# GEOMETALLURGICAL MODELLING TO HELP IN PREDICTING ZINC METALLURGICAL RECOVERY

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24th World Mining Congress
MINING IN A WORLD OF INNOVATION
October 18-21, 2016 • Rio de Janeiro /RJ • Brazil

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## ABSTRACT

A metal recovery at a mineral processing plant is affected by rock properties and process variables, including ore grain size liberation, head grades feeding the process, hardness among others. Additionally, metal recovery has not necessarily a linear relationship with those variables, which makes its prediction more complex. Geometallurgical tests results affect deeply the mining chain and economics and its correct modelling and prediction is of paramount importance to any mine operation. This study uses a multiple regression model to predict metallurgical recovery from geological variables, which is basically a statistical method that stablishes the relationship between a dependent variable $Y$, also known as response, and two or more independent variables $X(X_1,X_2,\dots,X_n)$, called explanatories, with the constraint that there is a correlation between these and that. The methodology is illustrated through a case study in a major zinc deposit. The explanatory variables used were zinc head grade feeding the processing plant and the main ore typologies from the deposit. The independent variables were estimated via ordinary and indicator kriging at every mined block. The regression model with its associated regression error provides the means to randomly draw a value for ore metallurgical recovery at each block. The results showed that the use of geological variables for metallurgical recovery prediction provides reliable estimates as the results reconcile well against production data.



## KEYWORDS

Geometallurgy, multiple regression, metallurgical recovery, indicator kriging, ordinary kriging.


## INTRODUCTION

A good performance of a mineral processing plant is close linked to a proper knowledge of geology, mining, processing and metallurgy. Studying these areas simultaneously, observing relationship between them, permits to understand and quantify the variability of the ore and, in the same way, predicts the response of different rock types during mineral processing. The link between all areas involved in the ore production chain is called geometallurgy (Beniscelli, 2011). Geometallurgy aims at controlling variables which interfere directly or indirectly in the process, such as concentration of gangue minerals, ore grade, work index, reagent consumption, grain size distribution, ore liberation, hardness, grindability, humidity, concentrate quality, among others, in order to add economic value to the resource (Rossi e Deutsch, 2014; Deutsch, 2013; Beniscelli, 2011). It also enables the identification of environmental impacts and their subsequent mitigation, ore recovery increase, contaminants and by-products detection and quantification, and product quality assurance. Geometallurgy assists the selection of the size of the selective mining unit, the direction of process routes, the ore comminution stages, in addition to quantifying and determining the products used for flotation, and in metallurgy.

Geometallurgical variables modeling is complex, as it involves nonlinear variables and, usually, large number of them. Often, what is available are indirect measures of metallurgical variables. To deal with nonlinearity problem, additive variables can be estimated first and by a transfer function, as a regression model for example, the non-additive variables are estimated. As for the multiple variables issue, they should either be grouped or even eliminated. In the first case, a supersecondary variable is created. It gather secondary variables that have some similarity in a single variable. In the second case, it is eliminated variables with very low correlation with the response variable and whose sampling is too sparse, as well as variables with high correlation with another one, as these are redundant and generate bias (Deutsch, 2013; Boisvert *et al.*, 2013).

In this work ordinary kriging of zinc grade and indicator kriging of one of the geometallurgical typologies were employed to predict metallurgical recovery through a regression model. Geometallurgical typology was used during the prediction because it was noticed that each typology has a different response in mineral processing, as is showed in the scatterplot of Figure 1 between metallurgical recovery and the two main mined typologies: dolomitc breccia, BXD, and willemitic breccia, BXW. This figure shows also the scatterplot between zinc head grade feeding the process and metallurgical recovery.

Different from willemitic breccia and zinc grade, that have positive correlation with zinc recovery, dolomitic breccia has negative correlation with this variable. This occurs because this typology generates very thin material and hinders flotation process: it induces slime coating effect, which is the coating of the bigger particles by a fine layer of smaller ones.

