TURBULENT AIR-FLOW AND DISPERSION OF POLLUTANTS IN THE ATMOSPHERE

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ABSTRACT

This study is a contribution toward the understanding and prediction of turbulent dispersion of pollutants in the atmosphere. The main underlying assumption suggestion is that the Lagrangian autocorrelation R_L used in computation of plume dispersion may be approximated by an Eulerian space-time correlation.

It is shown that the empirical models presently in use are valid in general only on the site where they were developed. Generalisation to other places may lead to errors of an order of magnitude in concentration predictions. This state of the art is attributed to the lack of a systematic discription of turbulence flow.

A semi-empirical tentative method to be used in the description of turbulent flows, is given. General models are given for the Eulerian time correlation and its Fourier transform, the spectral density. They are functions of the integral time scale of the motion, which in turn is a function of atmospheric stability. With their help it is shown that the Hay-Pasquill approximation to the Lagrangian autocorrelation underestimates the concentration far from the source, indicating that their correlation falls faster than the true Lagrangian autocorrelation.

Arguments are given in favour of the replacement of R_L by Eulerian space-time correlations for the calculation of horizontal dispersion of contaminants. No model exists for these space-time correlations, and which is valid at all times. A new model is proposed which is based on physical assumptions. The model is a function of the Eulerian time correlation, allowing for the computation of space-time correlations from a few measurements taken at a fixed point in space. This model is compared with experiments in channel flow and in the atmosphere. It is also used to replace R_L in the prediction of atmospheric diffusion. The results are in good agreement with experiment.

Assuming the validity of this model we can explain the divergent results obtained when Hay-Pasquill's model is used. The behaviour of the parameters in Sutton's formula is explained too.

Extension of the model to predict the dispersion of nuclear debris in the atmosphere is suggested. Other models are suggested briefly and without computation. They are designed to give a relationship between atmospheric stability, time scales and velocity variances. This should give us the turbulence intensity on the basis of velocity and temperature profiles only.

SYMBOLS

Cy, Cz	-	Sutton's coefficients
$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V}.\vec{\nabla}$		
C_p	-	Specific heat of air at constant pressure
E	-	Eulerian integral time scale
F(n), F(K)	-	Spectral density
K	-	Wave number
K_x, K_y, K_z	-	Eddy diffusivity
L	-	Monin-Obukhov length (always in the form z/L)
L	-	Lagrangian integral time scale
n	-	Frequency
Q	-	Source strength
Qo	-	Vertical heat flux
$R_L(t)$	-	Lagrangian autocorrelation
$R_E(t) \equiv R_E$	-	Eulerian autocorrelation
$R_E(x,y,z;t) \equiv R_{ST}$	-	Space-time Eulerian correlation
$R_i = \frac{g}{\Theta} \frac{\partial \Theta}{\partial z} \left(\frac{\partial U}{\partial z}\right)^{-2}$	-	Richardson number
T_*	-	Reference temperature
$\vec{V} = (U+u, v, w)$	-	Instantaneous velocity
u_*	-	Friction velocity
$\overline{v_L^2}$	-	Langrangian y velocity variance
(x, y, z)	-	Rectangular coordinate , x along mean wind, z vertical
β	-	Ratio of Lagrangian and Eulerian time-scale
ε	-	Rate of dissipation of kinetic energy per unit mass of air
ρ	-	Air density
σ_y^2	-	Variance (= $\overline{y^2}$)
τ_o	-	Horizontal shearing stress
χ	-	Concentration
ϕ_ε	-	Non dimensional dissipation rate = $\frac{z\varepsilon}{u_*^3}$
Θ	-	Potential temperature

1. INTRODUCTION

Atmospheric pollution has become a serious problem in a large number of industrial and urban communities. Atmospheric diffusion is a subject of considerable activity and research in the area of fluid dynamics and meteorology. At the present time our knowledge of transport mechanisms is far from being complete and, consequently, a great deal of faith cannot be placed in the quantitative predictions based on it.

Many research programmes have been carried out to determine the nature of atmospheric diffusion in order to correlate diffusion patterns to meteorological variables, and to develop models to describe such diffusion. Relationship between meteorological variables and plume behaviour has to be determined and incorporated into mathematical diffusion models to be used to predict atmospheric dispersion rates.

For example, rapid development of nuclear power throughout the world has been forecast. This, and a deepening world awareness of the necessity to preserve and improve the quality of the environment, requires that particular attention be given to the problems of release of radioactive contaminants into the atmosphere from various sectors of the nuclear fuel cycle. From a public-health point of view, the safe operation of nuclear research centres and power stations, or conventional industrial plants, can only be achieved if the diffusive properties of the first few hundred metres of the atmosphere over large areas around the sites, are adequately known.

Such knowledge of the diffusive capacities of the lower atmosphere would allow us:

- (a) to derive a working limit which may not be exceeded during routine release of an effluent into the atmosphere, and to check whether the resulting concentrations near the ground due to these substances are within safe limits, as regards the neighbouring population in the neighbourhood;
- (b) to take advantage of the most favourable meteorological conditions for special controlled release of pollutant material, the storage of which would otherwise entail considerable expense or other operational problems; and
- (c) to elaborate an effective environmental survey system, and to provide an infrastructure for intervening in case of major pollution accidents.

In most industrial countries the so-called "Gaussian plume" model is the basic method used for calculating ambient air pollution concentrations due to a point-source. It has the advantage of mathematical simplicity and easy applicability. But application of the Gaussian model requires knowledge of the standard deviations σ_y and σ_z of the concentrations, (see

Frenkiel (1953), Bochac et al (1974)).

$$\chi = \frac{Q}{2\pi U\sigma_y(x)\,\sigma_z(x)} \exp\{-\tfrac{1}{2}\left(\frac{y}{\sigma_y(x)}\right)^2 - \tfrac{1}{2}\left(\frac{z}{\sigma_z(x)}\right)^2\} \tag{1.1}$$

Taylor (1921) showed that for an ensemble average of particle displacements during conditions of stationarity and homogeneity, σ_y is given by

$$\sigma_y{}^2(t) \equiv \overline{y^2(t)} = 2\,\overline{V_L{}^2} \int_c^t \int_0^y R_L(q)\; d(q)\; d(y) \tag{1.2}$$

where v_L is the lateral component of the velocity affecting a particle, and $R_L(t)$ is the Lagrangian autocorrelation. A similar formula applies to σ_z.

The autocorrelation starts at 1 and approaches zero for large diffusion times. Exact knowledge of the behaviour of R_L at intermediate time is difficult to obtain for routine use in air pollution problems. Several methods have been suggested to determine σ_y and σ_z, which do not require the knowledge of R_L. These methods vary considerably in their development and application. Some rely more on empirical data than other ones. Separate methods are often recommended for elevated or ground sources. Frequently the different techniques in practical use are inconsistent with each other.

Therefore there exists a need for some relationship between the diffusive capacities of the atmosphere and a parameter, easily and continuously measurable, characteristic of the state of air stability within the layer wherein the effluents diffuse. The present work tries to contribute towards the determination of this relationship. Its three main results can be summarised as follows.

(a) It is shown that the Lagrangian autocorrelation can be replaced for practical purposes by an Eulerian space-time correlation following the mean flow;

(b) A model for the Eulerian space-time correlation in terms of the Eulerian time correlation is given;

(c) A model for the Eulerian time correlation as a function of amospheric stability is given.

This work is limited to the problem of horizontal dispersion of pollutants. Very few data on vertical diffusion are known to permit a deeper analysis in this case.

In Chapter 2 a literature survey is given.

A model for Eulerian time correlations $R_E(t)$ in terms of atmospheric or flow stability is given in Chapter 3. This model is used in Chapter 4 to test the Hay-Pasquill hypothesis, $R_L(\beta t) = R_E(t)$. This hypothesis is the most often used at the present time when R_E is available.

In Chapter 5 it is shown that a better approximation to R_L is needed, and a corresponding model is suggested.

Chapter 6 presents a model for space-time correlations, a field almost unexplored in fluid dynamics.

In Chapter 7 the dispersion coefficient σ_y is computed, replacing R_L by the model proposed in Chapter 6. This is the main point of this work which can be stated as "space-time correlations can be a workable approximation to the Lagrangian autocorrelations."

In Chapter 8 a tentative explanation is given for the variation of the factor in the Hay-Pasquill hypothesis, which leads to poor agreement between its predictions and measurements far from the source.

When R_E is not available, Sutton's formulae $\sigma_y = C_y x^n$ and $\sigma_z = C_z x^m$ are used with a great degree of em piricism since the parameters C_y, C_z, n and m vary with experiment site, atmospheric stability, direction of the wind, distance from the source, and season. In Chapter 9 a model is proposed to explain such behaviour.

A discussion concerning the results and propositions to new studies is given in Chapter 10.

Chapter 2 - LITERATURE REVIEW

2.1 Empirical Models for Turbulent Diffusion.

An unsolved problem of great importance in many fields is that of dispersion of a passive scalar in a turbulent fluid, e.g., the dispersion of pollutants in the atmosphere. A fundamental theory of turbulent dispersion is given by Taylor (1921) (see eq. (1.2)). This theory has not been put to common use as it is expressed in terms of Lagrangian (particle) correlations, which are considerably less accessible theoretically and experimentally than the corresponding Eulerian correlations. For Navier-Stokes equations are simpler in the Eulerian form, and the hot-wire anemometer provides a wealth of Eulerian data.

Several methods have been suggested for the determination of σ_y and σ_z, using Eulerian data. The methods are simple and quick to operate. Nevertheless, this application is empirical in the sense that the detailed micrometeorological process which carry and disperse the pollutant are not taken into account. (see Drexler (1976)).

Four main approaches were developed, leading to expression for σ_y:

(a) Fickian - $\sigma_y^2 = 2 K_y t$;

(b) Sutton - $\sigma_y^2 = C_y t^{2-n}$

(c) Hay-Pasquill - $R_L(\beta t) = R_E(t)$;

(d) Pasquill-Gifford Graphs.

2.1.1 Fick's Equation.

The differential equation which has been the starting point of most mathematical treatments of diffusion from sources is a generalisation of the classical equation for conduction of heat in a solid, and is essentially a statement of conservation of suspended material:

$$\frac{D\chi}{Dt} = \frac{\partial}{\partial x}\left(K_x \frac{\partial\chi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial\chi}{\partial y}\right) + \frac{\partial}{\partial y}\left(K_z \frac{\partial\chi}{\partial z}\right) \qquad (2.1)$$

If the K's are constant the simplified equation and the type of diffusion implied are Fickian and we obtain $\sigma_y^2 = 2\,K_y t$, etc. The coefficients K_x, K_y, K_z and the velocity are, generally speaking, variables, and the analytical solution is obtained only by making some particular assumptions about these coefficients. Analytical solutions of the three-dimensional diffusion equation with variable K and wind have been obtained only under restricted assumptions. Smith (1957) used power law variations for wind and K. Peters and Klinzing (1971) used power laws for K and constant wind. No analytical solution of the diffusion equation have been obtained taking a logarithmic wind profile. Numerical solution for this problem is presented by Ragland and Dennis (1975). More solutions of eq. (2.1) are given by Sutton (1953), Pasquill (1974) and Monin and Jaglom (1971), where the problem of the velocity of propagation of the pollutants is discussed. The exact behaviour of the K's is unknown and so the method has not been extensively used.

2.1.2 Sutton's Formulae.

On the basis of experimental data, Sutton (1953) proposed the relations

$$\sigma_y^2 = C_y t^{2-n} \quad \sigma_z^2 = C_z t^{2-m} \qquad m = n \qquad (2.2)$$

where C_y, C_z, n and m are assumed constants. These formulae are not free of theoretical difficulties. They give fixed values of n and m whereas we know from Taylor's theorem (eq. (1.2)) that they must tend to zero near the origin and to one when t is very large in homogeneous flows. This behaviour is observed in experimental data by Fuquay et al (1964). Sutton assumes too that n and m can be determined from the wind profile taking $U \sim z^q$. Barad and Haugen (1959) found that neither n or m can be determined by q. The data show some variation of n and m with distance from the source.

In the engineering use Sutton's formulae have come into widespread use in the design of stacks and environmental impact analysis. Three main sets of coefficients (n_y, n_z, C_y, C_z) are in use, each set giving the coefficients for all different states of atmospheric stability. Ragland (1976) calculates the highest possible ground-level ambient air concentration due to a stack, and finds that the three recommended sets of values, TVA (Carpenter et al (1971)), ASME (Smith (1973)) and EPA (Turner (1970)), give, under similar conditions of atmospheric stability results differing by a factor as high as 5. Different values for the coefficients were found by Bultynck and Malet (1972), Hogstrom (1964), Vogt (1974) and others cited in Slade (1968). From the published data one may conclude that the diffusion parameters in Sutton's formulae are functio of atmospheric stability, distance from the source, height of source, experimen site, and even season of the year, in an as yet unknown form.

It is clear that if Sutton's model may describe diffusion over some restricted range of distance, it should in some way be related to Taylor's theorem. There is urgent need for connecting the model with some well-founded theory, in order to correlate Sutton's cofficients with atmospheric and topographical features.

2.1.3 Hay-Pasquill's Hypothesis.

Hay and Pasquill (1959) proposed a simple relationship between the Lagrangian correlation and the Eulerian time correlation,

$$R_L(\beta t) = R_E(t) \tag{2.}$$

where the scale parameter β is supposed to be constant. In the original work the value $\beta = 4$ is suggested.

Haugen (1966) analysed selected Prairie Grass diffusion experiments (Barad (1958)). No correlation is found between β and atmospheric stability. Of the 35 experiments analysed, only 13 were found to give value of β between 1 and 10 In 9 cases $\beta>10$ (but less than 160); in the remaining cases $\beta<1$.

The results of various field and laboratory studies of diffusion yield a fairly wide scatter of β. For diffusion of helium in a neutral boundary layer Chandre (1972) found β ranging from 3.7 to 21. For turbulent diffusion in pipe flow Baldwin and Mickelsen (1963) found β between 4 and 18. Angell (1974) obtained β in the range 2 to 16 for atmospheric flows at high altitude.

It has been suggested to use a relationship between β and the intensity of turbulence of the flow under study (Corrsin (1963), Saffman (1963), Pasquill (1968), Pasquill (1974)), but experimental data do not seem to support this hypothesis (see Pasquill (1974), p. 92).

2.1.4 Pasquill-Gifford Graphs.

Gifford (1961) and Pasquill (1961) suggested that diffusion data could be most conveniently summarised in graphical form for various stability categories

But the graphs are not applicable to diffusion over water (Hosker (1974), Raynor et al (1975)). For the same forest Vogt (1974) concludes that different graphs are required in summer and in winter. Using his own results, Bowne (1974) shows that the graph predictions differ from Pasquill's results by a factor as high as 2 for the same class of stability. He proposes a new set of graphs for irregular terrains and urban areas.

Briggs (1973) has recently proposed revised dispersion coefficients for rural and urban areas which differ from Pasquill-Gifford's coefficients by a factor which approaches sometimes 3. Smith (1973 a) has attempted to separate the influence of roughness on the vertical dispersion coefficient σ_z by giving different graphs for different values of z_0. The main problem in turbulent dispersion is which graph gives the best results for a given site.

2.2 Eulerian Atmospheric Time Correlations and Spectral Densities.

For many years researchers looked for general formulae for the Eulerian time correlation $R_E(t)$ and its Fourier transform, the spectral density $F(n)$. Two main principles guided this search. Kolmogorov suggested (see Kolmogorov (1962)) that there could be a subrange of wave numbers $K(= 2\Pi n/U)$ in the spectral density, for which only the transfer of energy is important, if Reynold's number is large enough. This leads for large Reynolds number to

$$F(K) \sim \varepsilon^{2/3} K^{-5/3}$$

$$F(K) = \frac{U}{2\Pi} F(n) \tag{2.4}$$

ε is the energy-transfer rate, and n- the frequency. Next, the Monin-Obukhov similarity hypothesis (see Monin and Yaglom (1971)) predicts that, when turbulent fluxes of momentum and heat are constant, as they are in the atmospheric surface layer, the structure of turbulence is determined solely by the groups

$$u_* = \left(\frac{\tau_o}{\rho}\right)^{\frac{1}{2}}; \quad T_* = \frac{-Q_o}{\rho u_* C_p}; \quad L = -\frac{u_*^3 \rho \; C_p \; T}{0.4 \; g \; Q_o} \tag{2.5}$$

where u_* is the friction velocity and L- the Monin-Obukhov parameter. These lead to the following similarity relationships for the mean velocity and temperature distributions

$$\frac{z}{u_*} \frac{\partial U}{\partial z} = \phi_U \left(\frac{z}{L}\right)$$

$$\frac{z}{T_*} \frac{\partial T}{\partial z} = \phi_T \left(\frac{z}{L}\right) \tag{2.6}$$

where ϕ_U and ϕ_T are universal functions of $\frac{z}{L}$. Different forms for ϕ_U are given by Monin and Yaglom (1971), Plate (1971) and Hicks (1976) among others. Measurements to obtain L are difficult. Relationships between $\frac{z}{L}$ and the Richardson number R_i, which is easier to measure, are given by Busch et al (1968), Webb (1970), Busch (1973), Kaimal (1973), Lettau (1973) and Haugen (1973).

Extension of the Monin-Obukhov scaling to spectra of velocity, temperature, and humidity, leads to the assumption that the properly scaled spectrum nF(n) should be function of the reduced frequency $f = \frac{nz}{U}$ and $\frac{z}{L}$ only. The forms of these universal functions should either be determined by experiment or deduced theoretically, making further assumptions about the nature of the turbulence.

A steadily growing collection of micrometeorological spectra from the atmospheric surface layer is available in recent literature: Busch and Larsen (1972), Busch (1971), Kaimal et al (1972), Panofsky (1969), Busch and Panofsky (1968) Sharan and Wickerts (1974), Hogstrom and Hogstrom (1975), Kaimal et al (1976), McBean (1971), Davenport (1961), Pasquill (1972).

The most recent spectra of velocity in the atmosphere show a property: at high frequencies they may be described by(Kaimal et al.(1972))

$$\frac{\overline{v^2}\ n\ F(n)}{u_*^2} = C\ \left(\frac{z\varepsilon}{u_*^3}\right)^{2/3}\left(\frac{nz}{U}\right)^{-2/3} = C\ \phi_\varepsilon^{\ 2/3}\ f^{-2/3} \tag{2.7}$$

ϕ_ε, the dimensionaless dissipation rate for turbulent energy, is a function of $z/_L$ only. If we include ϕ_ε in the normalisation of F(n) for different states of atmospheric stability, all spectra, regardless of $z/_L$, coincide in the Kolmogorov (inertial) subrange. At low frequencies there is a systematic progression with $z/_L$ for a stable atmosphere, but no clear picture is available for unstable atmosphere. It is easy to see that eq. (2.7) obeys both Kolmogorov's and Monin-Obukhov's hypotheses.

Few data of Eulerian time correlations are given in literature. Barad (1958) and Panofsky (1962) present some results but no theoretical guide exists to describe them. Panofsky (1962) calls the attention to some situations where the time correlation drop to a plateau, which might be as high as 0.80. This is attributed to slow wave-like atmospheric oscillations which may contribute largely to the dispersion of pollutants. Caughey and Readings (1975) give an account of an observation of a "wave-like" phenomenon obtained during study of nocturnal inversions (see also Rayment and Readings (1974)).

All the spectra cited above refer to micrometeorological turbulence, with most energy in the range 10^{-2} - 10 hertz. The spectra of kinetic energy for synoptic scales (few hours to 20 days) are given by Hess and Clarke (1973). The spectrum falls off approximately as $n^{-2.6}$ whereas Kolmogorov's theory predicts $n^{-5/3}$ for spatial isotropic turbulence, and Kraichnan (1967) predicts one region with $n^{-5/3}$ and another with n^{-3} for plane turbulence.

Theoretical research in correlation problems is a much difficult subject. At the moment mainly very complicated formulas are known, and only for a few restricted cases. Theories, as well as experimental evidence, have shown that there exists no universal formula for Eulerian correlations but that the formula to be applied depends on the type, condition and stage of the turbulent flow.

Even for isotropic turbulence the equations are difficult to solve: there are more unknowns than equations available. The equation for the second-order correlation contains a third-order correlation; the equation for the third-order correlation contains a fourth-order correlation, and so on. Various attempts have been made to solve this so-called "closure problem." Most of these methods show negative parts in the spectral density of energy, a physically unacceptable situation. When we try to solve the equation for the spectral density we encounter the "energy-transfer function" which is related to the third-order correlation, and represents the interaction between eddies of different wave numbers. This is an unknown function too, for which various forms have been suggested. Most of these tentative solutions are found in Hinze (1975), Panchev (1971), Herring (1975) and Fox (1973).

Higher order closure models to be solved by computational techniques have been given by Lumley and Khajeh-Nouri (1974), Wyngaard et al (1974), Fox and Lilly (1972), Betchov (1975) and a review is given by Reynolds (1976), but no simple analytical form which can be used by meteorologists and air pollution researchers, has been published.

2.3 Space-time Correlations.

Taylor (1938) made the hypothesis that turbulence may be regarded as a frozen pattern of eddies being swept past the observer. This means that a signal received at one measuring position will be received at a second position x_1 directly downstream from the first, at a time t_1 later. Measurements of correlations involving two positions and two times, space-time correlations, have shown that this Taylor's hypothesis is not correct: the correlation between the pattern of eddies at $t = o$ and at $t = t$ decreases with distance downstream. Lumley and Panofsky (1964) summarise results from a number of field experiments in the atmosphere which suggest that Taylor's hypothesis is satisfied for "short times." Mizuno and Panofsky (1975) attempt to analyse the exact limits of validity of the hypothesis.

