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Spontaneous emission of electromagnetic radiation in turbulent plasmas

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Known radiation emission mechanisms in plasmas include bremmstrahlung (or free-free emission), gyro- and synchrotron radiation, cyclotron maser, and plasma emission. For unmagnetized plasmas, only bremmstrahlung and plasma emissions are viable. Of these, bremmstrahlung becomes inoperative in the absence of collisions, and the plasma emission requires the presence of electron beam, followed by various scattering and conversion processes. The present Letter proposes a new type of radiation emission process for plasmas in a state of thermodynamic quasi-equilibrium between particles and enhanced Langmuir turbulence. The radiation emission mechanism proposed in the present Letter is not predicted by the linear theory of thermal plasmas, but it relies on nonlinear wave-particle resonance processes. The electromagnetic particle-in-cell numerical simulation supports the new mechanism. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4861619]

Textbook radio emission mechanisms in plasmas include Bremsstrahlung (or free-free emission), gyro- and synchrotron radiation, cyclotron maser, and the plasma emission. The free-free emission takes place in collisional plasmas, and gyro- and synchrotron radiations operate in magnetized plasmas. The cyclotron maser and plasma emission require the presence of free energy source: The perpendicular electron population inversion with respect B field in the case of cyclotron maser and the field-aligned electron beam in the case of plasma emission. The plasma emission additionally involves nonlinear conversion of electron beam-generated Langmuir turbulence to radiation. Čerenkov emission in plasmas only involves longitudinal modes. As such, it does not lead to direct radiation emission. For a review of various radiation emission theories in plasmas, see Refs. 1 and 2.

For a field-free isotropic plasma, there is no known mechanism to produce electromagnetic (EM) radiation. In the present Letter, however, we introduce a novel radiation generation mechanism in unmagnetized plasmas with a thermal population of electrons. According to the proposed scenario, a spectrum of superluminous (i.e., phase speed greater than the speed of light *in vacuo*) transverse waves is generated by nonlinear processes, in a field-free plasma that can be described as a turbulent quasi-equilibrium state. Treumann[3] first discussed the notion of turbulent equilibrium in plasmas, which involves plasma particles constantly exchanging momentum and energy with enhanced electromagnetic fluctuations in steady-state fashion, but on average, the system is in dynamic equilibrium. Recently, we demonstrated that an explicit turbulent equilibrium solution can be obtained on the basis of plasma weak turbulence theory.[4]

The present work can be considered as an extension of Ref. 4 to include electromagnetic effects. It also involves nonlinear extension of the customary theory of thermal fluctuations.[5,6] According to the standard theory, unmagnetized thermal plasmas emit electrostatic fluctuation according to the formula (which is, in fact, the Čerenkov emission formula)

$$\langle \delta E_\parallel^2 \rangle_{\mathbf{k},\omega} = \frac{T}{2\pi^3 \omega} \frac{\mathrm{Im}\epsilon_\parallel(\mathbf{k},\omega)}{|\epsilon_\parallel(\mathbf{k},\omega)|^2}, \tag{1}$$

where the left-hand side represents the longitudinal electric field spectral wave energy, T represents the electron temperature (in the unit of eV), and $\epsilon_\parallel(\mathbf{k},\omega)$ is the longitudinal dielectric response function. Unmagnetized thermal plasmas can also spontaneously emit low-frequency subluminous ($ck/\omega > 1$) electromagnetic fluctuation,[7,8] of which, the dominant magnetic field spectral energy is given by

$$\langle \delta B^2 \rangle_{\mathbf{k},\omega} = \frac{c^2k^2}{\omega^2} \frac{T}{\pi^3 \omega} \frac{\mathrm{Im}\epsilon_\perp(\mathbf{k},\omega)}{|\epsilon_\perp(\mathbf{k},\omega) - c^2k^2/\omega^2|^2}, \tag{2}$$

where $\epsilon_\perp(\mathbf{k},\omega)$ is the transverse dielectric response of the plasma. Here, (ω, $\mathbf{k}$) are angular frequency and wave vector pair. Note that the angular frequency ω in the present discussion is considered real. However, in general, the Maxwellian plasma also supports aperiodic transverse fluctuations.[9] Note also that a recent Letter by the present authors demonstrated, by means of particle-in-cell simulation, that spontaneous electrostatic and magnetic field fluctuations are generated in a Maxwellian plasma, and that the simulated spectra are qualitatively similar to that described by Eqs. (1) and (2).[10]