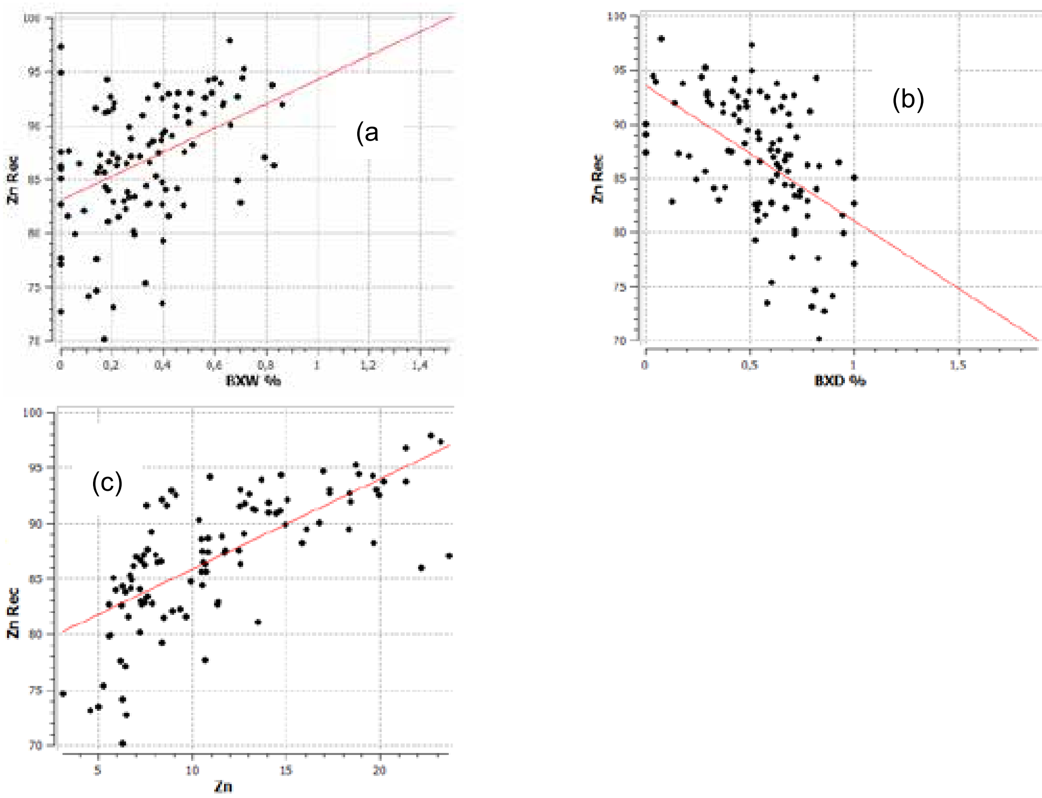



Figure 1 - Scatterplot of (a) zinc recovery versus BXW percentage, (b) zinc recovery versus BXD percentage and (c) zinc recovery versus zinc grade

The deposit being studied corresponds to the larger willemitic zinc deposit in the world, located at the northwest of Minas Gerais State, in Brazil. Mineralization is embedded in dolomitic rocks, in a hydraulic northeast-southwest breccia, and occurs in thin inclined to subvertical lenses. Zinc is present also in sphalerite ($ZnS$), hemimorphite ($Zn_4Si_3O(OH)_2$), smithsonite ($ZnCO_3$) and hydrozincite ($Zn_5(CO_3)_2(OH)_6$), but occurs at lower proportion in these minerals.

## METHODS

### Ordinary Kriging

Kriging is a generic term applied to several estimation methods that aim to minimize the estimation error variance (Sinclair e Blackwell, 2002). Through a variogram, optimal weights for nearby samples are established, thus avoiding bias and ensuring that the average error is zero. Therefore, kriging is known as the best linear unbiased estimator, BLUE (Isaaks & Srivastava, 1989).

Ordinary kriging is characterized for considering local fluctuations in average, limiting the stationarity to a local neighborhood. The estimate of a $Z^*(u_o)$ value is performed through a linear combination of near samples, as is shown in equation (1):

$$Z^*(u) = \sum_{\alpha=1}^{n(u)} \lambda_\alpha Z(u_\alpha) \quad \text{with} \quad \sum_{\alpha=1}^{n(u)} \lambda_\alpha = 1 \tag{1}$$

where $Z^*(u)$ is the point to be estimated, $\lambda_\alpha$ are the weights associate to samples and $Z(u_\alpha)$ are samples values at each point $\alpha$. To minimize the error variance $\sigma_\varepsilon^2(u)$ under the unbiasedness constraint that the mathematical expectation of the error is zero, the weights are obtained from the following ordinary kriging system:

$$\begin{cases} \sum_{\beta=1}^{n(u)} \lambda_\beta \, \gamma(u_{\alpha,} u_\beta) - \mu(u) = \gamma(u_{\alpha-} u) & \alpha = 1, \dots, n \\ \sum_{\beta=1}^{n(u)} \lambda_\beta(u) = 1 \end{cases} \tag{2}$$

where $\gamma(u_\alpha, u_\beta)$ is the semivariogram between two points $u_\alpha$, and $u_\beta$, $\gamma(u_i, u)$ is the semivariogram between one point $u_i$ and the point to be estimated $u$, and $\mu$ is Lagrange multiplier required for minimizing the error variance (Goovaerts, 1997; Yamamoto & Landim, 2013). It is calculated $N+1$ partial derivative with respect to $\lambda_\alpha$ and $\mu$, they are equated to zero and they lead to $N+1$ equations with $N+1$ unknown values whose solution results in $N$ weights $\lambda_\alpha$ under the unbiasedness constraint $\sum \lambda_\alpha = 1$, thus minimizing the error variance (Soares, 2006). The minimized error variance becomes:

$$\sigma_\varepsilon^2 = C(0) - 2 \sum_\alpha \lambda_\alpha \, C(u_\alpha - u) + \sum_\alpha \sum_\beta \lambda_\alpha \lambda_\beta \, C(u_{\alpha-} u_\beta) \tag{3}$$

where $C(0)$ is the covariance *a priori* of $Z(u)$.

Kriging is widely used in mining, agricultural science, environment, hydrogeology, among others, for it weights samples according to its location and clustering, which makes it the best and most accurate estimator of spatial variables .

**Indicator kriging**

Indicator kriging is usually employed to estimate non-linear variables and when the aim is to estimate the distribution rather than a value at some location. It offers flexibility when dealing with extreme values and with different continuity patterns, as it defines areas with greater or lesser probability of occurring a specific event. Indicator kriging can be applied to both continuous and categorical variables.

Indicators data of a random variable $I(u;z_k)$ generate a conditional probability function that is updated locally, from which is obtained a conditional cumulative distribution function, ccdf, at each unsampled location. This function depicts thus the possible values of an estimated point, being determined as

$$F(u, z_k|(n)) = \text{Prob}[Z(u) \le z_k|(n)] = \frac{1}{n} \sum_{\alpha=1}^{n} i(u_\alpha; z_k) \qquad k = 1, \dots, K \tag{4}$$

where $F(u,z_k)$ is the proportion of $z$ samples at $u$ below cutoff $z_k$, underlain in $n$ neighboring samples (Rossi & Deutsch, 2014; Yamamoto & Landim, 2013).

The indicator of a regionalized variable $I(u;z_k)$ has two possible values: zero, to values over a specific cutoff, or one, to values below this cutoff, according to equation (5):

$$i(u_\alpha; z_k) = \begin{cases} 1 & if \quad z(u_\alpha) \le z_k \\ 0 & if \quad z(u_\alpha) > z_k \end{cases} \qquad k = 1, \dots, K \tag{5}$$

Indicator kriging of a random variable gives the ccdf estimation at a given cuttof $z_k$. Data of a continuous attribute $z$ are discretized into $k$ classes, and is calculated to each class the proportion of $z$-

data that does not exceed a given cutoff $z_k$. The ccdf built from the kriging of $k$ classes represents a model of probability of uncertainty over the unsampled values $z(u)$ (Deutsch & Journel, 1998).

## GEOLOGY

The zinc deposit object of this case study is located in Vazante municipality and is part of the Brasilia Belt. This belt is an orogen formed by nappes and thrusts east vergent. It extends over thousand kilometers along the western border of São Francisco craton and shows north-south direction. Vazante group is a unit of Brasilia Belt. This group was first defined by Dardenne (1979) and has a pelitic-carbonate marine sequence, being Paleoprotezoic/Neoproterozoic.

This zinc deposit is classified as Vazante Deposit type for some authors (Monteiro, 2002; Hitzman *et* al, 2003) because it is hypogenic and non-sulphide, having thus a peculiar mineralization. Hot metalliferous fluids with low sulfur in a temperature around 250 ºC and meteoric fluids along the Vazante Fault Zone generated the willemite mineralization (Lemos, 2011).