A better understanding of the problem demands a better knowledge of space-time correlations. A model for these correlations in the atmosphere was given by Pielke and Panofsky (1970) and studied in more detail by Baldwin and Johnson (1973), Panofsky et al (1974), Panofsky and Mizuno (1975), Ropelewsky et al (1973), but no clear picture emerged and further studies and/or different models are needed.

Space-time correlations in isotropic turbulence behind a regular grid spanning a uniform airstream have been measured since the 1950's, and given by Fisher and Davies (1963), Favre (1965), Frenkiel and Klebanoff (1966), Comte-Bellot and Corrsin (1971). Measurements in pipe flow are given by Baldwin and

Mickelsen (1963), and Sabot et al (1973), among others. Most of the results concern the optimum space-time correlation, defined as the maximum reached by the correlation for a given distance x, at a certain time delay $t = t_{max}$.

Theoretical estimates of space-time correlations are given by Batchelor and Townsend (1948), Inoue (1951), Bass (1974), Kovasznay (1948), Favre (1965) and Kraichnan (1967). In general they give very complicated models with a restricted range of validity. It appears that none has ever been applied in pollution problems

2.4 Lagrangian Correlations R_L and Eulerian Space-time Correlations R_{ST}

Very little is known about Lagrangian autocorrelations. No completely satisfactory theory exists nor have any completely satisfactory experimental data been generated. It was suggested to replace the Lagrangian autocorrelations by space-time correlations for the calculation of dispersion in turbulent flow (Corrsin (1959), Sheppard (1959), Baldwin and Mickelsen (1963)). As the two correlations are determined by the same turbulent field, it is reasonable to expect a more or less close relationship between the two autocorrelations. Yet the problem of obtaining theoretically this relationship has not yet been solved.

Baldwin and Mickelsen (1963) used this idea taking the space-time correlation measured during their heat-diffusion experiment. Good agreement was obtained between measured and "calculated" diffusion. Similar results were obtained by Peskin (1974) in "computer experiments."

Corrsin (1963) showed that dimensional analysis leads to roughly equal integral time scales for both correlations. Shlien and Corrsin (1974) estimate from their measurements that $R_L(t) \geqq R_{ST}$. Computer experiments by Riley and Paterson (1972) show that $R_L \geqq R_{ST}$ for small values of t and $R_L \leqq R_{ST}$ for large values of t when similar experiments by Paterson and Corrsin (1966) show that $R_L < R_{ST}$ for small t and $R_L \approx R_{ST}$ for large t. Kraichnan (1970) shows theoretically that $R_L \leqq R_{ST}$. We may conclude that both correlations, the Lagrangian R_L and the space-time R_{ST} are roughly equal for all t. As computation of dispersion is relatively insensitive to the exact shape of the correlation, the replacement of R_L by R_{ST} should not alter substantially the results. The reason for the unfrequent use of this suggestion is the lack of a model to express space-time correlations at all t.

In relation to the microscales of both correlations (see Tennekes and Lumley (1972)), the prediction of Corrsin (1963) that they have nearly equal microscales, has not been confirmed by experiments (Shlien and Corrsin (1974)) and by a recent theoretical analysis (Tennekes (1975)).

Chapter 3 - A MODEL for AUTOCORRELATIONS and SPECTRAL DENSITIES in TURBULENT FLOWS.

3.1 Introduction

Recent years have seen a steady increase in the amount of data available concerning the detailed structure of atmospheric turbulence near the ground. Although much remains to be done and much confusion exists with respect to the interpretation of these data, a picture of the turbulence structure in thermally stratified media is emerging. Yet our knowledge about the characteristics and governing parameters of atmospheric turbulence is still limited, because of the complex nature of the turbulent flow field in the lower part of the earth's boundary layer.

In practical problems such as atmospheric pollution, structural design, and aviation, the knowledge of autocorrelations and their spectra is required. In this chapter we show how to obtain theoretically the spectrum as a function of the integral time scale E, a quantity which can easily and accurately be measured with some hints that it can be obtained from the Richardson number. The autocorrelations corresponding to the spectrum are computed and the results compared with experiment. This universal formula for the autocorrelation makes the use of Hay-Pasquill's hypothesis for computing $R_L(t)$ much easier. If this hypothesis were correct, the knowledge of the Richardson number would give us σ_y immediately.

3.2 Spectra and Correlations.

Assume a turbulent flow where the instantaneous Eulerian velocity is

$$\vec{V} = (U+u,\ v,\ w), \tag{3.1}$$

with the turbulent components satisfying

$$\overline{u} = \overline{v} = \overline{w} = 0. \tag{3.2}$$

The Eulerian time autocorrelation is defined by

$$R_{Ev}(t) = \frac{\overline{v(o)v(t)}}{\overline{v^2}} \tag{3.3}$$

with similar expressions for the u and w components. Stationary conditions are implied in (3.3), R_E is assumed to be dependent only on time-lag, not on the actual time at which the sequence of values begins.

Regarding the turbulent flow as composed of a spectrum of fluctuations, we define the quantity F(n), the energy spectrum (power spectrum, spectral density) which represents the fractional contribution to the total energy of frequencies between n and n+dn, so that

$$\int_0^\infty F(n)\ d(n) = 1 \tag{3.4}$$

The spectral density and the autocorrelation are the Fourier Transforms of each other (Taylor (1938)).

$$F(n) = 4 \int_0^\infty R_E(t) \cos 2\Pi nt \ dt$$

$$R_E(t) = \int_0^\infty F(n) \cos 2\pi nt \ dn \quad (3.5)$$

A convenient overall representation is provided by the Eulerian integral time scale E

$$E = \int_0^\infty R_E(t) \ dt \quad (3.6)$$

which is related to the spectral density by

$$\lim_{n \to o} F(n) = 4\int_0^\infty R_E(t) \cos (o) \ dt = 4E \quad (3.7)$$

In general the product nF(n) is plotted against ln n in view of the wide range of frequencies involved. Also, this method of plotting produces in the graph a peak at a frequency n_m related to the integral time scale E, as I will show later.

3.3 A Simple Model for the Spectrum.

Exact analytical expressions for F(n) are not known. There exist more unknowns than equations to be solved. So empirical models are given as function of n_m (Kaimal (1973)); as the spectra are very flat, the assessment of n_m is difficult. In neutral stratification, for example, Busch and Panofsky (1968) found for the u component n_m z/U somewhere between 0.025 and 0.06.

To get a simple model for F(n) in terms of E, we use three properties that the spectrum should satisfy: eq. (3.4), eq. (3.7), and Kolmogorov's (1962) hypothesis. He predicts a range of n, where (see eq.(2.4))

$$nF(n) \propto n^{-2/3}, \quad (3.8)$$

Experiment has shown that this law is obeyed over a large range at high frequencies (Hinze (1975)). In order to obtain a simple formula for the spectra, I suggest expressions, functions of E, which satisfy eqs. (3.7) and (3.8) and resemble empirical formulae for $S_v(n)=\overline{V^2}\ F(n)$:

$$F(n) = \frac{\nu E}{1 + (\eta En)^{5/3}} \quad (3.9a)$$

and

$$F(n) = \frac{\delta E}{(1 + \lambda En)^{5/3}} \quad (3.9b)$$

From eq. (3.7) we conclude at once that $\nu = \delta = 4$. Eq. (3.4) gives values for η and λ , resulting in

$$nF(n) = \frac{4En}{1 + 31.5(En)^{5/3}} \quad (3.10a$$

and

$$nF(n) = \frac{4En}{(1 + 6En)^{5/3}} \tag{3.10b}$$

The difference between the two expressions is not great. Eq. (3.10a) gives a smaller contribution at low frequencies. Only comparison with experiment can show which formula fits better the data.

Searching for the maximum of nF(n) given by eq. (3.10) gives a relationship between E and n_m, the frequency at which nF(n) attains its highest value

$$E = \frac{0.160}{n_m} \tag{3.11a}$$

$$E = \frac{0.25}{n_m} \tag{3.11b}$$

Eq. (3.11a) corresponds to eq. (3.10a). Writing the spectrum in terms of n_m we get for (3.10a) and (3.10b)

$$nF(n) = \frac{0.64\ (n/n_m)}{1+1.5(n/n_m)^{5/3}} \tag{3.12a}$$

$$nF(n) = \frac{(n/n_m)}{1+1.5(n/n_m))^{5/3}} \tag{3.12b}$$

These formulas agree well with the empirical expressions of Kaimal (1973) and Pasquill and Butler (1964).

3.4 Relation to Monin-Obukhov Similarity Hypothesis.

One approach, widely accepted in the description of spectra, is the "similarity hypothesis" of Monin and Obukhov (see Monin and Jaglom (1971)). It is assumed that all spectra can be written in the form

$$nF(n) = G\ (\frac{nz}{U}, \frac{z}{L}) \tag{3.13}$$

$\frac{nz}{U}$ is the nondimensional "reduced frequency"and z the height above the ground. L is the Monin-Obukhov length scale, related to the vertical heat flux carried by turbulence, and consequently to atmospheric stability. It appears always in the form z/L so that there is no confusion with the Lagrangian time scale L.

Take eq. (3.10a) for example. It may be re-written as a function of $\frac{nz}{U}$

$$nF(n) = \frac{4\ \frac{EU}{z}(\frac{nz}{U})}{1 + 31.5\ \{(\frac{EU}{z})(\frac{nz}{U})\}^{5/3}} \tag{3.14}$$

In addition, Kaimal et al (1972) show that for stable atmosphere $\frac{EU}{z}$ is a function of $\frac{z}{L}$. This proves that our model satisfies eq. (3.13). One important point to be noted is the possibility of knowing F(n) from simple measurements of mean velocity and temperature at several heights. From both profiles we determine the Richardson's number (Turner (1973)), from which we can get z/L (Busch (1973), Lettau (1973)), E and F(n).

The similarity hypothesis is valid in the layer of constant heat flux. The layer height in the atmosphere is not well defined but 100 - 150m seems to emerge as the true value (Counihan (1975), Panofsky (1974)). For unstable atmosphere, experiments show that E is constant with height, and determined in the surface layer by the large convective systems (see Kaimal (1973), p. 581, and Wyngaard (1973)).

3.5 Autocorrelations.

The next step is the calculation of autocorrelations corresponding to different spectral forms. Eqs. (3.5) and (3.10) lead to

$$R_E(t) = \int_o^\infty \frac{4}{1 + 31.5(nE)^{5/3}} \cos\left(2\pi nE \frac{t}{E}\right) d(nE) \tag{3.15a}$$

and

$$R_E(t) = \int_o^\infty \frac{4}{(1 + 6En)^{5/3}} \cos\left(2\pi nE \frac{t}{E}\right) d(nE) \tag{3.15b}$$

Both autocorrelations are functions of $\frac{t}{E}$. Our model predicts that knowledge of E is enough to describe the Eulerian time autocorrelation.

Unfortunately (3.15a) and (3.15b) cannot beexpressed in terms of known or tabulated functions. They have been computed numerically and are given in Table 3.1 and fig. 3.1. The results are compared with the exponential correlation which is often taken as an approximation in analytical solutions involving $R_E(t)$ and $R_L(t)$ (see Csanady(1973)).

Table 3.1 - Autocorrelations given by eq. (3.15a) (column 1), by eq. (3.15b) (column 2) and by $R_E(t) = \exp(-t/E)$ (column 3).

t/E	Column 1	Column 2	Column 3
0.0625	0.86	0.80	0.94
0.125	0.79	0.70	0.88
0.25	0.67	0.56	0.78
0.5	0.52	0.40	0.61
1	0.32	0.25	0.37
2	0.14	0.12	0.13
4	0.03	0.04	0.01
8	0.00	0.01	0.00

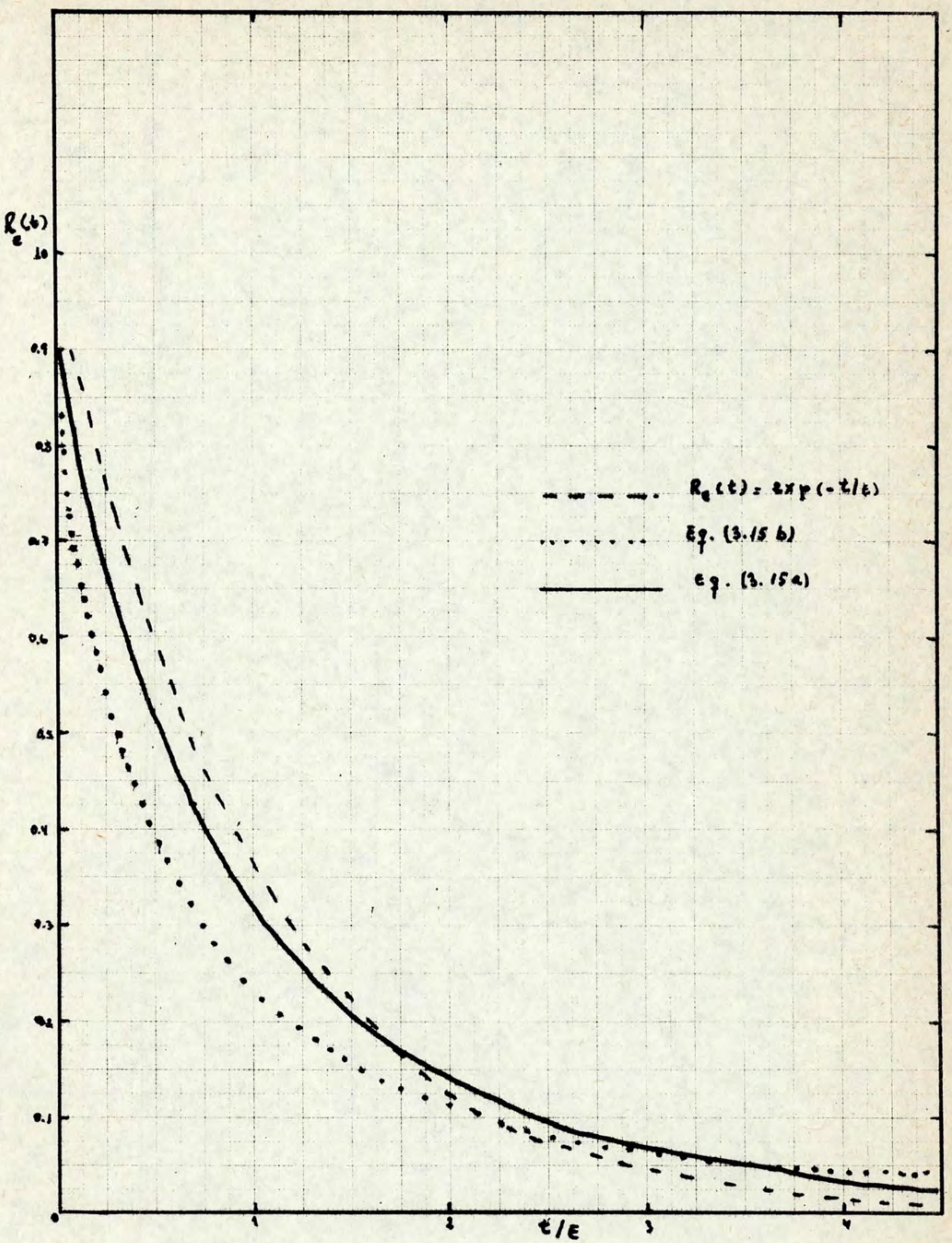

Fig. 3.1 - Models for Eulerian autocorrelation $R_E(t)$ compared with the exponential correlation.

From fig. 3.1 we see that the exponential autocorrelation is a poor approximation to the computed autocorrelations since when they agree in a given interval t/E (with a different E for each correlation), there is a difference of tens of percent between them outside this interval. In respect to computation of pollution dispersion the situation is much better since Taylor's theorem eq. (2.1), is not sensitive to the exact shape of the autocorrelation. It will be shown later that all three correlations of fig. 3.1 give approximately the same shape for $\overline{y^2}$.

In the computation of space-time correlations this is not the case. In Chapter 6 I show that space-time correlations generated from exponential time correlations have a shape very different from that obtained when we take the correlations given by eqs. (3.15). For this reason, the Hay-Pasquill's hypothesis can be tested using R = exp (-t/E) but for the computation of space-time correlations we must take eqs. (3.15).

In fig. 3.2 we compare our computed predictions with time autocorrelations measured in laboratory turbulent flows behind a grid. Good agreement is obtained between preditions given by eq. (3.15a) and experimental data. The value of E is taken to give the best agreement. In all cases where F(n) is given, this E agrees well with the value of F(o)/4.

In fig. 3.3 our computed results are compared with autocorrelations measured in the lower atmosphere (z = 2m). There is agreement between experiment and eq. (3.15b).

One tentative explanation for this difference between correlations in the atmosphere and in laboratory is as follows: there is no space in laboratory flow for eddies of small frequency, so the spectral density has a smaller contribution from this region. This is exactly the difference between eqs. (3.15a) and (3.15b).

We conclude that the present model for spectra and correlations is simple to use and describes satisfactorily measured data, when E is known. The model has potential use in air pollution problems where formulae simple to understand and to apply are in demand.

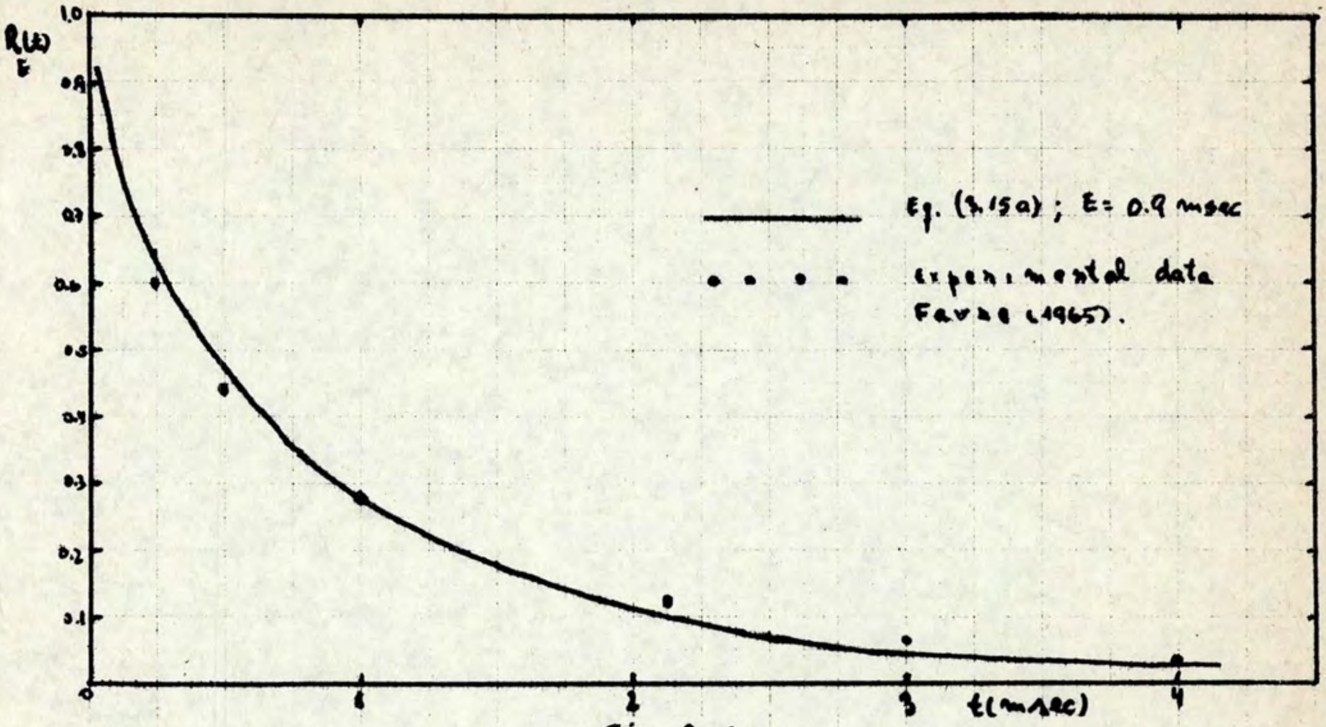

Fig 3.2a

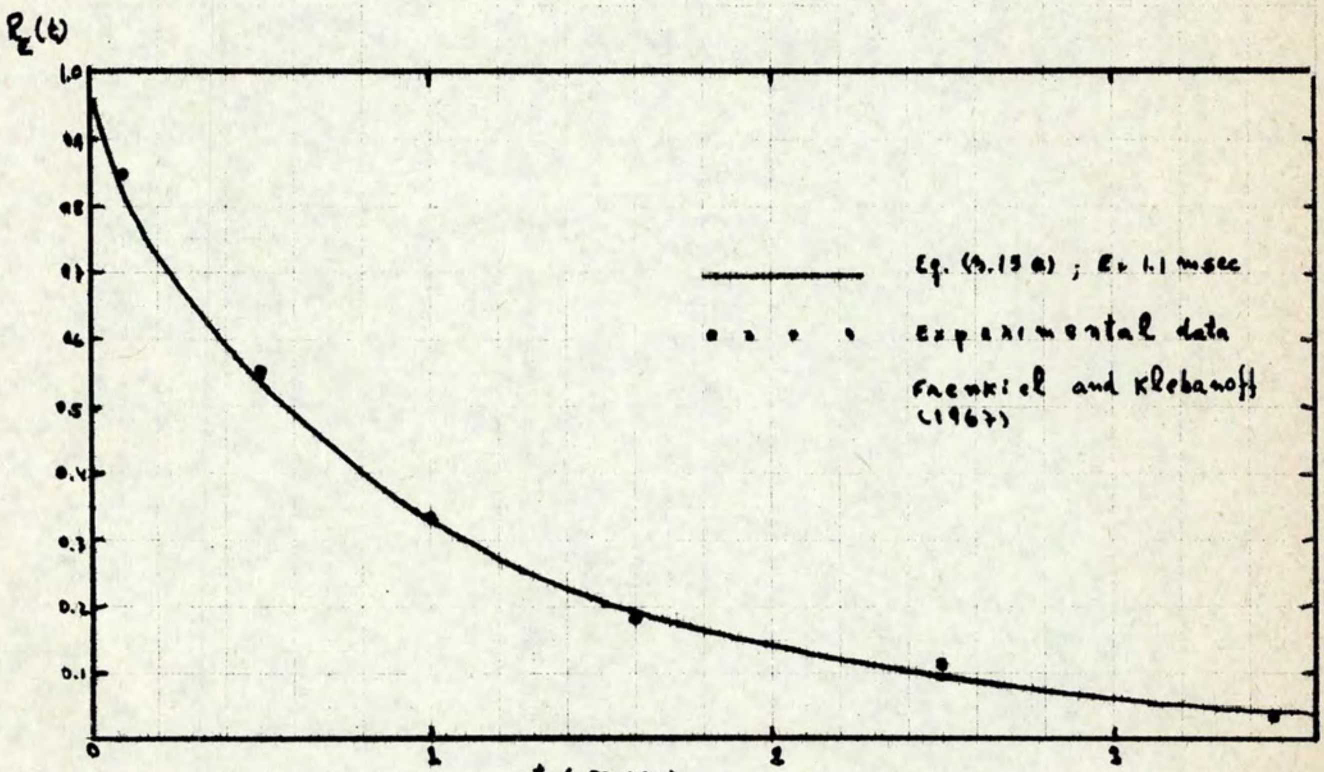

Fig 3.2.b

Fig. 3.2 - Comparison between predicted and measured Eulerian time correlations in turbulence behind grids. In fig. 3.2.c the exponential correlation is added for comparison. The value of E is chosen to give the best fit.

