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**The Role of Heterogeneity in Social  
Problem-Solving**

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of the requirements for the degree of  
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*“The roots of education are bitter, but the fruit is sweet.”*

— ARISTOTLE

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## ABSTRACT

This thesis reviews and investigates social problem-solving with a particular focus on artificial and heterogeneous systems. More specifically, we not only compile and comprehensively examine recent research results, but also discuss future directions in the study of such heterogeneous complex systems. Given their complex nature, such systems often defy analyses. Even computationally simple models can behave unpredictably after a few iterations. Therefore, one central issue in Social Computing is to devise models of social interaction that are amenable to investigation. This way, one can understand the complex relationships among the components and the outcome of the social process. This thesis surveys scientific inquiries concerned with fundamental aspects in social problem-solving systems and their impact in ability and performance of such systems. These aspects include modeling, communication structure and individual problem-solver traits. This thesis also reports the student endeavour during the period of research and summarizes several already published contributions. Among them there is (i) the study of general frameworks for the study of social problem-solving, (ii) the investigation of the role of centrality in individual and collective outcomes, and (iii) the exploration of heterogeneous models of social problem-solving. These three points, in an integrated perspective underpin the understanding of network and communication structures, adjust the strategic systems' composition, and exploit problems' structures and patterns in social problem-solving systems.

**Keywords:** Computational social systems. computational social science. social computing.

## **Sistemas Heterogêneos de Resolução Social de Problemas**

### **RESUMO**

Metódos analíticos de investigação são usualmente ineficazes para sistemas computacionais sociais já que apenas algumas iterações do sistema já são suficientes para que o sistema se torne imprevisível. Portanto, uma das principais questões na Computação Social é o desenvolvimento de modelos sociais passíveis de investigação. Assim é possível que se compreenda o relacionamento complexo entre os componentes de sistemas sociais computacionais e o resultado. Este aspecto incluem a modelagem, a estrutura de comunicação e características individuais do agentes envolvidos na resolução dos problemas do processo social. Esta tese explora sistemas computacionais de resolução de problemas com foco em sistemas artificiais e heterogêneos. Nela é feita uma compilação extensiva da literatura relacionada em sistemas complexos onde as contribuições do candidato são expostas dentro de contextos específicos da área. Entre elas está o estudo de modelos abstratos e gerais de resolução social de problemas, a investigação do impacto da centralidade no resultado individual e coletivo, a análise experimental de modelos heterogêneos de resolução social de problemas. Quando integradas, estas contribuições reforçam o entendimento sobre a importância da rede e das estruturas de comunicação, a composição estratégica do sistema, a estrutura do problema e possíveis padrões gerais na resolução social de problemas.

**Palavras-chave:** Sistemas Sociais Computacionais, Computação Social, Inteligência Artificial.

## **LIST OF ABBREVIATIONS AND ACRONYMS**

DSR	Dynamic Search Range
FIPS	Fully-Informed Particle Swarm
LF	Lazer-Freidman Model
MN	Memetic Networks
NBA	Barabási-Albert generative network model
PSO	Particle Swarm Optimization

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## 1 INTRODUCTION

Behavioral research has shown that humans can excel at solving complex problems where there are strong constraints and interdependencies (KEARNS, 2012). Yet, recent research results are far from answering why (CHEN et al., 2016). One way to peer into why humans excel into some complex problems is by building computational models of social problem-solving that mimic fundamental aspects of the real systems (LAZER et al., 2009; MASON; WATTS, 2012). However, there are many challenges in doing so. Even simple systems composed of homogeneous parts can defy analyses after a few iterations, while more sophisticated ones provide results which are often hard to generalize (MACY; FLACHE, 2009).

One of the first attempts to understand social problem-solving in a controlled laboratory-based setup can be traced back to early experiments at the Massachusetts Institute of Technology, led by Bavelas (BAVELAS; BARRETT, 1951). Bavelas and his group were interested in the eventual effects that different network structures could have in a group's communication pattern. Their experiments on networked communication and social problem-solving were organized within physical laboratories where people would solve problems and exchange information following rigid protocols. The network structures they investigated included a star formation with a central individual by whom all information passed through or a complete clique where anyone could exchange information with anyone else in the experiment. Their hypothesis considered that subjects more "connected", i.e. in a more privileged position within the internal network's information flow, would fare better when compared to non-privileged subjects. This kind of structural property evolved under the general concept of centrality in graph theory (FREEMAN; ROEDER; MULHOLLAND, 1979).

Centrality is one of the first concepts developed to formally describe the relevance of positioning within a structured network (BURGESS, 1969; FREEMAN, 1978; FREEMAN; ROEDER; MULHOLLAND, 1979; BORGATTI, 2005). Experimental evidence has suggested that centrality is positively correlated with individual's performance in human social problem-solving experimental settings (MASON; WATTS, 2012). Moreover, there is also evidence that *long-distance connectivity*, another structural network property, may elicit opposing outcomes depending on the problem humans are trying to solve (JUDD; KEARNS; VOROBAYCHIK, 2010). In such case, the position of an individual alone may not account for her influence on neighbors and *individual traits* may play a

more important role than previously thought. The authors also suggest some benefits of broadening the research on social systems to include individual aspects of solvers instead of focusing solely on network structure.

By knowing patterns and features of the communication structure, one can avoid resource wasting and develop more flexible and efficient collaborative group work flows (NEWELL; SIMON, 1972; MARCH, 1991; LAVIE; STETTNER; TUSHMAN, 2010). In this search, Network Science techniques have become the standard tool to do so (BARABÁSI; ALBERT, 1999; KEARNS; SURI; MONTFORT, 2006; HOGG; HUBERMAN, 2008; LAZER et al., 2009). Studies concerning how individual and group influence drives diversity<sup>1</sup> in social networks abound in literature for the same reason (FLACHE; MACY, 2011).

Not only there is a lack of understanding of diversity effects in real-world scenarios, but there are difficulties at the modeling and the analytical levels too (CHEN et al., 2016). For instance, one of the most cited papers in social problem-solving and diversity itself<sup>2</sup> published in the Proceedings of the National Academy of Science (HONG; PAGE, 2004) has received some strong criticism. In this paper, the authors claim that “diversity trumps ability” in the sense that a group of diverse problem solvers is more likely to outperform a group of highly-able homogeneous solvers. However, (THOMPSON, 2014) presented a critical mathematical analysis of their results. Such kind of dispute represents an expression of how complex and fertile the matter of diversity in social problem-solving can be. One consequence of her work is the cautionary warning about the claims of research using computational methods. Although her arguments have been recently criticized on qualitative grounds (KUEHN, 2017), her main conclusion remains relevant to the field of computational social science. Fortunately, there is a current research trend that seeks to provide a rigorous mathematical foundation to Social Computing (CHEN et al., 2016). One of the objectives of this line of research is to determine whether the results from different Social Computing system generalize (or not) to other social problem-solving scenarios.

In a recent paper in *Nature Communications* (BARKOCZI; GALESIC, 2016) investigated a classic problem in Social Computing: the relationship between the communication network and the system’s performance. Using agent-based simulations, they investigated the interplay between network structure, how information is aggregated from different sources, and the complexity of the problem being solved. Among their findings,

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<sup>1</sup>The terms diversity and heterogeneity are used interchangeably throughout this thesis.

<sup>2</sup>As of 2017, the article has 25,559 total downloads and 583 citations according to Google Scholar.

they found that well-connected networks favor solution aggregation by averaging as many solution sources as possible while poorly connected networks favors copy of the single best source. The authors used a synthetic problem space called the NK model (KAUFFMAN; WEINBERGER, 1989) and concluded that findings have “broad implications for organizational learning, technological innovation, cultural evolution and the diffusion of innovations”. Even though this is a recent result, their research question is not distant from the 15 years old question asked by Mendes when he developed the FIPS model (MENDES; KENNEDY; NEVES, 2003).

When investigating social problem-solving systems, one can experiment with roughly two types of parameters: (i) problem-dependent parameters, or (ii) problem independent parameters. Given a specific problem to be solved, problem-dependent parameters can be grouped — but are not limited to — as:

**The Problem to be Solved:** This parameter defines what the agents are going to solve.

For instance, a search for the best transit route in a large set of possible routes.

**The Problem’s Difficulty Level:** The problem’s difficulty level can be measured in many ways, and the asymptotic analysis of the best know algorithm is the accepted limit of the problem’s difficulty. However, not all problems’ difficulty can be measured in an centralized fashion. Social problem-solving is done in a networked, distributed and collaborative manner from relatively local information and interaction (KEARNS, 2012). There is currently no variant to computational complexity that handles difficulty in social problem-solving. For instance, the problem’s difficulty may depend on the number of agents solving it in addition to the size of the problem and the time given to solve it.

**The Strategies Employed to Solve the Problem:** This parameter refers to the methods and strategies employed individually by solvers to process *and* communicate information about solutions. One example is trying to employ agents that excel at solving chess to play Sudoku instead: they would find the Sudoku problem rather difficult, even if the given Sudoku instance could be easily solvable by a backtracking procedure.<sup>3</sup>

Some parameters that do not depend on the problem at hand are the following:

**The Number of Solvers:** This parameter describes the number of autonomous yet inter-dependent agents employed to solve the same problem.

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<sup>3</sup>Here one must assume that no individual learning occurs.

**The Solver’s Interconnectedness:** This parameter describes the network’s topology that connects the solvers. Since communication in the form of solution exchange *only* happens among “connected” solvers. In other words, collaboration is highly dependent on the network. This parameter in turn can be divided in more parameters depending on the generative network model employed.

It is necessary to point to similar efforts on heterogeneous social problem-solving systems in the related field of Computational Intelligence (ENGELBRECHT, 2007). However, such efforts had different objectives. In this approach, the model itself is the object of study instead of a means to an end. In other words, there is no intent to help understanding how human social problem-solving systems <sup>4</sup> converge to good solutions. A brief description of such line of investigation is presented in the next subsection.

Computational Intelligence is concerned, among other things, with population-based heuristics and blending different heuristics into a single, homogeneous population-based heuristic. This has been investigated for many years under the name of hybridization (TALBI, 2002). Hybridization can be understood as blending different heuristics at the individual level.

It is, however, interesting that at the same time that there was an explosion in the number of heuristics in the last two decades (SÖRENSEN, 2015), there is a methodological approach that “blends” different heuristics at the *collective level*, i.e. networked and interdependent automatons collaborating in a search problem. This approach is at the root of any heterogeneous collaborative system and its motivation lies in the exploitation of a diverse population of solvers. This motivation is similarly employed in the development of collaborative hybrid meta-heuristics (TANESE, 1989; ADAMIDIS; PETRIDIS, 1996; TALBI, 2002; OUELHADJ; PETROVIC, 2010).

Blending different meta-heuristics into one at the collective level can be traced back to (LESSER, 1990). Many other researchers in the field of Computational Intelligence (ENGELBRECHT, 2007) and Swarm Intelligence (ENGELBRECHT, 2010) are converging to this approach amid hundreds of new heuristics proposed each year, see e.g. (KRINK; LØVBJERG, 2002; ALBA et al., 2004; ENGELBRECHT, 2011; OCA et al., 2009b; ENGELBRECHT, 2010; LEONARD; ENGELBRECHT; WYK, 2011; OCA et al., 2009a; NEPOMUCENO; ENGELBRECHT, 2012; RIBEIRO; ENEMBRECK, 2013; HARA; SHIRAGA; TAKAHAMA, 2012; CHEUNG; DING; SHEN, 2013; PINCIROLI et al., 2010; OLORUNDA; ENGELBRECHT, 2009; ATYABI; PHON-AMNUAISUK,

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<sup>4</sup>An exception is the PSO model that will be further explained in the next section.

2008; EBERHART, 2008; NEPOMUCENO; ENGELBRECHT, 2013; OLORUNDA; ENGELBRECHT, 2008; OCA et al., 2009c).

Considering all that was previously elaborated, this thesis seeks to understand social problem-solving by investigating the impact of the main parameters that were historically studied in different areas of research in social systems. By integrating distinct yet related work, finding the gaps in our knowledge, we can proceed to fill in the most urgent and relevant gaps. Therefore we seek to contribute to the development of more sophisticated computational models that could both (1) be used to guide research in real-world settings and that (2) could be objects of research by themselves.

Even though the thesis is organized as a compilation of self-contained investigations, the main line of research was the study of how different parameters play a role in social problem-solving. The following list enumerates the thesis' specific objectives.

1. To investigate a method where one could employ different social problem-solving models in such a way that individuals could exchange information seamlessly, i.e. collaborate.
2. To investigate how the network's structure impacts individual and collective outcome.
3. To investigate the dynamics behind a diverse pool of solvers and their network structure.

The most relevant results so far have already been published in international conferences broadly associated with the Artificial Intelligence field. Table 1.1 presents them, their respective publication year, the conference name, and the section in this thesis document where it is described in more detail. There is a direct relationship between the previous cited objectives and these papers, and such relationship information is presented in the table.

The remainder of this thesis is organized as follows: Chapter 2 presents the study of a general model of social problem-solving, Chapter 3 describes our recent results on investigations of the network's impact in social problem-solving (NOBLE et al., 2015b), Chapter 4 points recent results from heterogeneous models (NOBLE et al., 2015a) and Chapter 5 summarizes the main contributions from this thesis with respect to the author's published results. Finally, a summary of the thesis is presented in Portuguese, to comply with formal university regulations.

Table 1.1: Published works.

<b>Objective</b>	<b>Chapter</b>	<b>Conference</b>	<b>Year</b>	<b>Title</b>
1	2	AINA	2013	<i>Investigating a Socially Inspired Heterogeneous System of Problem Solving Agents</i>
2	3	GECCO	2015	<i>The Impact of Centrality on Individual and Collective Performance in Social Problem-Solving Systems</i>
3	4	AAAI	2015	<i>Collaboration in Social Problem-Solving: When Diversity Trumps Network Efficiency</i>



## **2 ON THE FEASIBILITY OF A GENERAL FRAMEWORK FOR COMPUTATIONAL SOCIAL PROBLEM-SOLVING**

Many problem-solving methods in the field of Computational Intelligence draw inspiration from nature (BROWNLIE, 2011). From the observation of a natural or physical phenomena, one can derive computational methods that solve a problem in a analogous way as the original natural system does. These methods usually employ stochastic heuristics. One would wonder how can two socially-inspired methods collaborate in a networked scenario. The first study during our research contemplated such question and combined two models at the collective level: the Memetic Networks model and the PSO model. This approach did not create a new heuristic but combined the two cited models to compose a population of individuals that could “communicate” seamlessly. In other words, this means that particles from the PSO model could collaborate with memetic nodes and vice-versa.

