$). Initially, at the clamped end ($\theta = 0°$), the moment in the $\boldsymbol{x_2}$ direction is maximum and is evaluated as $M_y = 11.72Nm$, the linear behavior presented by this component was already expected. The moment M_z has the maximum equals, in modulus ($M_z = 0.5Nm$), to the minimum value at $\theta = 90°$ and $\theta = 270°$, respectively. As the load applied is parallel to the $\boldsymbol{x_1}$ direction, there is no moment generated in that component in any point, therefore, $M_x = 0Nm$.

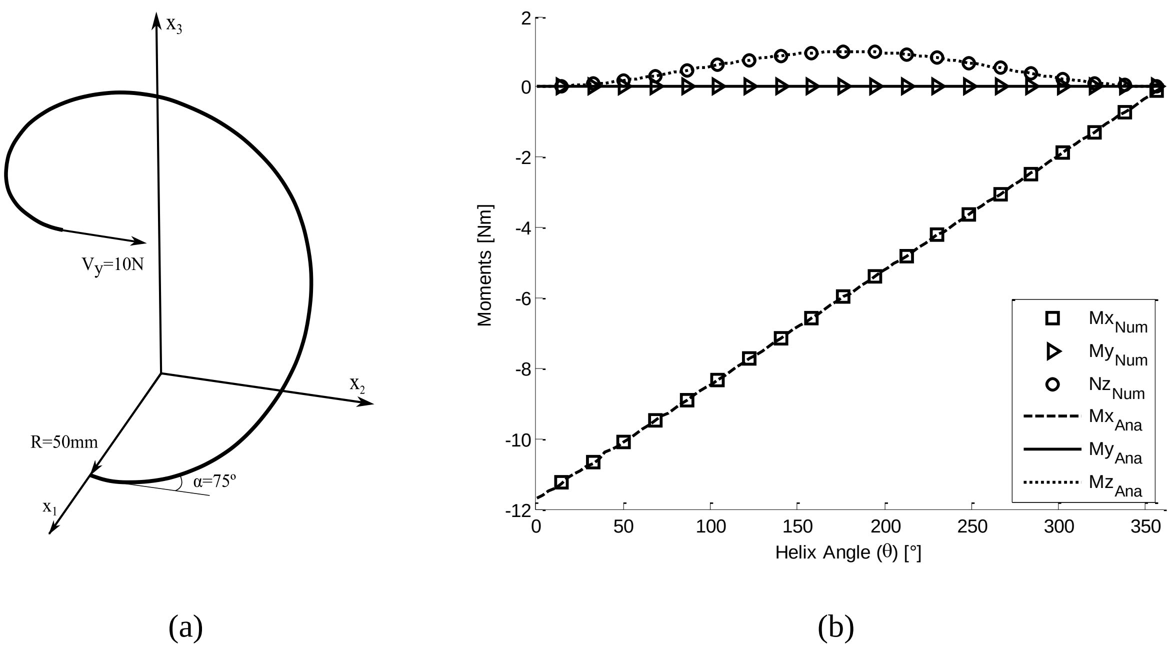


Figure 2 - Case 2, V_y applied parallelly to the x_2 direction and (b) the diagram of moments.