In Eq. (2), it is important to note that only subluminal magnetic field fluctuation characterized by $ck > \omega$ can be generated since the quantity $\mathrm{Im}\epsilon_\perp(\mathbf{k},\omega)$ contains linear wave-particle resonance condition delta function, $\delta(\omega - \mathbf{k}\cdot\mathbf{v})$, which cannot be satisfied for superluminal fluctuation satisfying $\omega/k > c$, especially in relativistic formalism. This is why thermal unmagnetized plasmas cannot emit fast electromagnetic radiation, according to the customary theory.

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However, as we shall discuss subsequently, once we include nonlinear effects, it is possible to show that radiation can be generated even in thermal field-free plasmas. The starting point of our discussion is the standard weak turbulence theory in unmagnetized plasmas in which fully electromagnetic interaction is taken into account.[11] The set of equations describe the time evolution and interactions among Langmuir (L), ion-sound (S), and transverse (T) waves, as well as among plasma particles. We assume that the system is in quasi-steady state, $\partial/\partial t \approx 0$, and that the electrons and ions are distributed in velocity space according to thermal equilibrium velocity distribution functions (VDF)

$$F_e(v) = (\pi v_e^2)^{-3/2}\exp(-m_e v^2/2T),$$
$$F_i(v) = (\pi v_i^2)^{-3/2}\exp(-m_i v^2/2T), \tag{3}$$

where $v_e = (2T/m_e)^{1/2}$ and $v_i = (2T/m_i)^{1/2}$ are electron and ion thermal speeds, respectively, and m_e and m_i being the electron and ion (proton) masses, respectively. Note that by assuming the above form of particle VDFs, we eliminate the need for particle kinetic equations among the set of weak turbulence equations.

If we are only interested in the thermal emission of plasma normal mode (in this case, Langmuir or L mode), then the spontaneously emitted spectral Langmuir wave intensity can be obtained from Eq. (1) by approximating $\langle \delta E_\parallel^2\rangle_{\mathbf{k},\omega} \approx I_L(k)\,\delta(\omega - \omega_{\mathbf{k}}^L)$, which leads to the following expression:

$$I_L(k) = \frac{T}{4\pi^2}\frac{1}{1+3k^2\lambda_{De}^2} = \frac{T}{4\pi^2}\frac{\omega_{pe}^2}{(\omega_k^L)^2}, \tag{4}$$

where $\lambda_{De}^2 = T/(4\pi n e^2) = v_e^2/(2\omega_{pe}^2)$ is the square of the electron Debye length, n being the ambient plasma number density, and e being the unit electric charge. The quantity $\omega_{pe}^2 = 4\pi n e^2/m_e$ is the square of the plasma frequency, and the Langmuir wave dispersion relation $(\omega_{\mathbf{k}}^L)^2 = \omega_{pe}^2(1+3k^2\lambda_{De}^2)$ is used in the second equality in Eq. (4). The equilibrium ion-sound wave intensity, $I_S(k)$, can also be obtained from Eq. (1), but we do not need to concern ourselves with the low-frequency electrostatic fluctuations in the present analysis. For Maxwellian thermal plasmas with equal electron and proton temperatures, the ion-sound mode intensity is ignorable. By assuming the spontaneously emitted L and S mode intensities, $I_L(k)$ and $I_S(k)$, we further eliminate the need for L and S mode wave kinetic equations in the general set of weak turbulence equations.