There are two main questions to be answered in this study. They are the following:

1. Can we use the principle of networked communication of socially-inspired systems to create an heterogeneous collaborative system?
2. Do this approach hold some benefit in comparison to the homogeneous models?

Subsection 2.1 contains explanations on how we performed such investigation on the framework followed by the results achieved 2.2, and a discussion in subsection 2.3.

### **2.1 Methods**

This section describes in detail the two models used and then proceeds to describing the proposed general model which is being investigated.

#### **2.1.1 Particle Swarm Optimization**

The model of Particle Swarm Optimization had its inception in the work of Heppner and Grenander (HEPPNER; GRENANDER, 1990). The authors originally investigated simple social analogues of a population of interacting automatons which were modeled as flocks of birds searching for food. Kennedy and Eberhart (KENNEDY; EBER-

HART, 1995) evolved their ideas intending to produce computational intelligence from the interactions of such automatons (POLI; KENNEDY; BLACKWELL, 2007). The resulting model was then used with relative success with optimization problems. In 2008, Poli (POLI, 2008) analyzed over 1100 publications concerning improvements and applications of the PSO model. He contributed to the understanding that the PSO model is a mature and quite successful model of social problem-solving.

The PSO model basically defines how particles will interact collectively and the individual effect of such interaction. At the collective level, particles are allowed to interact if there is a communication channel between. This level is usually modeled using undirected graphs, where particles are represented by nodes and communication channels by edges of the graph.

Particles interact by copying solutions. More specifically, a particle will copy the solution of its best ranked network neighbor and combine this solution with its own inner solution. The interplay between a particle own solution the particle's best neighbor solution characterizes a motion in search space of valid solutions. This motion is described by a velocity vector.

Previous studies recognized the importance of restraining the particle's velocity in some way (POLI; KENNEDY; BLACKWELL, 2007). Thus, Kennedy (KENNEDY, 1998) developed the constriction factor mechanism. Equations (2.1) and (2.2) define analytically the process of velocity update considering these constriction factors.

$$\begin{aligned} \vec{v}_{i+1} &\leftarrow 0.7298(\vec{v}_i + U(0, 2.05) \\ &\otimes (\vec{p}_i - \vec{x}_i) + U(0, 2.05) \\ &\otimes (\vec{p}_g - \vec{x}_i)) \end{aligned} \quad (2.1)$$

$$\vec{x}_{i+1} \leftarrow \vec{x}_i + v_{i+1} \quad (2.2)$$

where  $v_i$  and  $x_i$  represent the respective the current velocity and solution of a given particle,  $U(0, 2.05)$  represents a random vector with components uniformly distributed between  $[0, 2.05]$ ,  $\otimes$  is the component-wise multiplication,  $p_g$  is the best previous position know by the best ranked neighbor — the collective influence, and  $p_i$  is the best previous position know by the own particle — the individual influence. The constant 2.05 was experimentally derived in Kennedy (KENNEDY, 1998).

Yet, improvement was achieved when the particle's speed was limited in each dimension by the maximum range of such (EBERHART; SHI, 2000). The PSO with

constriction factors combined with speed limits is considered to be the canonical version of PSO (POLI; KENNEDY; BLACKWELL, 2007).

### 2.1.2 Memetic Networks

Memetic Networks is a recent model inspired by how humans collectively solve problems (ARAUJO; LAMB, 2008b). The model determines guidelines for the development of a system that can be used as means to solve an optimization problem or when one seeks to investigate the effects of modeling real social features into artificial social systems. Memetic Networks provide a framework from which it is possible to objectively determine the impact of features such as network structure influence in the collective performance of the system along the search process.

The model defines simple elements such as nodes, memes and links. It also defines rules that state how these elements relate to each other. The concepts behind elements and rules are problem invariant; however, their instantiation is not: one needs to encode information about the problem's domain in order to instantiate an effective problem-solving system. In this sense, the Memetic Networks model is not a purely blackbox method for optimization, but rather a design guideline for the development of algorithms with embedded information about the problem domain.

A *memenet* is the final construct when one assembles the Memetic Network's elements and then describes the rules by which the elements will interact. The *memenet*'s main objective is to achieve better solutions in less iterations than independent and isolated nodes would achieve. To do so, nodes in a *memenet* are allowed to exchange information.

The computing mechanism behind Memetic Networks is represented by nodes. Each node contains a complete solution to the problem at hand. This solution can be (i) evaluated using a global evaluation function, and (ii) compared or ranked against other solutions. Nodes are responsible for retrieving, aggregating and processing *memes*, which are the minimal meaningful pieces that compose complete solutions. The meme concept is partly based on ideas from Dawkin's book, *The Selfish Gene* (DAWKINS, 2006). Links connect nodes and ultimately determine a supporting "social" network. If there is a link between two nodes, then these nodes are allowed to exchange memes. Therefore, this network allows communication of memes between nodes.

The Memetic Network elements alone are not self-sufficient to perform search as

it remains necessary to define how these elements will behave when assembled together. This is accomplished by three rules, which roughly capture how humans process information in a social context (ARAUJO; LAMB, 2008b). These rules detail how nodes choose to connect to each other, how information is retrieved from the social network and how individual nodes contribute to a solution being sought. Each rule is detailed in what follows.

**Connection Rule:** Defines how the nodes should connect, i.e. how links are to be established. This rule can be performed once before the search takes place, characterizing a static network topology. Alternatively, the connection rule can be executed during the search process so the network topology can evolve together with the system. As in other socially-inspired models (e.g. Particle Swarm Optimizers), there is currently no conclusive evidence about what topology is generally better for specific types of problems (POLI; KENNEDY; BLACKWELL, 2007).

**Aggregation Rule:** This rule controls the interaction between connected nodes, that is, how nodes retrieve and aggregate information from their network neighbors. Typically, a node retrieve information from better ranked neighbors only. After retrieving memes, a node must aggregate - i.e. combine - them to assemble a new solution. The aggregation rule must also describe algorithmically how these memes are to be combined. Aggregation is highly dependent on the network topology <sup>1</sup>.

**Appropriation Rule:** In possession of the recently aggregated solution, a node may add some novelty to it. In this step, local information is added to the solution, allowing memenets to explore the search space, much in the same way mutation works in genetic algorithms. Appropriation can happen by simple random changes to the solution or by applying some deterministic local search to it (in a similar fashion to Genetic Algorithms (GOLDBERG, 1989) and Memetic Algorithms (MOSCATO, 1999) respectively).

Although the aggregation and the appropriation rules depend on the problem at hand, the structure of a Memetic Network is the same across different problems.

### 2.1.3 An Heterogeneous Framework Composed by Particles and Memetic Nodes

The PSO and Memetic Networks models rely on the concept of social network to regulate the internal information flow in a structured way. Such structure allow a node

---

<sup>1</sup>This feature is where socially-inspired optimization algorithms differ the most from Evolutionary Algorithms.

---

**Algorithm 1:** Structure of our heterogeneous system. Note that the *build\_network()* sub procedure is performed once only and that the *stop condition* can be the number of iterations or a desired solution quality threshold.

---

```

Input :  $Population_{size}, stopcondition, Problem_{size}$ 
Output:  $Solution_{best}$ 

Population  $\leftarrow$  InitializePopulation ( $Population_{size}, Problem_{size}$ )
InitializeNetwork (Population)
repeat
  foreach  $node \in Population$  do
    UpdateState ( $node$ )
    EvaluateState ( $node$ )
    if  $node's_{current}_{score} < node's_{best}_{score}_{yet}$  then
      // Store new solution as best solution found so far by  $node$ .
  until  $stopcondition$ 
 $Solution_{best} \leftarrow$  RetrieveBestSolutionFrom (Population)
return  $Solution_{best}$ 

```

---

to retrieve and send information to as many sources as possible. Additionally, socially inspired models need a few modifications from their canonical form to work in collaboration, since communication and information processing are conceptually separated. This contrasts with the canonical GA, for instance, which communication is done indirectly through crossover operations using two individuals.

Communication between a particle and a memetic node should be as seamlessly as possible in order to avoid overhead. To facilitate such feature, all agents must encode the solution in the same way. For example, if memetic nodes encode their solutions as arrays of real values, particles shall encode their solutions as arrays of values as well. To solve the problem, we developed a simple interface that must be followed by all nodes, particles and memetic nodes. This interface follows from three premises: a node must have a routine (1) to update its internal state by performing its respective search operations, (2) evaluate its current internal state, and (3) store the internal state if it leads to a better solution than any state.

Algorithm 1 describes the structure of the proposed framework including references to initialization routines. The network structure is initialized only once. Hence, the resulting system possess a static network. The *stop condition* can be a maximum number of iterations, running time, or a desired solution quality threshold.

The next decision is about the communication between particles and memetic nodes and vice versa. Each particle retrieves a solution from its best ranked neighbor

while a memetic node retrieves multiple solutions from the set of better ranked neighbors. However, if a memetic node is the best ranked neighbor of a particle, it must provide its previous best position to the particle. Therefore, a memetic node *must* store its previous best position even if this was originally not sought in the Memetic Networks model. If a memetic node has a one or more particles in its neighborhood it can then retrieve their current solution directly.

It remains necessary to implement the search operators from both particles and memetic nodes through the *UpdateState* routine for the problem of function minimization. We will start by the particle *UpdateState* routine which is depicted in Algorithm 2.

---

**Algorithm 2:** Update state procedure for a particle.

---

**Input:** *node*

$BestNeighbor \leftarrow \text{RetrieveBestNeighbor}(node)$

$\text{UpdateVelocity}(node, BestNeighbor)$

$\text{UpdatePosition}(node)$

---

Our implementation follows the canonical PSO form stated in Poli's review work (POLI; KENNEDY; BLACKWELL, 2007); more details can be found in the original work (EBERHART; SHI, 2000).

The *UpdateState* routine of a memetic node is depicted in Algorithm 3.

---

**Algorithm 3:** Update state procedure for a memetic node.

---

**Input:** *node*

$BetterNeighbors \leftarrow \text{RetrieveBetterNeighbors}(node)$

$aggregated_{solution} \leftarrow \text{Aggregate}(BetterNeighbors)$

$\text{Appropriate}(aggregated_{solution})$

---

The aggregation procedure retrieves and then combines memes from a set of solutions from better ranked neighbors to create a new solution.

For real function minimization this process is described in Algorithm 4 and work as follows: the memetic node generates an array of arrays called *MemePool*. The purpose of this structure is to store solutions that came from neighbors with better scores and also the node's own solution. Each component of the solution is replaced with a respective meme of the *MemePool* list, that is a meme from the same position of the available arrays. All solutions from *MemePool* have equal chances to be chosen but the meme index is incrementally computed.

The appropriation procedure is the mechanism which allows nodes to exploit their current solution, that is perform small modifications to it. For real-function minimization



Table 2.1: Benchmark Functions

Name	Definition	Initialization Range $[x_{min}, x_{max}]^n$	Search Range $[x_{min}, x_{max}]^n$
Ackley	$20 + e - 20e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2}} - e^{\frac{1}{n}\sum_{i=1}^n \cos(2\pi x_i)}$	$[15.0, 30.0]^n$	$[-30.0, 30.0]^n$
Griewank	$\frac{1}{4000}\sum_{i=1}^n (x_i - 100)^2 - \prod_{i=1}^n \cos\left(\frac{x_i - 100}{\sqrt{i}}\right) + 1$	$[300.0, 600.0]^n$	$[-600.0, 600.0]^n$
Rastrigin	$\sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10)$	$[2.5, 5.1]^n$	$[-5.1, 5.1]^n$
Rosenbrock	$\sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	$[15.0, 30.0]^n$	$[-30.0, 30.0]^n$
Sphere	$\sum_{i=1}^n x_i^2$	$[50.0, 100.0]^n$	$[-100.0, 100.0]^n$

both solution components and cost value.

Since four functions have the global minimum at the very center of the search space <sup>2</sup>, all particles and nodes were initialized following a uniform distribution inside a limited search range to avoid any advantage a model may have over the other (ANGELINE, 1998). The specific information about these functions is described in Table 2.1.

The total population size was set to 30, distributed as half particles and half memetic nodes. The distribution of the population remained static along 10,000 iterations that were used as the stop condition criteria.

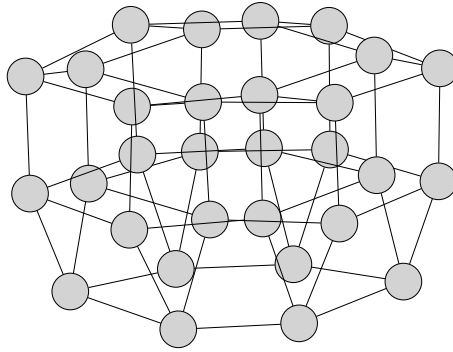
The network topology chosen for all experiments was a  $6 \times 5$  “Von Neumann” neighborhood. In this setting, the population is arranged in a rectangular matrix where each node is connected to one individual above, one below, and one at both sides, wrapping the edges of the matrix (KENNEDY; MENDES, 2006). Previous works found supporting evidence that this topology is better suited to the kind of problem we are tackling (KENNEDY; MENDES, 2002; MENDES; KENNEDY; NEVES, 2003; KENNEDY; MENDES, 2006). Moreover, this topology is balanced in terms of density and average path length when compared to more traditional network topologies — like the fully connected network or the ring network. Therefore, in the grid topology each node is at same average distance from every other node. This allows one to cancel any advantage or disadvantage that network position alone may cause. Figure 2.1 provides a three dimensional representation of the topology used.

In each experiment, the heterogeneous model was compared against the original PSO and Memetic Network models. Also, an improved version of PSO algorithm, the Fully Informed Particle Swarm (FIPS) algorithm (MENDES; KENNEDY; NEVES, 2003) was included for comparative purposes. The FIPS algorithm improves over the original PSO by gathering information of all neighbors the particle have and not only from the best neighbor. Each experiment was repeated 50 times and the results were averaged. The  $p$  probability used by memetic nodes in appropriation was 0.02. This value was

<sup>2</sup>The exception is the Rosenbrock Function.



