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Micromechanics Approach to the Effective Permeability of a Jointed Rock

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Abstract. The use of macroscopic models in numerical approaches for studying subsurface flow has become common practice in hydrogeology and petroleum engineering since the late 1960's. Particularly in rock masses, where the presence of joints constitutes a key weak point along which the mechanical, physical and hydraulic properties of rock matrix degrade, the presence of discontinuities, at different scales, represent a fundamental component of transport of fluid or contaminants through rock masses. However, one of the major questions that still poses a problem is what parameters to introduce into the models. From a transport properties viewpoint, joints within rock masses represent preferential channels for fluid flow and, as such, may be contributors to rapid transport of fluid and contaminants through rock masses, particularly when the permeability of the rock matrix is low. This article aims to present a micromechanical approach to the derivation of jointed rock permeability. Several examples are present for different numbers of joint sets crosscutting rock masses, showing the flow anisotropy introduced by their presence.

Keywords: Micromechanics, Homogenization, Permeability, Hydraulic conductivity, Jointed rock

1 Introduction

Product of geological phenomena in the history of rock masses, discontinuities, usually referred to as joints, are commonly present at different scales, modifying and degrading mechanical and other physical properties of the rock matrix. They are surfaces of small thickness, along which sliding is likely to occur and strongly affect the overall behavior of rock masses. In what concerns hydraulic and transport properties, joints represent preferential channels for fluid flow, contributing to rapid transport of fluid and/or contaminants through the medium, especially when rock matrix permeability is low. Since the late 1960's the use of macroscopic approaches in numerical models simulating subsurface flow has become common practice in hydrogeology and petroleum engineering, as well as attempts towards upscaling approaches, searching to understand the macroscopic behavior from the microscopic one [1–3]. Numerical simulations as well as experimental data show changes of several orders of magnitude caused by the existence of joints in a porous medium [4]. The presence of long crosscutting joints in a rock mass usually leads, for instance, an originally isotropic material – as normally is the case for the rock matrix – to behave anisotropically with respect to a number of properties.

The macroscopic approach for determination of hydraulic properties is a classical and useful branch in engineering, considering a continuum medium for which every physical is defined for all points in the medium. Instead, this article aims to present a micromechanical approach to determination of the homogenized permeability for jointed rock media, giving an alternative approach to the determination of input data for numerical and other physical models.

2 Macroscopic and Microscopic Darcy's Law

The macroscopic theory used to model fluid flow in permeable media is based in the so called Darcy's Law. An useful micromechanical approach is to use the Darcy's Law in the microscopic scale (`REV` scale). In such scale the fluid flow is exactly described by the Navier-Stokes Equations, which governs the equilibrium and constitutive behavior of the fluid. Ene and Sanchez-Palencia [7] and Auriault [8] have shown in the context of periodic homogenization that the concept of permeability express on the macroscopic scale the physics of the flow for a viscous Newtonian fluid at the scale of the pores. At the macroscopic scale, Darcy's Law writes:

$$\underline{Q} = -\frac{1}{\mu}\underline{\underline{K}}' \cdot \underline{G}, \tag{1}$$

where $\underline{Q}$ [m/s] is the macroscopic filtration velocity vector, $\underline{G}$ [Pa/m] is the macroscopic pressure gradient, μ [Pa.s] is the fluid viscosity, and the second order tensor $\underline{\underline{K}}'$ [m^2] is the macroscopic intrinsic permeability tensor, which is entirely determined by the morphology of the porous space. In a micromechanics point of view $\underline{\underline{K}}'$ is the result of a previous homogenization process ($\underline{\underline{K}}' \equiv \underline{\underline{K}}'^{\,\mathrm{hom}}$).