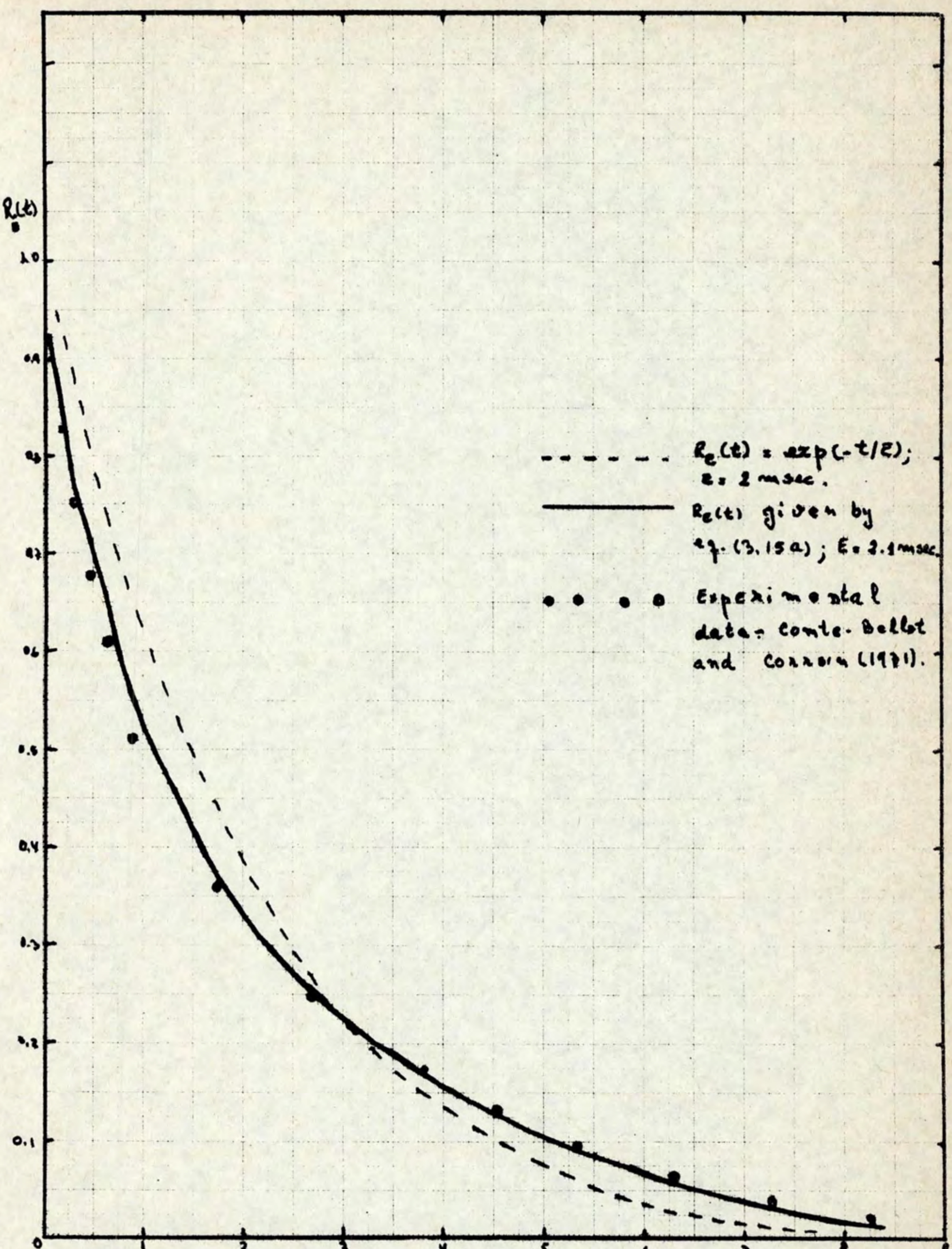
R(t)
$R_E(t) = \exp(-t/\tau)$; $\tau = 2$ msec.
$R_E(t)$ given by eq. (3.15a); $\tau = 2.1$ msec.
Experimental data - Comte-Bellot and Corrsin (1971).
t (msec)

Fig 3.2.c

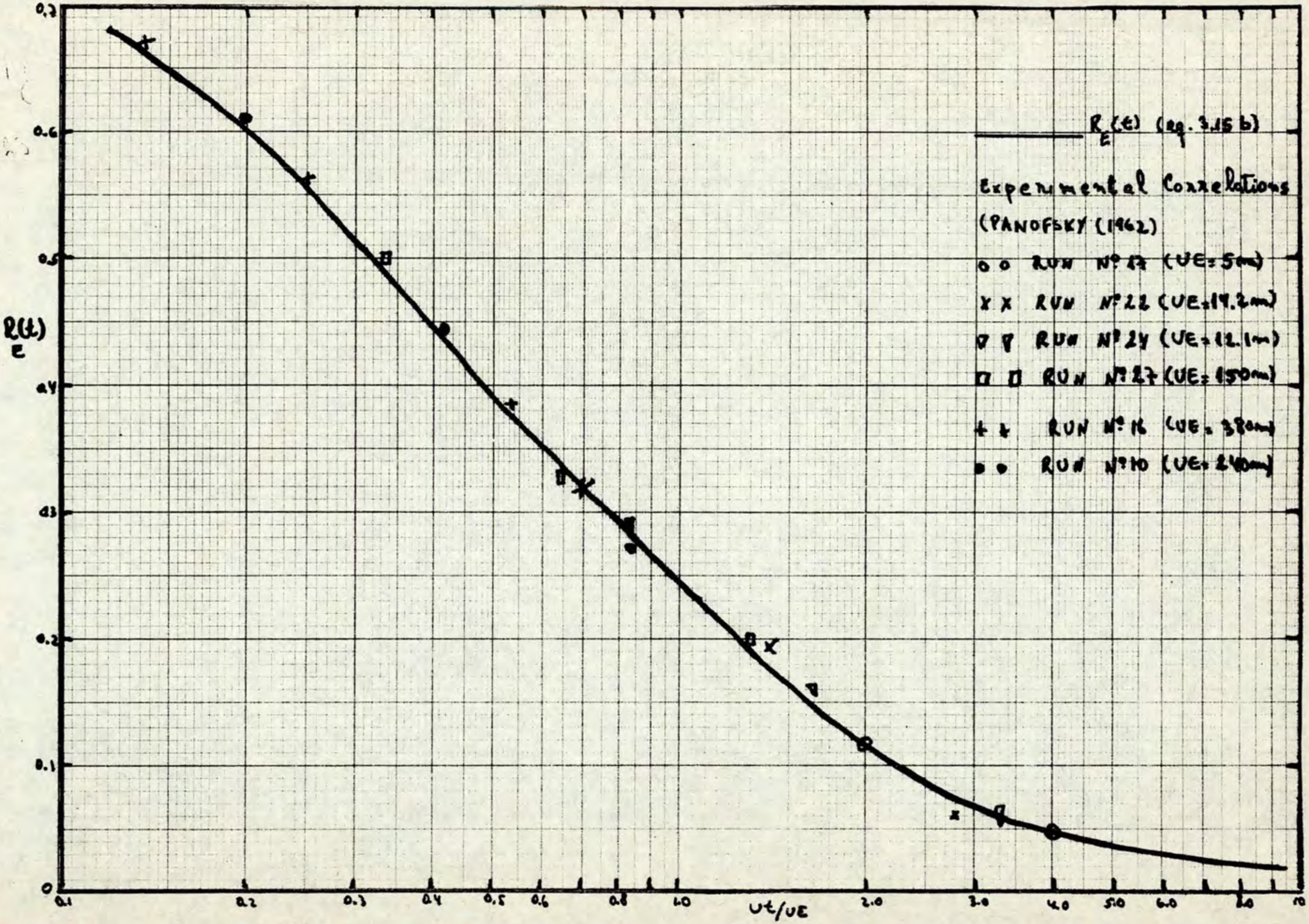

Fig. 3.3 - Comparison between predicted and measured Eulerian time correlation

for atmospheric flow. Z = 2m.

Chapter 4 - ARE THE LAGRANGIAN AND THE EULERIAN TIME AUTOCORRELATIONS OF SIMILAR SHAPE?

4.1 Introduction

In the preceding Chapter different possible shapes of the Eulerian time autocorrelation were displayed. Hay and Pasquill (1959) assume that one of these correlations can represent the Lagrangian autocorrelation $R_L(t)$ by a simple change of scale,

$$R_L\ (\beta t) = R_E\ (t) \tag{4.1}$$

In this Chapter we test, this hypothesis comparing its predictions with observations. The method used here has the advantage that it does not require the knowledge of the variance of the particle velocity, $\overline{V_L^2}$.

4.2 Theory

For steady, homogeneous turbulence, the spread of particles released serially from a fixed point is described by the well-known formula, due to G.I. Taylor (1921)

$$\sigma_y^2\ (t) \equiv \overline{y^2} = 2\ \overline{V_L^2} \int_0^t \int_0^\tau R_L(\nu)\ d\nu d\tau \tag{4.2}$$

or alternatively, an expression given by Kampé de Fériet in 1939 (see Kampé de Fériet (1974)). ν and τ are integration variables.

$$\sigma_y^2\ (t) = 2\ \overline{V_L^2} \int_0^t (t - \tau) R_L(\tau) d\tau \tag{4.3}$$

y(t) is the displacement of a particle along the y-axis, V_L is the y component of the particle velocity ($\overline{V}_L = 0$), and $R_L(\tau)$ is the Lagrangian correlation,

$$R_L(t) = \frac{\overline{V_L\ (t)\ V_L(o)}}{\overline{V_L^2}} \tag{4.4}$$

$V_L(o)$ and $V_L(t)$ refer to the velocity of the same particle, not to the velocity at a given point in space. The bar denotes an ensemble average.

Although the turbulence near the ground is not homogeneous in any vertical plane, it may be assumed to be homogeneous if we confine our attention to properties in a horizontal plane. Thus, for horizontal dispersion over a uniform level ground we may assume approximate homogeneity and applicability of eq. (4.3).

Hay-Pasquill's hypothesis is tested introducing eq. (4.1) into eq. (4.3),

$$\sigma_y(t) = 2\overline{V_L^2} \int_0^t (t - \tau)\ R_E\ (\tfrac{\tau}{\beta})\ d\tau. \tag{4.5}$$

In Chapter 3 we showed that R_E is a function of t/E, and

$$R_L(t) = R_E(\frac{t}{\beta E}) = R_E(\frac{Ut}{\beta UE}) \tag{4.6}$$

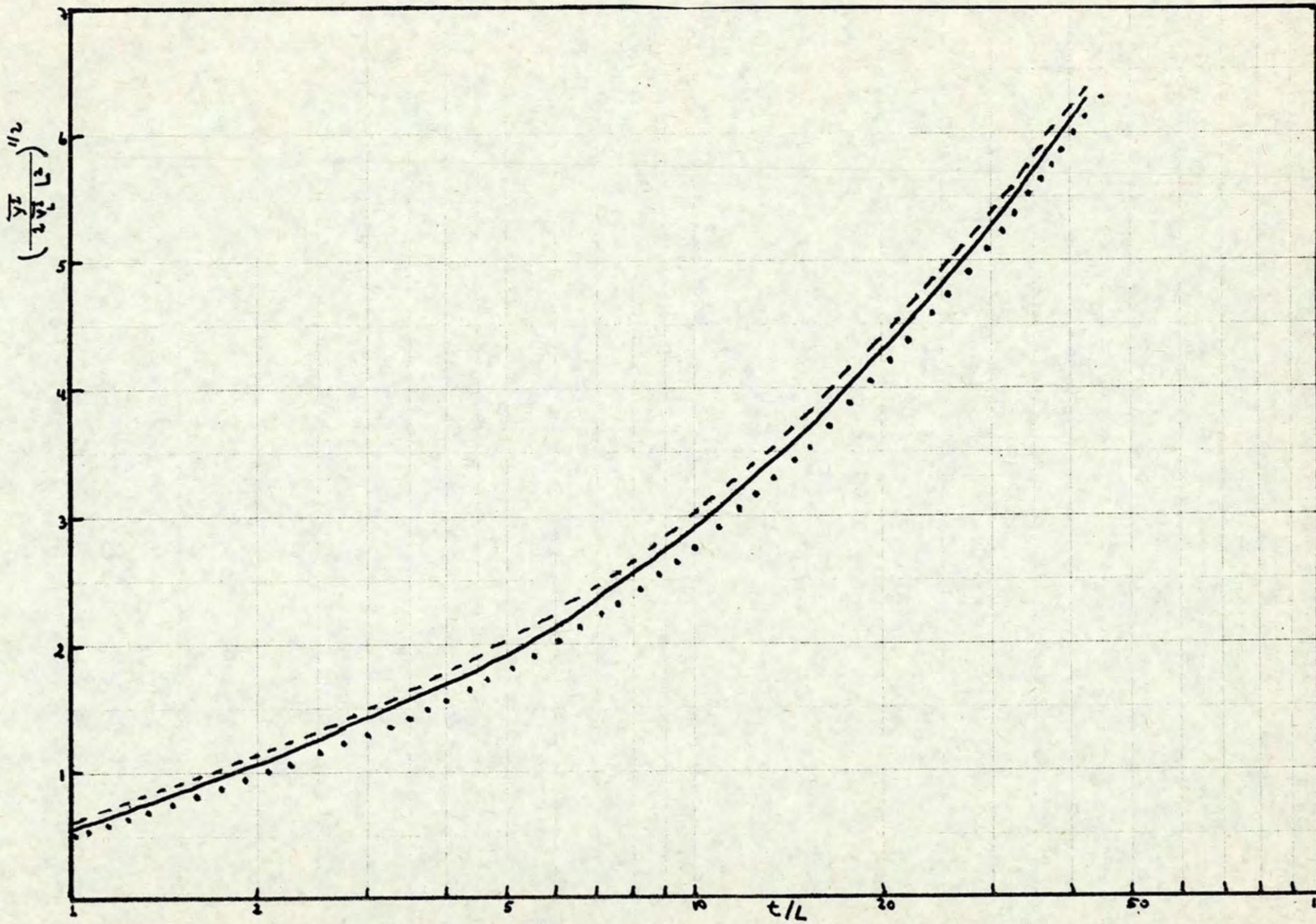

Fig. 4.1 - Dispersion predicted assuming Hay-Pasquill's hypothesis, $R_L(\beta t) =$ $= R_E(t)$, with $R_E(t)$ given by the models of Chapter 3. ....R_E given by $R_E = \exp(-t/E)$; ——— $R_E(t)$ given by eq. (3.15a); ... $R_E(t)$ given by eq. (3.15b). $L = \beta E$.

resulting in (4.7)

$$\sigma_y^2 (x) = 2(\frac{\overline{V_L^2}}{U^2}) \int_0^X (x-U\tau)R_E(\frac{U\tau}{\beta UE})\, d(U\tau) = 2\overline{V_L^2}\, L^2 \int_0^{\frac{X}{UL}} (\frac{X}{UL} - \frac{U\tau}{UL})R_E(\frac{U\tau}{UL})d(\frac{U\tau}{UL})$$

where the Lagrangian integral time scale L,

$$L = \int_0^\infty R_L(t)\, dt \tag{4.8}$$

is related to the Eulerian integral time scale E by

$$L = \beta E \tag{4.9}$$

The plume standard deviations σ_y obtained from eq. (4.7), using the three models for R_E presented in Chapter 3, are shown in fig. 4.1 and Table 4.1. They were obtained by numerical integration. The difference between the results is not significant. Once we know E and β, or L, we may use the exponential autocorrelation to obtain approximate results for $\overline{y^2}$. As pointed out before (Frenkiel (1953)), the standard deviation is not very sensitive to the exact shape of the autocorrelation. All these conclusions are valid if and only if the Hay-Pasquill's hypothesis, eq. (4.1) is correct, i.e., if β is constant for a given experiment.

Table 4.1 - $\overline{y^2}/(2\overline{V_L^2}\, L^2)$ predicted by the Hay-Pasquill similarity hypothesis. The Eulerian autocorrelations are given in Chapter 3. Column 1 - $R_L = \exp(-t/L)$; column 2 - R_E given by eq. (3.15a); column 3 - R_E given by eq. (3.15b).

t/L	Column 1	Column 2	Column 3
0.125	0.0075	0.0070	0.0066
0.5	0.106	0.095	0.085
1.0	0.367	0.324	0.276
4.0	3.018	2.73	2.26
8.0	7.	6.58	5.60
16.0	15.	14.50	12.91
32.0	31.	30.46	28.21
64.0	63.	62.40	59.47
12.80	127.	126.35	122.77

4.3 A Test of the Hay-Pasquill Hypothesis

In this section we compare with experiment the theoretical preditions assuming $R_L(\beta t) = R_E(t)$. Taking the ratio

$$\frac{\overline{y^2}(X_B)}{\overline{y^2}(X_A)} = \frac{\int_0^{X_B} (X - U\tau)R_E(\frac{U\tau}{UL})\, d(U\tau)}{\int_0^{X_A} (X_A - U\tau)R_E(\frac{U\tau}{UL})\, d(U\tau)} \tag{4.10}$$

and searching for the value of $L(=\beta E)$ which satisfies eq. (4.10)for all (x_B, x_A), th pairs of distance from the source, we obtain L and β for a given experiment. If the Hay-Pasquill scheme is correct, in a given run, L is constant for all pairs (x_B, x_A)

The advantage of the ratio method used in eq. (4.10) consists in the eliminations of $\overline{V_L^2}$, about which we know only a rough value of its magnitude, $\approx\overline{v_E^2}$.

Assuming $R_E(t) = \exp(-t/E)$, we obtain

$$\frac{\overline{y^2}(X_B)}{\overline{y^2}(X_A)} = \frac{\exp(-\frac{X_B}{UL}) + \frac{X_B}{UL} - 1}{\exp(-\frac{X_A}{UL}) + \frac{X_A}{UL} - 1} \qquad (4.11)$$

which is a transcendental equation for UL. For the remaining correlations, (3.15a) and(3.15b), the same steps may be taken but the complexity increases since no analytical expression exists for them. We show later that eq. (4.11) gives nearly the same results we would obtain with the remaining correlations.

Two sets of data on atmospheric dispersion were chosen. They give $\overline{y^2}$ for dispersion near the ground over large flat fields. More detail is found in the given references or in Slade (1968) (Chapter 4).

4.3.1 Project Prairie Grass (Barad and Haugen (1959), Haugen (1966)): data are given for x = 50, 100, 200, 400 and 800 m. from the source. The experiments were adopted among the general set because they have nearly Gaussian concentration distributions. Roots UL of eq. (4.11), with the left-hand side given by experiment, are given in Table 4.2. TS is the thermal stability index defined by

$$TS = \frac{T_{4m.} - T_{0.5m.}}{U_{2m}^2} \times 10^5 \ {}^oC\ cm^2$$

large positive values of TS mean a strong inversion; large negative values of TS mean thermal instability.

Table 4.2 - Roots UL (in meters) of eq. (4.11). The paris are x_A/x_B. Data from Barad and Haugen (1959).