The remaining equation of interest is the wave kinetic equation for transverse (T) waves. Among terms of the T wave kinetic equation, the spontaneous and induced emission terms dictated by linear wave-particle resonance condition, $\omega_{\mathbf{k}}^T - \mathbf{k}\cdot\mathbf{v} = 0$, are unimportant since the resonance condition cannot be satisfied due to the fact that the wave phase velocity is higher than the light speed *in vacuo* c. Nonlinear decay terms in T wave kinetic equation, dictated by three-wave resonance conditions, $\omega_{\mathbf{k}}^T - \omega_{\mathbf{k}'}^L - \omega_{\mathbf{k}-\mathbf{k}'}^L = 0$ and $\omega_{\mathbf{k}}^T - \omega_{\mathbf{k}'}^L - \omega_{\mathbf{k}-\mathbf{k}'}^S = 0$, turn out to play only secondary roles in the process of radiation generation, as these processes involve exchange of momentum and energy only among the waves and no particles are involved. Consequently, the only relevant terms are induced and spontaneous scattering terms, which shall be shown below. Expressing the T mode intensity by $\langle \delta E_\perp^2\rangle_{\mathbf{k},\omega} = I_T(k)\ \delta(\omega - \omega_{\mathbf{k}}^T)$, the time-asymptotic steady-state wave kinetic equation for T mode that only includes induced and spontaneous scattering processes is given by

$$\begin{aligned}\frac{\partial I_T(k)}{\partial t} \approx 0 \approx &\int d\mathbf{k}'\int d\mathbf{v}\, U_{\mathbf{k},\mathbf{k}'}\\ &\times\left(\frac{1}{4\pi^2}\left[2\omega_k^T I_L(k') - \omega_{k'}^L I_T(k)\right](F_e+F_i)\right.\\ &\left.+\frac{1}{m_i} I_L(k')\, I_T(k)\,(\mathbf{k}-\mathbf{k}')\cdot\frac{\partial F_i}{\partial \mathbf{v}}\right),\end{aligned} \tag{5}$$

where we have assumed isotropic spectra for the waves, $I_L(\mathbf{k}) = I_L(k)$ and $I_T(\mathbf{k}) = I_T(k)$, and the nonlinear scattering coefficient is defined by

$$U_{\mathbf{k},\mathbf{k}'} = \frac{\omega_k^T}{8n}\frac{(\mathbf{k}\times\mathbf{k}')^2}{k^2k'^2}\delta[\omega_k^T - \omega_{k'}^L - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]. \tag{6}$$

It should be noted that even though the linear wave-particle resonance condition cannot be satisfied for fast EM waves, the nonlinear wave-particle resonance, $\omega_k^T - \omega_{k'}^L - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v} \approx 0$, can be satisfied if T and L wave frequencies are sufficiently close to each other, $\omega_k^T \sim \omega_{k'}^L$. The wave momenta for the two modes can be different. In fact, for most cases, $|\mathbf{k}'| \gg |\mathbf{k}|$, which can easily be verified on the basis of the transverse wave dispersion relation, $\omega_k^T = (\omega_{pe}^2 + c^2k^2)^{1/2}$. Note that the second line of Eq. (5) describes the spontaneous scattering process, while the third line corresponds to the induced scattering term, where the electron contribution was ignored when compared with that from the much heavier ions. The induced scattering process, therefore, can also be called "scattering off thermal ions."

Equations (5) and (6) can be derived on the basis of a semi-classical method. See Eqs. (6.58) and (6.59) in Ref. 2 (p. 90). Upon identifying the subscript M to designate the transverse mode and P to represent the Langmuir mode, and making use of an unnumbered equation at the top of p. 99, one can establish the equivalence of the present Eq. (5) with that derived in the literature.

Since the steady-state condition in Eq. (5) is achieved by balancing the spontaneous and induced scattering terms, which are nonlinear, the resulting steady-state spectrum for transverse wave (to be determined subsequently) represents the "turbulent equilibrium" radiation intensity in a thermal plasma. For thermal particle VDFs, upon carrying out the velocity integration by virtue of the delta function resonance condition, we obtain

$$\begin{aligned}0 \approx &\int d\mathbf{k}'\frac{(\mathbf{k}\times\mathbf{k}')^2}{k^2k'^2|\mathbf{k}-\mathbf{k}'|}\left\{\frac{T}{4\pi^2}\left[2\omega_k^T I_L(k') - \omega_{k'}^L I_T(k)\right]\right.\\ &\times F_e(v_{\mathbf{k},\mathbf{k}'}^{\rm res}) + \left(\frac{m_i}{m_e}\right)^{1/2}\left[\omega_k^T\left(\frac{T}{2\pi^2} - I_T(k)\right)I_L(k')\right.\\ &\left.\left.-\omega_{k'}^L\left(\frac{T}{4\pi^2} - I_L(k')\right)I_T(k)\right]F_i(v_{\mathbf{k},\mathbf{k}'}^{\rm res})\right\},\\ v_{\mathbf{k},\mathbf{k}'}^{\rm res} = &\frac{\omega_k^T - \omega_{k'}^L}{|\mathbf{k}-\mathbf{k}'|}.\end{aligned} \tag{7}$$