In many geological formations where the rock matrix have low permeability, the fluid flow occurs predominately through joints. In some cases, most of the flow occurs in a single joint, while in other cases a joint network exists for fluid flow [9]. A classical approach to compute the permeability of a single joint, commonly used for hydromechanical behavior of fractured rocks, considers a joint as a system of two smooth parallel plates separated by a mechanical aperture e^m, resulting in a linear relation between flow rate and pressure gradient [5]. This makes possible to define a fictional porous medium equivalent to the real joint conserving the link between flow rate and pressure gradient through a microscopic second order intrinsic permeability tensor $\underline{\underline{k}}'$ [m^2]. Therefore, the microscopic flow through a single fracture can be written as

$$\underline{q} = -\frac{1}{\mu}\underline{\underline{k}}' \cdot \underline{g} \tag{2}$$

where $\underline{q}$ [m/s] is the microscopic filtration velocity vector and $\underline{g}$ [Pa/m] is the microscopic pressure gradient. Mathematically, use of eq. (2) simplifies the problem, allowing homogenization of a darcyan medium instead of a flow governed by the Navier-Stokes equations. For simplicity, it is also useful to define a microscopic conductivity tensor as $\underline{\underline{\kappa}} = \underline{\underline{k}}'/\mu$ [m^{2}Pa^{-1}s^{-1}].

2.1 Homogenized hydraulic behavior

In order to determine the macroscopic hydraulic behavior of a fractured rock mass from the microscopic one or, in other words, to determine an homogenized permeability tensor $\underline{\underline{K}}'^{\,\mathrm{hom}}$, which links $\underline{Q}$ and $\underline{G}$, the micromechanics theory states that a volume Ω of the rock mass can be taken where the joint network is statistically represented. With this concept in mind, if an elementary volume is taken, where the characteristic size d of the heterogeneities (joints) is supposed to be small with respect to the dimension l of the volume Ω, which in turn is supposed to be sufficiently smaller than the characteristic dimension L of the rock mass, this volume is said to be a representative elementary volume (REV). The precedent conditions ($d \ll l \ll L$) are called scale separation conditions. It the context of jointed rocks, where the joints are long and crosscut the REV, d is identified as the average spacing between the joints.

The rock matrix as well as the joints are considered to be permeable. The rock matrix is usually taken as isotropic with respect to its conductivity ($\underline{\underline{\kappa}}_m = \kappa_m \underline{\underline{1}}$). The conductivity tensor for the joints $\underline{\underline{\kappa}}_f$, referred to its local frame ($\underline{t}_j, \underline{t}'_j, \underline{n}_j$), with $\underline{n}_j$ being the outer unit normal to the plane of any joint set j, can be expressed as:

$$\underline{\underline{\kappa}}_f^j = \kappa_t^j \left(\underline{t}_j \otimes \underline{t}_j + \underline{t}'_j \otimes \underline{t}'_j\right) + \kappa_n^j \underline{n}_j \otimes \underline{n}_j, \tag{3}$$

where κ_n^j and κ_t^j are, respectively, the normal and tangential conductivity components for the joint and j indicates the referred joint set. The tangential component κ_t is classically associated with the hydraulic aperture (e^h) of the joint by some version of the cubic law, and the normal component κ_n is usually taken as equal to the rock matrix conductivity ($\kappa_n = \kappa_m$).

Considering a REV with joints as layers of small thickness, as shown in Figure 1, a localization problem can be stated by applying an uniform pressure gradient on the boundary $\partial\Omega$ of the REV, and the problem imposed to Ω writes:

$$\forall \underline{x} \in \begin{cases} \Omega: \ \mathrm{div}\, \underline{q}\,(\underline{x}) = 0 & \text{(a)} \\ \Omega: \ \underline{q} = -\underline{\underline{\kappa}}\,(\underline{x}) \cdot \underline{g} & \text{(b)} \\ \partial\Omega: \ p = \underline{g}\,(\underline{x}) \cdot \underline{x} & \text{(c)} \end{cases} . \tag{4}$$