Figure 2.1: The  $6 \times 5$  periodic grid graph used as the network topology in our experiments.



experimentally derived as a value that allowed convergence in all tested functions.

## 2.2 Results

Figure 2.2 presents the evolution of the best solution found by all algorithms averaged over 50 independent trials. The heterogeneous model is represented by *Hybrid*, the original memenet by *MN*, the original PSO by *PSO* and the Fully Informed Particle Swarm by *FIPS*.

The heterogeneous model with the population distribution of half particles and half memetic nodes presented no statistically difference from the best performing model in the Griewank, Rosenbrock and Sphere functions. In such cases, the PSO was the best performing model that composed the heterogeneous models. This result is positive because if we suppose that one is to solve a novel problem that no exploitable structure is known at moment, then a heterogeneous model can perform sometimes as better than the worst performer or as good as the best performing model. The insight of which model performs best is a valuable one, for it allows the designer to concentrate the efforts into a single model. Moreover, such result confirms previous findings by Montes de Oca et al. (OCA et al., 2009b). The extended version of PSO, the FIPS algorithm, presented slightly advantage in cited functions.

In the Ackley and Rastrigin functions, both highly multimodal, the heterogeneous model presented a remarkable advantage. We performed unpaired t-tests comparing all the 50 last generations of the heterogeneous model against PSO, Memetic Networks and FIPS, finding significant results ( $p = 0, t < 0$ ). The hypothesis for such result is the heterogeneous model presents a higher solution diversity in these functions. A higher solution diversity translates into a more exploratory and less exploitative search behavior,

which in turns increases the likelihood of finding a good solution. To test this hypothesis, we extracted the 30 individuals from the last generation of all the 50 runs and analyzed the distribution of the population according to their solution quality. Figure 2.3 presents the boxplots of this distribution.

As hypothesized, the heterogeneous model presented the highest solution diversity in the tested functions: Ackley and Rastrigin. This result was expected because the two sorts of individuals<sup>3</sup> that compose our heterogeneous model can be seen as two driving forces that guide the search to different regions. It is likely that particles may find a specific region of the search space more promising than memetic nodes as well as the opposite. From this lack of agreement, a favorable search pattern may have arisen in these two functions. Nevertheless, we are further investigating the effect and developing a visualization tools to analyze the population dynamics across all generations. Another result was that the pure PSO model eventually found the global optimum in the Ackley function — this can be seen as the lower whisker mark. However, the expected result is much above it. The pure Memetic Networks model achieved similar results in most runs, this fact is depicted by the equally distributed quartiles from the median.

### 2.3 Discussion

We proposed an initial heterogeneous problem-solving system based on socially inspired models and we were concerned with the investigation of the benefits of a heterogeneous problem-solving system composed of two socially inspired models: Particle Swarm Optimization and Memetic Networks. Our approach does not change how individuals behave, as it is commonly tackled by hybridization mechanisms, rather it allowed different individuals, with different search heuristics, to communicate and collaborate to solve the problem at hand – which we framed as minimization of five real valued functions.

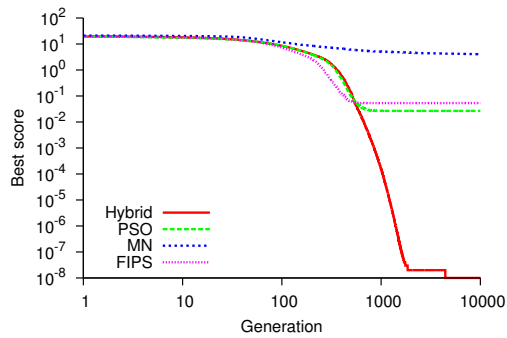
Individuals were connected by links in Von Neumann network and they were allowed to communicate information about their solution through a communication medium designed for that. Our results showed that at one hand the heterogeneous system was able to outperform the Memetic Network model in all scenarios and considerably outperform the PSO model in two of the most difficult (highly multimodal) scenarios. On the other hand, in simpler scenarios (mostly unimodal functions), the heterogeneous model was

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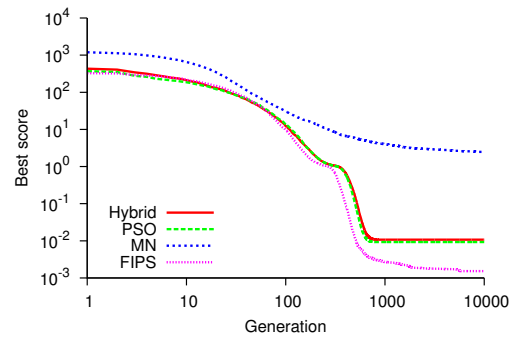
<sup>3</sup>Represented by their search heuristics.

able to perform equally well as the homogeneous models. We further investigated why our heterogeneous model performed well in highly multimodal scenarios and elaborated the hypothesis that heterogeneous models increase diversity of solutions. From the analysis of distribution of solution quality we were able to confirm our hypothesis. Together, these results are evidence that heterogeneous systems are promising problem-solving tools but their inner working mechanisms must still be better understood.

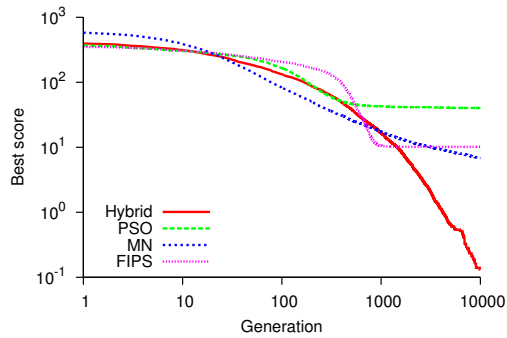
Figure 2.2: The evolution of the best solution found by all algorithms averaged over the 50 independent trials. Our heterogeneous model with the population distribution of half particles and half memetic nodes presents better results for the Ackley and Rastrigin Functions after one thousand iterations. In the other cases, the heterogeneous model was equivalent to PSO and FIPS. Both axis are in logarithmic scale.



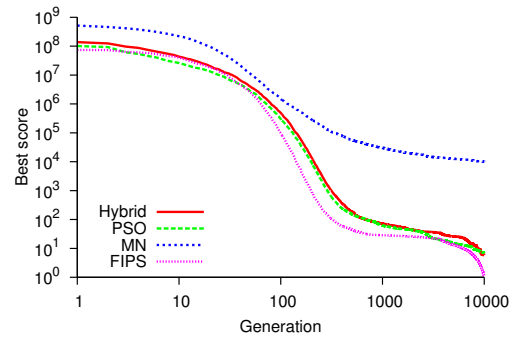
(a) Ackley Function.



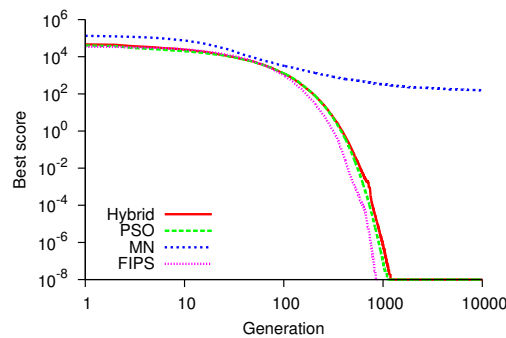
(b) Griewank Function.



(c) Rastrigin Function.

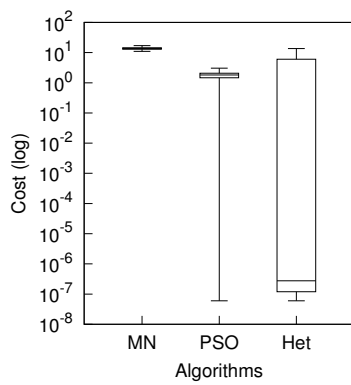


(d) Rosenbrock Function.

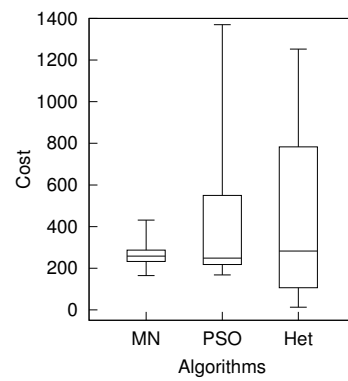


(e) Sphere Function.

Figure 2.3: Solution quality distribution extracted the 30 individuals from the last generation of all the 50 runs. The heterogeneous model presented the highest solution diversity among all as we hypothesized.



(a) Ackley Function.



(b) Ackley Function.

### 3 THE IMPACT OF CENTRALITY IN COMPUTATIONAL SOCIAL PROBLEM-SOLVING

The concept of centrality has a long history within Graph Theory, Social Networks, and, more recently, in Network Science. Earlier researchers recognized that the structure which people formed to collaborate could be beneficial to individuals at one level and detrimental to the collective at the other level (BARONCHELLI et al., 2013; STADLER; RAJWANI; KARABA, 2014). One question thus is posed: “How can one determine optimal communication structures in collaborative networks that make use of social problem-solving?”. One way to understand communication structure is through the concept of centrality (BAVELAS; BARRETT, 1951; BURGESS, 1969; FREEMAN, 1978; FREEMAN; ROEDER; MULHOLLAND, 1979; BORGATTI, 2005; BORGATTI; EVERETT, 2006).

One may then ask whether computational models of social problem-solving can give us a better understating of the impact of centrality in social problem-solving. More specifically:

1. Are central positions better to the individual in a social problem-solving system?
2. Do centralized networks lead to better collective outcomes?
3. Do results at both individual and collective levels in one model generalize to different models of social problem-solving?
4. Which measure of centrality best correlates with individual or collective good outcomes in Social Problem-solving systems?

The next section exposes the methods that respond to these questions.

#### 3.1 Methods

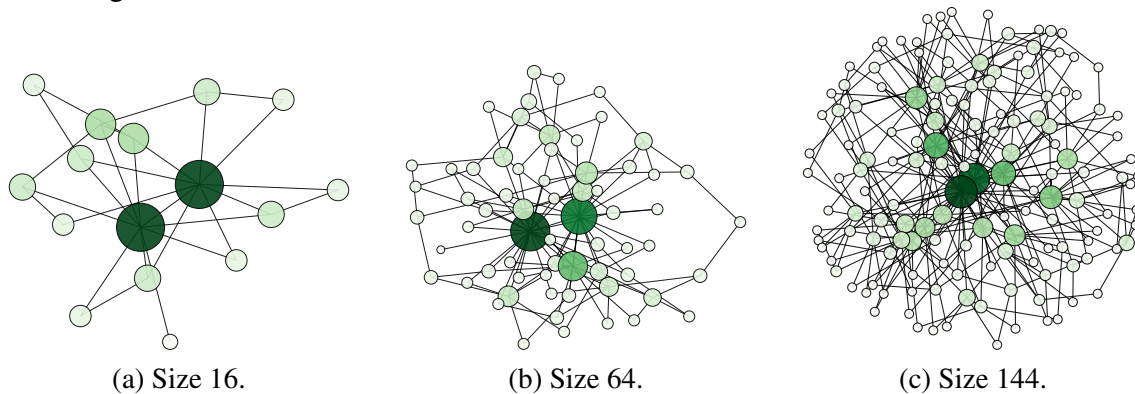
Many decisions have to be made when investigating centrality’s impact in social problem-solving. Those decisions include what networks would be used, what problems the agents would solve and how the agents themselves would be modeled. After such decisions, we gathered statistical correlations at the individual and the collective level. This section provides details about those decisions.

The first decision pertains to the choice of an adequate network topology. While regular or random networks are often used in sociological studies, we choose to use

topologies where centrality distribution follows what is observed in social settings. To generate such topologies, we employ the generative network process developed by Barabási and Albert (BARABÁSI; ALBERT, 1999). The process starts by adding one new node at each iteration and connecting this node to  $m$  other nodes.

The population sizes used in experiments were of 16, 64 and 144 individuals (nodes). These values represent a small, an intermediate, and a big population size when one considers previous, standard experimental works from the literature.

Figure 3.1: Networks generated by the Barabási-Albert model where new nodes connect to  $m = 2$  following a preferential attachment mechanism of connection. The network sizes were 16 a, 64 b, and 144 c. The node size in these pictures is directly proportional to its degree.



The next decision was the selection of the problem to be solved. We decided to frame problem-solving as a minimization problem of real-valued functions. We have chosen five functions: the Ackley function (here referenced as F1), the Griewank function (F2), the Rastrigin function (F3), the Rosenbrock function (F4) and the Sphere function (F5). By using a diversity of functions with different properties, regarding modality and separability, we aimed at capturing features of distinct real-world problems. Interested readers can refer to (LIANG et al., 2006) for more detailed information about these functions. All functions were set to have 30 dimensions - i.e. 30 real-valued independent variables.

Later, we elaborate our measure of individual's performance (success of the agent). We consider two kinds of individual performance, one global and the other local. Global performance of a node is defined by the number of times it has found a solution which was better than the previous best solution held by the whole system (i.e. all nodes) — we call this the *individual global contribution*. The local performance is defined as the number of times that a node held a solution which was better than those held by any of its neighbors — we call this the *individual local contribution*.

---

**Algorithm 6:** Algorithm structure used by all models.

---

```

Input :  $Population_{size}, stop\_condition, Problem_{size}, m$ 
Output:  $Solution_{best}$ 

// Initialize the population with random solutions.
Population  $\leftarrow$  InitializePopulation ( $Population_{size}, Problem_{size}$ )

// Establish social ties among individuals.
BarabasiAlbert (Population,  $m$ )

repeat
  | foreach node  $\in$  Population do
  |   | UpdateState (node,  $Problem_{size}$ )
  |   | EvaluateState (node)
until stop_condition

Solutionbest  $\leftarrow$  RetrieveBestSolution (Population)
return Solutionbest

```

---

Four agent-based models were used in the experiments: (i) Particle Swarm Algorithm (PSO), (ii) the Fully Informed Particle Swarm (FIPS), (iii) an instance of Memetic Networks (MN) model, and (iv) the Dynamic Search Range (DSR). All models share a similar structure as depicted in Algorithm 6. All these models satisfy the social problem-solving property we highlighted earlier: agents work better when they collaborate, despite being capable of solving the problem by themselves. They start by initializing the population with random solutions, yielding a uniform distribution of solutions within the search space. After initializing the individuals' solutions, we connect them according to the Barabási-Albert model, using different random seeds for each independent trial. We also used a bi-dimensional periodic grid as control network – i.e. all agents have the same degree and have the same average distance from all nodes. Both network topologies were static.