The rocks of the deposit belong to Serra do Garrote and to Serra do Poço Verde Formations – this last being the host of the mineralization. Serra do Garrote Formation has a thick package of carbonaceous slates with thin quartzite intercalated and Serra do Poço Verde, which is at the top, has mainly dolomites, besides slates and metasiltstone. The mineralization occurs in a breccia associated to the Vazante fault zone. There are three breccia types: willemitic breccia (BXW), which is the main mineralized rock type, dolomitic breccia (BXD) and hematitic breccia (BXH).

## PROCEDURE

Prediction of zinc metallurgical recovery involved ordinary kriging of zinc grade and indicator kriging of dolomitic breccia. Regression model was established after factor analysis and hypotheses test based on $F$ statistics.

First, metallurgical tests were performed at a pilot plant, in order to verify lithotypes behavior in the mineral processing route. The tests consisted of grinding, classification via wet sieving, grade analysis, froth flotation and mineralogical characterization via MLA (mineral liberation analyzer). A total of 207 boreholes were tested comprising 104 geometallurgical samples.

Besides ore grade, lithotypes samples were also used for building the geometallurgical model. For this, it was utilized 5,297 boreholes with 337,000 samples. The deposit lithotypes were grouped into six typologies, according to their behavior at processing plant:

- Typology I: willemitic breccia;
- Typology II: dolomitic breccia;
- Typology III: hematitic breccia;
- Typology IV: dolomite;
- Typology V: clay material (weathered rocks, fractures filling) and marl;
- Typology VI: slate, phyllite, shale and metabasic rock.

### Multiple linear regression

To define the regression model, it was built a correlation matrix between geometallurgical typologies percentage in a stope and metallurgical recovery. Then, a regression was adjusted with the prior defined explanatories variables and, after this, an analysis of variance was conducted to test the significance of the regression. Another test was run to examine the importance of each regression coefficient and, thus, a final regression model was established. The regression model was obtained using the software *sabor.exe* from the GSLib library (Deutsch and Journel, 1998; Zagayevskiy and Deustch, 2011). *Sabor.exe* does sensitivity analyses based on linear regression and quantifies the influence of each variable in the response model.

### Zinc grade ordinary kriging

It was built a zinc grade model using ordinary kriging. The distances in search ellipsoid were the same found in analysis of spatial continuity. Variograma showed maximum continuity along azimuth 55 º.

### Dolomitic breccia indicator kriging

To estimate dolomitic breccia ocurrence, the lithotype information was transformed into indicators according equation 6, where BXD presence is coded as 1 and its absence, as 0:

$$i(u_\alpha ; s_l) = \begin{cases} 1 & if \;\; s(u_\alpha) = s_l \\ 0 & otherwise \end{cases} \tag{6}$$

being $i(u_\alpha{:}s_l)$ the indicator of category $s_l$ at location $u_\alpha$.

Analysis of spatial continuity showed maximum continuity along azimuth 50º. After indicator kriging estimate, data were post processed for correction of order relation deviations. Both dolomitc breccia indicator kriging and zinc grade ordinary kriging were estimated at a 12x12x3 m blocks.

## RESULTS AND DISCUSSION

### Linear regression

The correlation matrix (Figure 2) showed that the geometallurgical typologies that are correlated to the metallurgical recovery are dolomitc breccia and willemitic breccia. Knowing this, a prior regression was done with these typologies, and next it was processed an analysis of variance (ANOVA) to test the significance of the regression (Table 1). In ANOVA, total variance of a response variable is partioned in variance between the group average and the variance of the experimental error, i.e., variance within the group. The test checked the null hypothesis $H_0$ that all regression coefficients are zero ($\beta_1 = \beta_2 = ... = \beta_k = 0,\ k > 1$) against the alternative hypothesis $H_1$ that at least one coefficient is nonzero. As the calculated $F$ (16.28) is greater than the tabulated $F$ (2.09), and the $p$-value is less than the stablished significance level $\alpha$ (0.05), the null hypothesis was rejected and the proposed model was accepted.