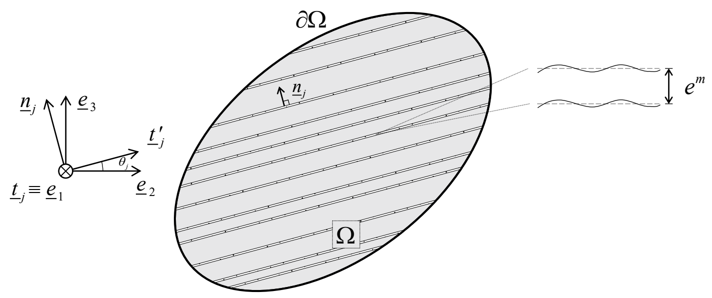

Figure 1. REV for a jointed rock

A classical micromechanics reasoning indicates that for an admissible microscopic pressure field p, solution of (4), the average rule applies[1]:

$$\underline{G} = \langle \underline{g} \rangle \,. \tag{5}$$

The macroscopic filtration velocity vector is defined as

$$\underline{Q} = \langle \underline{q} \rangle \tag{6}$$

and the continuity of the microscopic filtration velocity vector crosscutting any joint is expressed as $[\![\underline{q}]\!] \cdot \underline{n} = 0$.

A microscopic pressure gradient $\underline{g}\,(\underline{x}) = \underline{G}$ is solution of the localization problem if it satisfies (4) and $\underline{q}\,(\underline{x}) = -\underline{\underline{\kappa}}\,(\underline{x}) \cdot \underline{G}$ respects the continuity of $\underline{q}$ through the joints. This is true $\forall \underline{G}$ considering $\kappa_n = \kappa_m$ and, in this case, the relation between the microscopic and macroscopic Darcy's Law is given by:

$$\underline{Q} = \frac{1}{\mu} \underline{\underline{K}}'^{\,\mathrm{hom}} \cdot \underline{G}, \text{ with } \underline{\underline{K}}'^{\,\mathrm{hom}} = \mu \, \langle \underline{\underline{\kappa}} \rangle \,. \tag{7}$$

3 Hydromechanical Behavior of Joints

The application of the Navier-Stokes equations for laminar incompressible flow between two parallel smooth plates results in the following well known expression for the tangential conductivity component [10]:

$$\kappa_t = \frac{\left(e^h\right)^2}{12\mu}. \tag{8}$$

Experimental results presented by Barton [11] shows that values of e^h are, in fact, different from the measured mechanical opening e^m of the joints, given that natural fractures are dissimilar to the ideal parallel plates model. The author gives an empirical expression that takes into account the joint roughness in the relation between e^h and e^m, as follows:

$$e^h = \frac{\left(e^m\right)^2}{JRC^{2.5}}, \tag{9}$$

where *JRC* [dimensionless] is the joint roughness coefficient, easily estimated by tilt and Schmidt hammer tests [12]. In eq. (9) e^h and e^m are in µm.

The mechanical aperture, in turn, can be estimated by means of an linear relationship as:

[1] $\langle \cdot \rangle = \frac{1}{|\Omega|} \int_\Omega (\cdot) \, d\Omega$

$$e^m = e_0^m - [\![\xi_n]\!] \tag{10}$$

where e_0^m is the initial mechanical aperture existing *in situ* and $[\![\xi_n]\!]$ is the joint closure due to a normal load. The initial mechanical aperture can be estimated by the empirical relationship proposed by Barton and Bakhtar [13] as

$$e_0^m \approx \frac{JRC}{50}\left[2\left(\frac{\sigma_c}{JCS}\right) - 1\right], \tag{11}$$

where e_0^m is given in mm, σ_c [MPa] is the rock matrix compressive strength and *JCS* [MPa] is the joint wall compressive strength. The joint closure $[\![\xi_n]\!]$ is calculated following the model of Bandis et al. [14] as:

$$[\![\xi_n]\!] = \frac{\sigma}{\sigma/\xi_0 - k_{n,0}} \tag{12}$$

where σ [MPa] is the acting compressive stress, ξ_0 [m] is the maximum closure and $k_{n,0}$ [MPa/m] is the initial normal stiffness of the joints.

3.1 Illustrative Examples of Homogenized Hydromechanical Behavior

The anisotropy and non-linearity induced in the permeability by the presence of the joints is evidenced here.