With the individuals in place and the network ties formed, the models start their iterative stage, where each agent evaluates and updates its current solution according to her own search operations. After all individuals have updated and evaluated their state, the algorithm proceeds to the next round until the stop condition is found. We used 5,000 rounds as the stop criterion and four decimal digits of precision.

In the PSO model, particles hold a velocity vector which they update at each iteration. This vector is used to update the particle's current position. This is calculated by summing over two components: the cognitive component, which points to the own particle's best previous position; and the social component, which points to the particle's



---

**Algorithm 7:** UpdateState procedure of a particle. Operations  $\{+, \times, -\}$  are in the matrix notation.

---

```

Input:  $\text{node}, \text{Problem}_{size}$ 
// Update particle's velocity.
 $\text{neighbor} \leftarrow \text{FindBestNeighbor}(\text{node}_{neighbors})$ 

 $\mathbf{V1} \leftarrow \text{RandomVector}(\text{min} = 0.0, \text{max} = 2.05, \text{size} = \text{Problem}_{size})$ 
 $\mathbf{V2} \leftarrow \text{RandomVector}(\text{min} = 0.0, \text{max} = 2.05, \text{size} = \text{Problem}_{size})$ 

 $\text{Cognitive} \leftarrow \text{node}_{best\_previous\_position} - \text{node}_{current\_position}$ 
 $\text{Social} \leftarrow \text{neighbor}_{best\_previous\_position} - \text{node}_{current\_position}$ 

 $\text{node}_{velocity} \leftarrow 0.7298 * (\text{node}_{velocity} + (\text{Cognitive} \times \mathbf{V1}) + (\text{Social} \times \mathbf{V2}))$ 

CheckVelocityLimits( $\text{node}_{velocity}$ )

// Update particle's position.
 $\text{node}_{solution} \leftarrow \text{node}_{solution} + \text{node}_{velocity}$ 
CheckSolutionLimits( $\text{node}_{solution}$ )

```

---

best ranked neighbor's previous solution. After adding some perturbation to these components, the particle sums its velocity vector to its current solution. If this new solution is better evaluated than the previous best solution (used by the cognitive component), the particle updates its previous best solution.

Our PSO implementation follows Clerc and Kennedy's work (CLERC; KENNEDY, 2002). This implementation is considered the canonical form of the PSO model (POLI; KENNEDY; BLACKWELL, 2007). Velocities are limited to the maximum absolute range of each variable in each dimension (EBERHART; SHI, 2000). Any particle's solution component that leaves the boundary of the problem's space is set to the maximum valid value and its respective velocity component is reflected back by multiplying it by  $-1.0$ . The Algorithm 7 depicts the PSO algorithm we used. Note that most operations in pseudocode are in the matrix notation to simplify and clarify understanding.

The first step is to find the particle's best ranked neighbor. The second is to generate two random vectors that will be multiplied by the cognitive and the social components. The cognitive component is given by the difference between the particle's previous best position visited and its current position, whereas, the social component is calculated from the difference from the position of the best ranked neighbor and the particle's current position. Both components are multiplied by the random vectors and summed. The resulting value is summed with the previous particle velocity, then it is multiplied by a constant

$c = 0.7298^1$ . Afterwards, the model performs a double boundary check during each iteration: one on the velocity and other on the position. Finally, particles update their current solution with the new one and, if it is the case, update their best previous solution.

Among the PSO extensions, there is the Fully-Informed Particle Swarm (FIPS) by (KENNEDY; MENDES, 2006). The main difference is that FIPS's particles use the information from all of its neighbor particles to update their position, averaging over all possibilities instead of relying only on the best ranked neighbor as the canonical PSO. We can say that a FIPS particle receives more influence from its neighborhood than the other models.

Recall that the MN is a class of socially-inspired networked problem-solving models. It was developed to test the impact of social network topology on a system's performance (ARAÚJO; LAMB, 2008a). Memetic Networks are not exactly a model, but rather a meta-model with guidelines for the design of an agent-based problem-solving system. This model resembles the PSO model in the sense that both models make use of the social network concept to structure communication, but the exchange of information is made more explicit in MNs.

A MN makes use of the concept of memes (DAWKINS, 2006), in order to represent the basic units of information. Recall that its dynamics arise from three simple rules: *connection, aggregation and appropriation*. The Memetic Networks model used here follows the version from (NOBLE; LAMB; ARAÚJO, 2012) which in turn is inspired by Araújo and Lamb's version (ARAÚJO; LAMB, 2008a). The Algorithm 8 depicts how a memetic node updates its current state. Note that the appropriation rule does not depend on any exogenous information and that the better ranked set of nodes include the node itself.

Each node in a memenet performs aggregation by condensing a pool of better ranked nodes. With this pool filled, the memetic node composes its solution by retrieving randomly memes from it, one at a time, and conserving their respective positions. The node's own solution is added to the pool to guarantee that its solution is not lost. The appropriation rule applies a small perturbation in order to introduce novel memes. As memes flow through a social network, they are likely to change their original form and the appropriation step incorporates such behavior, similarly to a mutation operator in Genetic Algorithms. A system where individuals only aggregate information would converge to a combination of the initial memes.

---

<sup>1</sup>This constant is originally part of the model

---

**Algorithm 8:** UpdateState procedure of memetic node. Note that the appropriation rule is purely endogenous and the better ranked set of nodes include the node itself.

---

```

Input: node, Problemsize, Upperlimit, Lowerlimit

// Aggregation rule.
Better_Neighbors ← RetrieveBetter(nodeNeighbors) ∪ node
for i ← 1 to Problemsize do
    Chosen_Neighbor ← RandomChoose(Better_Neighbors)
    nodeSolution[i] ← Chosen_NeighborSolution[i]

// Appropriation rule.
for i ← 1 to Problemsize do
    With  $1/Problem_{size}$  probability
        nodeSolution[i] ← Random(min = Lowerlimit, max = Upperlimit)

```

---

The Dynamic Search Range (DSR) model is an extension of the original Memetic Networks model that includes a fundamental human trait: the individual ability to adapt to the social context. The authors suggest the idea that individuals could be modeled as self-adaptive regarding their social problem-solving strategies (NOBLE; LAMB; ARAUJO, 2012). The original idea stems from prior works that relate humans to motivated tacticians (FISKE; TAYLOR, 1991; SCHWARZ, 1998). In the context of social problem-solving, it represents an individual mechanism that balances exploration and exploitation during the search process.

To perform this balance, the individual relies on network's feedback information. If a majority of its neighbors hold better solutions, then this focal node expands its search range during the appropriation step. Hence, it can explore more of the search space in hope of increasing the likelihood of finding a better solution than the status quo. Otherwise, the node may gain little from intensifying the search around its relatively poor solution. The DSR model basically allows nodes to count how many neighbors are better than themselves and, with this information, adjust their search range during the appropriation step. Thus, the appropriation rule is not purely endogenous and is called R\* appropriation (NOBLE; LAMB; ARAUJO, 2012). The aggregation rule used here is the same used in the canonical MN.

The R\* appropriation embodies the idea of social adaptation through a mechanism that controls the search range of a individual. This mechanism receives as input a ratio called Social Relative Performance (SRP) before performing any operation on the memes. The SRP is the ratio between the number of better ranked neighbors and the total number

---

**Algorithm 9:** UpdateState procedure of memetic node employing a R\* appropriation (DSR). SRP stands for Social Relative Performance.

---

```

Input:  $node, Problem_{size}, Upper_{limit}, Lower_{limit}$ 

// Aggregation rule.
Better_Neighbors  $\leftarrow$  RetrieveBetter( $node_{Neighbors}$ )

for  $i \leftarrow 1$  to  $Problem_{size}$  do
    Chosen_Neighbor  $\leftarrow$  RandomChoose(Better_Neighbors)
     $node_{Solution}[i] \leftarrow$  Chosen_Neighbor $_{Solution}[i]$ 

// R* Appropriation rule.
SRP  $\leftarrow$  Count(Better_Neighbors)/Count( $node_{Neighbors}$ )
 $\Delta \leftarrow (|Upper_{limit}| + |Lower_{limit}|) / 2 \times (SRP) + \alpha$ 

for  $i \leftarrow 1$  to  $Problem_{size}$  do
    With  $1/Problem_{size}$  probability
         $node_{Solution}[i] \leftarrow$  Random( $min = node_{Solution}[i] - \Delta, max =$ 
             $node_{Solution}[i] + \Delta$ )

```

---

of neighbors. An SRP of 1.0 means that all neighbors of the node possess better solutions while a SRP of 0.0 means that no neighbor has a better solution. This value is multiplied by half the maximum interval for each meme. The resulting value is the search range for the node, being this interval centralized at the current meme so that the new solution is within this range. The  $\alpha$  constant represents the minimum interval that a node with SRP of 0.0 is to search and for our experiments we used  $\alpha$  as 0.0001 for all problems.

The more neighbors a node have, the more information it has to adjust its search range. That is because the SRP can assume a larger set of values. This observation has some parallel with real social systems in the sense that well connected nodes can make more informed decisions if they pay attention to how their peers are faring.

Our experiments were composed by 1,000 independent trials for each configuration (model, problem, population size, network topology), calculating individuals' contributions (global and local) and centrality values (degree, betweenness, closeness, eigenvector), normalized to the range [0, 1] as proposed by Freeman (1979) to allow comparison of different networks sizes.

Pearson's correlation was used to measure the degree of linear dependency between each contribution (global and local) and each centrality measure (degree, eigenvector, betweenness and closeness).

A correlation close to one indicates a strong relation between the individual centrality and its contribution to their peers or to the whole system. On the other hand, a

correlation value close to minus one shows a strong inverse correlation (the higher is the contribution the more peripheral is the position of the individual in the network). Values near zero for correlation denote a nonexistent or weak correlation between the metrics.

### 3.2 Results

Experimental results are summarized in Table 3.1 and Table 3.2. The former regards global contribution, and the latter, local contribution. They show the averaged correlation grouped by the model and then by the problem, for each centrality measure. Each cell condenses three thousand experimental trials, being a thousand for each size of population.

Each pair composed by a model and a problem presented very distinct results. Nevertheless, we observed high positive correlations between local contribution and every centrality measure in the MN data. In this case, closeness was slightly better to estimate local contribution.

The DSR model showed high positive correlation between global contribution and every centrality measure, being the degree centrality slightly higher than the others. The PSO model presented a moderate positive correlation between both kinds of contribution and all measures. The FIPS model had a weak negative correlation with all the measures, meaning that a central node is likely to contribute less than a peripheral node.

In addition, while DSR and MN did not present big differences in correlation values between different problems, a central position seems more strongly correlated in FIPS and PSO in the Sphere (F5) and the Rosenbrock (F4) problems. Moreover, standard deviations for FIPS correlation values are very high (close to 0.3 on most cases) while all other models are relatively low (close to 0.1), showing a strong noise and randomness inherent to FIPS model regarding individual's network position.

Another factor that can be highlighted is that local and global contribution, as we defined, resulted in diverse correlation values with the centrality measures, especially for MN and DSR models, indicating that it plays an important role on the analysis.

The strong negative correlations shown by FIPS model (especially with F5 problem, in both contributions measures) was an unexpected result. We hypothesize this happens because central FIPS particles receive high amounts of influence, impairing any possible convergence behavior. More importantly, it points that there is a strong dependence between the problems and how each individual solves it. Such strong dependence

calls attention to hasty associations of centrality with individual success.

The results also call attention to the fact that very similar models, the MN and the DSR, presented distinct correlations for different kinds of contribution: centrality (all measures) is useful to predict the MN individual ratio of local contribution while in DSR it is useful to predict individual likelihood of global contribution. Central DSR nodes have more sources of information to balance their own exploration/exploitation rate, i.e. their problem-solving behavior, thus they had an advantage when in central positions. MN nodes being unable to change their exploration/exploitation balance, fared better locally only. It may be the case that when in central positions, thus in a high information flow position<sup>2</sup>, a solver must be flexible to adapt its behavior and careful to select what information must be useful to achieve high global contribution rates. However, further investigation is required to confirm this hypothesis.

We also tested whether centrality really accounted for differences in contribution rates. To do so, we measured the standard deviations of contributions of a control topology (Grid) and compared to the centralized networks generated using the Barabási-Albert model (NBA). Table 3.3 shows the comparison. A small standard deviation means that contribution is uniformly distributed across the population at the end of the solving process.

Each cell of Table 3.3 presents an average over three thousand trials (a thousand for each size of population), except the third and sixth valued columns, which represent how much (percentage) is the NBA's standard deviation value higher than that of the Grid.

The Grid presented lower standard deviation than the NBA topology whenever there was a high discrepancy between central nodes contribution and peripheral ones, and also, the correlation values between centrality measures and the individuals' contribution are more extreme (closer to -1.0 or 1.0).

The opposite is also true, when the disparity amid the networks standard deviation is low, correlation values between centrality measures and individuals' contribution are closer to zero — especially when NBA network standard deviation values are not more than 50% of the ones shown by the Grid network. It also shows that whenever there is a different behavior (being it an advantage or disadvantage) caused by individuals' positions, centrality measures are good to predict which individuals will contribute more.

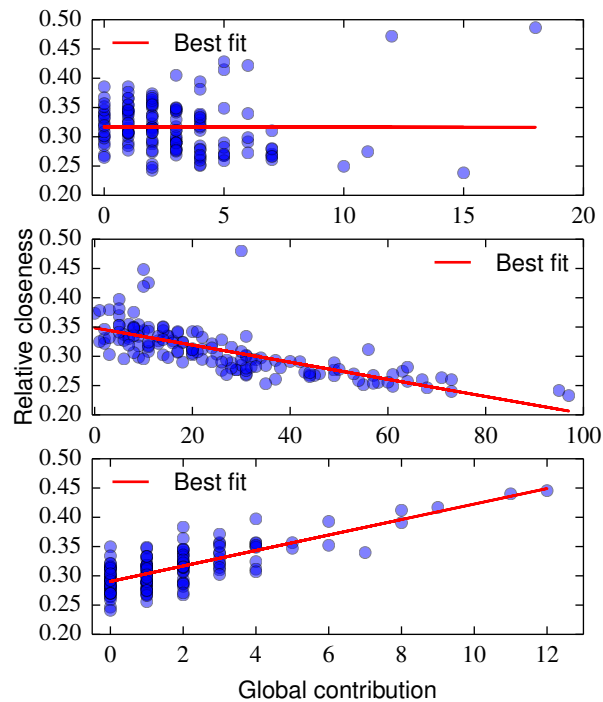
Figure 3.2 depicts three trials samples taken from our experiments of different correlation values. The figure reinforces that centrality can be negatively correlated, weakly

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<sup>2</sup>Here information is meant in the broad sense, it can be regarded as pieces of solution themselves as well as information about how neighbors solvers are faring.

correlated and also positively correlated with individual contribution. These trials were picked to show extreme results and the variations found in our experimental data regarding centrality prediction of success in social problem-solving.