Run No.	TS	50/100	50/200	50/400	50/800
53	5.79	3	9	20	28
39	3.30	29	41	37	-
18	0.77	44	71	-	-
28	0.71	34	61	54	-
54	0.44	16	24	-	-
41	0.39	23	32	45	46
17	0.37	12	21	32	52
65	0.31	13	21	28	48
35	0.30	34	57	86	85
60	0.27	-	27	36	44
38	0.27	10	20	27	37
67	0.23	31	41	52	73
56	0.20	24	38	43	46
37	0.18	56	12	-	-
42	0.13	27	-	122	-
55	0.12	13	27	42	64
21	0.09	36	59	83	-
22	0.07	23	35	40	44
24	0.06	28	42	52	59
57	-0.08	61	134	255	243
6	-0.18	120	167	203	332
34	-0.21	31	52	77	104
11	-0.22	41	61	108	143
30	-0.36	63	115	214	-
27	-0.47	120	134	133	-

We conclude that UL is not constant for a given run. Similar conclusions were obtained by Haugen (1966). He assumed $\overline{V_L^2} = \overline{V_E^2}$, which might be a source of error. The present results seem to prove that this is not the case.

Choosing arbitarily one of the values of UL may lead to serious errors in estimating $\overline{y^2}$. For the run 56, for example, we get 24<UL<46, resulting in

$$\overline{y^2}\ (x = 800\text{m},\ UL = 24\text{m}) = 37200\ (\overline{V_L^2} / U^2)$$

$$\overline{y^2}\ (x = 800\text{m},\ UL = 46\text{m}) = 69400\ (\overline{V_L^2} / U^2)$$

a difference of 86% in $\overline{y^2}$.

In Table 4.3 we give some results for UL obtained for R_E given by eq. (3.15a). In this case the solution of eq. (4.10) involves numerical integrations and transcendental equations.

No sensible difference is observed relative to values given in Table 4.2, justifying the use of the exponential correlation in this test.

Table 4.3 - UL (in meters) which satisfy eq. (4.10). R_E given by eq. (3.15a). Data source: Prairie Grass.

Run No.	50/100	50/200	50/400	50/800
56	22	34	38	42
6	120	150	225	340
17	10	18	27	45

4.3.2 Project Green Glow (Fuquay et al (1964)): $\overline{y^2}$ are given for x = 200, 800, 1600 and 3200 m. downstream from the source. Table 4.4 gives the roots UL of eq. (4.11)

Table 4.4 - roots UL (in meters) of eq. (4.11). Experimental $\overline{y^2}$ from Project Green Glow (Fuquay et al (1964)). Exponential autocorrelation.

Run No.	Richardson No. R_i	200/800	200/1600	200/3200
67	0.130	-	12500	1090
7	0.119	6820	560	400
9	0.112	1310	-	940
5	0.097	185	400	2350
57	0.089	95	165	460
46	0.086	235	-	815
55	0.084	510	1180	-
70	0.083	720	695	410
13	0.078	220	-	355
43	0.067	160	470	2725
65	0.054	7230	10790	5500
69	0.053	2200	1250	980
50	0.051	1060	6995	-
68	0.048	290	285	165
10	0.037	300	370	580
26	0.036	115	200	335
17	0.032	420	-	11000
22	0.031	250	390	460

Run No.	Richardson No. R:	200/800	200/1600	200/3200
23	0.018	165	330	755
19	0.011	655	960	1040
33	0.005	280	235	190
60	-0.010	270	265	270
41	-0.015	200	315	620
32	-0.023	525	590	650
52	-0.026	145	140	195
45	-0.076	28000	3080	1360
61	-0.085	625	410	-
40	-0.117	300	660	1450
30	-0.176	220	-	715

Although some results cannot be considered of "good quality", we see clearly that UL is not constant for a given run. There is a trend for increase of UL with distance from the source. This means that β is not constant, as required by the Hay-Pasquill hypothesis.

Another point deserving attention is the value of UL for the pair 200/3200. It changes from 150 to 11000, with a mean value of UL = 1100m., as suggested by Pasquill (1974) (p. 197). But this value is valid only for the ratio y(200) / y(3200). For other pairs of distance, different results are obtained, and the value UL = 1100m. cannot be taken for practical computations.

4.4 Analysis of the Tests

The immediate result is that the Hay-Pasquill hypothesis is not obeyed in atmospheric turbulent dispersion near the ground. The factor β tends to increase with distance from the source. The same result was obtained in laboratory experiments (see Chapter 2 for references). After we accept this conclusion, we can observe it even in the original work of Hay and Pasquill (1959) (p. 361). They show clearly that β increases with distance!

One explanation for the variation of β may be found in fig. 4.2, giving the results for Run. No. 55 of Project Prairie Grass. $\overline{y^2}$ predicted by Hay and Pasquill increases much slower than the measured value. The disagreement may be traced to the rate of decrease of the autocorrelation assumed. The real value seems to decay more slowly than the value proposed by Hay and Pasquill, as shown in fig. 4.3.

Another explanation of the failure of the Hay-Pasquill hypothesis could be the existence of shear flow near the ground. In Project Prairie Grass some data were obtained with source and samplers at 1.5m. height (Runs Nos. 65 and 67), and the remaining data with source at z = 0.5m. and samplers at z = 1.5m.

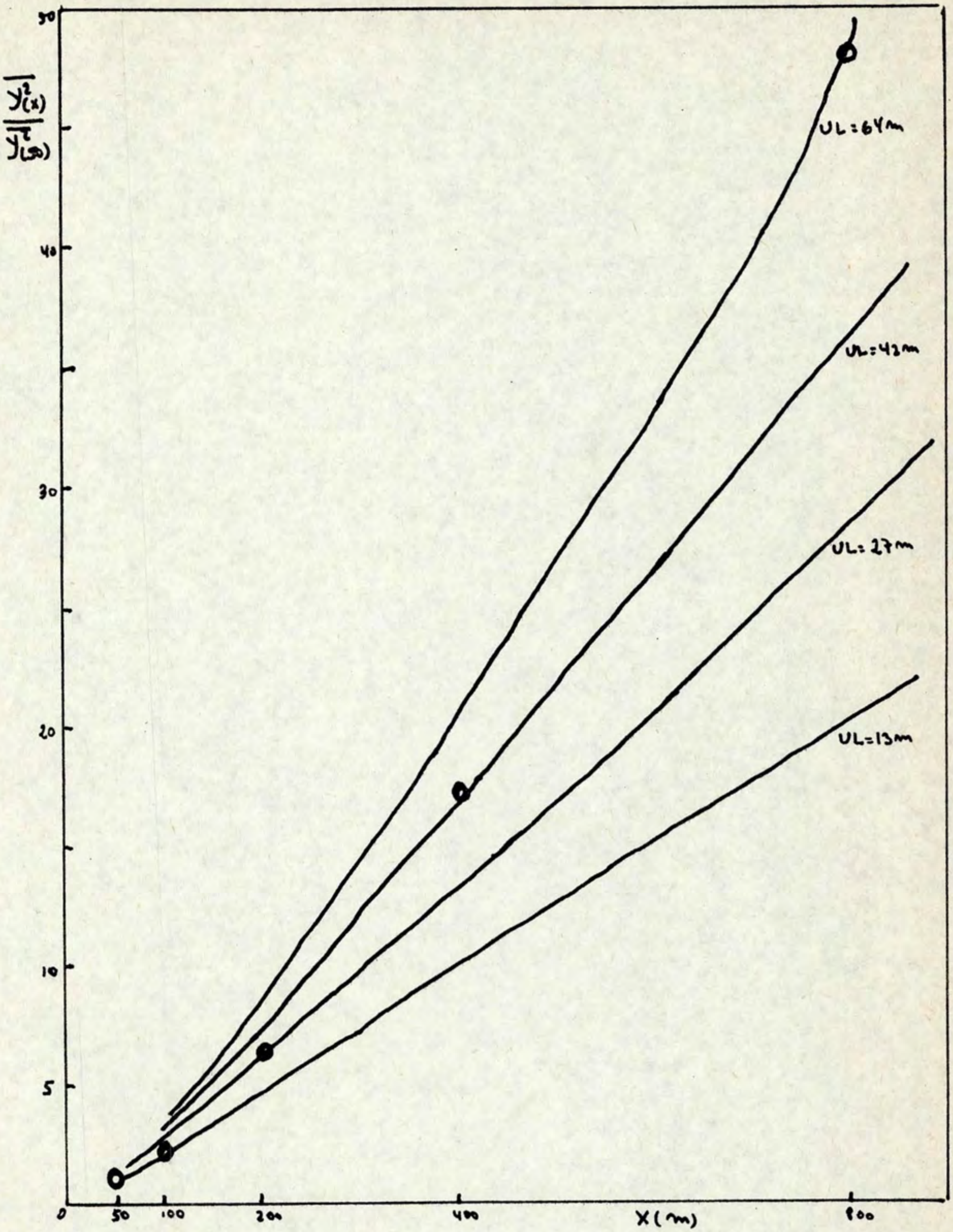

Fig. 4.2 - $\overline{y^2}$ (x) / $\overline{y^2}$ (50) for run 55 of Project Prairie Grass (Barad and Haugen (1959)). ——— Prediction of Hay-Pasquill's theory. UL - βUE is given near the curves. Points are the experimental data.

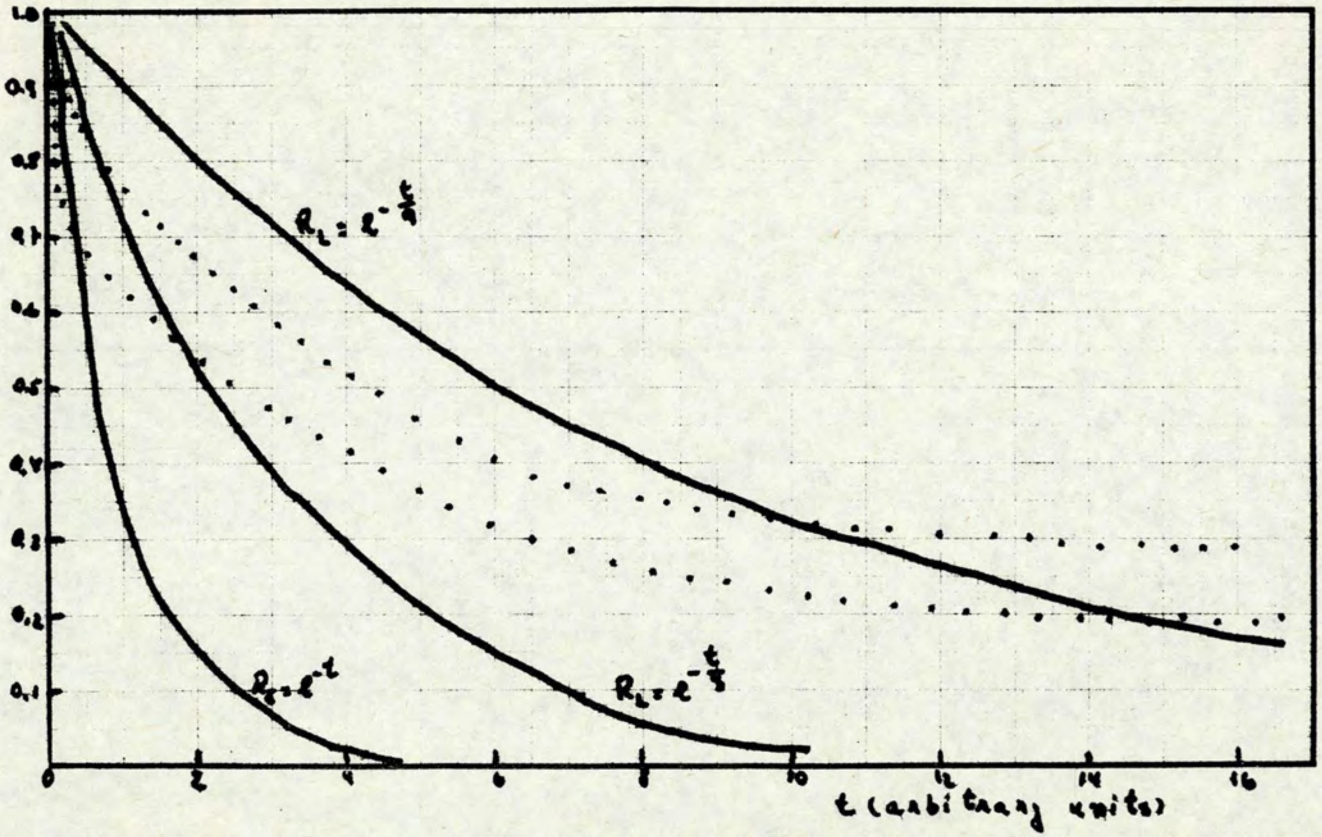

Fig. 4.3 - R_L given by Hay-Pasquill theory ($R_L = e^{-t/\beta}$) compared with what is probably the correct shape of R_L (····). Note that if the lower curve (···) represents R_L, β may be smaller than 1.

Pasquill (1974) showed that the shear produces no apreciable change in the predictions. Besides this, all the results show a similar trend for the variation of β.

Chapter 5 - A BETTER APPROXIMATION FOR $R_L(t)$.

5.1 Introduction

The main problem in the theory of turbulent dispersion - an expression for the Lagrangian autocorrelation - started with Taylor's theorem, eq. (2.1), and remains unsolved. No scheme seems to give a good approximation to real data. Partial agreement is obtained for short distances from the source, but generalisation is a dangerous step which may lead to errors of orders of magnitude (see Slade (1968) p. 133).

The Hay-Pasquill model for R_L presented in Chapter 4 continues to receive wide attention, mainly because of its simplicity. The model is probably incorrect, as its correlations decay faster than the real Lagrangian autocorrelation, underestimating the dispersion far from the source. Besides this, in principle there is no reason for its being correct. A better approximation may probably be obtained as follows.

If a particle is released from the origin at t = 0, (x, y, z;t) = (o, o, o;o), there is high probability that after a time t the particle will be in the close vicinity of the point (Ut, o, o;t) if the intensity of turbulence is not too large. One approximation to R_L may be given by the correlation between the Eulerian velocity at (o,o,o; o) and (Ut,o,o: t). If the "space-time correlation" is defined by

$$R_E\ (x, o, o; t) \propto \overline{v\ (o, o, o; o)\ v\ (x, o, o; t)} \qquad (5.1)$$

our suggestion consists in taking for R_L the "space-time correlation following the mean flow"

$$R_L\ (t) \simeq R_E\ (Ut, o, o; t) \qquad (5.2)$$

which is the correlation between the initial velocity and the velocity measured by a probe travelling at the velocity of mean flow. On the other hand, Hay-Pasquill's maybe written as

$$R_L(\beta t) = R_E(o,o,o;\ t) \equiv R_E(t) \qquad (5.3)$$

It will be shown later that the two suggestions yield different results.

The idea of using space-time correlations is not new, but is seldom applied. At the 1958 Symposium on Atmospheric Diffusion and Air Pollution at Oxford (Frenkiel and Sheppard (1959)),the idea was mentioned many times (Corrsin pp. 162, 442), Palm (p. 398)), but no practical scheme was given for the calculation of space-time correlations. In addition to this correlation for all x and t the proposed models require information about the probability of displacement of fluid

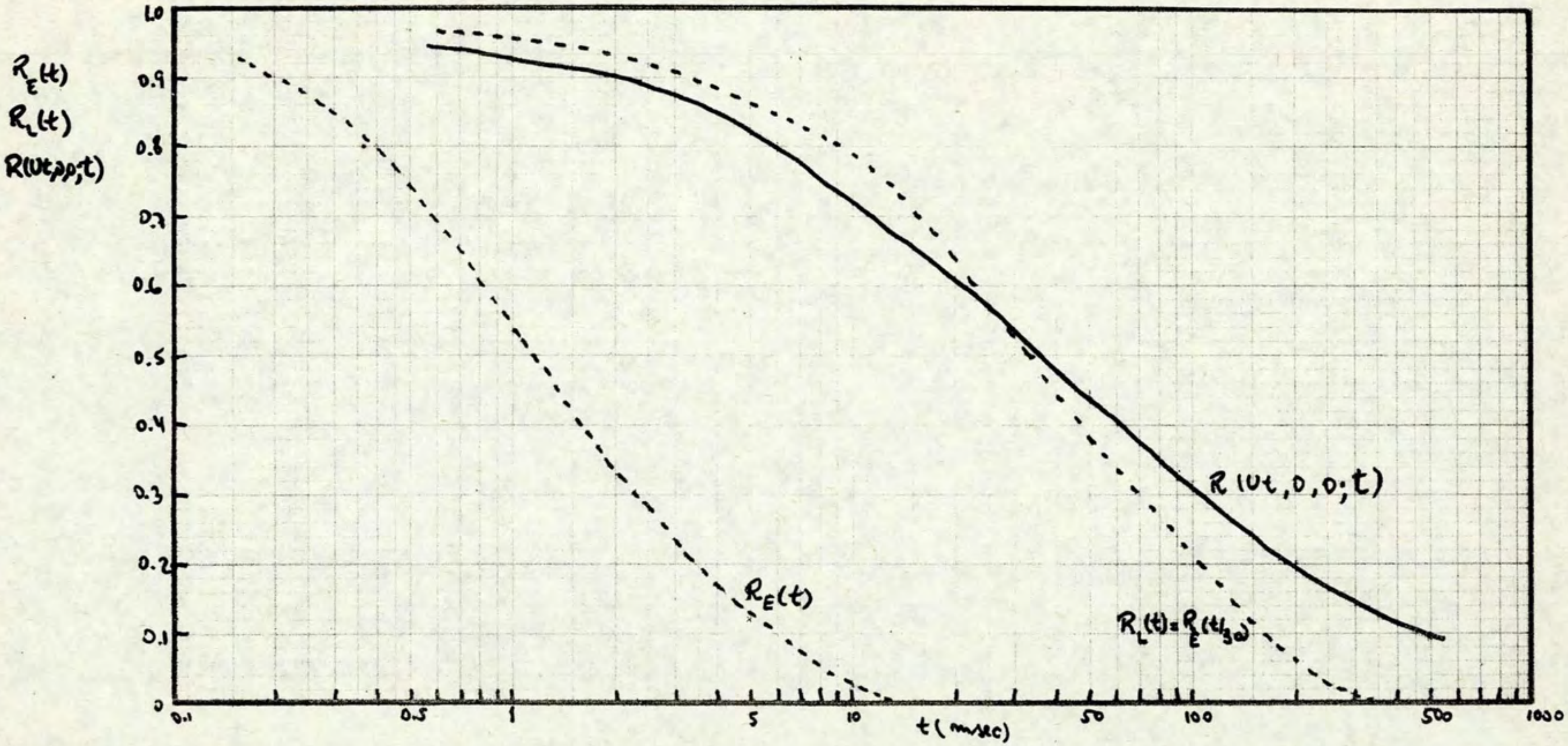

Fig. 5.1 - $R_E(t)$ and R(Ut,o,o;t) measured by Comte-Bellot and Corrsin (1971) compared with R_L predicted by the Hay-Pasquill theory, $R_L(t) = R_E(t/30)$.

particles, which is an unknown function (see Saffman (1963), Peskin (1974)).

The model suggested here is simpler because we need the space-time correlation for $x = Ut$ only. The difficulty is that even for this "simple" correlation no analytical expression exists which is valid at all times t. Chapter 6 presents such a model.

5.2 A Preliminary Test.

One preliminary test for the suggestion given here is shown in fig. 5.1. The Eulerian time correlation and the Eulerian space-time correlation following the mean flow, both measured by Comte-Bellot and Corrsin (1971), are compared with the Lagrangian autocorrelation for this flow, as proposed by Hay and Pasquill, eq. (5.3). R(Ut, o,o; t) falls more slowly resulting in a larger y far from the source. This is in qualitative agreement with the results of Chapter 4.

What we need now is a theoretical model for R(Ut, o, o; t).

Chapter 6 - A MODEL FOR SPACE-TIME CORRELATIONS FOLLOWING THE MEAN FLOW.

6.1 Introduction

The Eulerian space-time correlation for the y-component of the velocity in a steady flow is defined by

$$R_E\,(x,y,z;t) = \frac{\overline{v(x,y,z;t)\; v(o,o,o;o)}}{\{\overline{v(x,y,z,t)^2}\;\; \overline{v(o,o,o;o)^2}\}^{\frac{1}{2}}} \tag{6.1}$$

It correlates the velocity at two points, (o, o, o) and (x, y, z), and at two times, with an interval t between the measurements. When t = o we get the Eulerian space correlation

$$R(\vec{x}) = R(x,\; y,\; z;\; o) \tag{6.2}$$

The Eulerian time correlation (autocorrelation) $R_E(t)$ is the most easily observed

$$R_E(t) = R(o,\; o,\; o;\; t). \tag{6.3}$$

The hypothetical correlation we would obtain in the case of one probe fixed at the origin and another probe travelling downstream in the x direction at the mean flow velocity U, is called "space-time correlation follwing the mean flow." This is the object of the present Chapter,

$$R(x = Ut,\; 0,\; 0;\; t) \equiv R(Ut,\; o,\; o;\; t). \tag{6.4}$$

6.2 The Frozen Pattern.

Taylor (1938) made the hypothesis that turbulence may be regarded as a frozen pattern of eddies being swept past the observer. This means that a signal received at one measuring position will be received at a second point, distance x directly downstream from the first, at a time t later The velocity $U_c = x/t$ is called velocity of convection of the pattern (~U) (Fisher and Davies (1963)). Thus the value of R(x, o, o; t) will attain its maximum (= 1) for x = $U_c t \sim Ut$ and so

$$R(Ut, o, o; t) = 1 \tag{6.5}$$

Fig. (6.A) shows the space-time correlation we would obtain if the frozen pattern were correct.