• *Case* 1:

Consider the rock matrix crosscut by a single joint set within the $(\underline{e}_1, \underline{e}_2)$ plane, uniformly distributed with spacing d and under an uniaxial macroscopic compressive stress $\Sigma_0 \underline{e}_3 \otimes \underline{e}_3$, as shown in Figure 2(a). The volumetric fraction η of the joints is given by

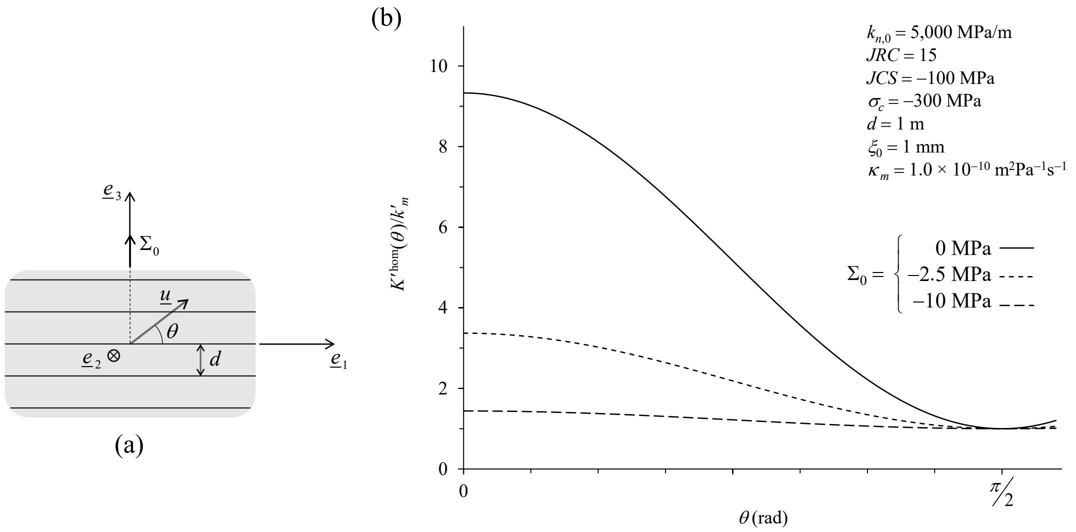

Figure 2. Permeability variation as a function of the considered direction for one joint set, considering the acting normal stress in the joints

$$\eta = \frac{e^m}{d}, \tag{13}$$

The homogenized permeability tensor (7) is then given by

$$\frac{\underline{\underline{K}}'^{\,\text{hom}}}{\mu} = \langle \underline{\underline{\kappa}} \rangle = \underline{\underline{\kappa}}_m + \eta \left(\kappa_t - \kappa_m\right) \left(\underline{\underline{1}} - \underline{e}_3 \otimes \underline{e}_3\right). \tag{14}$$

The permeability anisotropy of this jointed rock may be illustrated by calculating the homogenized permeability component in the direction $\underline{u} = \cos\theta \underline{e}_1 + \sin\theta \underline{e}_3$. This permeability component, function of the angle θ, is given by

$$\frac{K'^{\,\text{hom}}(\theta)}{\mu} = \underline{u} \cdot \left(\frac{\underline{\underline{K}}'^{\,\text{hom}}}{\mu} \right) \cdot \underline{u} = \kappa_m + \eta \left(\kappa_t - \kappa_m\right) \cos^2\theta. \tag{15}$$

Figure 2(b) shows the variation of the ratio $K'^{\,\text{hom}}(\theta)/k'_m$ with respect to the considered direction for different values of Σ_0 and the following input data: $k_{n,0} = 5{,}000$ MPa/m, *JRC* $= 15$, *JCS* $= -100$ MPa, $\sigma_c = -300$ MPa, $d = 1$ m, $\xi_0 = 1$ mm and $\kappa_m = 1.0 \times 10^{-10}$ m^{2}Pa^{-1}s^{-1} (considering viscosity of water $\mu_w = 9.97 \times 10^{-4}$ Pa.s at 295 K). The tangential conductivity κ_t is function of the normal stress level and may be calculated by use of eq. (8) to (12).