Figure 3.2: Sample trials depicting a close to zero correlation (FIPS, F4, population size 144), a negative correlation (-0.72, FIPS, F1, population size 144) and a positive correlation (0.79, DSR, F5, population size 144) respectively top-down.



In summary, specific cases demonstrate the disparity among different models and tell us that individual's traits and problem's features are more relevant than the network flow itself. Furthermore, this is a first and important step to identify how and when centrality is relevant and can be successfully applied in social problem-solving contexts.

### 3.3 Discussion

This main objective of the above was to investigate how centrality correlates with individual performance in social problem-solving systems. In order to do so we performed a series of experiments with agent-based models. These experiments sought to compare these models and understand the relationship between different centrality measures and individual performance.

The overall results present some of the difficulties in generalizing a centrality measure for very distinct problems and models, reinforcing that each kind of individual be-

havior and problem being solved are important factors to determine which individual will take more advantage from its position in the network. Sometimes central individuals get a substantial advantage over the others, but that cannot be assumed true for all the cases (we show one case where peripheral positions are advantageous).

In addition, we have shown that there is not a big difference between distinct centrality measures, probably because larger variations caused by the models and problems to overcome the measures variations and/or the power law network generation method excludes topologies of networks that will present larger divergence among centrality measures.

When nodes are able to systematically exploit their network positions, we observed that there is a higher correlation between centrality and individual's contribution. In this case, we observed that some measures have significantly better correlations than others. This suggests that one should not rely on only one centrality measure when investigating individual's performance in social problem-solving systems. We also provided evidence that central nodes tend to "collaborate" more with their neighbors, but that did not necessarily translated into higher global contribution (with the exception of the DSR model).



Table 3.1: Pearson's Correlation Between Individual Global Contribution and Centrality.

Model	F	Centrality			
		Betweenness	Closeness	Degree	Eigenvector
MN	F1	0.38	0.40	0.40	0.40
	F2	0.37	0.39	0.39	0.39
	F3	0.45	0.47	0.48	0.48
	F4	0.33	0.35	0.35	0.36
	F5	0.38	0.39	0.40	0.40
	Mean	0.38	0.40	0.40	0.41
	Std.Dev.	0.0398	0.0392	0.0417	0.0391
DSR	F1	0.74	0.71	0.78	0.73
	F2	0.75	0.72	0.78	0.74
	F3	0.69	0.67	0.73	0.68
	F4	0.76	0.73	0.80	0.75
	F5	0.75	0.72	0.78	0.74
	Mean	0.74	0.71	0.78	0.73
	Std. Dev.	0.0240	0.0201	0.0219	0.0243
PSO	F1	0.45	0.47	0.48	0.48
	F2	0.44	0.46	0.47	0.47
	F3	0.38	0.39	0.40	0.40
	F4	0.62	0.65	0.66	0.67
	F5	0.73	0.77	0.78	0.78
	Mean	0.52	0.55	0.56	0.56
	Std. Dev.	0.1324	0.1404	0.1414	0.1431
FIPS	F1	0.43	0.22	0.36	0.33
	F2	0.25	0.03	0.17	0.15
	F3	-0.29	-0.46	-0.34	-0.39
	F4	-0.20	-0.46	-0.29	-0.33
	F5	-0.44	-0.72	-0.54	-0.58
	Mean	-0.05	-0.28	-0.13	-0.16
	Std. Dev.	0.3304	0.3459	0.3369	0.3457

Table 3.2: Pearson's Correlation Between Individual Local Contribution and Centrality.

Model	F	Centrality			
		Betweenness	Closeness	Degree	Eigenvector
MN	F1	0.72	0.82	0.79	0.81
	F2	0.72	0.82	0.79	0.81
	F3	0.76	0.85	0.83	0.83
	F4	0.68	0.79	0.75	0.77
	F5	0.72	0.83	0.79	0.81
	Mean	0.72	0.82	0.79	0.80
	Std. Dev.	0.0239	0.0194	0.0240	0.0203
DSR	F1	0.12	0.14	0.14	0.13
	F2	0.09	0.11	0.10	0.10
	F3	0.06	0.09	0.08	0.08
	F4	0.10	0.13	0.12	0.12
	F5	0.12	0.14	0.14	0.13
	Mean	0.10	0.12	0.12	0.11
	Std. Dev.	0.0230	0.0193	0.0223	0.0208
PSO	F1	0.23	0.23	0.24	0.23
	F2	0.22	0.22	0.23	0.23
	F3	0.24	0.27	0.25	0.27
	F4	0.62	0.65	0.66	0.67
	F5	0.74	0.78	0.80	0.80
	Mean	0.41	0.43	0.43	0.44
	Std. Dev.	0.2257	0.2367	0.2436	0.2428
FIPS	F1	0.02	-0.14	0.01	-0.09
	F2	-0.03	-0.19	-0.04	-0.13
	F3	-0.15	-0.32	-0.17	-0.25
	F4	-0.20	-0.36	-0.22	-0.31
	F5	-0.41	-0.70	-0.51	-0.57
	Mean	-0.15	-0.34	-0.19	-0.27
	Std. Dev.	0.1511	0.1971	0.1824	0.1675

Table 3.3: Averaged Standard Deviation of Individuals' Contributions.

Model	F	Global Contribution			Local Contribution		
		Grid	NBA	%	Grid	NBA	%
MN	F1	1.10	1.26	14.25	13.62	33.32	144.68
	F2	1.09	1.25	14.66	13.69	33.35	143.55
	F3	1.21	1.45	19.34	13.16	37.24	182.90
	F4	1.10	1.21	10.67	14.06	29.61	110.66
	F5	1.08	1.25	15.26	13.69	33.31	143.37
DSR	F1	2.20	3.99	81.82	23.05	25.24	9.50
	F2	2.23	4.07	82.51	24.89	26.84	7.84
	F3	2.12	3.49	64.55	21.04	24.08	14.45
	F4	2.42	4.52	86.43	26.61	28.25	6.16
	F5	2.25	4.11	82.54	23.62	25.85	9.46
PSO	F1	3.62	3.84	6.08	45.58	40.08	-12.05
	F2	3.90	4.15	6.54	40.54	36.29	-10.49
	F3	7.23	6.79	-5.98	82.10	58.55	-28.68
	F4	16.75	21.77	29.97	43.81	54.97	25.47
	F5	9.60	16.28	69.51	25.87	42.87	65.70
FIPS	F1	2.45	3.91	59.61	64.00	137.69	115.13
	F2	3.61	5.79	60.68	71.74	129.81	80.96
	F3	3.98	7.48	87.91	127.48	185.36	45.40
	F4	6.02	16.37	171.91	53.31	114.53	114.84
	F5	10.79	50.28	365.80	11.46	56.98	397.15

## 4 HETEROGENEOUS MODELS COMPUTATIONAL OF SOCIAL PROBLEM-SOLVING

Experiments with human social problem-solving showed that “results of both artificial simulations and artificial experiments should be generalized with caution” (MASON; WATTS, 2012). The main argument in favor of this assertion was the empirically verified richness and complexity of human problem-solving behavior that current artificial models of social problem-solving lacked. The existence of this knowledge gap may jeopardize results from long standing computational models.

When comparing humans collaboratively solving a problem in a networked experiment and a previous model of human problem-solving, it became clear that recent models such as LF model<sup>1</sup> at moment are “insufficiently sophisticated and heterogeneous to reflect real human responses to changing circumstances”.

Considering such observations, it became clear that there was an urgent need to study diversity in artificial social problem-solving systems. Such feature is present in real human groups and needed to be *integrated* into the current models despite the increased complexity brought about.

Following this line of thought, (NOBLE et al., 2015a) designed a study where the simple and homogeneous LF model was exchanged by a diverse and heterogeneous model. Their main objective was to understand the effects that such feature would bring to both the individual level, or micro, and the collective level, or macro.

The main questions tackled in (NOBLE et al., 2015a) work were:

1. What is the interplay among (1) collective diversity (heterogeneity), (2) dynamic strategic behavior, and (3) network efficiency (in the form of network centralization)
2. Which of these variables impact more in the collective outcome: diversity, network centralization, or dynamic strategies.
3. What happens when exploration and exploitation are mutually exclusive?

The methods and findings resulting from this study are described in the sequel.

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<sup>1</sup>The Lazer-Friedman model (LAZER; FRIEDMAN, 2007)

## 4.1 Methods

The heterogeneous model builds upon the one employed by Mason and Watts (MASON; WATTS, 2012), which in turn draws inspiration from (LAZER; FRIEDMAN, 2007). The variation in the Mason and Watts' model was derived mainly to support heterogeneity and dynamicity in the population, enabling different agents to have different search radii (heterogeneity) and also enabling them to change their radii during execution (dynamicity). The agents are tasked with finding the maximum output value from a bi-dimensional real valued function. Therefore, problem-solving was framed as a maximization problem.

The search space used in this study derives from previous works in early works in computer graphics whose focus was to generate procedurally noise as a modeling technique of realistic landscapes (PERLIN, 1985). This idea can be adapted to generate a combination noise plus signal where the problem is therefore finding the coordinates of the signal among all the noise. The bigger the signal, the easier it gets to solve it since the signal becomes unambiguous. Framing problem-solving through the use of a procedural noise generation function plus adding a signal to noise was first proposed by Mason and Watts (MASON; WATTS, 2012) and is replicated here. The signal in this case, was modeled as a single bivariate Gaussian as well.

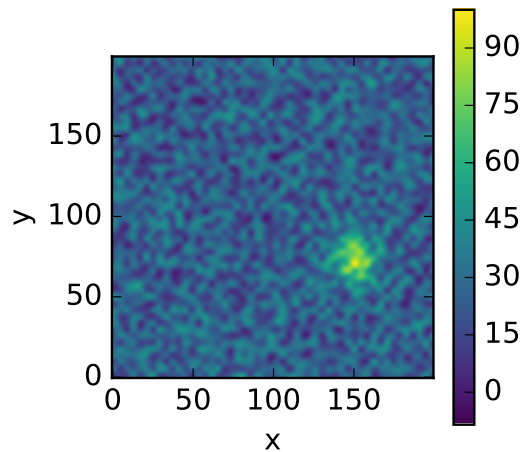
The size of the search spaces was fixed at  $200 \times 200$  integer points - i.e. each solution was a pair  $(x, y)$  where  $x$  and  $y$  are integers within the range  $[0, 200)$ . Each combination of  $(x, y)$  represented a solution which has a respective score and the problem consists basically of finding the pair that maximizes the function output, ie. the signal. The signal has score of a 100.0. Successful trials are the ones in which any agent achieve a score equal to or higher than 95.0. Moreover, trials where any agent's initial score was higher than or equal to 55.0 were discarded in order to avoid the case where an agent is positioned too close to the signal.

As in (MASON; WATTS, 2012), there were 3 parameters that need to be set to generate the search space. Two parameters ( $\omega$  and  $\varpi$ ) described how "rugged" the resulting search space will be and one ( $\sigma$ ) described how outspread the signal will be. The signal coordinates are generated randomly for each new instance generated. The parameters to generate problem instances are:  $\omega = [2, 3, 4, 5, 6]$ ,  $\sigma = 9$ , and persistence parameter  $\varpi = 0.7$ . Figure 4.1 depicts a sample search space generated using these parameters.

The size of the search spaces is fixed at  $200 \times 200$  integer points - i.e. each solution

is a pair  $(x, y)$  where  $x$  and  $y$  are integers within the range  $[0, 200)$ . Each combination of  $(x, y)$  represents a solution which has a respective score and the problem consists basically of finding the pair that maximizes the function output. For each independent trial, a new random instance was generated. A total of a 1000 independent trials were simulated. Moreover, our search space was larger than the search space employed by Mason & Watts that is of  $128 \times 128$ . The peak (best solution) has score of a 100.0. Successful trials are the ones in which any agent achieve a score equal to or higher than 95.0. Moreover, we discarded trials where any agent's initial score score was higher than or equal to 55.0 in order to avoid the case where an agent is positioned too close to the peak (best solution).

Figure 4.1: Sample problem instance: The peak is represented by the light green region while the noise is represented in darker green.



The precise meaning of the terms mentioned here and the details of the function solved by the agents are detailed below.

**Search Radius:** The search radius is an agent parameter responsible for establishing the boundaries of that agent's search space. Each agent's search space is bounded by a circle whose center is determined by the coordinates of that agent's current best solution and whose radius is that particular agent's search radius. It is only within the boundaries of this circle that the agent can search for new solutions in the *exploration* phase.

**Myopic Search:** A myopic search is a task performed by agents in the Lazer-Friedman (LF) model. It consists of selecting a point in the parameter space respecting the boundaries determined by the search radius.

**Lazer-Friedman Model:** (LAZER; FRIEDMAN, 2007) is a model of social problem-solving in which agents are connected according to a particular network topology. The problem is solved after a number of iterations. At each iteration, every agent updates its own state; if the agent in question (focal agent) holds the best solution it performs *myopic*

*search*; otherwise it copies the solution from the best neighbor agent.

**Heterogeneity (diversity):** An agent population is *heterogeneous* when agents have different search radii. A population is *homogeneous* when all agents have the same search radii.

**Dynamicity:** When agents are able to change their search radius when solving a problem, they are dynamic. In a static population all agents are endowed with predetermined search radii throughout the entire problem-solving task.

In a myopic search, agents select a point in the parameter space respecting the boundaries determined by their search radii. As our results will show, this approach can be problematic: the fact that some agents have bigger circles than others puts the second group in an unfair disadvantage towards the first one, as its agents have their search space reduced in comparison. The problem with this disadvantage is that it generates a bias toward homogeneous populations of high-ability agents and against populations of diverse ones. It is always possible for a homogeneous population to outperform a heterogeneous one - the only necessary condition is that the global radius of the homogeneous population is greater than every single radii of the heterogeneous one.