Owing to the exponential factor associated with the particle distributions,

$$F_a(v^{\rm res}_{\mathbf{k},\mathbf{k}'}) \sim \exp[-(\omega^T_k - \omega^L_{k'})^2/|\mathbf{k}-\mathbf{k}'|^2 v_a^2],$$

the most significant region of integration is around $k' = k_*$, where $\omega^L_{k_*} = \omega^T_k$, which implies $k_* = \sqrt{2/3}(c/v_e)k$, so that $k_* \gg k$, as anticipated. The exponential factor can therefore be approximately represented by a delta function centered around k_*. Upon employing this approximation to the $\mathbf{k}'$ integral in Eq. (7) and making use of the equilibrium L mode intensity given by Eq. (4), we obtain the desired "turbulent" equilibrium radiation intensity

$$I^T(k) = |\delta E^T_{\mathbf{k}}|^2 = \frac{T}{2\pi^2}\frac{1}{1+(c^2/\omega^2_{pe})k^2}. \quad (8)$$

The associated magnetic field intensity, $|B_{\mathbf{k}}|^2$, can be written as follows:

$$|\delta B_{\mathbf{k}}|^2 = \frac{T}{2\pi^2}\frac{(ck/\omega_{pe})^2}{(1+c^2k^2/\omega^2_{pe})^2}, \quad (9)$$

where we have made use of the relation $3k^2_*\lambda^2_{De} = c^2k^2/\omega^2_{pe}$.

Note that Eq. (8) for transverse electric field intensity resembles the Langmuir wave intensity (4) even though its derivation invokes nonlinear or turbulent processes, which is why Eq. (8) represents a turbulent equilibrium radiation intensity. For $k \simeq 0$ both Langmuir and transverse waves should have comparable intensities, and scale in the same way with plasma density and temperature. However, for higher values of k (smaller wavelength), the transverse mode intensity falls off much more rapidly than the corresponding L mode intensity as a function of k.

In order to verify the theoretical prediction, we have carried out a particle-in-cell (PIC) numerical simulation. The present numerical scheme is based on the one-dimensional electromagnetic PIC code KEMPO (Kyoto ElectroMagnetic Particle Code),[12] but in a modified version.[13] We use 8192 spatial grid points with distances normalized by λ_{De}, the grid spacing being $\Delta x = 1.0\lambda_{De}$, and we have run the simulation for a total of 16384 time steps, with $\Delta t = 0.01\omega^{-1}_{pe}$, which means that the system evolves until $\omega_{pe} = 163.84$. The ambient magnetic field is assumed to be zero, frequencies are normalized by plasma frequency, velocities are normalized by c. We consider $T_e = T_i = T$, thermal velocity $v_e = 0.02c$, zero drift velocity ($v_d = 0.0$), realistic ion-to-electron mass ratio, $m_i/m_e = 1836$, and 512 super-particles per grid cell.

Figure 1 shows the (ω, k) diagram obtained from the x (longitudinal) component of the electric field by Fourier transforming in space along the x axis and in time. This result is essentially the same as that of Ref. 10. For reference, we also plot the theoretical Langmuir (Bohm-Gross) dispersion relation $\omega^2 = \omega^2_{pe}(1+3k^2\lambda^2_{De})$ in dotted line. The simulated spontaneously emitted electrostatic fluctuation by thermal plasma is in agreement with that based on Eq. (1). Figure 1 confirms the known results.[5,7,8]

Shown in Fig. 2 is the dispersion diagram, ω-k, generated from the magnetic field fluctuation. We also plot the theoretical electromagnetic plasma wave ($\omega^2 = \omega^2_{pe} + k^2c^2$) in dotted line. For low frequency regime ($\omega/\omega_{pe} < 1$), the simulation produces the low-frequency magnetic field fluctuation,[7,8] which can be discussed on the basis of Eq. (2).