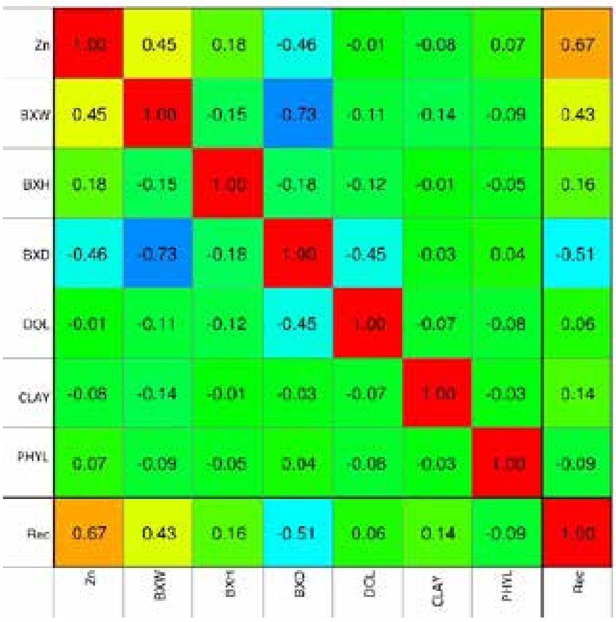



Figure 2 - Correlation matrix between geometallurgical typologies percentage, zinc grade and metallurgical recovery

Table 1– Analysis of variance to test significance of the regression (ANOVA table)

| | Sum of squares | Degrees of freedom | Mean of squares | $F$-value | $p$-value | Accept in α=0.05 |
|---|---|---|---|---|---|---|
| Regression | 1774.272 | 7 | 253.467 | 16.277 | 0 | yes |
| Residual (Error) | 1762.242 | 97 | 18.167 | | | |
| Total | 3536.515 | 104 | | | | |

Since the model was considered plausible, each regression coefficient were assessed to determine their individual influence in the model. In this test, the null hypothesis $H_0$ is that the regression coefficient is zero against the alternative hypothesis $H_1$ that the regression coefficient is not zero, with a significance level $\alpha$ of 0.05. The test results can be seen in Table 2, showing that the model can be more effective with the removal of willemitic breccia, BXW, and keeping dolomitic breccia, BXD, and zinc head grade.

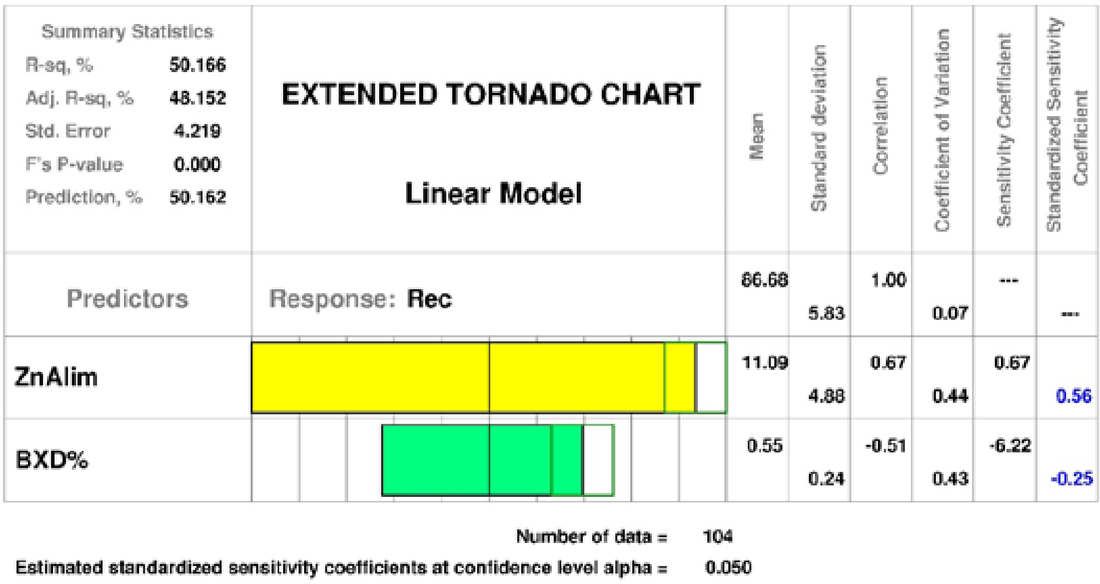
Table 2– Test for individual regression coefficients

| Predictor | Coefficient | Std Coefficient | $F$ | Accept at α = 0.05 |
|---|---|---|---|---|
| Zn grade | 0.667 | 0.559 | 46.352 | yes |
| BXW% | -0.272 | -0.010 | 0.008 | no |
| BXD% | -6.377 | -0.258 | 5.812 | yes |

After the regression coefficients were determined, linear regression was adjusted and an Extended Tornado Chart was plotted. This chart is a graphical plot that summarizes sensitivity analysis results. Sensitivity coefficients showed in the chart capture each predict variable influence in the response model while standardized sensitivity coefficients show the influence of uncertainty from each predict variable on model response uncertainty. The sensitivity coefficients correspond to the regression coefficients.