This conclusion is logical, but is nevertheless inconsistent with the generally accepted idea that, at least to some extent, *diversity trumps ability*. It is particularly inconsistent with the findings of (HONG; PAGE, 2004) who show that high ability agents are often outperformed by diverse ones. Naturally, the inconsistency is not due to the fact that diversity actually does not trump ability - it does - but rather to the fact that using search radii to determine circular search spaces creates a bias against heterogeneous populations. This happens because circular search spaces fail to include the exploration-exploitation dichotomy in the problem-solving task, as agents with larger circles are just as capable of exploiting their current solutions as agents with smaller circles, but additionally capable of exploring new solutions in a larger area. In short, although we can create heterogeneous populations by diversifying their radii, we cannot create a heterogeneous population in which agents have varied inclinations towards exploration and exploitation - we can only diversify their search.

A strategy to solve this problem is to restrict the search space of exploration-inclined agents to outside their close neighborhood, making it impossible for them to exploit their current solutions. Exploitation-inclined agents should also have their search space restricted, but this time to include *only* their close neighborhood. This strategy can be implemented by replacing the circles in the myopic search by rings. Thus, when

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**Algorithm 10:** General algorithm structure used by all models.

---

```

Input :  $Population_{size}, stop\_condition, Problem_{size}$ 
Output:  $Solution_{best}$ 

// Initialize the population with random solutions.
1 Population  $\leftarrow$   $InitPopulation(Population_{size}, Problem_{size})$ 

// Establish social ties among individuals.
2 GenerateNetwork (Population)

3 repeat
4   foreach  $node \in Population$  do
5     // Execute the model search procedures.
6      $NextState(node, Problem_{size})$ 
7   foreach  $node \in Population$  do
8     // Decide to accept or not the new state.
9      $UpdateState(node, Problem_{size})$ 
until  $stop\_condition$ 
10  $Solution_{best} \leftarrow RetrieveBestSolution(Population)$ 
11 return  $Solution_{best}$ 

```

---

we enlarge a particular agent’s search radius, we are not only enlarging the ring’s outer circle but the ring’s inner circle as well. This forces this agent to explore in a region that becomes increasingly distant of the agent’s current solution as its search radius grows. By manipulating an agent’s search radius, we can place this agent wherever we want in the exploration-exploitation continuum. We refer to our model as the *Ring Model*.

In our experiments, we tested the Mason & Watts instance of the LF model (MASON; WATTS, 2012) as well as our proposed *Ring Model* and its variations. Although these variations implement different logical rules to solve a problem, their algorithmic structure is the same - as described in Algorithm 10.

Each agent is instantiated with a valid solution (line 1) then it is embedded in a social network (line 2). Each social tie establishes that an agent will perform bidirectional communication with another agent to whom it is connected. After these two stages, each agent performs the search. The agent samples a new solution according to the procedure *NextState* (line 5). This procedure is where each model implements its own search rules (i.e. if the sampling will be done within a full circle or within a ring shaped area). If the new solution improves the current one, then the agent accepts it, otherwise it will hold the old solution (line 7). An iteration is complete after all agents have their search turn. The process is repeated until there is no more iterations to be done. We limited the number of iterations to one hundred.

Algorithm 11 details how each individual executes the *NextState* procedure in our



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**Algorithm 11:** Standard Ring Model procedure.
 