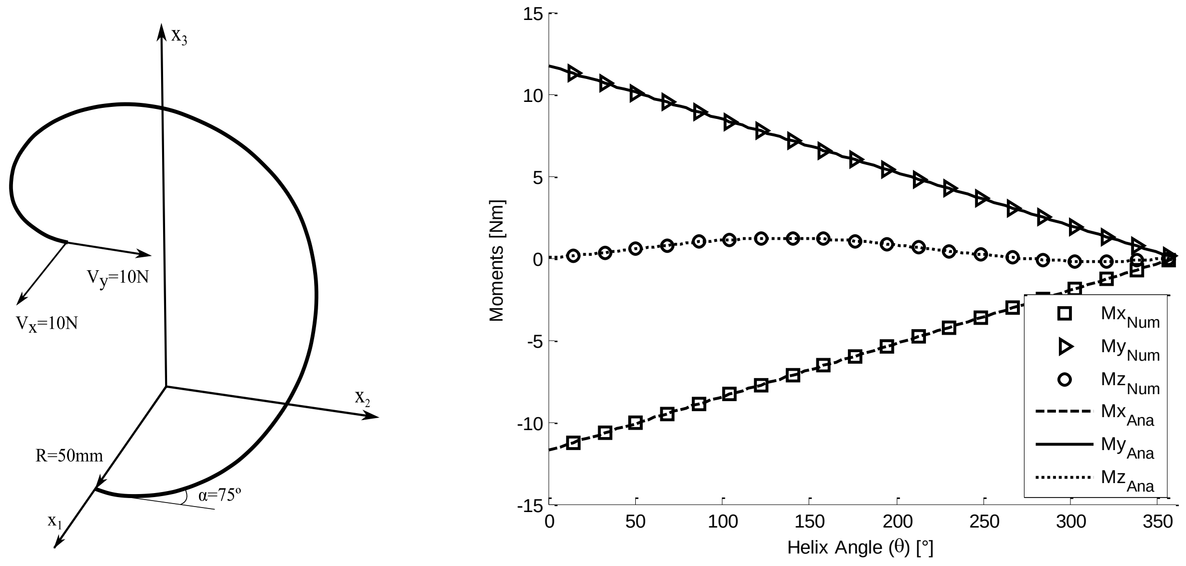
In order to analyze the quality of results, a FEM model was developed, using a commercial software, and the responses compared. For the first case, all the curves are coincident to the numerical points generated by the finite element analysis. This result can be used to validate the model.

For the second case, Fig. X(a), the load applied is now parallel to the x_2 direction and set again as $10N$. Once more the bending maximum value is $M_x = -11.72Nm$, it is also linear, but is negative, due to the notation used. For M_z and M_y it is possible to note that the curves have a similar behavior as M_z and M_x in the first case, because of the same reasons.

When two loads are applied to the wire, V_x and V_y, as demonstrated in Fig. 3(a), the bending behavior has the same features as cases one and two. This is not only expected but also indicates that the methodology used is respecting the superposition effect, since the system solved is linear and must present this characteristic. Adding the responses generated by each load alone provide the same result when both loads are analyzed together.

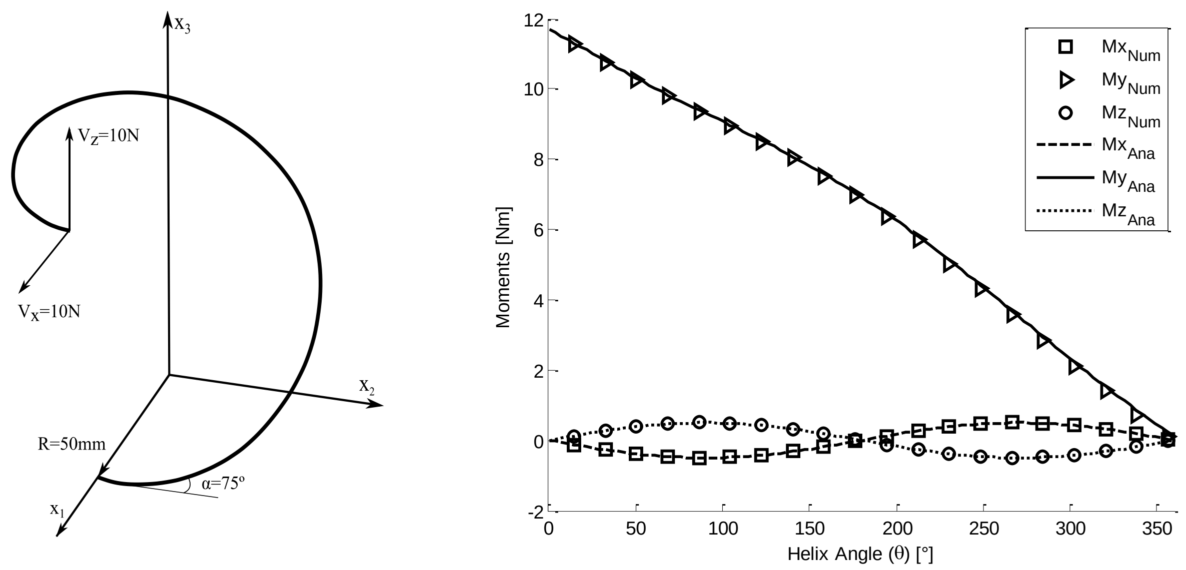
It is interesting to observe that the moment in the x_3 direction is generated by the resultant of the loads, indicated as V. Therefore the maximum moment occurs not at one of the axis, but in the middle. The higher value is set at 135° while the angle where it is zero is 270°, those points are at the vector V direction (no distance) and parallel to it (top distance), respectively.