However, the unexpected result, according to linear theory anyway, is the generation of fast transverse EM fluctuation, as can be seen in Fig. 2. Note that Ref. 10 only

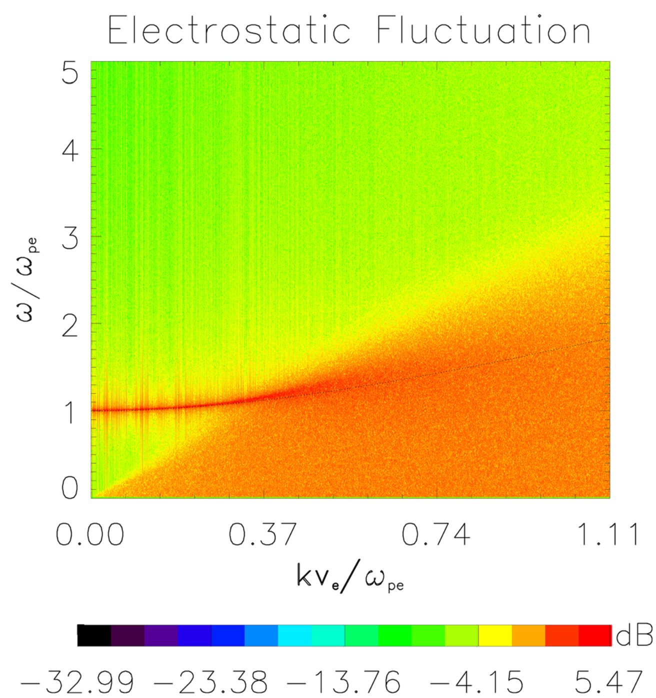

FIG. 1. ω-k diagram obtained from the time and space evolution of the x (longitudinal) component of the electric field, E_x. The theoretical Langmuir wave relation is shown by a dotted line.

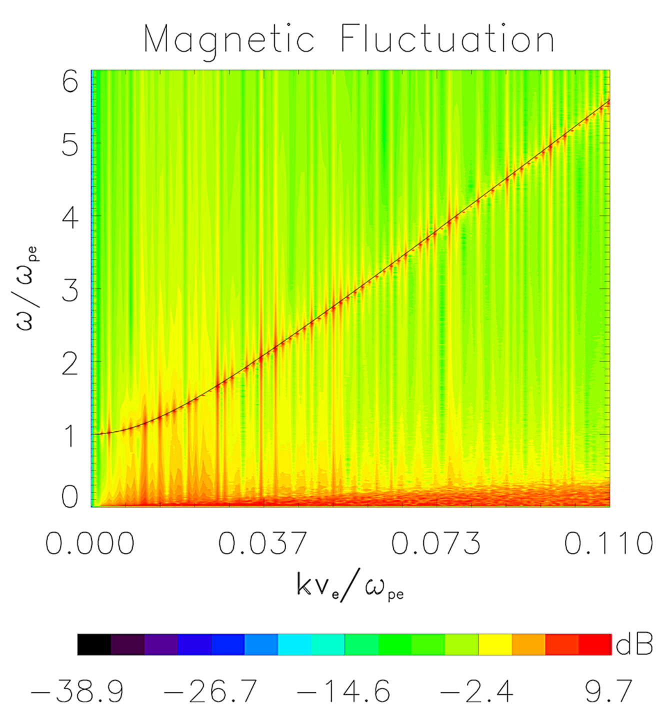

FIG. 2. ω-k diagram obtained from the time and space evolution of the magnetic field, B_y. Theoretical electromagnetic plasma wave is shown by a dotted line.

shows the subluminal low-frequency magnetic field fluctuation. According to linear theory, thermal electrons cannot emit fast EM fluctuations satisfying the dispersion relation $\omega^2 = \omega_{pe}^2 + k^2c^2$ since the electron speed cannot exceed the speed of light. However, as elaborated above, nonlinear theory of turbulent equilibrium predicts the generation of just such fast EM fluctuations by thermal electrons. Incidentally, in our simulation, the particles are not allowed to exceed the speed of light. We thus conclude that the spontaneously emitted electromagnetic fluctuation for $\omega/\omega_{pe} > 1.0$ and $ck/\omega < 1$ regime can only be explained by the turbulent spontaneous emission mechanism proposed in the present Letter.