In Figure 2, a strong normal stress dependency for the permeability component in the direction of the joints ($\theta = 0$ rad) can be noticed, caused by use of the non-linear elastic law proposed by Bandis et al. [14], associating the joint closure to the acting normal stress. As expected, the ratio $K'^{\,\text{hom}}(\theta)/k'_m$ is maximum at $\theta = 0$ rad, given that the tangential joint permeability is orders of magnitude bigger than the matrix permeability, decreasing when θ increases until, for any normal stress, the permeability is the same as that of the matrix at $\theta = \pi/2$ rad.

The procedure can be generalized for any number M of joint sets considering the contribution of each set, summing their effects in the permeability tensor by rewriting eq. (14) as:

$$\frac{\underline{\underline{K}}'^{\,\text{hom}}}{\mu} = \langle \underline{\underline{\kappa}} \rangle = \underline{\underline{\kappa}}_m + \sum_{j=1}^{M} \left[\eta_j \left(\kappa_t^j - \kappa_m\right) \left(\underline{\underline{1}} - \underline{n}_j \otimes \underline{n}_j\right) \right]. \tag{16}$$

• *Case* 2:

Consider now two sets of joints inclined by 0 and $\pi/4$ rad, respectively, with respect to the $(\underline{e}_1, \underline{e}_2)$ plane, with identical uniaxial macroscopic compressive stresses and parameters for both two sets as the previous case (Figure 3(a)).

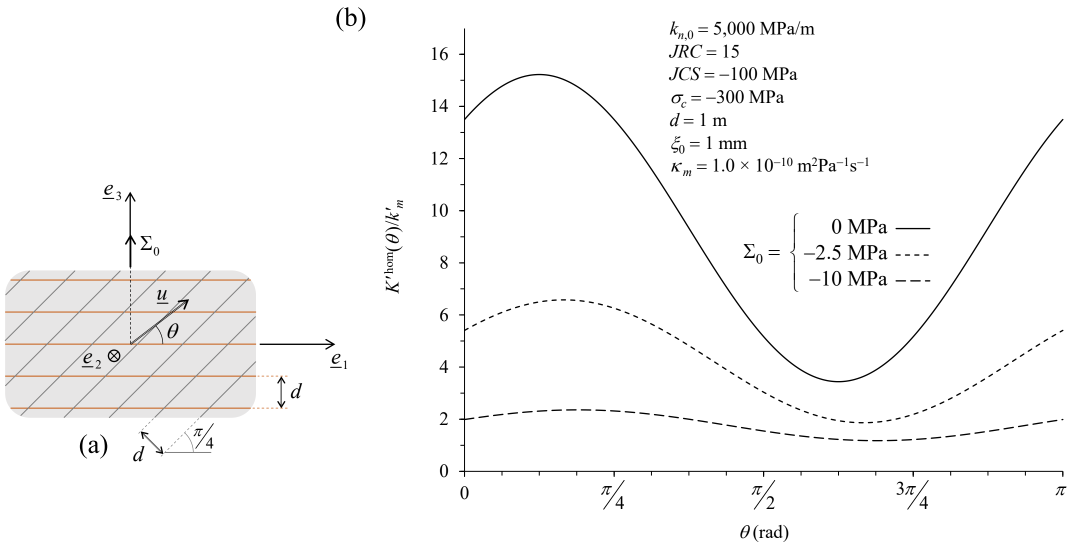

Figure 3. Permeability variation as a function of the considered direction for two joint sets considering the acting normal stress in each set

In Figure 3(b) the global increase in the permeability is evident, as well as the different effect of the normal stresses acting in each joint set. The effect of the different normal compressive stress ($\sigma_j = \underline{n}_j \cdot (\Sigma_0 \underline{e}_3 \otimes \underline{e}_3) \cdot \underline{n}_j$) acting on each set can be seen in the (slight) deviation in the maxima and minima values of $K'^{\,\mathrm{hom}}(\theta)/k'_m$ for each Σ_0. This results are in accordance with the common understanding that, the more fractured the rock mass, the bigger its permeability in any direction.