Case 4, in Fig. 4, demonstrates that due to V_z the M_y component is no longer linear. Once again the superposition is present. Considering the first case, since V_z does not produce moment in the x_3 direction, adding this load only generates effects in M_x and M_y. In other words, M_z does not change when V_z is taken into account.



(a) (b)

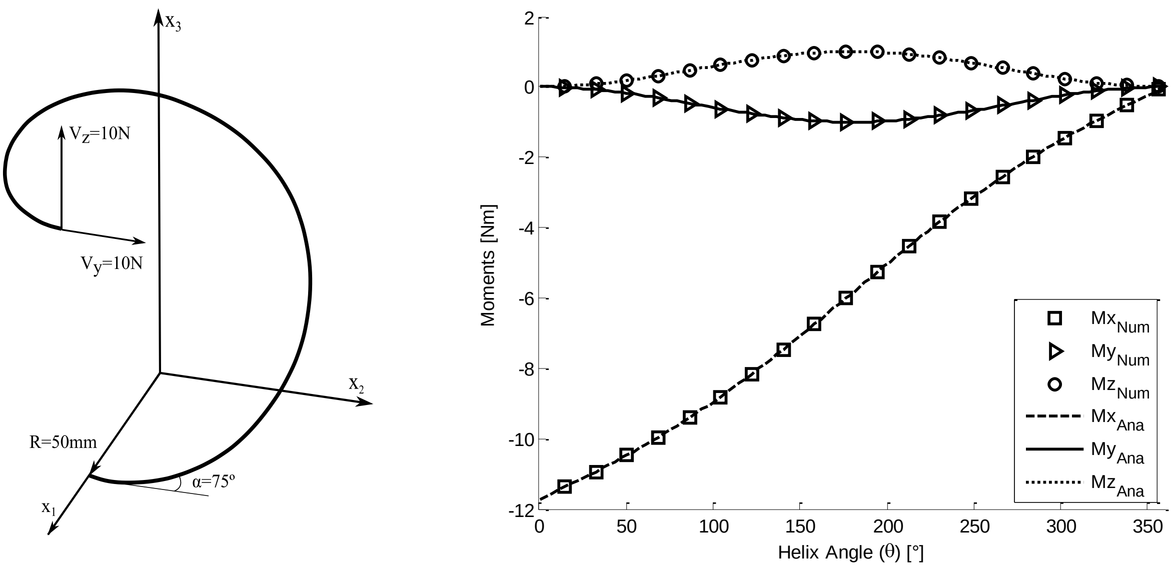
Figure 3 - Case 3 with V_x and V_y applied parallelly to the x_1x_2 plane and (b) the diagram of moments.