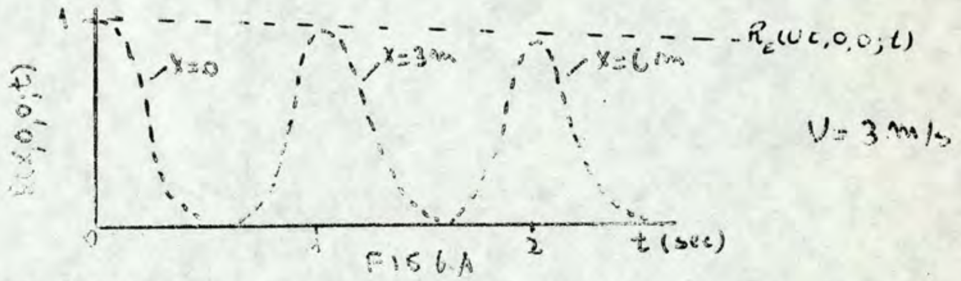

FIG 6.A

6.3 The Non-Frozen Pattern.

In practice the frozen pattern scheme shown in fig. 6.A is not observed. Fig. 6.B shows more realistic results and, although each correlation curve rises to a maximum at some value of the time delay, the amplitude of this maximum is a function of the separation x. For an observer travelling at the velocity of convection of the eddies (which is approximately the velocity of the mean flow), the autocorrelation in his frame of reference is exactly our R(Ut, o, o; t) and is given by the envelope of the space-time correlations, as shown in fig. 6.B.

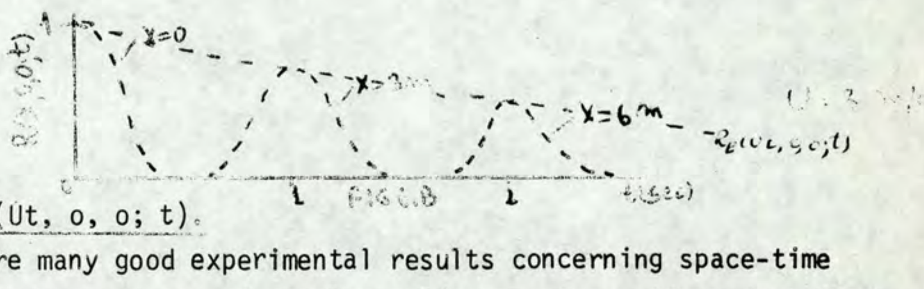

FIG 6.B

6.4 A Model for R(Ut, o, o; t).

Today there are many good experimental results concerning space-time correlations measured downstream of regular grids spanning uniform duct flows (Comte-Bellot and Corrsin (1971), Favre (1965), Frenkiel and Klebanoff (1966)), and in pipe flow (Sabot et al (1972), Baldwin and Mickelsen (1963)).

All these experiments show one important characteristic: the narrow band correlation R(Ut, o, o; t; n) which involves only the frequencies in the vicinity of n, behaves differently for different frequency bands. It decays much faster for high frequencies than for low frequencies. This can be seen in Comt-Bellot and Corrsin (1971) (fig. 14). This led us to propose the present model, based on two hypotheses.

Hypothesis #1: each eddy lives during a time proportional to its period (~1/n) (or, a distance proportional to its wavelength). After this time its energy is dissipated or transferred to other frequencies.

Hypothesis #2: starting with a given eddy configuration at t = o, the eddies created or transferred from a given frequency to another frequency at t > o have no correlation with the original configuration.

As a consequence, after a time t we have eddies and energy for all frequencies but only the lower frequencies contribute to the space-time correlation: the original high frequency eddies are dissipated and are replaced by new eddies uncorrelated to the initial configuration. If the pattern were frozen, all eddies non-dissipated at (Ut, o, o; t) would be correlated to the eddies at the origin,

$$\overline{v(Ut, o, o; t)\ v(o,o,o; o)} = (\overline{v(Ut, o, o; t)^2}\ \overline{v(o, o, o; o)^2})^{\frac{1}{2}} \qquad (6.6)$$

As in reality only part of the spectrum at (Ut, o, o; t) is correlated to the configuration at (0, 0, 0; 0) we have to modify eq. (6.6).

Each original eddy maintains its identity for a time proportional to its period of oscillation ($\sim 1/n$). After a time t all eddies of frequency larger than γ/t are no more correlated with the initial configuration So we obtain

$$\overline{v(Ut,o,o;t)\ v(o,o,o;o)} = \{\overline{v(Ut,o,o;t)^2}\ \overline{v(o,o,o;o)^2}\}^{\frac{1}{2}} \int_0^{\gamma/t} F(n)\ dn \qquad (6.7)$$

where the integral represents the "fraction" of eddies correlated with the initial configuration. With this basic assumption we obtain from eq. (6.1)

$$R(Ut, o, o; t) = \int_0^{\gamma/t} F(n)\ dn \qquad (6.8)$$

$2\pi\gamma$ is the ratio between the lifetime of an eddy and its period of oscillation.

The possibility of predicting R(Ut, o, o; t) from F(n) is an important result, since the spectral density may easily and cheaply be obtained from correlations measured at a fixed point.

Eq. (6.8) is valid for steady flow and whenever the change of spectral density F(n) downstream is caused by internal factors such as dissipation and energy transfer, and not by external factors such as obstacles and change of roughness. The reason is that we do not know how they modify the original spectrum.

6.5 Comparison With Laboratory Flows

In Chapter 3 it is shown that Eulerian time correlations for turbulent flows behind grids can be well represented if we assume for the spectral density

$$F(n) = \frac{4E}{1 + 31{,}5\ (En)^{5/3}} \qquad (3.10a)$$

resulting in

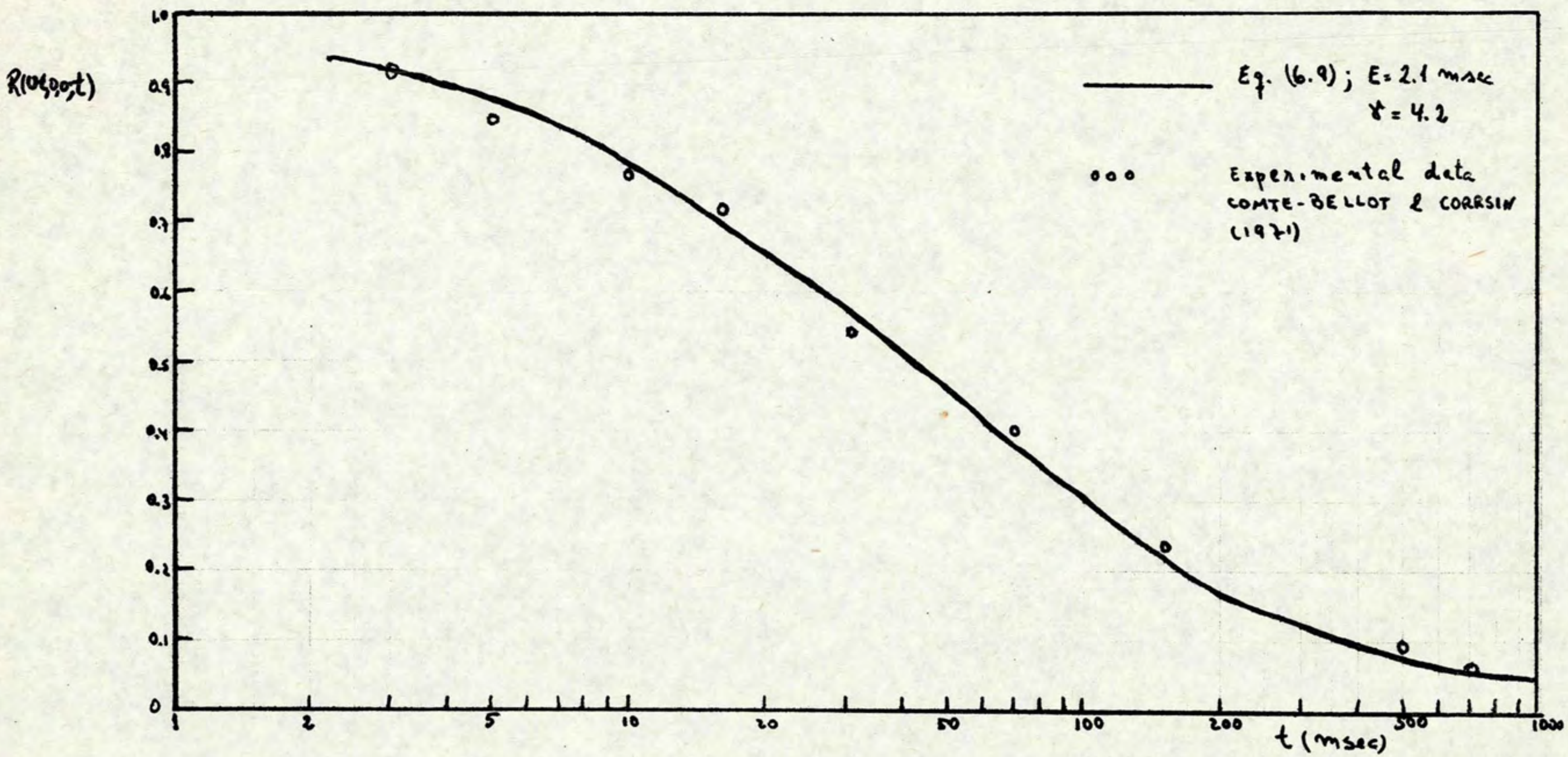

Fig 6.1a

Fig. 6.1 (a,b,c) - Comparison between predicted and measured space-time correlations following the mean flow, R(Ut,o,o;t). Values of E from Chapter 3. Values of γ chosen to give the best fit.

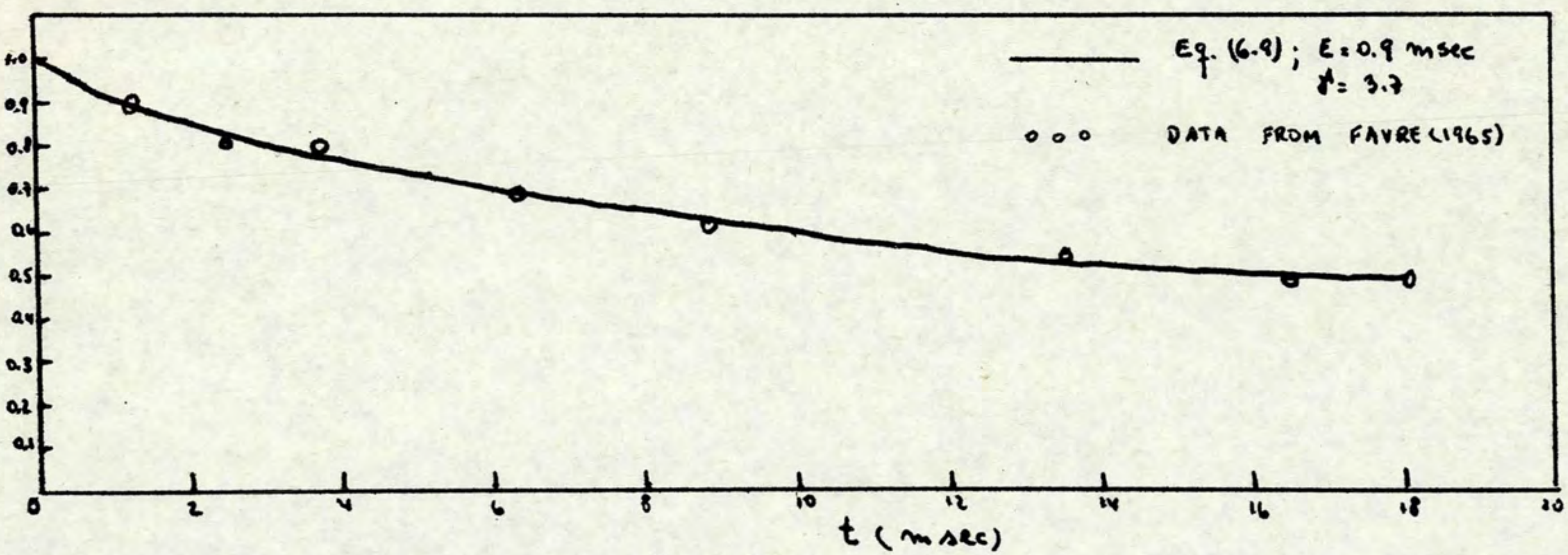

Fig 6.1b

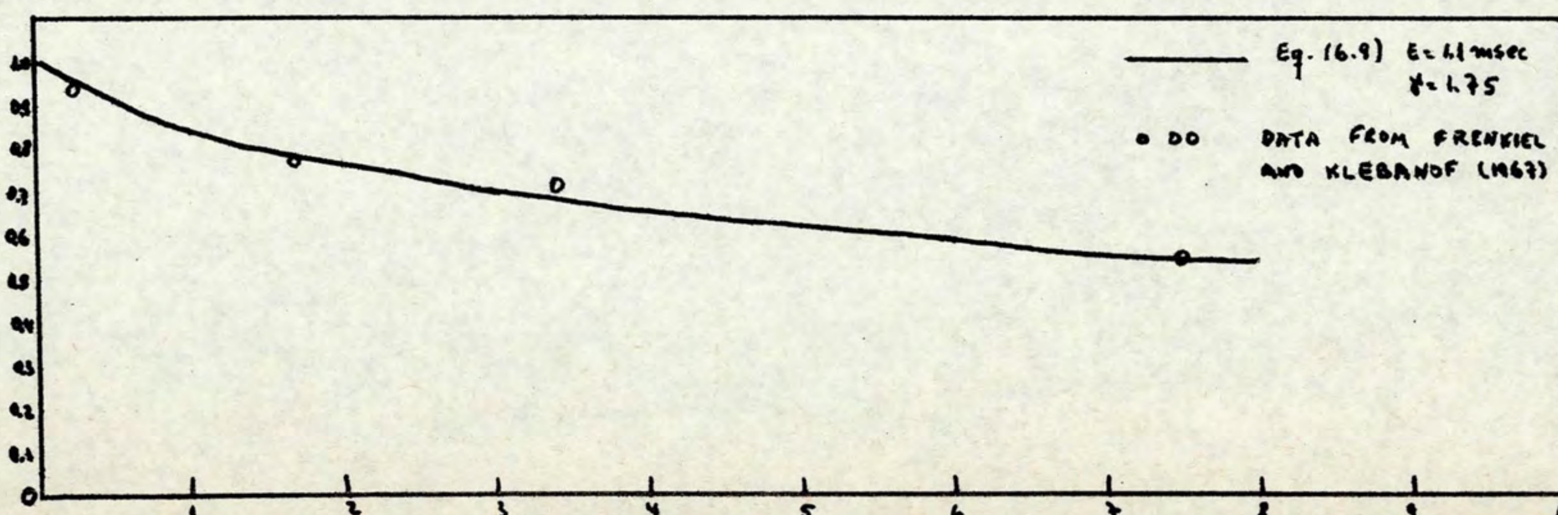

Fig 6.1c t (msec)

$$R(Ut,o,o;t) = 1.516\{\ln \frac{(1+C)^{0.2}\ (1-1.618C+C^2)^{0.0618}}{(1+0.618C+C^2)^{0.1618}} + 0.38 \arctan \frac{C-0.809}{0.5877} -$$

$$- 0.235 \arctan \frac{C+0.309}{0.951} + 0.4319\} \tag{6.9}$$

where

$$C = 1.993\ (\gamma E/t)^{1/3} \tag{6.9a}$$

In fig. 6.1 the predictions of eq. (6.9) are compared with experimental data. Three experiments are considered:

(a) The data by Comte-Bellot and Corrsin (1971) are the best test for any "space-time correlation model." The mean flow velocity is 12.70 m/s, the grid mesh M is 5.08 cm. and data are given for time delays up to 700 msec. No other experiment gives such a large time delay. The integral time scale E as obtained in Chapter 3 is 2.1 msec.

(b) Frenkiel and Klebanoff (1966) use a grid with mesh M = 2.54 cm. so as to generate turbulence in a flow of U = 15.4 m/s. E as obtained in Chapter 3 is 1 msec.

(c) Favre (1965) presents correlations for a flow with U = 12.2 m/s, M = 2.54 cm. and E = 0.9 msec.

The model suggested agrees very well with the data when an appropriate value of γ is assumed, as shown in fig. 6.1.

It has been suggested that the exponential correlation may reasonably represent Eulerian autocorrelations. To R = exp(- t/E) it corresponds

$$F(n) = \frac{4E}{1 + (2\pi nE)^2} \tag{6.10}$$

Introducing this spectral density in eq. (6.8), we get

$$R(Ut,\ o,\ o;\ t) = \frac{2}{\pi} \arctan\ (\frac{2\pi\gamma E}{t}) \tag{6.11}$$

Although eq. (6.11) is simple and tempting, it does not fit experimental data. This is shown in fig. 6.2.

6.6 Comparison With Atmospheric Data.

Chapter 3 shows that the spectral density fitting atmospheric data is

$$F(n) = \frac{4E}{(1 + 6En)^{5/3}} \tag{3.10b}$$

When introduced in eq. (6.8) results in

$$R(Ut,\ o,\ o;\ t) = 1 - \frac{1}{(1 + \frac{6\gamma E}{t})^{2/3}} \tag{6.12}$$

This is a simple expression for the space-time correlation following the mean flow. This expression will be used in Chapter 7 as an approximation to $R_L(t)$. In

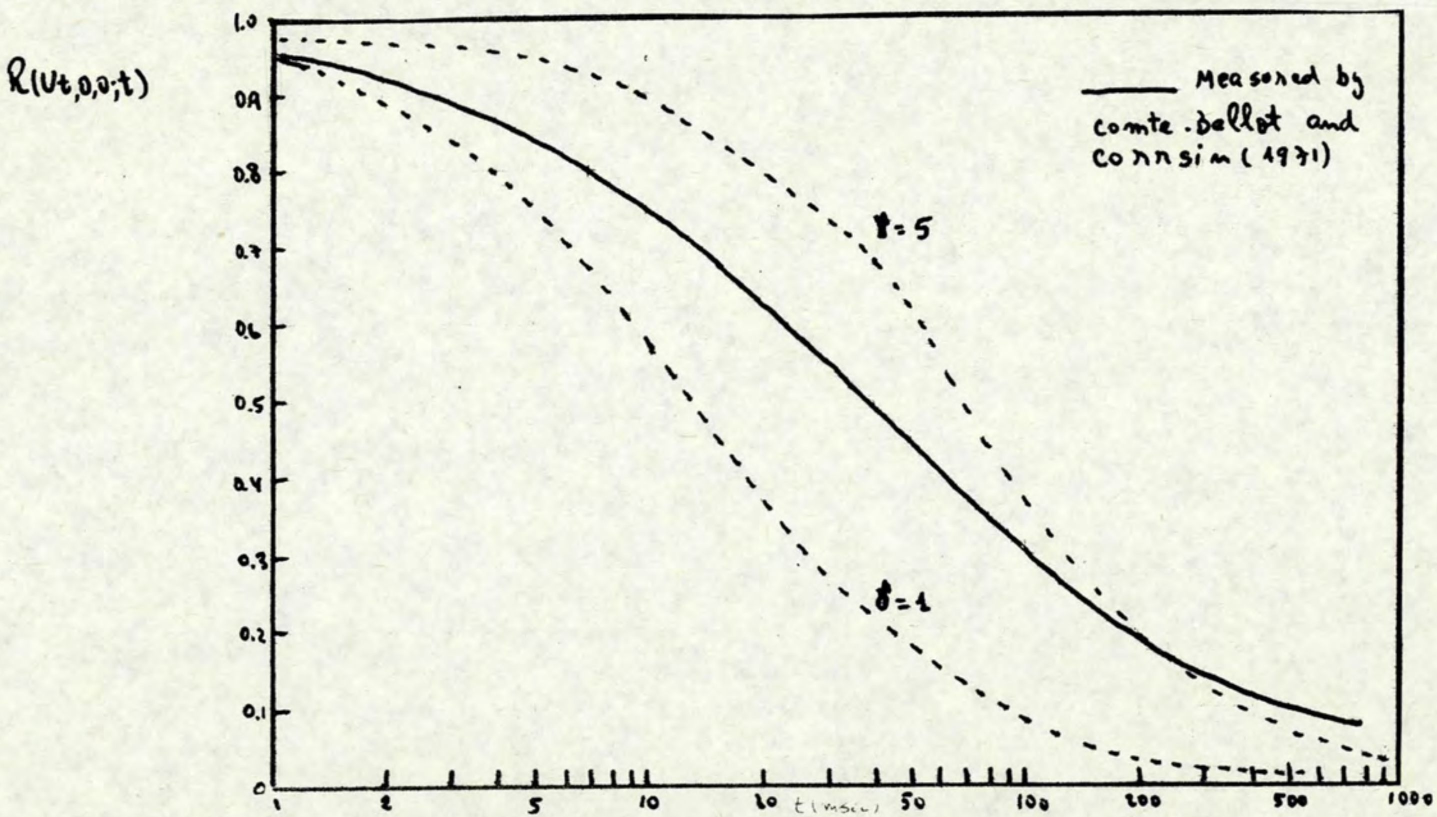

Fig. 6.2 - R(Ut,o,o;t) predicted by eq. (6.11) (assuming $R_E(t) = \exp(-t/E)$, E = 2 msec) compared to experimental data given by Comte-Bellot and Corrsin (1971).