4 Conclusions

A micromechanical formulation for a preliminary determination of the macroscopic permeability tensor for jointed rock is presented. Most of the classic macroscopic approaches are based in expensive and time consuming laboratory and field testing programs, giving as result an unique average permeability coefficient and has as output that any rock mass is ultimately considered as isotropic with respect to its permeability. The micromechanical approach derived allows to take into account the anisotropic hydraulic behavior introduced by the presence of joint sets and their orientations.

The effects of normal stresses acting on the joints are considered by an hydromechanical coupling, linking the normal stress to the tangential joint permeability/conductivity coefficient using micromechanical reasoning together with the parallel plate model, empirical expressions and a non-linear elastic law. This stress dependency is evidenced by examples for one e two joint sets, which results are consistent with experimental and practical observations.

Besides, the present approach is suitable for use in numerical analyses. The quality and computational costs of numerical analyses are strongly dependent of their input data and numerical method used. An evident benefit of the present approach is the use in finite element analyses, for instance, where an homogeneous medium can be considered using the permeability tensor components as inputs to allow anisotropy consideration, instead of the explicit consideration of existing joint sets.

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References

[1] Warren, J. E. & Price, H. S., 1961. Flow in Heterogeneous Porous Media. *Soc. Pet. Eng. J.*, vol. 1, n. 03, pp. 153–169.

[2] Freeze, R. A. & Witherspoon, P. A., 1966. Theoretical analysis of regional groundwater flow: 1. Analytical and numerical solutions to the mathematical model. *Water Resour. Res.*, vol. 2, n. 4, pp. 641–656.

[3] Meyboom, P., 1966. Current trends in hydrogeology. *Earth-Science Rev.*, vol. 2, pp. 345–364.

[4] Pouya, A. & Courtois, A., 2002. Définition de la perméabilité équivalente des massifs fracturés par des méthodes d'homogénéisation. *Comptes Rendus Geoscience*, vol. 334, n. 13, pp. 975–979.

[5] Witherspoon, P. A., Wang, J. S. Y., Iwai, K., & Gale, J. E., 1980. Validity of Cubic Law for fluid flow in a deformable rock fracture. *Water Resources Research*, vol. 16, n. 6, pp. 1016–1024.

[6] Tsang, Y. W. & Witherspoon, P. A., 1981. Hydromechanical behavior of a deformable rock fracture subject to normal stress. *Journal of Geophysical Research*, vol. 86, n. B10, pp. 9287–9298.

[7] Ene, H. I. & Sanchez-Palencia, É., 1975. Équations et phénomènes de surface pour l'écoulement dans un modèle de milieu poreux. *Journal de Méchanique*, vol. 14, n. 1, pp. 73–108.

[8] Auriault, J.-L., 1987. Comportement des milieux poreux saturés.

[9] Zimmerman, R. W. & Bodvarsson, G. S., 1996. Hydraulic conductivity of rock fractures. *Transport in Porous Media*, vol. 23, n. 1, pp. 1–30.

[10] Benjelloun, Z. H., 1991. *Etude expérimentale et modélisation du comportement hydromécanique des joints rocheux*. Thesis, Université Joseph Fourier, Genoble.

[11] Barton, N. R., 1982. Modelling rock joint behavior from in situ block tests: implications for nuclear waste repository design. Technical Report March.

[12] Barton, N. R. & Choubey, V. D., 1977. The Shear Strength of Rock Joints in Theory and Practice. *Rock Mechanics*, vol. 10, pp. 1–54.

[13] Barton, N. R. & Bakhtar, K., 1983. Rock joint description and modelling for the hydrothermechanical design of nuclear waste repositories.

[14] Bandis, S. C., Lumsden, A. C., & Barton, N. R., 1983. Fundamentals of rock joint deformation. *International Journal of Rock Mechanics and Mining Sciences*, vol. 20, n. 6, pp. 249–268.