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```

1 if  $\text{node}_{\text{solution}} < \max(\text{Neighbors}'\text{solutions})$  then
  // Local search procedure (or myopic search).
2   repeat
3      $r \leftarrow \text{Random}(low = \max(\text{node}_{\text{radius}} - 1, 0), high = \text{radius})$ 
4      $angle \leftarrow \text{Random}(low = 0, high = 2 * \pi)$ 
5      $x \leftarrow \text{RandomInt}(r * \cos(angle) + \text{node}_x)$ 
6      $y \leftarrow \text{RandomInt}(r * \sin(angle) + \text{node}_y)$ 
    until Until a valid solution is found
  else
    // Copy procedure.
7    $(x,y) \leftarrow \text{RetrieveBestSolution}(\text{node}_{\text{Neighbors}})$ 
8 node}_{\text{nextSolution}} \leftarrow (x,y)

```

---

### Ring Model.

The first step is to detect if the solver (agent) holds the best solution among its neighbors (line 1). If this is the case, then the agent performs the myopic search procedure, that is randomly selecting the coordinates of the next solution to be sampled. The coordinates in lines 5 and 6 are bound by a ring shaped area with a constant radius and constant thickness (lines 2 to 6). If the agent is not among the best in its neighborhood, the agent copies both coordinates from her neighbor (line 7). A solution is valid if it is within the search space coordinates.

The Ring Model can be divided into four types: **Static Homogeneous** — All agents have the same fixed radius throughout the search process. **Static Heterogeneous** — Though fixed, there may be diverse radii within the same population. **Dynamic Social** — Agents copy not only their neighbor’s solutions, but their radius at each iteration. **Dynamic Random** — At each iteration, the agent chooses a random radius from a limited pool of choices.

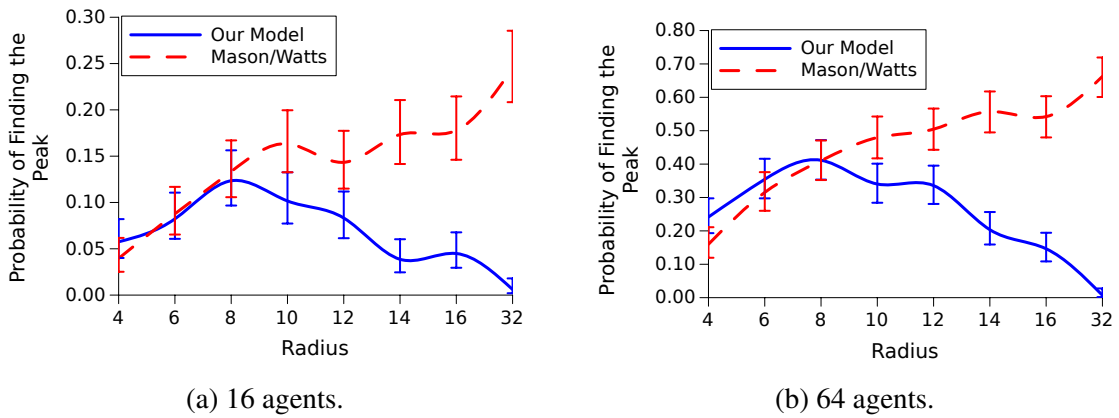
## 4.2 Dissociating Exploitation from Exploration

One of our aims is to compare the effects of exploration and exploitation in agent behaviors in social problem-solving systems. We also investigate when such effects are mutually exclusive. This means that we consider the effects of using current information to solve a problem, while keeping or not the chance of finding rather novel solutions.

In this experiment, we compare the LF model instantiated by (MASON; WATTS, 2012), with our proposed Ring Model. The probability of finding the peak for both mod-

els is shown in Figures 4.2a and 4.2b. The main parameters used are the radius and the population size. We considered the first 32 rounds for this experiment and arranged individuals in a bi-dimensional periodic grid network where all agents have the same degree and are at the same average distance from any other node. By controlling the network, we remove any effect that a network position may have on the agent behavior (KEARNS; SURI; MONTFORT, 2006; JUDD; KEARNS; VOROBAYCHIK, 2010). Vertical bars represent 99% confidence intervals using the Wilson score interval method.

Figure 4.2: Probability of finding the peak for various radius sizes in our Ring Model and in the Mason & Watts model.



Both models presented different behaviors when individuals have bigger radii. The optimal behavior, w.r.t. radius size, is around 8 (4% of the grid's size) in the Ring Model, but in the Mason & Watts model, the bigger the radius the better. Thus, even if these models do not significantly differ for radius equal to or less than 8, for bigger radii they clearly depart. This trend is valid for both population sizes. Therefore, when one has a homogeneous agents' population that have traits associated with exploratory behavior, one must pay attention to which kind of exploratory behavior the agents employ. If exploration and exploitation are packed together (as in the Mason & Watts' models), the highest the exploratory bias, the better. However, if this is not the case, as it is in the Ring Model, that exploratory and exploitative behaviors co-exist separately; one must fine tune the agent group to the optimal strategy. We were able to reproduce the effect we described before in the Mason & Watts's version of the LF model: as the individual radius increased, so did the collective performance.

### 4.3 Diversity, Strategy and Network Efficiency

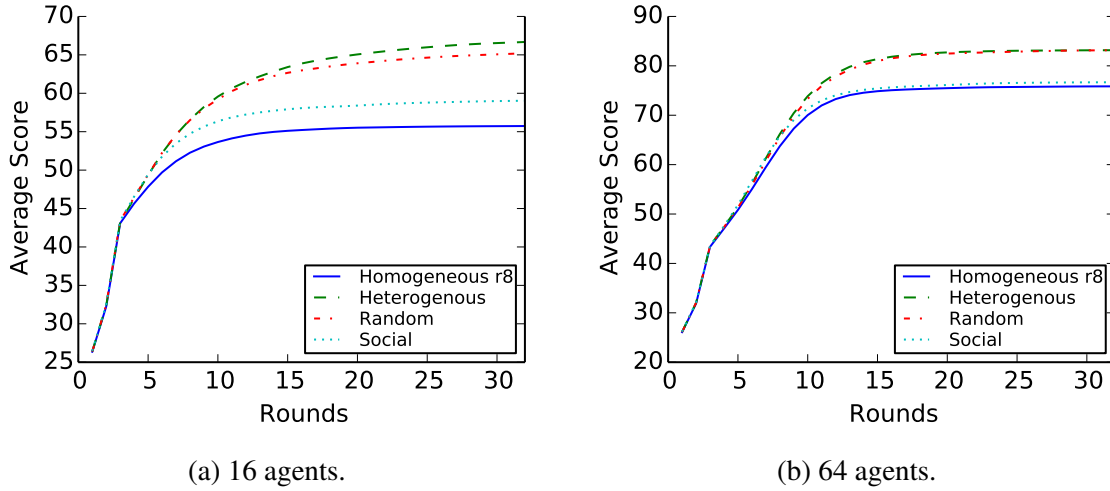
Until now, we experimented with the Ring Model and a homogeneous population where every agent held the same radius during the entire solving process. From now on, we will incorporate diversity into the Ring Model, with both static and dynamic diversity. Static diversity is characterized by agents starting with different radii and holding such radii until the end of the simulation, while in dynamic diversity they start with different radii, but can change their own radius during the simulation. Recall that the Ring Model with static diversity is known as the Static Heterogeneous Model. In this model, agents start with a radius of 4, 8, 16, 24, or 32 units. Each radius has the same chance of being chosen (the distribution is uniform).

Dynamic diversity invites an interesting question: *how can strategies be changed on an individual basis?* To answer this question we designed two versions of the Ring Model with dynamic diversity. The first is called the *Social Model*, where agents copy not only the solution of their best neighbor, but also their radius at each iteration; the second one is the *Random Model*, where agents always change their radius randomly after performing search procedures. In both models, agents start with a random radius. In the random model, the radii to be chosen are from the same radii pool used in the Static Heterogeneous model. Copy is considered fundamental in social learning systems (RENDELL et al., 2010).

Figure 4.3 depicts the average score of the population over all trials. We consider 4 models: (1) the homogeneous Ring Model with radius 8, (2) the Static Heterogeneous Model, (3) the Random Model, and (4) the Social Model. The average score includes all trials, whether or not the trial is successful. We isolated the effect of the individual position in the network by using a bi-dimensional periodic grid topology.

For both population sizes, we observed that the Random Model and the Heterogeneous Model presented the same average behavior. The difference between the Heterogeneous Model and the Random Model is not significant for population sizes of 16 and 64 ( $p=0.2$  and  $p=0.9$ , respectively). Thus, random dynamic diversity does not statistically differ from static diversity in this context. We can also see that the Social Model never outperformed the Static Heterogeneous, nor the Random Model; it is not significantly better than the Static Homogeneous model for a bigger population ( $p=0.57$ ). Even if agents are dynamic, and diverse, the Social Model is not better than the Static Homogeneous Model. When we remove the influence of network topology our results suggest that di-

Figure 4.3: Average scores over time for different population sizes and models.



versity could be more relevant than individual strategy dynamics which supports results of (JUDD; KEARNS; VOROBAYCHIK, 2010).

We then test how network efficiency impacts each model. We define *network efficiency* as the speed that solutions flow through the network, which is related to the average shortest path between any two network nodes (MASON; WATTS, 2012). An efficient network has small average path lengths while an inefficient has large ones; i.e. an efficient network can be seen as a decentralized network where a good solution spreads faster on average when compared to a centralized network where the average distance between peripheral and central agents tend to be higher. The inefficient (centralized) network was built using the Barabási's generative model (1999). This model produces a network topology with a power-law distribution that guarantees a centralization of the network as a minority of agents will hold most connections. The periodic grid topology was used as the efficient (decentralized) network.

Table 4.1 presents data from the experiments for all models in efficient and inefficient networks. Values between parenthesis represent 99% confidence interval using the Wilson score. We used the population size 64. We observe that a small population of 16 agents would present a less clear difference between an efficient and an inefficient network. We measured these probabilities after the 32nd round, therefore if any agent found the best solution (peak), we considered it a successful trial.

The difference among the above models was much stronger and clear than the difference regarding network efficiency. With the exception of the Social Model, all models performed slightly better in efficient (decentralized) networks when compared to inefficient (centralized) networks. However, the likelihood of finding the best solution

Table 4.1: Probabilities of finding the peak for efficient (decentralized) and inefficient (centralized) networks.

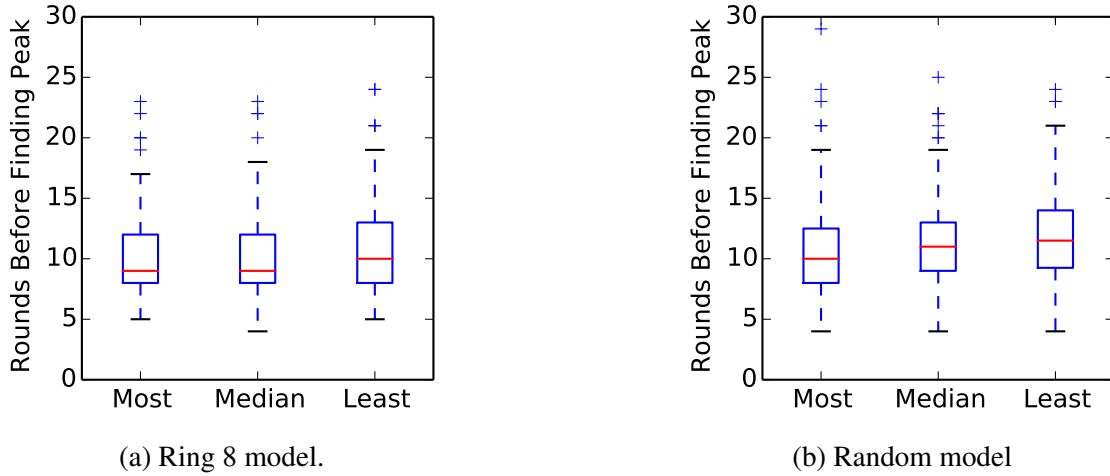
Model	Network	
	Efficient	Inefficient
Homogeneous radius 8	0.41 ( $\pm$ 0.06)	0.33 ( $\pm$ 0.06)
Homogeneous radius 16	0.15 ( $\pm$ 0.04)	0.13 ( $\pm$ 0.04)
Homogeneous radius 32	0.01 ( $\pm$ 0.01)	0.03 ( $\pm$ 0.02)
Static Heterogeneous	0.62 ( $\pm$ 0.06)	0.59 ( $\pm$ 0.06)
Dynamic Random	0.62 ( $\pm$ 0.06)	0.54 ( $\pm$ 0.06)
Dynamic Social	0.23 ( $\pm$ 0.05)	0.27 ( $\pm$ 0.05)

varied considerably among the models. The Dynamic Random and the Static Heterogeneous models presented the highest odds of finding the solution. The small difference between these models is evidence that static diversity is as effective as dynamic diversity. Among the Static Homogeneous versions, the Ring Model with radius 8 was the best ranked model in both networks. In this case, the difference between network efficiency accounted for approximately 20%, while between the Ring Model with radius 16, the difference was approximately 64%. The model with radius 32 presented the worst results for both networks. The Social Model was the only one that took advantage of an inefficient network; its performance increased by approximately 17% when compared to the efficient network. These results provide evidence - by means of artificial agents simulations - that individual behavior may have a stronger influence in the collective outcome than network topologies themselves. This evidence, as previous researchers have shown, reinforce that “the intrinsic behavioral traits of human actors may play a stronger role in their relative influence than purely structural properties of their network position” (JUDD; KEARNS; VOROBAYCHIK, 2010).

After that, we tested if the most central agent would reach the best solution earlier than the median agent or the least central one. In order to do so, we retrieved the round that each of the above agents found the solution, from all independent trials. One could expect central agents to find the best solution before the peripheral ones. Results are depicted in Figure 4.4. We used betweenness (BORGATTI; EVERETT, 2006) as a centrality metric and used the Barabási-Albert (BARABÁSI; ALBERT, 1999) generative model to generate the networks. Results are depicted in Figure 4.4.

For this problem, an agent’s individual position in a network was irrelevant. On average, the least central agent, the median and the most central agent achieved the best solutions at the same round as seen in Figure 4.4.

Figure 4.4: Rounds before finding the peak (solution) for the most, median, and least central node in: (a) Static Homogeneous with radius 8, and the (b) Dynamic Random model.



#### 4.4 Discussion

In homogeneous populations, all agents are restricted to a fixed search radius. As a result, all of them see the search space through the lens of exactly the same ring. For example, a possible setup for an homogeneous population consists of all agents having a search ring delimited by an outer circle of radius 5 and an inner circle of radius 1. We could also envision a setup in which all agents have an outer radius of 10 and an inner radius of 8.7. Although the rings used by agents in both examples would have approximately the same area ( $24\pi$  and  $24.3\pi$ ), they would render two completely different strategies. The first setup would render the population highly exploitation-oriented, as each agent would be able to see its surroundings in great detail, while not being granted access to further areas which would usually be searched by exploration-oriented agents. In contrast, the second setup above (radius 10 and inner radius of 8.7) would render the population highly exploration-oriented, being unable to exploit their current solution and thus would be forced to search for other, far away solutions.

The search space we used in our experiments was a bi-dimensional Gaussian function with additional random noise. Let us consider an one-dimensional analogue of this search space. For simplicity, let us also assume we have a sinusoidal wave of fixed frequency instead of random noise. This search space has a global maximum near  $x = 0$ , but several well-distributed local maxima. As a result, agents that rely solely on exploitation will have a hard time finding the global maximum, often getting stuck at local ones. As each agent's ring is limited (agents cannot see through the ring's cavity), agents will

sometimes be unable to grasp the general behavior of the curve in a given point, and would refuse to accept a solution that would bring them closer to the global maximum, even though they can see it with their annular vision.

In a homogeneous population, the situation described above is bound to happen very often. As all agents have the same search radius, the population as a whole will not be able to overcome the pitfalls of their particular search ring, with its agents finding themselves stuck on local maxima a lot more often than they should. Conversely, in a heterogeneous population, the diversity of shapes agents' rings can take assures that no pitfall is more common than any other. As a result, the probability that at least one search ring will meet the necessary criteria to get its agent climbing the search space without getting stuck in any point is maximized. Therefore the social dynamics of the agent model can then spread this agent's solution throughout the entire population.

## 5 CONCLUSIONS

There is a plethora of arguments both in favor and against diversity in social collaboration and social problem-solving. In a recent meta-analysis (EAGLY, 2016), it was observed that the enhancement in performance brought about by a diverse group of human subjects can have limited or, at least, negligible effect. In addition, current empirical results are mixed. It is advised, therefore, that future researchers “conduct research to identify the conditions under which the effects of diversity are positive or negative” (see *Loc. cit.*). Even when dealing with conflicting agendas, there is a call for an improved understanding of the effects of diversity before the implementation of policies that favor such a predicate. Despite years of research by organizational scientists, psychologists, sociologists, and economists, open questions regarding diversity are abound.

The main topic throughout this thesis was the investigation of how different aspects of diversity impacted social problem-solving. Under this topic, we contributed with the following results and analyses:

**The feasibility of a general framework for computational social problem-solving:** We developed a meta-model which allowed to blend seamlessly two different social problem-solving models — Memetic Networks and PSO — into the same population to collaboratively solve the same problem. In most problem cases tested, the heterogeneous population performed as good as the best model, sometimes even outperforming it and converging to the best solution.

**The impact of centrality in computational social problem-solving:** Can the individual network position in a social problem-solving system be decisive in the solver’s performance? We answered this question with a positive answer and therefore concluded that the individual network position does indeed represents a form of heterogeneity. However, results were not clearcut, different models had different relationships with different centrality measures. Generalizing results in this setting seemed to be more challenging than expected.

**Heterogeneous models of computational problem-solving:** Under certain conditions — especially regarding how exploitation of the search space is modeled within each individual — heterogeneous systems could achieve better results than their homogeneous counterparts. Results seemed to be sensitive to project modeling decisions and this results begs to more deeper investigations on our current models of social problem-solving.



When integrated, these results point out that there are many complex aspects of diversity in social problem-solving that still need further investigation. Therefore, we are far from completely understand this problem. Fortunately, there is a current trend in Social Computing that seeks to provide a rigorous mathematical foundation to Social Computing (CHEN et al., 2016). This foundation can provide formal definitions about what problems are interested to investigated, how one can model a diverse set of solvers and yet derive general results, among other contributions. Whether results from one system could be *generalized* to another system are still open endeavors in Social Computing (KEARNS, 2012; CHEN et al., 2016).