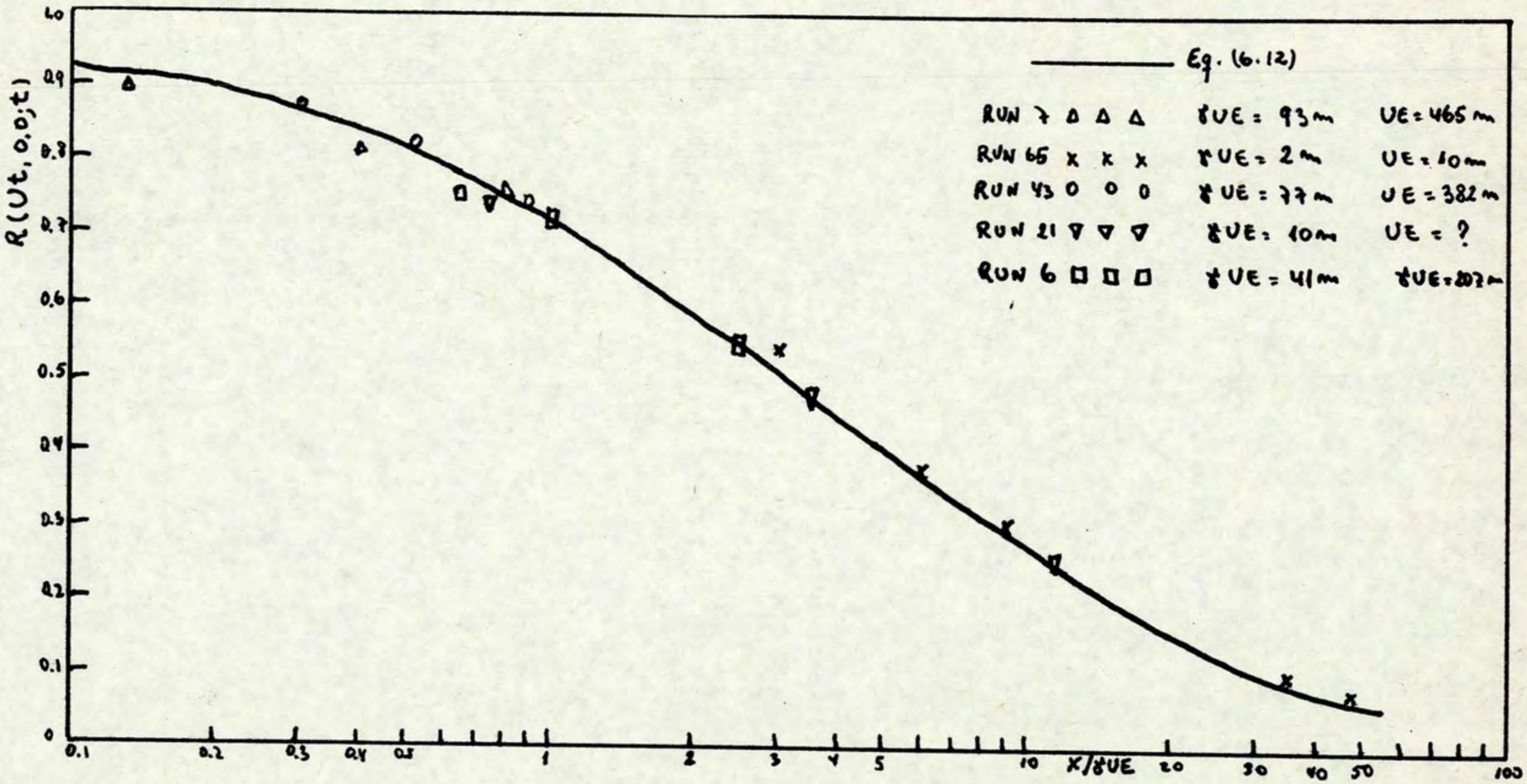

Fig. 6.3 - Predicted space-time correlation following the mean flow compared to measurements in the atmosphere (z = 2m). Data from Barad (1958) and Haugen (1959). UE obtained independently from $R_E(t)$.

fig. 6.3 we compare eq. (6.12) with data in Project Prairie Grass (Barad (1958), Haugen (1959)). There is good agreement. The interesting fact is that now γ is nearly constant for all states of atmospheric stability, γ = 0.18 to 0.20. In Chapter 7 it will be shown that when the value of γ is not known, a good approximation for diffusion prediction is γ 0.2.

Chapter 7 - SPACE-TIME CORRELATIONS AND TURBULENT DISPERSION IN THE ATMOSPHERE.

7.1 Introduction.

In Chapter 6 an analytical expression for the space-time correlation following the mean flow in terms of micrometeorological quantities is suggested. This expression is used (instead of $R_L(t)$) to compute $\overline{y^2}$. The theoretical results are compared with experimental data obtained in diffusion experiments in the lower atmosphere. The test is made by the method explored in Chapter 3, which has the advantage of not assuming the equality between the Lagrangian and the Eulerian velocity variances. It is shown indeed that these two quantities are in general different.

One important characteristic of the model proposed is its dependence on the Eulerian integral time scale E, which is itself a function of height above the ground, of the temperature and velocity profiles, and of the terrain roughness. This could explain why, under the same atmospheric conditions, the dispersion behaves differently in different places, and even at the same place, but in different seasons (Vogt (1974a)).

As in the preceding Chapters, here we confine ourselves to horizontal process only where the velocity field is assumed to be homogeneous and steady in the plane of dispersion.

7.2 The Standard Deviation $\overline{y^2}$.

The Eulerian space-time correlation following the mean flow for atmospheric turbulence is given by

$$R(Ut, o, o; t) = 1 - \frac{1}{(1 + \frac{6\gamma UE}{Ut})^{2/3}} \qquad (6.12)$$

The computation of $\overline{y^2}$ is not sensitive to small changes in R_L. Eq. (6.12) is proposed as an approximation to the Lagrangian autocorrelation. This results in

$$\overline{y^2}(t) \overset{?}{=} 2\, \overline{V_L^2} \int_0^t (t - \tau)\, R\,(U\tau, o, o; \tau)\, d\tau \qquad (7.1)$$

Introducing (6.12) into (7.1) we obtain

$$\overline{y^2}(x,\gamma UE) = 2\, \frac{\overline{V_L^2}}{U^2} (6\gamma UE)^2 \left\{\frac{1 - P}{2(1-P^3)^2} + \frac{5}{6}\frac{P}{(1-P)^3} - \left(\frac{5}{6} + \frac{1}{(P^3-1)}\right)\left[\frac{1}{3} \ln\frac{(1-P)^2}{P^2+P+1} - \right.\right.$$

$$\left.\left. - \frac{2}{\sqrt{3}} \arctan \frac{\sqrt{3}\, P}{2 + P} + 1.209\right]\right\} \equiv 2\, \frac{\overline{V_L^2}}{U^2} (6\gamma UE)^2\, H\left(\frac{x}{\gamma UE}\right) \qquad (7.2)$$

where

$$P = (1 + \frac{6\gamma UE}{Ut})^{1/3} \qquad (7.3)$$

Eq. (7.2) may probablybe simplified,using some approximations. This is avoided here.

The relationship between $\overline{V_L^2}$ and the measured $\overline{V_E^2}$ is unknown, so eq. (7.2) cannot be tested directly. We may take the ratio of eq. (7.2) for two distinct distances and introduce the value of γUE obtained from space-time correlations measurements,

$$\frac{H(x_A, \gamma UE)}{H(x_B, \gamma UE)} = \frac{\overline{y^2}(x_A)}{\overline{y^2}(x_B)}\Bigg|_{\text{experimental}} \qquad (7.4)$$

If eq. (7.4) is satisfied for all pairs (x_A, x_B,) in a given run, R(Ut, o, o; t) is similar in shape to R_L. Then we may use eq. (7.2) for obtaining $\overline{V_L^2}$.

7.3 Comparison With Experimental Data.

Comparison between physical models and measured data on diffusion is a difficult task: with one known exception (Project Prairie Grass) the data are unaccompanied by turbulence characteristics (time correlations or spectral densities). Consequently the value of γUE cannot be extracted.

7.3.1 Project Prairie Grass (see Haugen 1966)).

For some runs of this project we have γUE (see fig. 6.3). For others the direction of the wind was different from that of the anemometer line, so that we are unable to find γUE but we have UE. We may guess the value of γUE by taking $\gamma = 0.2$. For the remaining cases, γUE is chosen to give the best fit. Table 7.1 gives the predicted dispersion and compares it with experimental data. Some results are presented in fig. 7.1.

Table 7.1 - ($\overline{y^2}(x_B)$ / $\overline{y^2}(x_A)$) predicted by eq. (7.2). The experimental ratios are given within parentheses. Source of data: Barad and Haugen (1959) and Haugen (1959). Pairs indicate x_B/x_A, distances from the source in meters. Runs 6, 21 and 65 - γUE from measurements; runs 54 to 27 - γUE guessed from measured UE and $\gamma = 0.2$; runs 53 - 30 - γUE chosen to give the best fit.

Run No.	γUE(m)	50/100	200/100	400/100	800/100
6	41.	0.52(0.53)	1.88(1.87)	3.39(3.34)	5.95(6.30)
21	8.	0.56(0.58)	1.73(1.74)	2.90(2.97)	-
65	2.	0.59(0.65)	1.63(1.58)	2.60(2.46)	4.07(4.11)
54	1.	0.61(0.64)	1.59(1.60)	-	-
17	1.	0.61(0.65)	1.59(1.59)	2.49(2.55)	3.85(4.29)
38	1.1	0.61(0.66)	1.60(1.59)	2.50(2.46)	3.87(3.86)
56	1.	0.61(0.61)	1.59(1.66)	2.49(2.58)	3.85(3.82)
55	1.8	0.60(0.64)	1.63(1.64)	2.58(2.71)	4.03(4.52)
22	2.5	0.59(0.61)	1.64(1.64)	2.64(2.55)	4.15(3.82)
24	2.4	0.59(0.60)	1.64(1.67)	2.64(2.68)	4.14(4.11)
27	20.8	0.53(0.53)	1.81(1.82)	3.18(3.05)	5.41(-)
53	0.6	0.62(0.69)	1.57(1.50)	2.43(2.41)	3.72(3.75)
39	1.5	0.60(0.60)	1.61(1.66)	2.54(2.45)	-
57	54.	0.52(0.56)	1.89(1.90)	3.48(3.65)	6.11(6.11)
34	12.5	0.54(0.60)	1.77(1.74)	3.03(2.97)	5.04(4.96)
11	18.5	0.53(0.58)	1.80(1.74)	3.15(3.16)	5.32(5.33)
30	70.	0.52(0.55)	1.90(1.86)	3.54(3.56)	-

Good agreement is obtained between predictions and experiment with a difference in general of less then $\pm$ 5%.

In fig. 7.2 and 7.4 we compare the results of this Chapter with those predicted by Hay and Pasquill's hypotheses. These figures may be considered as the main result of this thesis for they show clearly that Hay-Pasquill's model underestimates extrapolations when the model proposed here predicts quite correctly the shape of $\overline{y^2}(x)$.

7.3.2 Project Green Glow (Fuquay et al 1964)).

No time correlations are given, so we assume that the spectral density is the same as for Project Prairie Grass, and that R(Ut, o, o; t) is given by eq. (6.12). This means that $\overline{y^2}$ is given by eq. (7.2). Table 7.2 compares prediction with measured values. γUE is chosen to give the best fit.

Table 7.2 - $\overline{y^2}(x_B)$ / $\overline{y^2}(x_A)$ predicted by the present model (eq. (7.2)). Data inside parentheses from Project Green Glow. Pairs, indicate x_B / x_A in meters.

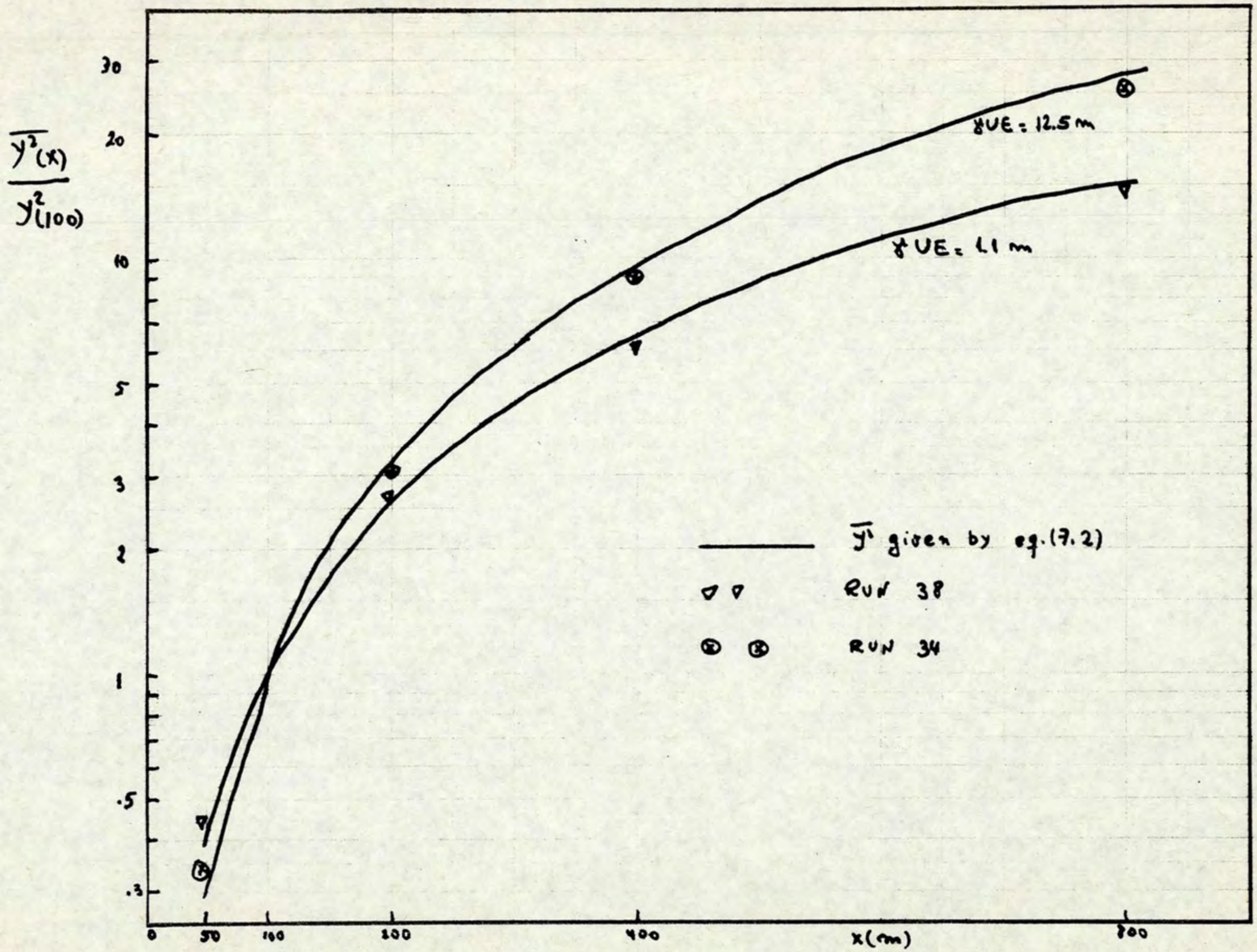

Fig. 7.1 - $\overline{y^2}$ (x) / $\overline{y^2}$ (100) predicted by the present model (eq. (7.2)) compared to experimental values from Project Prairie Grass (Barad and Haugen (1959)).

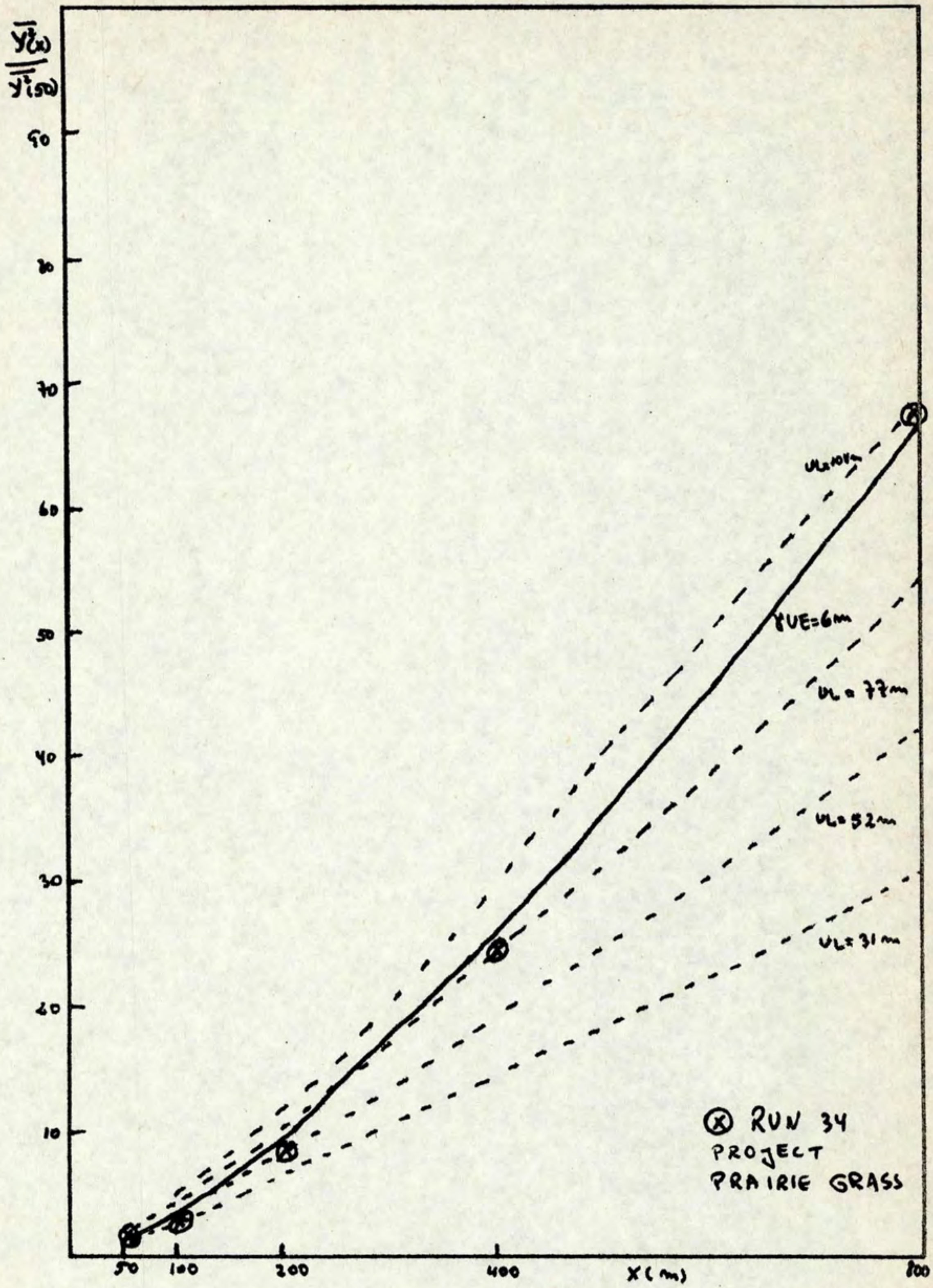

Fig. 7.2 - $\overline{y^2}$ (x) / $\overline{y^2}$ (50) predicted by the present model (eq. (7.2)) (———) and by Hay-Pasquill theory (----), both compared to data for Run No. 34, Project Prairie Grass.

Run No.	γUE(m)	800/200	1600/200	3200/200
9	180	3.62(3.72)	(-)	11.57(10.76)
57	15	2.89(2.46)	4.71(4.15)	7.50(8.54)
70	60	3.30(3.54)	5.70(6.10)	9.54(8.18)
13	19	2.95(2.93)	(-)	7.79(7.79)
43	80	3.38(3.38)	(-)	10.05(10.05)
44	160	3.58(3.63)	6.48(6.52)	11.33(11.26)
69	200	3.63(3.83)	6.65(6.78)	11.72(10.88)
68	12	2.85(3.10)	4.60(4.86)	7.20(6.05)
10	40	3.18(3.11)	5.33(5.22)	8.86(9.22)
26	13	2.86(2.56)	4.62(4.37)	7.33(7.62)
22	32	3.11(3.00)	5.22(5.29)	8.53(8.53)
25	5	2.64(2.84)	4.16(4.39)	6.44(5.64)
53	41	3.18(2.71)	5.40(5.29)	8.90(9.41)
23	31	3.09(2.76)	5.17(5.06)	8.45(10.06)
19	138	3.45(3.50)	6.37(6.50)	11.07(11.07)
33	9	2.78(3.07)	4.45(4.57)	7.00(6.35)
60	17	2.93(3.05)	4.79(4.74)	7.65(7.10)
41	31	3.10(2.87)	5.19(5.00)	8.46(9.44)
32	68	3.34(3.40)	5.80(5.88)	9.76(9.60)
52	4.4	2.62(2.68)	4.11(3.96)	6.35(6.36)
40	138	3.53(3.11)	6.34(6.04)	11.00(12.04)
30	44	3.20(2.92)	(-)	9.01(9.88)

Although there is some scatter, the difference between measured and predicted values is within ± 10% in most cases, in support of our model.