The chart reveals that adjusted $R^2$ of model is 48.15%; which is a reasonable value; the standardized error is 4.22; $p$-value is 0 and prediction power of the model, i.e., the percentage ratio of standard deviation of predicted and actual values, $predic = \hat{\sigma}/\sigma$, is 50.16%. More statistics can be viewed on Chart 1.

Chart 1- Extended Tornado Chart




Regression model was defined as

$$Recovery = 86.68266 + 0.66548*(Zn\_grade - 11.09231) - 6.21621*(BXD\% - 0.5493876) \quad (2)$$

**Kriging**

Both zinc grade ordinary kriging and BXD% indicator kriging honored the statistical summary of input data (Table 3). The decreased variance in the block model estimated is due to smoothing caused by kriging.

Table 3 – Statistics of original declustered data, estimated zinc grades and BXD percentage

| Variable | Zn declustered | Zn estimated | BXD declustered | BXD% estimated |
|---|---|---|---|---|
| Number of samples | 23,481 | 4,947,542 | 5.464.269 | 4.878.873 |
| Minimum | 0.05 | 0.59 | 0.00 | 0.00 |
| Maximum | 57.99 | 49.50 | 1.00 | 1.00 |
| Mean | 19.99 | 20.65 | 0.15 | 0.15 |
| Variance | 178.45 | 57.81 | 0.36 | 0.31 |

**Recovery**

Metallurgical recovery was calculated according to equation 2, using Zn grade estimated and BXD% estimated in each stope available. The recovery estimate histogram is shown in Figura 3. It shows a mean recovery of 86.78% for the stopes estimated.

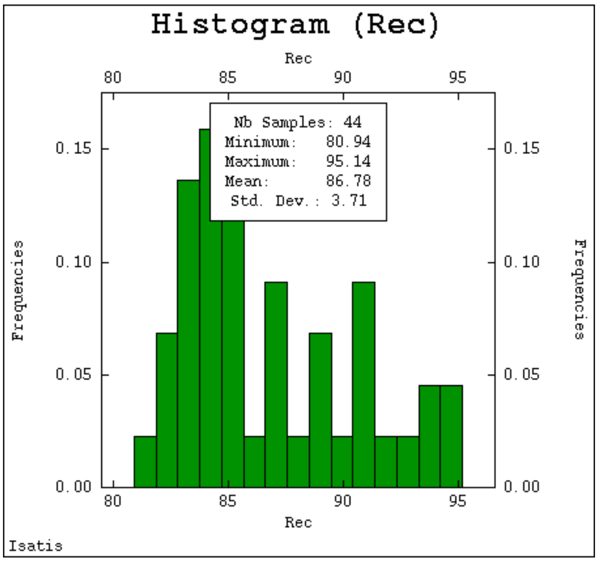



Figure 3 - Histogram of zinc metallurgical recovery estimated

Metallurgical recovery calculated values were compared against laboratorial tests of recovery for validation. The mean calculated recovery usually presented similar values when compared to the mean recovery from laboratorial test (Figure 4). The result showed an average residual close to zero, as desirable (Table 4, Figure 5). It showed, either, that 50% of the data have a residual lower than 2.47%, and 75% of the data have a residual lower than 4.66%, with a mean percentage of 3.92% of residual. Therefore, it was obtained a small residual and the regression model is suitable for estimating the metallurgical recovery.