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## APPENDIX A — RESUMO DA TESE

Sabe-se que humanos podem resolver problemas complexos em pouco tempo com respostas quase ótimas (KEARNS, 2012; CHEN et al., 2016). Identificar o porquê deste fato ainda é uma questão em aberto que requer a integração de diferentes áreas do conhecimento humano. A Computação Social busca responder esta e outras questões pertinentes a forma com que as pessoas resolvem problemas colaborativamente.

Uma forma de expandir a compreensão sobre o fenômeno social é através da experimentação com sistemas sociais computacionais. Estes sistemas são construídos através da modelagem computacional do ambiente social humano (LAZER et al., 2009; MASON; WATTS, 2012). Modelos computacionais não substituem o estudo de sistemas sociais reais; eles podem, sim, informar direções, variáveis e novas perspectivas que auxiliem no estudo e desenvolvimento de experimentos em condições reais. Além disso, sistemas computacionais permitem o teste rápido de hipóteses em ambientes controlados (MACY; FLACHE, 2009).

Entretanto, existem desafios no uso de sistemas computacionais sociais. Sistemas simples podem se comportar de forma imprevisível após algumas iterações assim como sistemas mais sofisticados podem ter resultados difíceis de serem generalizados. Entender as características destes sistemas e os padrões que governam a comunicação é fundamental para o desenvolvimento de sistemas mais sofisticados e precisos (NEWELL; SIMON, 1972; MARCH, 1991; BARABÁSI; ALBERT, 1999; KEARNS; SURI; MONTFORT, 2006; HOGG; HUBERMAN, 2008; LAZER et al., 2009; LAVIE; STETTNER; TUSHMAN, 2010; FLACHE; MACY, 2011).

Este resumo da tese está organizado da seguinte forma: a seção A.1 apresenta os objetivos, a seção A.2 descreve um modelo computacional geral de resolução social de problemas, a seção A.3 apresenta os resultados da investigação sobre a relevância da centralidade na resolução social de problemas, a seção A.4 aborda sistemas heterogêneos de resolução de problemas e a seção A.5 faz uma discussão geral dos principais resultados em cada investigação. Com exceção da seção de conclusões, cada uma das seções a seguir representa uma investigação autocontida que resultou em um trabalho científico. Cada trabalho foi publicado em conferências ligadas ao problema abordado conforme descrito na tabela 1.1.

## A.1 Principais Objetivos e Trabalhos Publicados

No contexto desta tese, resolução social de problemas pode ser definida como a atividade onde indivíduos, humanos ou computacionais, trocam informações para resolver um mesmo problema de forma que eles podem resolver o problema em um tempo menor ou então com uma resposta melhor do que o caso onde cada indivíduo resolveria de forma isolada e independente o problema em questão. Esta tese está focada na extensão e investigação de modelos e sistemas computacionais de resolução social de problemas. Mais especificamente, os dois principais objetivos desta tese são os seguintes:

1. Encontrar combinações paramétricas que favoreçam o processo de resolução social de problemas.
2. Otimizar o uso de recursos (tempo, conexões de rede, etc) a partir de um sistema social computacional já existente.

Parâmetros podem incluir o problema em si a ser resolvido, a dificuldade<sup>1</sup> das instâncias do problema, as estratégias dos indivíduos, a quão conectada é a estrutura de comunicação, o número de indivíduos, etc.

## A.2 Estudo de Viabilidade de um Modelo Geral de Resolução Social de Problemas

Existem inúmeros modelos de resolução social de problemas. A concepção de modelos inspirados nos mais diversos fenômenos naturais extrapolou a nossa habilidade em saber de todos, muitos menos entendê-los (TALBI, 2002; SÖRENSEN, 2015). Considerando esta dificuldade, surge a pergunta se é viável combinar indivíduos de diferentes modelos em apenas um modelo de forma que eles possam colaborar entre si de forma transparente. Além disso, supondo que seja viável, o modelo resultante apresenta algum benefício?

Foram usados três modelos originais para testar a viabilidade de um modelo heterogêneo:

**MN** Modelo de Redes Meméticas inspirado na forma com que humanos colaborativamente resolvem problemas (ARAÚJO; LAMB, 2008a; ARAÚJO; LAMB, 2008b).

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<sup>1</sup> A dificuldade do problema não é necessariamente a ideia de complexidade algorítmica já que o problema é resolvido de forma distribuída e pode depender de variáveis endógenas ao sistema todo, como o tipo de informação trocada

Este modelos pode ser construído a partir de três regras que descrevem o comportamento dos indivíduos e como eles trocam informações.

**PSO** Modelo de Enxame de Partículas inspirado em sistemas naturais coletivos como enxames de insetos ou bandos de aves procurando alimentos (KENNEDY, 1998; EBERHART; SHI, 2000; POLI; KENNEDY; BLACKWELL, 2007). Usa o conceito de velocidade das partículas para dar origem ao sistema coletivo.

**FIPS** Evolução do modelo PSO considera uma dinâmica de comunicação diferente (MENDES; KENNEDY; NEVES, 2003).

A rede usada foi um *grid* periódico de tamanho  $6 \times 5$ . Os problemas a serem resolvidos foram todos problemas de minimização de funções contínuas frequentemente usadas para teste de novos algoritmos.

A figura A.1 mostra os dados de convergência de todos modelos testados ao longo de 100000 iterações. A população heterogênea é composta de metade partículas do modelo de enxame (PSO) e o resto de indivíduos do modelo de Redes Meméticas.

O modelo heterogêneo convergiu, na média, da mesma forma que o modelo homogêneo que obteve os melhores resultados, isto é, que convergiu mais rapidamente para a solução ótima.

### A.3 A Relevância da Centralidade na Resolução Social de Problemas

O conceito de centralidade em resolução social de problemas é bastante antigo (BAVELAS; BARRETT, 1951; BURGESS, 1969) e parte do pressuposto que indivíduos em posições centrais da rede possuem vantagens durante o processo de resolução em relação a indivíduos em posições periféricas. O primeiro problema histórico foi sobre a necessidade de formalização do conceito de centralidade (FREEMAN, 1978; FREEMAN; ROEDER; MULHOLLAND, 1979), problema este que permanece em aberto (BORGATTI, 2005).

A partir da vasta quantidade de modelos computacionais disponíveis, surge a oportunidade de investigar o impacto da centralidade em diferentes modelos. Esta investigação está descrita nas próximas subseções.

### A.3.1 Métodos e Principais Resultados

Qualquer indivíduo em um sistema social de resolução de problemas pode, eventualmente, contribuir tanto para seus vizinhos imediatos na rede quanto para todo o coletivo quando este encontra uma solução melhor que a melhor solução já descoberta pelo sistema. No primeiro caso, temos uma *contribuição local* e, no segundo, uma *contribuição global*. Uma forma de medir a relevância ou não de posições centrais para o indivíduo é medir a correlação entre a contribuição global, a contribuição local e as diferentes métricas de centralidade selecionadas. A tabela A.1 apresenta as correlações entre a contribuição individual e centralidade para cinco funções de minimização<sup>2</sup> e quatro modelos de resolução social de problemas.

Os resultados para a contribuição local e para a contribuição global apontam para a inexistência de um padrão claro. Algumas métricas mostraram correlações positivas altas com alguns modelos enquanto que outras métricas tiveram, inclusive, correlações negativas com a contribuição.

### A.4 Modelos Heterogêneos de Resolução Social de Problemas

Ao mesmo tempo que experimentos com humanos apontem a nossa habilidade para resolver problemas de forma colaborativa, fica evidente que não dispomos de modelos sofisticados para recriar o que é observado (KEARNS; SURI; MONTFORT, 2006; JUDD; KEARNS; VOROBAYCHIK, 2010; KEARNS, 2012; MASON; WATTS, 2012). Duas características que foram observadas em humanos mas que modelos computacionais de resolução social de problemas na época não possuíam eram a diversidade de estratégias entre os indivíduos de uma mesma população e a forma com indivíduos adaptavam a própria estratégia de busca (MASON; WATTS, 2012).

#### A.4.1 Métodos e Principais Resultados

A investigação partiu do modelo e problema original propostos no artigo (MASON; WATTS, 2012). A partir destes sistemas originais, foram adicionados e testados mecanismos que permitissem a heterogeneidade de estratégias, isto é, a diversidade, e

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<sup>2</sup>F1 representa a função Ackley; F2, Griewank; F3, Rastrigin; F4, Rosenbrock; e F5, Sphere.

a dinamicidade. Características comportamentais observadas em experimentos com humanos no mesmo problema.

O problema que os indivíduos resolveram foi o mesmo: a maximização de uma função composta por ruído de *Perlin* adicionado de um sinal. O sinal foi modelado como uma gaussiana bidimensional conforme pode ser observado na figura A.2. A solução é construída individualmente através da amostragem direta de pontos nesse espaço. Cada ponto  $(x, y)$  representa uma solução nesse espaço, o problema está em encontrar o ponto que maximiza o valor da função.

Uma das observações a respeito do trabalho original (MASON; WATTS, 2012) é que o modelo usado lá associa a estratégia de busca como sendo o tamanho do raio. Isso implica que diferentes tamanhos de raios de de busca representam diferentes estratégias de busca. Uma das características dessa interpretação é que um raio maior não exclui a amostragem de pontos mais próximos à solução atual.

A figura A.3 mostra a comparação entre o modelo original e uma variação do modelo que propomos onde a estratégia que usa raios maiores exclui a possibilidade de busca por soluções próximas à solução. O resultado dessa variação foi significativo o bastante para que os modelos mostrassem um comportamento bastante diferente para raios de tamanho médio a grande. Ou seja, o modelo original é bastante sensível a pequenas e os resultados originais precisam ser generalizados com cautela.

Outro experimento foi a comparação de modelos com distribuições de raios homogêneas e heterogêneas assim como modelos onde indivíduos alteravam o próprio raio de forma aleatória ou então copiavam o raio do melhor vizinho na rede social. Além disso, foram comparadas redes eficientes com redes ineficientes. Os resultados estão na tabela A.2

Os dados apontam que a distribuição de estratégias tem um impacto maior no resultado coletivo final do que a rede. Essa evidência dá suporte à tendência de acrescentar ao foco da Computação Social dados sobre as estratégias e características individuais (KEARNS; SURI; MONTFORT, 2006; KEARNS, 2012).

## A.5 Conclusões Gerais

Este trabalho de tese apresentou resultados novos sobre sistemas computacionais de resolução de problemas. As suas principais contribuições foram:

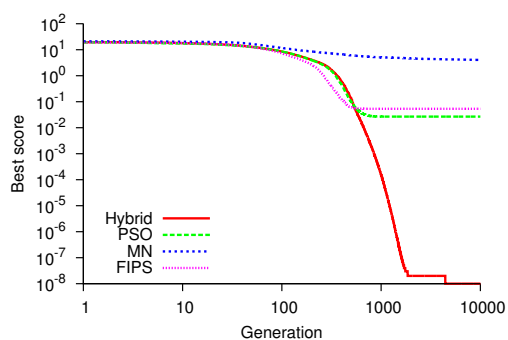
**O estudo da viabilidade de um modelo geral de resolução de problemas** Foi desenvolvido um modelo que permitiu com que indivíduos de dois modelos diferentes de resolução social de problemas — Redes Meméticas e Enxame de Partículas (PSO) — colaborassem para resolver o mesmo problema. Na maioria dos problemas de otimização testados, populações heterôgeneas tiveram resultados melhores ou tão bons quanto os resultados das melhores populações homogêneas originais em termos de convergência para soluções ótimas.

**A investigação do impacto da centralidade no resultado individual e coletivo** Diferentes métricas de centralidade tiveram diferentes impactos em diferentes modelos. Em outras palavras, a investigação mostrou que existe muito o que ser estudado sobre o impacto da centralidade em diferentes modelos e o porquê algumas métricas tiveram mais poder preditivo em alguns modelos computacionais de resolução de problemas, mas não em outros. A principal contribuição, portanto, é que há uma dificuldade em generalizar os resultados iniciais da importância e da vantagem da centralidade na resolução social de problemas.

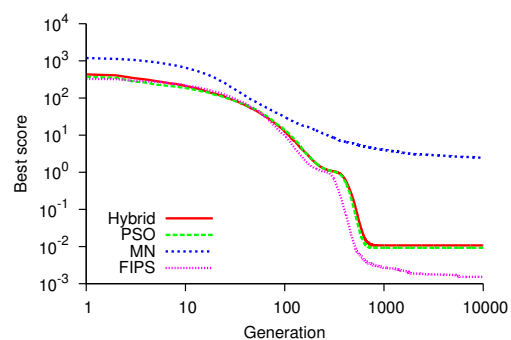
**A análise experimental de modelos heterogêneos de resolução social de problema** Os resultados dos experimentos apontam que uma população heterogênea em relação às estratégias usadas pelos indivíduos pode ter resultado positivo mas desde que certas condições estejam presentes. Estas condições estão relacionadas à forma com que os indivíduos realizam a exploração de novas soluções no espaço de busca.

Quando integradas, estes resultados apontam que ainda existem muitos aspectos a serem investigados na resolução social de problemas. O primeiro passo é a formalização da área de Computação Social (CHEN et al., 2016), a iniciar pela definição de um conjunto de problemas que contemplem várias aspectos que possam usar usados para descrever estratégias que humanos frequentemente usam para resolver problemas. Esta tese aponta que a carência de modelos heterogêneos só pode ser resolvida a partir de mais estudos com humanos. Além disso, existe a necessidade de definições mais precisas sobre a resolução social de problemas, uma definição de diversidade que faça sentido tanto para sistemas computacionais quanto para sistemas reais.

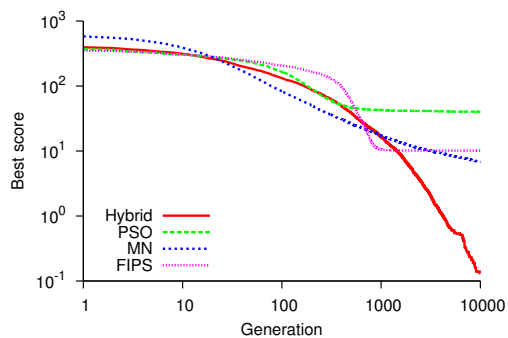
Figure A.1: Convergência dos modelos para a melhor solução em diferentes modelos para uma média de 50 execuções independentes, cada execução limitada a 100000 iterações. A população heterogênea é composta de metade partículas do modelo de enxame (PSO) e o resto de indivíduos do modelo de Redes Meméticas.



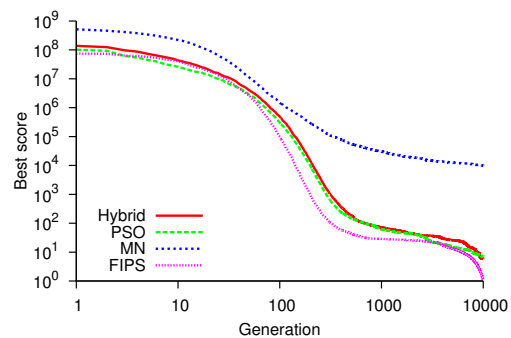
(a) Função Ackley.



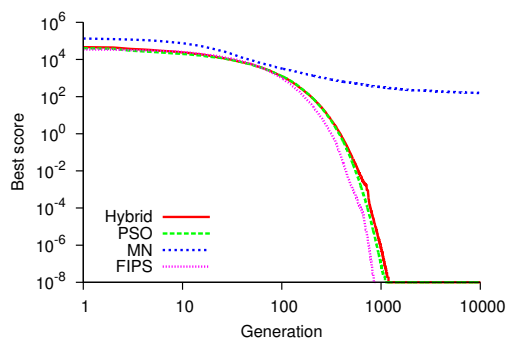
(b) Função Griewank.



(c) Função Rastrigin.



(d) Função Rosenbrock.



(e) Função Sphere.

Table A.1: Correlação de Pearson entre a contribuição individual ao coletivo e diferentes métricas de centralidade.

Modelo	Função	Métrica			
		Intermédio	Closeness	Grau	Eigenvector
MN	F1	0.38	0.40	0.40	0.40
	F2	0.37	0.39	0.39	0.39
	F3	0.45	0.47	0.48	0.48
	F4	0.33	0.35	0.35	0.36
	F5	0.38	0.39	0.40	0.40
	Média	0.38	0.40	0.40	0.41
Desvio	0.0398	0.0392	0.0417	0.0391	
DSR	F1	0.74	0.71	0.78	0.73
	F2	0.75	0.72	0.78	0.74
	F3	0.69	0.67	0.73	0.68
	F4	0.76	0.73	0.80	0.75
	F5	0.75	0.72	0.78	0.74
	Média	0.74	0.71	0.78	0.73
Desvio	0.0240	0.0201	0.0219	0.0243	
PSO	F1	0.45	0.47	0.48	0.48
	F2	0.44	0.46	0.47	0.47
	F3	0.38	0.39	0.40	0.40
	F4	0.62	0.65	0.66	0.67
	F5	0.73	0.77	0.78	0.78
	Média	0.52	0.55	0.56	0.56
Desvio	0.1324	0.1404	0.1414	0.1431	
FIPS	F1	0.43	0.22	0.36	0.33
	F2	0.25	0.03	0.17	0.15
	F3	-0.29	-0.46	-0.34	-0.39
	F4	-0.20	-0.46	-0.29	-0.33
	F5	-0.44	-0.72	-0.54	-0.58
	Média	-0.05	-0.28	-0.13	-0.16
Desvio	0.3304	0.3459	0.3369	0.3457	

Table A.2: Probabilidades de encontrar a solução ótima para redes eficientes (descentralizadas) e redes ineficientes (centralizadas)

Modelo	Rede	
	Eficiente	Ineficiente
Homogêneo raio 8	0.41 ( $\pm$ 0.06)	0.33 ( $\pm$ 0.06)
Homogêneo raio 16	0.15 ( $\pm$ 0.04)	0.13 ( $\pm$ 0.04)
Homogêneo raio 32	0.01 ( $\pm$ 0.01)	0.03 ( $\pm$ 0.02)
Heterogêneo estático	0.62 ( $\pm$ 0.06)	0.59 ( $\pm$ 0.06)
Dinâmico Aleatório	0.62 ( $\pm$ 0.06)	0.54 ( $\pm$ 0.06)
Dinâmico Social	0.23 ( $\pm$ 0.05)	0.27 ( $\pm$ 0.05)



Figure A.2: Exemplo de instância de otimização: a solução ótima está localizada na região mais clara enquanto que a região escura representa o ruído. Cada ponto  $(x,y)$  do espaço de busca representa uma solução cujo valor objetivo da função está associado à cor na figura.

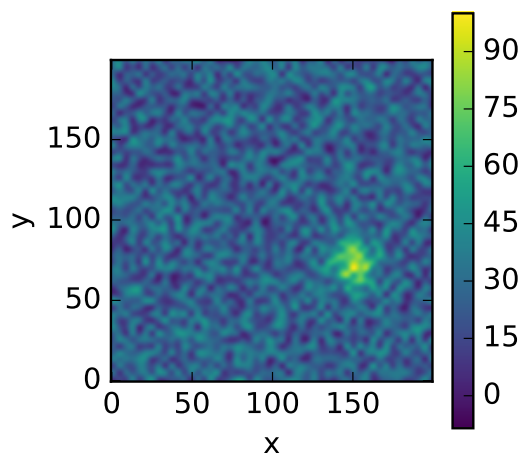
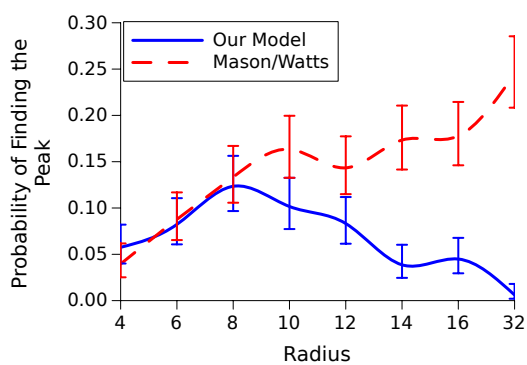
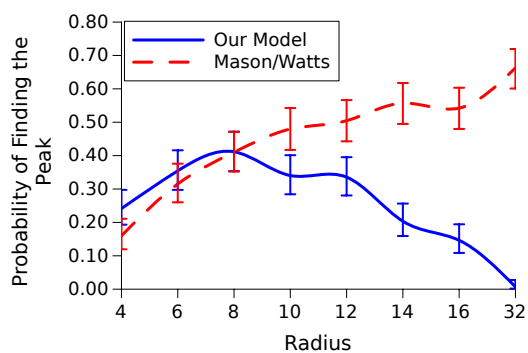


Figure A.3: Probabilidade de encontrar a solução ótima para tamanhos de raios variados no nosso modelo e no modelo original de Mason e Watts.



(a) 16 indivíduos.



(b) 64 indivíduos.