For this project we have data for $\overline{V_E^2}$ and $\overline{y^2}(x)$ (not ratios of $\overline{y^2}$, as in Prairie Grass). This enables us to find a relationship between $\overline{V_E^2}$ and $\overline{V_L^2}$. In Table 7.3 we give $\overline{y^2}(x)$ as predicted by eq. (7.2), whereas γUE is obtained from Table (7.2), and the ratio of velocity variances is as indicated. The agreement with experimental dispersion seems to be excellent, supporting our model and leading to the conclusion that in general $\overline{V_E^2} \neq \overline{V_L^2}$.

Table 7.3 - ($\overline{y^2}(x)$) predicted by eq. (7.2). γUE from Table 7.2. Experimental $\overline{y^2}$ and $\overline{V_E^2}$ given by Fuquay et al (1964). Ratio ($\overline{V_L^2}$ / $\overline{V_E^2}$) taken to obtain the best agreement with experimental (inside parentheses).

Run No.	$(\overline{V_L^2}/\overline{V_E^2})^{\frac{1}{2}}$	x(m) 200	800	1600	3200
57	1.23	13.6(13)	39 (32)	62(54)	100(111)
13	1.45	29 (29)	85.4(85)	-	225(226)
43	0.96	21 (21)	71.5(71)	-	212(211)
44	1.21	46.5(46)	164 (167)	297(300)	520(518)
69	0.92	40.4(41)	149 (157)	274(278)	485(446)
68	1.50	19.5(21)	56 (65)	90(102)	141(127)
10	1.33	18 (18)	56.9(56)	96(94)	158(160)
22	1.06	17 (17)	52.5(51)	88(90)	143(145)
25	2.00	30.6(31)	91 (88)	127(136)	197(175)
53	1.08	17.4(17)	55 (46)	94(90)	156(160)
19	1.10	28 (28)	98.5(98)	177(182)	308(310)
60	0.93	18.6(19)	55 (58)	89(90)	142(135)
41	1.25	16 (16)	49.5(46)	83(80)	135(151)
32	1.18	25 (25)	84 (85)	146(147)	247(240)
52	1.46	27.7(28)	73 (75)	114(111)	176(178)
30	0.81	26 (26)	84.1(76)	-	237(257)

Some results of Table 7.3 are displayed in figs. 7.3 and 7.4. We may draw from fig. 7.4 the conclusion that Hay-Pasquill's hypothesis is reasonable when it provides interpolated data, but it may not be used in extrapolation or in giving confident prediction when diffusion experiments are difficult or impossible, and when we require results with 50 - 60% confidence. We shall try to explain in the next Chapter why their hypothesis fails at large distances from the source.

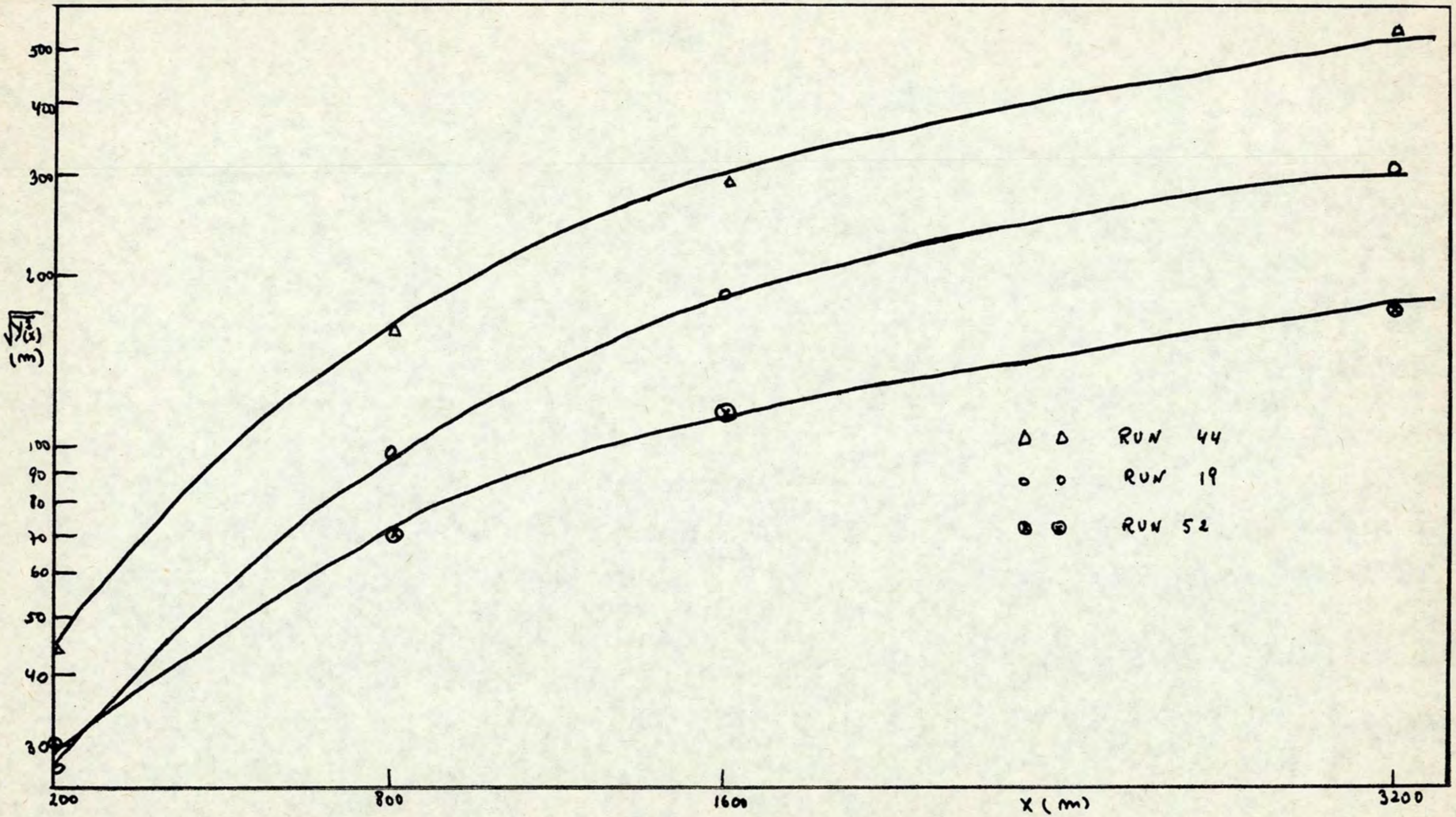

Fig. 7.3 - $\sqrt{\overline{y^2(x)}}$ predicted by the present model, eq. (7.2), compared to experimental data from Project Green Glow. Parameters from Table (7.3).

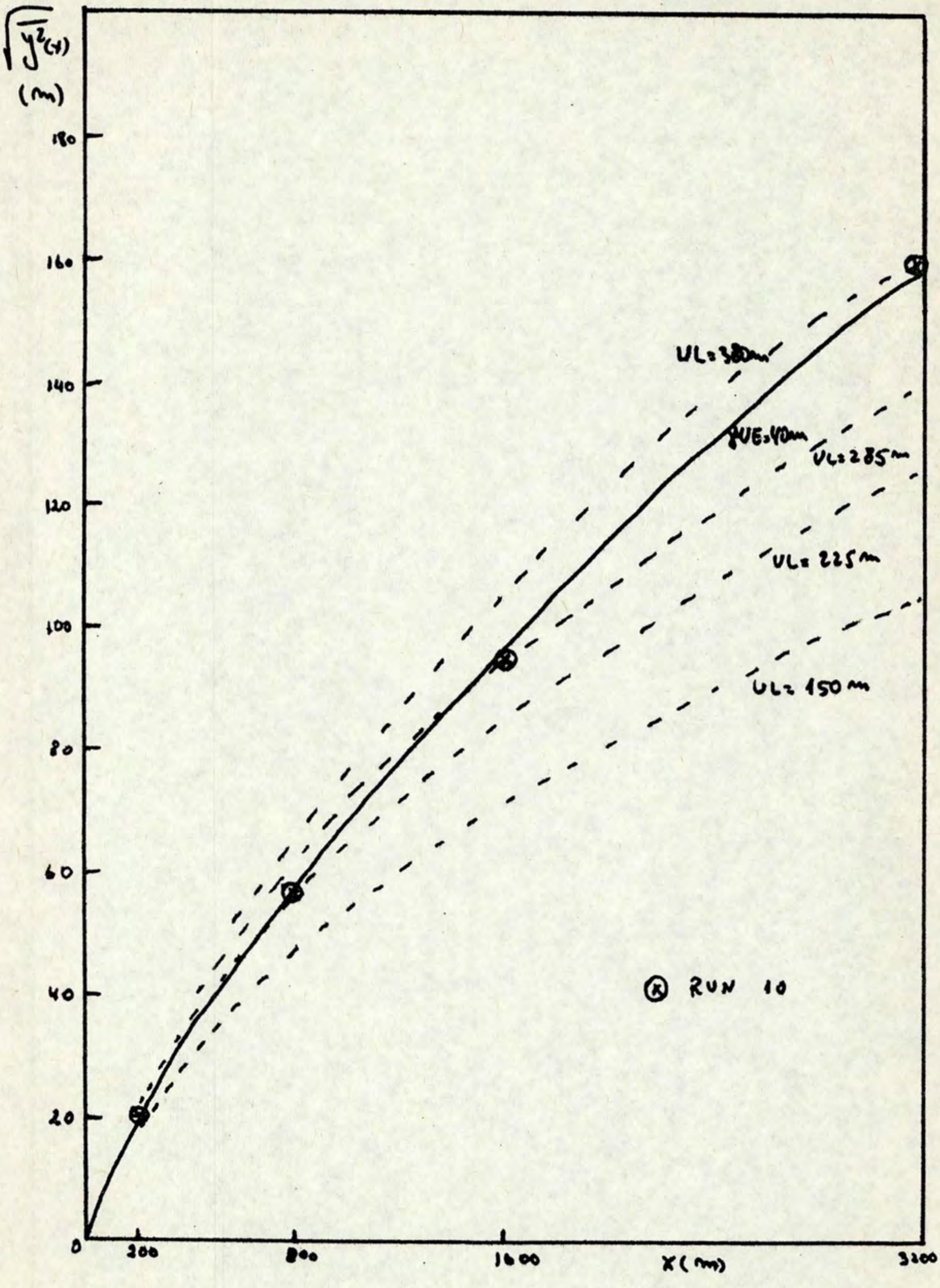

Fig. 7.4 - $\overline{y^2}$ (x) predicted by the present model (——), γUE = 40m, and by Hay-Pasquill theory (----), both compared to Run No. 10 of Project Green Glow (Fuquay et al (1964)). See Table (7.3).

Chapter 8 - THE VARIATION OF PASQUILL'S β WITH DISTANCE.

It was shown in Chapter 3 and by many authors that the factor β in Hay-Pasquill's hypotheses $R_L(\beta t) = R_E(t)$ changes with distances in an unknown way.

One tentative explanation may be found when we compare the approximation for the Lagrangian autocorrelation proposed by Hay and Pasquill with that proposed here, R(Ut, o, o; t). Both are shown in fig. 8.1.

Let us assume that our suggestion is correct. For the dispersion at x = 50m., the Hay-Pasquill is a good approximation, taking β = 2. But to get the correct dispersion at x = 500m. we have to take β = 4 or 5, showing that β changes with distance. The reason is that the Eulerian time correlation $R_E(t)$ and consequently $R_E(t/\beta)$ tend to zero much faster than the real Lagrangian autocorrelation. This explains too, why β<1 is sometimes obtained. It can be shown that for certain Values of UE and the space-time correlation falls below the time correlation for certain intervals of time, resulting in 0<β<1.

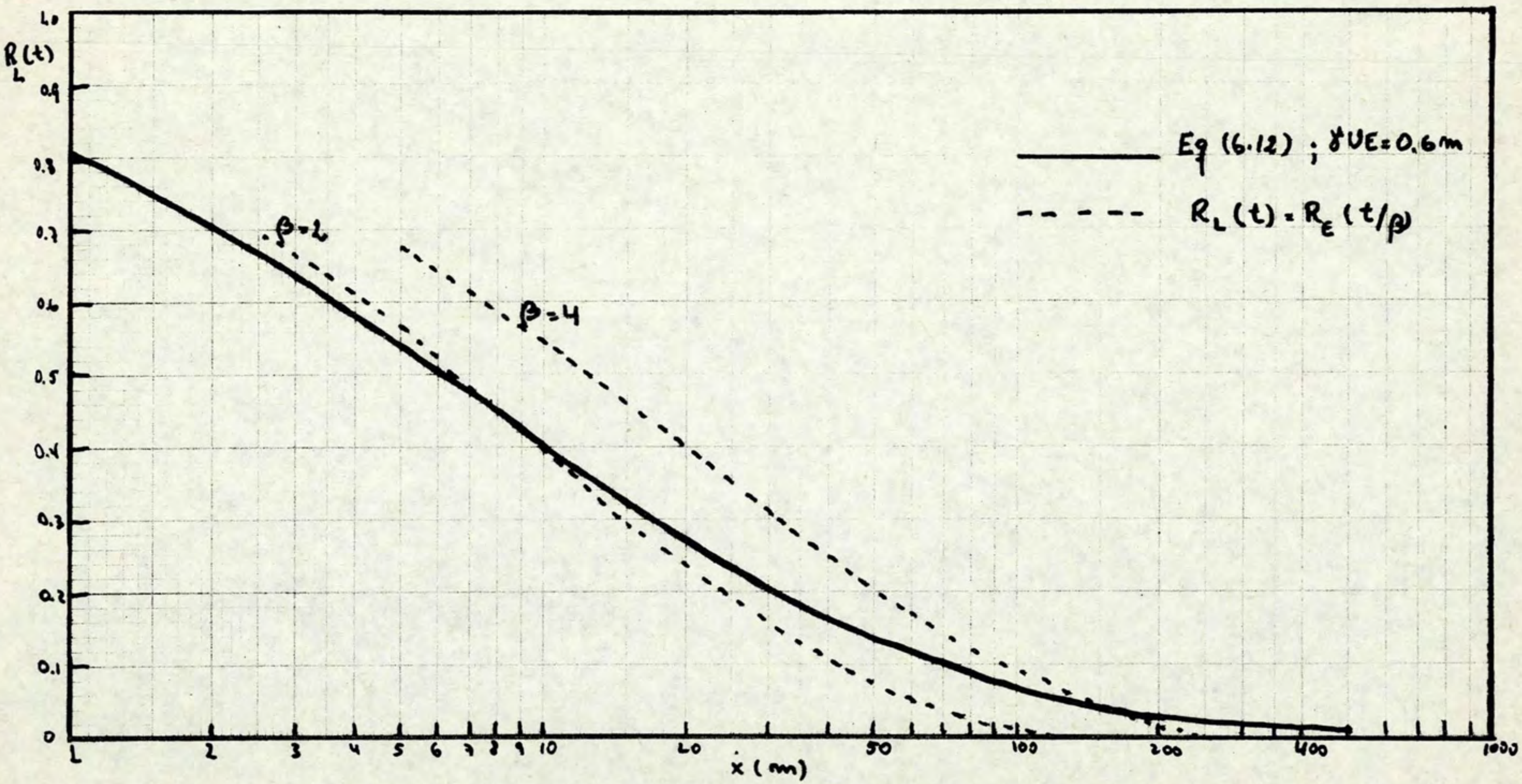

Fig. 8.1 - Approximations for $R_L(t)$. —— assuming $R_L(t) \simeq R(Ut,o,o;t)$ (eq. 6.12) with UE = 3m, γUE = 0.6m. ---- taking $R_L(\beta t) = R_E(t)$. This R_E is the generator of $R(Ut,o,o;t)$.

Chapter 9 - A NOTE ON SUTTON'S FORMULA.

9.1 Introduction.

The simple power laws suggested by Sutton (see Sutton (1953))

$$\overline{y^2}(x) = C_y^2 \, x^{2-n} \tag{9.1}$$

and

$$\overline{z^2}(x) = C_z^2 \, x^{2-m} \tag{9.2}$$

are analytically convenient, and therefore attractive. But problems may arise whenever we try to use them in practical applications. First, the constants C_y, C_z, n and m are not really constant: they change with distance downstream from the source, with atmospheric stability, with height above the ground and site of the experiment. Sutton (1953) proposed an autocorrelation R_L which leads to eqs. (9.1) and (9.2), but his correlation has an infinite integral time scale! In this Chapter it is shown how we may use the approximation to R_L proposed in this work to explain the behaviour of the "constants" in Sutton's formula, eq. (9.1).

9.2 A Model for the Parameters.

In Chapter 7 we saw that the space-time correlation following the mean flow can be a good workable approximation to R_L and that is a function of x/UE only:

$$R_L = R_L\left(\frac{x}{UE}\right) \tag{9.3}$$

consequently,

$$\overline{y^2} = \overline{y^2}\left(\frac{x}{UE}\right) = 2\overline{V_L^2}\, E^2 \int_0^{x/UE} \left(\frac{x}{UE} - \frac{U\tau}{UE}\right) R_L\left(\frac{U\tau}{UE}\right) d\left(\frac{U\tau}{UE}\right) \equiv 2\overline{V_L^2}\, E^2\, G\left(\frac{x}{UE}\right) \tag{9.4}$$

and this can be written in "Sutton's form"

$$\overline{y^2}\left(\frac{x}{UE}\right) = 2\overline{V_L^2}\, E^2\, G\left(\frac{x}{UE}\right) \equiv C_y^2 \left(\frac{x}{UE}\right)^{2-n} \tag{9.5}$$

resulting in

$$C_y^2 = 2\overline{V_L^2}\, E^2\, G(1) \tag{9.6a}$$

and

$$2-n = \frac{\ln G\left(\frac{x}{UE}\right) - \ln G(1)}{\ln\left(\frac{x}{UE}\right)} \tag{9.6b}$$

This shows that n is a function of x, as we know from Taylor's theorem (see Csanady (1973)). Given a reasonable shape for R_L we can obtain the behaviour of Cy and n.

9.3 A Simple Example.

As an instructive example we may assume

$$R_L(x) = \exp\left(-\frac{x}{UL}\right) \tag{9.7}$$

This results

$$\overline{y^2}(x) = 2\,\overline{V_L^2}\,L^2(e^{-\frac{x}{UL}} + \frac{x}{UL} - 1) \tag{9.8}$$

$$C_y^2 = 2\,\overline{V_L^2}\,L^2\,(e^{-1}) \tag{9.9}$$

$$2-n = \frac{\ln(\exp(-\frac{x}{UL}) + \frac{x}{UL} - 1) - \ln(e^{-1})}{\ln(\frac{x}{UL})} \tag{9.10}$$

Eq. (9.10) is shown in fig. 9.1. The following general behaviour of (2-n) can be extracted:

(a) When instability increases, UL increases too, and for a fixed distance x from the source, $\frac{x}{UL}$ decreases and (2-n) increases. So this model predicts an increase of (2-n) with instability, in qualitative agreement with experiment (see Slade (1968) and I A E A (1974)).

(b) A Change in site and/or season implies a change of the environmental roughness. This means a variation in L consequently, for a given distance x, the valued (2-n) changes through a change in x/UL.

(c) UL probably changes with height, which implies a change of (2-n) with z. This is in agreement with experiment (Pasquill (1974)).

9.4 A Realistic Model.

The exponential correlation and eq. (9.6) are not good modles to represent atmospheric diffusion processes. Eq. (7.2) gives what is supposed to be a better approximation to $\overline{y^2}$,

$$\overline{y^2}(\frac{x}{\gamma UE}) = 2\,\frac{\overline{V_L^2}}{U^2}\,(6\gamma UE)^2\,H(\frac{x}{\gamma UE}) \tag{7.2}$$

where γ is the factor related to the lifetime of an eddy. From eq. (9.6b) we obtain

$$C_y^2 = 2\overline{V_L^2}\,(6\gamma\,E)^2\,H(\frac{x}{\gamma UE})\Big|_{\frac{x}{\gamma UE}=1} = 2\overline{V_L^2}\,(6\gamma E)^2\,0.012 \tag{9.11}$$

and

$$2-n = \frac{\ln H(\frac{x}{\gamma UE}) - \ln 0.012}{\ln(\frac{x}{\gamma UE})} \tag{9.12}$$

(2-n) given by (9.12) is shown in fig. 9.2. One advantage of eq. (9.12) is that we can obtain (2-n) knowing Eulerian quantities easy to measure. Presently it can be obtained only by means of diffusion experiments in situs, expensive and difficult to perform.