(a) (b)

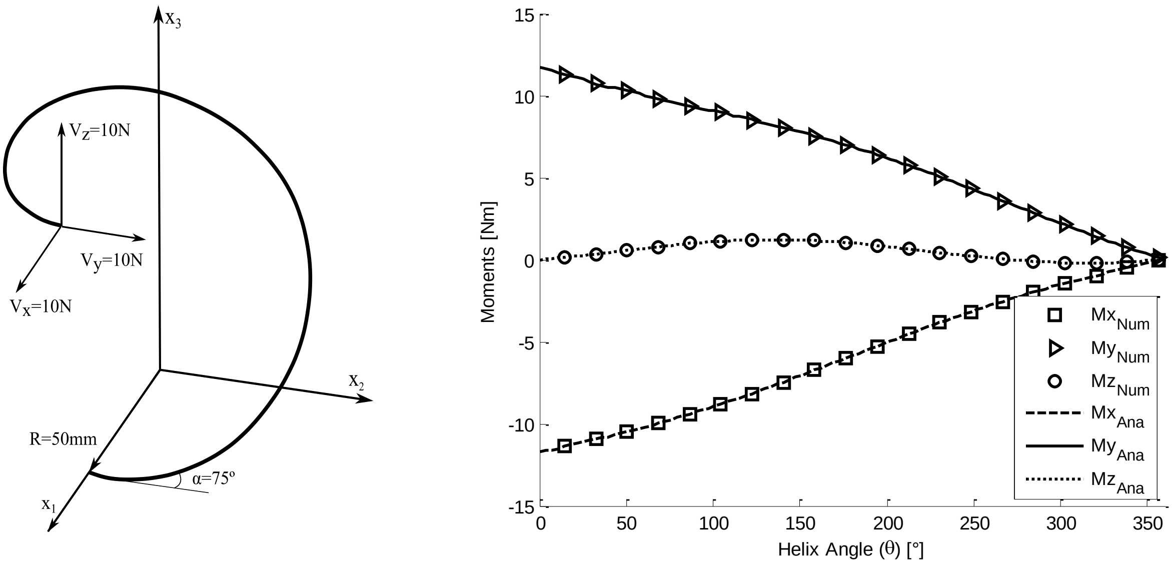
Figure 4 - Case 4, V_x and V_z applied parallelly to the x_1x_3 plane and (b) the moments diagram.

For the fifth case where V_y and V_z are applied, see Fig. 5, the component M_x is not linear for the same reasons of the forth case. Once more, the analysis of two loads were identical to the addition of separated cases, which demonstrates the linearity of the system.



(a) (b)

Figure 5 - Case 5, V_y and V_z applied parallelly to the x_2x_3 plane and (b) the moments diagram.



(a) (b)

Figure 6 - Case 6, V_x, V_y and V_z applied and (b) the moments diagram.

The last case studied is an application of loads in all the directions. As expected there are two points where the moment is maximum and they are equal in modulus because the loads and distances are the same. Both M_x and M_y are set as $11.72Nm$ and have the same behavior (nonlinear) as cases 5 and 4, respectively. The other component (M_z) has the same result as in case 3, reaching its maximum value at 135° as $0.5Nm$ for the same reasons.

Therefore, it is possible to realize that this last case is a combination of the other cases and that the methodology is coherent to the expected. In fact, the application of this procedure leads to a linear system.

For all the cases the comparison between the analytic model and the numerical analysis were presented. The methodology used to generate the results was validated by the Fem model. No significant difference was found in any case analyzed when the solutions were compared.

4 CONCLUSIONS

An analytical expression to study the mechanical behavior of a cable, subjected to bending, was obtained by solving the spatial beam theory. The wire was modeled as a helix and a parameterization was used to describe the cable geometrically. As the results of one wire can be used to study the whole cable, since the responses are similar, only one component was analyzed.

Using the transformation matrix to decompose the loads and moments, the boundary conditions were used to determine the integration constants. The results obtained in the local system were transformed back to the global system. Six cases of loading were analyzed, using the methodology applied, and compared to a FEM model. The differences between the results were not significant and the theory developed was validated.

Another feature of the theory was the superposition. It was expected, since the system is linear, that this theory could be used to analyze any loading case. The third case showed that the methodology applied could generate the same result as the first two cases combined. The other cases also presented this characteristic.

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