As a further confirmation, we plot the magnetic field intensity shown in Fig. 2, as a function of wave number, and compare the result against the theoretical formula, (9). The result is shown in Fig. 3. Notice how the two results are in qualitative agreement. The reason for the slightly higher B field intensity near short wavelength limit, $kv_e/\omega_{pe} \sim 1$ in the simulated spectrum is because of the contamination by the spontaneously emitted low-frequency magnetic fluctuation,[7,8] whose approximate B field intensity can be obtained from Eq. (2) by[7,14]

$$|B_{\mathbf{k}}^{\text{low-freq}}|^2 \approx \frac{T}{2\pi^{5/2}\omega_{pe}} \frac{v_e^2}{c^2} \frac{1}{k^3\lambda_{De}^3}. \qquad (10)$$

If we admit that the simulated magnetic field intensity is a superposition of Eqs. (9) and (10), while the theoretical curve represents purely Eq. (9), then the small discrepancy can easily be accounted for.

To summarize, in the present Letter, we have shown the existence of a radiation mechanism associated with nonlinear interactions, which can be described by weak turbulence theory. According to our findings, energetic electrons should generate enhanced EM radiation near the plasma frequency, even without the presence of any free energy source, such as the beam.

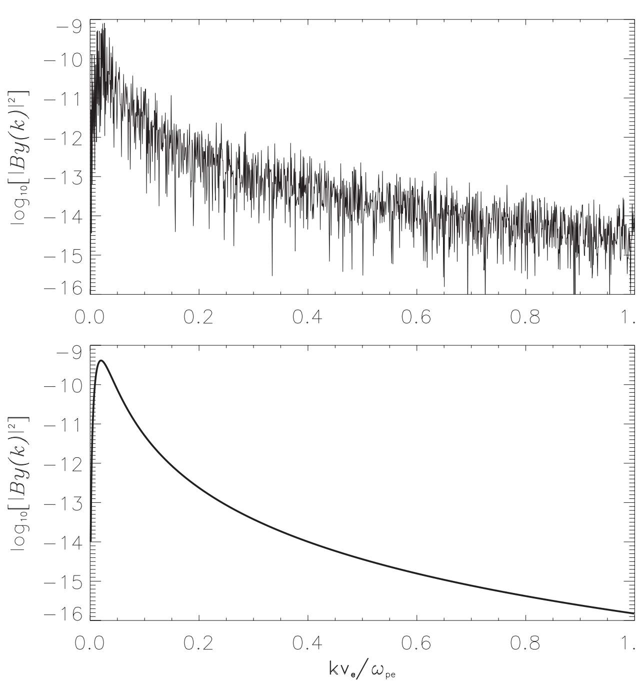

FIG. 3. Comparison of simulated magnetic field fluctuation associated with the fast transverse mode versus the theoretical magnetic field intensity [Eq. (9)]. The vertical axes are in arbitrary units.

Even though the purpose of the present Letter is to elucidate the essential physics of the new radio emission mechanism, one may conceive of a number of potential applications. One of them is the so-called quasi-thermal noise (QTN).[6] It is well known that the solar-terrestrial environment is replete with various radio signals and noises. QTN is often interpreted as electrostatic Langmuir-type fluctuations, which is reasonable given the antenna geometry and dimensional limitations, but there is a likelihood that EM component may exist in such fluctuations [N. Meyer-Vernet, private communication]. Future, better designed instruments may be employed to reanalyze the EM polarization associated with QTN. We should note, however, that if the present theory is applied to the solar-terrestrial environments in general, we do not expect that the proposed mechanism can result in high level of radiation since the emission process is attributed to thermal particles, which have low energies.

To reiterate, however, the main focus of the present Letter is not any immediate application, but to report a novel mechanism of spontaneous radiation by thermal plasmas. The new mechanism is distinct from all other known radio emission mechanisms including the Bremmstrahlung (radiation emitted during electron collision with ions), gyro- and synchrotron emission by spiraling relativistic electrons in magnetic field, cyclotron maser (collective radiation emitted by inverted electron population), and the plasma emission (nonlinear conversion of electron beam-generated Langmuir waves to radiation).

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