We can easily extract prediction from fig. 9.2. Assume $\gamma = 0.2$ as we got in Chapter 7, and a distance x = 800m. from the source. For neutral atmosphere,

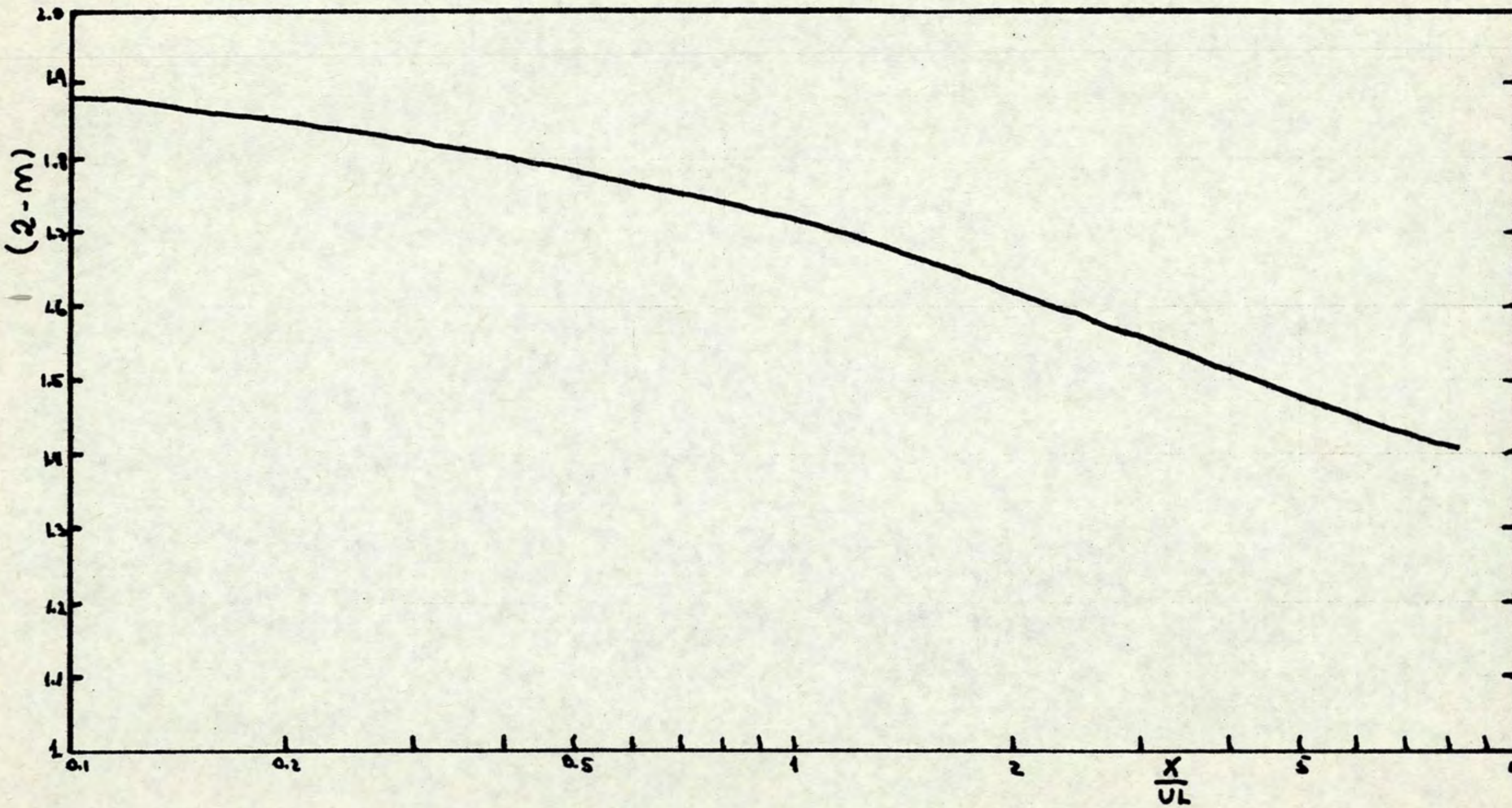

Fig. 9.1 - (2-n) predicted by eq. (9.10).

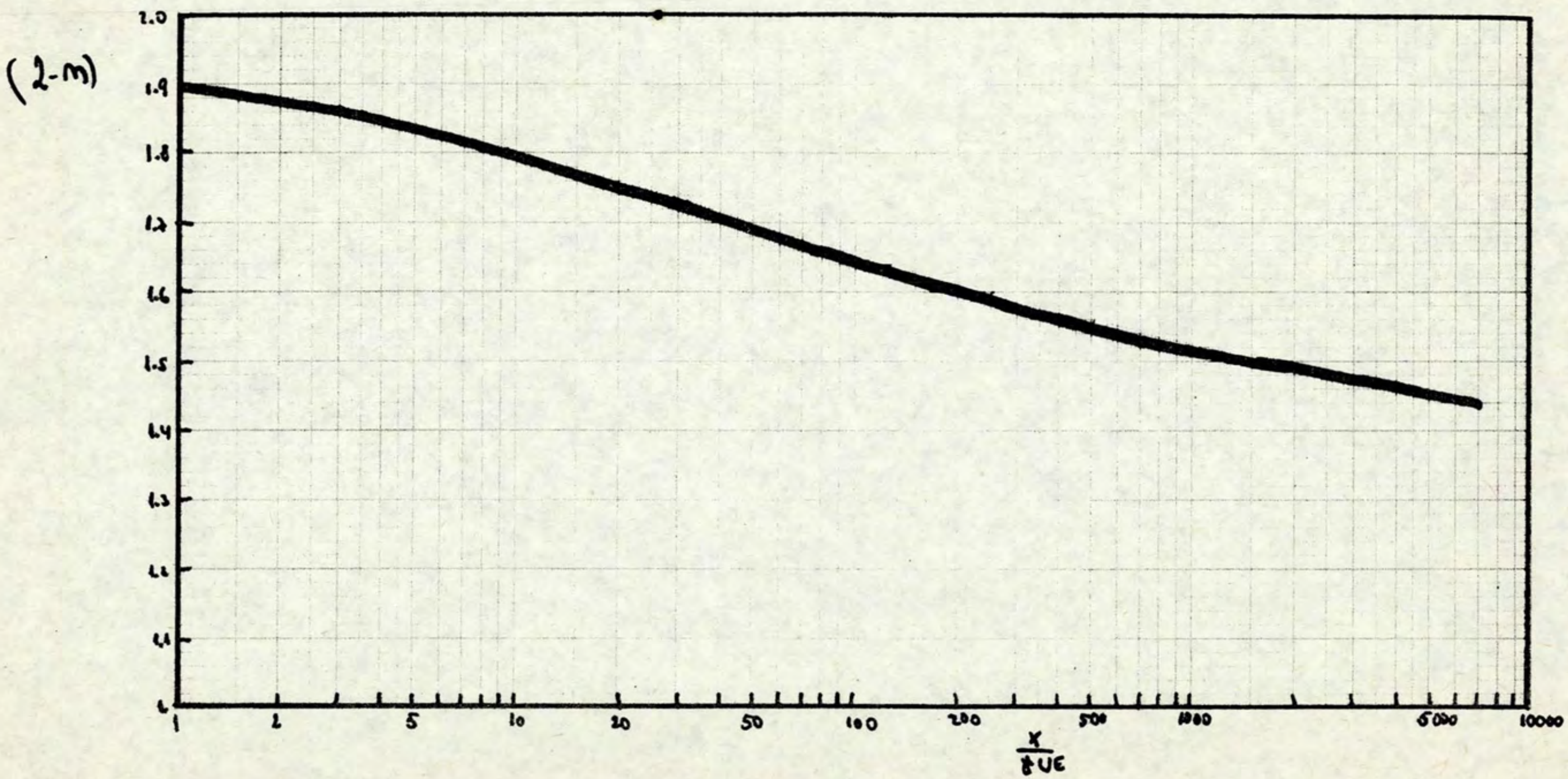

Fig. 9.2. - (2-n) predicted by eq. (9.12).

UE ≈ 10 - 12m. in Project Prairie Grass. This means $\frac{x}{\gamma UE}$ = 320 - 400 and (2-n) = 1.57 - 1.59, which is near the observed value 1.60. For very stable atmosphere, UE ≈ 0.5 m, $\frac{x}{\gamma UE}$ ≈ 8000 and (2-n) = 1.45, to be compared with experimental values, in the range 1.25 - 1.50. For unstable atmosphere, UE is in the range 100 - 350m., resulting in (2-n) = 1.70 - 1.80. The experimental value is within the range 1.65 - 1.80! The experimental values are found in Slade (1969) and Bysova (1973).

Chapter 10 - DISCUSSION AND CONCLUSIONS.

Models describing turbulence, to be applied in air pollution problems should be simple in their use and easy to understand, so that they can be adopted to any new situation. Simple and general models describing turbulence and turbulent dispersion are given here, leading to a new model for the computation of pollution dispersion in any thermally stratified fluids in homogeneous and steady flows.

It is shown that a few conditions to be satisfied by the spectral density of turbulence lead to simple expressions. The model satisfies both the Kolmogorov and the Monin-Obukhov principles. The corresponding Eulerian time autocorrelations $R_E(t)$ are computed and compared with experiment. Good agreement is obtained. Atmospheric and pipe flows require somewhat different spectral expressions. This is probably due to the different ranges of frequencies excited in the flow. The spectra and the correlations are functions of the energy frequency distribution, not of the turbulence intensity. The models depend on the Eulerian integral time scale E. Future development of experimental techniques may lead to a state of the art such that measurements of mean velocity and temperature at two or three different heights would give us the Richardson's number, and consequently E.

With the help of the model proposed for $R_E(t)$ and of a method which does without the knowledge of the Lagrangian velocity variance v_L, it is shown that the Hay-Pasquill model for the Lagrangian autocorrelation $R_L(\beta t) = R_E(t)$ is not a good approximation. In general the computed value falls to zero much faster than the observed value, underestimating pollutant dispersion far from the source. Hay-Pasquill's model may be used only in interpolations over short distances, but not in extrapolation.

A different model is proposed for replacing $R_L(t)$. It is suggested that a better approximation is obtained if we take the correlation between the signal at (o, o, o; o) and the signal of a probe travelling downstream at the velocity of the mean flow. A simple model is here proposed for this "space-time correlation follwing the mean flow." A few assumptions about the behaviour of the eddy pattern in turbulent flow lead to a model for $R_E(Ut, o, o; t)$ as function of the spectral density and consequently of E. This model is in good agreement with experiment, for both channel and atmospheric flows.

Replacing R_L by $R_E(Ut, o, o; t)$ in Taylor's theorem for computing $\overline{y^2}$ leads to excellent agreement between theoretical predictions and experiment, showing that Lagrangian autocorrelations and space-time correlations following the mean flow have approximately the same shape. Assuming this to be correct, we can confirm what was previously found empirically: that R_L as proposed by Hay and Pasquill (1959) has a shape different from the real autocorrelation which requires different values of β at different distances from the source. It is shown too that in general $\overline{V_L^2} \neq \overline{V_E^2}$.

Our model is also capable of explaining the behaviour of Sutton's parameters for different environmental and atmospheric conditions.

In conclusion, simple and general models are given for Eulerian time and space-time correlations, the Hay-Pasquill scheme is shown to underestimate pollutant dispersion and a new approximation is proposed for R_L. All results agree with experiment within 10%.

Some problems remain unsolved. The factor γ, related to the eddy lifetime, has to be determined empirically. It is not even known whether γ is a universal constant (e.g., 0.2 for atmosphere) or a local constant only. A study of the process of eddy transfer from one frequency to another will probably throw some light on the problem, but this is outside the scope of this study.

Another problem is the variation of UE with height. In Chapter 7 values of UE measured at a height of 2m. were used for the computation of dispersion at 1.0 and 1.5m. above the ground. Good agreement with experimental results was achieved. This may mean that at the experimental site, the value of UE corresponding to the v component of the velocity is nearly constant up to a given height exceeding 2m. Not too much can be said about this because experimenters do not agree as to the behaviour of UE versus Z.

The relationship between Eulerian and Lagrangian velocity variances has yet to be discovered before our model can be used in practical applications. This is the difficulty with statistical models involving particle velocity.

This study may be extended to the dispersion of nuclear debris in the atmosphere. The Eulerian time correlation for large scale atmospheric motions has been measured recently (see Hess and Clark (1973)) and data on nuclear clouds dispersion are being published (Randerson (1972)), but at present the data are too scarce to permit an exact analysis.

Study of diffusion over hills, which is a tempting subject, turned out to be impracticable, as no data on diffusion and on turbulence over hills are available. Theoretical models for air flow over gentle slopes have been criticized (see Jackson and Hunt (1975), Frost et al (1974) and Peterson (1975)) since there exist no supporting observations. Diffusion models for flow over obstacles in the atmosphere are over simplifications (Kao (1976)) and are without experimental support.

As to the model for Eulerian time correlations $R_E(t)$, two directions may be considered for improving our results. First, we may assume that the model is the zero order approximation to the Navier-Stokes equations. This would give an approximation for the triple correlation, and subsequent solution of the equation by iteration.

The second direction is to explore the theoretical results of Gage (1971) and Gage and Reid (1968). They show that in a thermally stratified fluid the rate of increase of a perturbation introduced into the flow is a function of the Richardson number and of the perturbation frequency. At sufficiently large Reynolds number,has no influence on the process. Assuming that the energy is fed into the spectrum at the frequency of the largest rate of increase of a perturbation, we can obtain a relation between the Richardson number and E, because E is related to the region of highest energy in the spectrum. Another possibility to be studied is to assume that the energy fed at each frequency is proportional to the rate of increase of the corresponding perturbation. This would generate a spectrum directly dependent on the Richardson number.

One additional consequence is obtained from the experimental relation eq. (2.7) and the model eq. (3.10b):

$$\frac{\overline{V^2}\ n\ F(n)}{u_*^2} = C\left(\frac{z\varepsilon}{u_*^3}\right)^{2/3}\left(\frac{nz}{U}\right)^{-2/3} \qquad (2.7)$$

$$nF(n) = \frac{4En}{(1+6En)^{5/3}} \qquad (3.10b)$$

In the inertial (Kolmogorov) subrange our model reduces to

$$nF(n) = 0.2\ (En)^{-2/3} = 0.2\ \left(\frac{EU}{Z}\right)^{-2/3}\left(\frac{nZ}{U}\right)^{-2/3} \qquad (10.1)$$

Comparing (10.1) with (2.7) we get

$$\overline{V^2} = u_*^2 (0.2)^{-1} C\ \phi_\varepsilon^{\ 2/3} \left(\frac{EU}{z}\right)^{2/3} \qquad (10.2)$$

This seems to be an important result: the variance is a linear function of u_*^2, and of the atmospheric stability through the remaining terms of (10.2). The linear relationship between $\overline{V^2}$ and u_*^2 has been confirmed experimentally (see Kicks (1976)). The relationship between $\overline{V^2}$ and z/L or Ri is presently under study (see Sethuraman and Brown (1976), McBean and McPherson (1976), Binkowsky (1975), Deloach et al (1976)). When we shall be able to find such a relation, the determination of the Richardson number will suffice to describe entirely the dispersion process in turbulent flow.

Chapter 11

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זרימת אוויר טורבולנטית
ופיזור מזהמים באטמוספרה

חיבור על מחקר

לשם מילוי חלקי של הדרישות לקבלת התואר

דוקטור למדעים

מאת

מרין אפשטיין

הוגש לסנט הטכניון - מכון טכנולוגי לישראל
סיוון תשל"ז חיפה 1977

ת ו ד ו ת

מחקר זה נתמך על ידי הקרן "ליידי דייויס", מר שמואל, מ. ברנשטיין, CNPq, CNEN, ו - UFRGS, ברזיל.

המחקר בוצע בפקולטה להנדסה אזרחית בהנחייתו של פרופ' ש' אירמאי. המחבר אסיר תודה לפרופ' אירמאי על הדרכתו ועזרתו הבלתי פוסקים במשך כל תקופת לימודיו בבית הספר ללימודי מוסמכים ובכתיבת התיזה.

תודה נתונה גם לפרופ' י' נוימן וד"ר י' זבירין על התעניינותם במחקר ועל היעוץ שנתנו למחבר; לפרופסורים: א' פראניו, מ' פורה נ - ע' סגיבר על עזרתם במשך לימודיו ועבודת המחקר.

תודה לד"ר נ' אבואף על הדרכתו בשנת תשל"ה.

המחבר מודה גם לגבי טריקסי הודלי וגבי מרים רוזנברג על הדפסת העבודה.

תקציר

מחקר זה מהווה תרומה להבנת וחיזוי פיזור טורבולנטי של מזהמים באטמוספרה. הוא מושתת על ההנחה, שהאוטוקורלציה הלאגרנג'ית R_L, המשמשת לחישוב פיזור פלומה, ניתנת לקביעה בקירוב ע"י קורולציה אוילרית של מקום וזמן.

חיבור זה מראה, שהמודלים האמפיריים הנמצאים בשימוש יפים להכללה רק במקום שבו הם פותחו. החלת ההכללה על מקומות אחרים עלולה להוליך לטעויות בחיזוי ריכוזים של בסדר גודל אחד יותר גדולות. מצב זה נובע מהיעדר תיאור שיטתי מספיק של הזרימה הטורבולנטית.

מודלים המתארים טורבולנטיות ושנועדו לשימוש בבעיות זיהום אויר, חייבים להיות קלים בשימוש ונוחים להבנה, כדי שאפשר יהיה לאמצם לכל מצב חדש. ניתנים כאן מודלים פשוטים וכלליים, המתארים טורבולנטיות ופיזור טורבולנטי והמובילים לקראת מודל חדש לחישוב הפיזור בזורמים המרובדים תרמית, בזרימות הומוגניות ותמידיות.

מוסבר כאן שמספר תנאים, שהצפיפות הספקטרית הטורבולנטית חייבת להישמע להם, מוליכים לביטויים פשוטים. המודל מספק הן את עקרון קולמוגורוב והן את עקרון מונין-אובוכוב. חושבו האוטוקורלציות האוילריות של זמן $R_E(t)$ והושוו עם הניסוי. זרימות באטמוספרה ובצינורות דורשות ביטויים ספקטריים שונים במקצת. דבר זה נובע, כנראה, מתחומי תדר של גלים שונים המתעוררים בזרימה. הספקטרות והקורלציות הינן פונקציות של התפרסות האנרגיה בין אורכי גל שונים, ולא של עצמת הטורבולנטיות. המודלים תלויים בסקלת הזמן האינטגרלית E לאוילר. פיתוח עתידי של טכניקות ניסוייות עשוי להוליך למצב, שבו מדידות מהירות וטמפרטורה ממוצעות בשניים או שלושה רומים שונים, יתנו לנו את מספר RICHARDSON ומכן את E.

בעזרת המודל המוצע עבור $R_E(t)$ ושיטה שאינה נזקקת למידע של שונות המהירות הלאגרנג'ית V_L, אפשר להראות שמודל האי-פאסקויל לאוטוקורלציה לאגרנג'ית $R_L(\beta t) = R_E(t)$ אינו קירוב טוב. בדרך כלל, הערך המחושב יורד עד 0 במהירות רבה יותר מאשר הערך הנצפה, תוך אומד-חסר של פיזור המזהמים בריחוק מהמקור. מודל האי-פאסקויל ניתן ליישום רק באינטרפולציה (ביון) לגבי מרחקים קצרים, אך לא לאקסטרפולציה (חיוץ).

מוצע כאן מודל שונה להחלפת $R_L(t)$. כן מוצע, שקירוב טוב יותר יושג, אם נקח את הקורלציה בין האות ב(0;0,0,0) והאות של המַבְחָן הנע במורד הזרימה במהירות הזרימה הממוצעת. מוצע מודל פשוט עבור קורלצית מקום וזמן בעקבות הזרימה הממוצעת. מספר הנחות לגבי התנהגות דגם הערבולים בזרימה טורבולנטית מביאות לידי מודל $R_E(Ut,o,o;t)$ כפונקציה של הצפיפות הספקטרית ומכן של E. מודל זה מתיישב היטב עם הניסוי, הן עבור זרימה בתעלות והן עבור זרימה באטמוספרה.

החלפת R_L ע"י R_E (Ut,0,0;t) במשפט טילור עבור חישוב $\overline{y^2}$, מוליכה לקראת חפיפה מצוינת בין החיזוי התיאורטי לניסוי ומראה, שהאוטוקורלציות הלאגרנג'יות והקורלציות של מקום וזמן בעקבות הזרימה הממוצעת, יש להן כמעט אותה צורה. אם נניח שהדבר נכון, נוכל לאשר את אשר מצאנו קודם לכן אמפירית, כלומר, שה R_L כפי שהוצע ע"י האי-פאסקויל (1959), יש לו צורה שונה מהאוטוקורלציה הממשית הדורשת ערכים שונים של β במרחקים שונים מהמקור.

כמו כן מראה המחקר, שבאופן כללי $\overline{V_L^2} \neq \overline{V_E^2}$

המודל שלנו יכול להסביר את התנהגות הפראמטרים של סאטון לגבי תנאים סביבתיים ואטמוספריים שונים.

בסיכום, ניתנו כאן מודלים פשוטים וכלליים עבור קורלציית זמן לאוילר וקורלציית מקום וזמן. הוכח כי התיאוריה של האי-פאסקויל אומדת בחסר את פיזור המזהמים והוצע קירוב חדש עבור R_L.

הסתבר שהפיזור בהרים שהוא נושא מגרה ביותר, איננו עדיין בר-ביצוע בגלל היעדר נתונים לגבי הפיזור והטורבולנטיות בהרים. מודלים תיאורטיים על זרימת אויר במדרונות לא תלולים הועברו תחת שבט הביקורת, בעיקר משום שאין להם תימוכין אמפיריים. מודלי פיזור לגבי זרימה מעל מכשולים באטמוספרה, הם פשוטים יתר על המידה וגם להם אין תימוכין ניסויים.

כמו כן מוצע להרחיב את המודל לחיזוי פיזור נשורת גרעינית באטמוספרה. מודלים אחרים הוצעו בקיצור וללא חישובים והם אמורים לתאר את היחס בין יציבות אטמוספרית, סקלות זמן ושונות המהירות. זה צריך לתת לנו את עצמת הטורבולנטיות על בסיס פרופילי מהירות טמפרטורה בלבד.