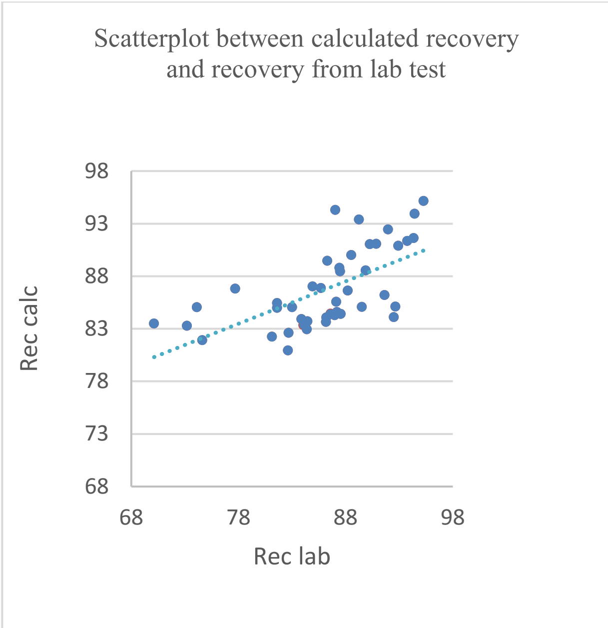



Figure 4 - Scatterplot depicting the calculated recovery *versus* recovery from laboratorial tests

Table 4 – Statistcs of Residual

| | Minimum | Maximum | Mean | Q1 | Median | Q3 |
|---|---|---|---|---|---|---|
| Absolute residual | -13.36 | 8.39 | -0.60 | -2.07 | 0.1 | 2.14 |
| % Residual | 0.05% | 19.05% | 3.92% | 1.38% | 2.47% | 4.66% |

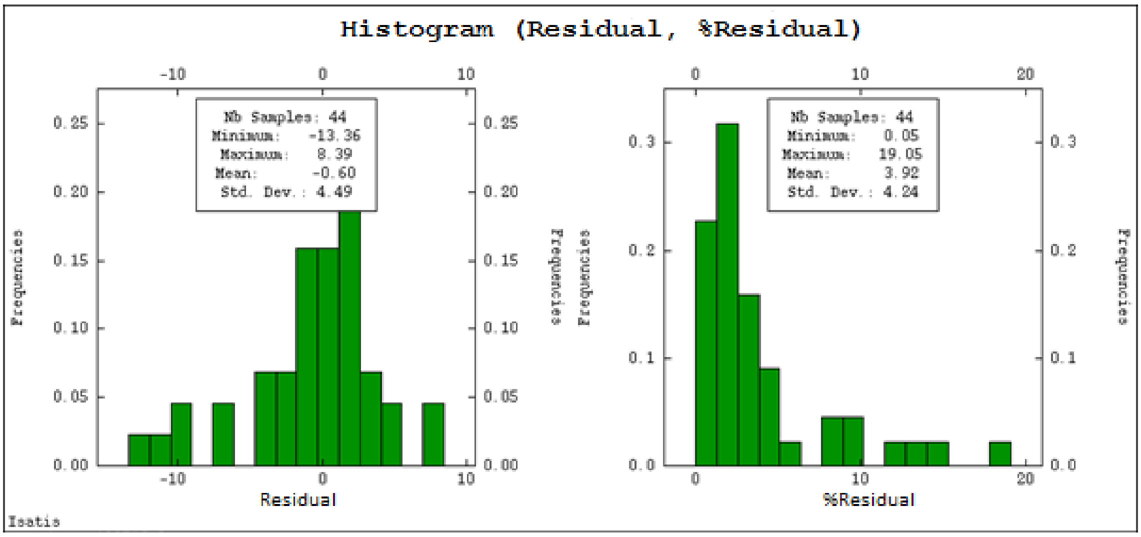



Figure 5 – Histogram of residual between calculated and lab test recovery (absolute values and percentage, respectively)

## CONCLUSIONS

Geometallugy studies are highly important for defining the ore to be processed, how it should be processed and what would be the expected recovery. The use of geological variables, including the metal feeding grade provided a more adequate model to predict ore metallurgical recovery instead of considering only the metal grade, since the residual obtained showed a mean percentage of 4%, and 75% of data have residual lower than 5%. Therefore, by considering the variables that significantly affect recovery, whether in a negative or positive way, led to a more accurate and precise prognosis model for metallurgical recovery. As a result, the model can help in mine planning, beneficiation and metallurgical processes, minimizing unexpected results during mineral processing.

The estimation via kriging corroborated the concept that it provides a good unbiased estimator, as the estimated model reconcile well with original data. The indicator kriging map derived of lithotype BXD was obtained faster and at less effort compared to manual modelling through vertical sections.

## AKNOWLEDMENTS

The authors thank Dr. Vanessa Koppe and Dr. Diego Marques for their comments and suggestions; Votorantim Metais for providing support and the data used in this research, especially the geologist Artur Saldanha, and Capes for providing financial support.

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