Universidade Federal do Rio Grande do Sul
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O Princípio da Incerteza entre Posição e Momentum na Teoria Quântica Não-Relativística: Gênese e Estabilização no Contexto Pedagógico-Científico

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Dissertação apresentada como requisito parcial à obtenção do título de mestra em Ensino de Física pelo Programa de Pós-graduação em Ensino de Física do Instituto de Física da Universidade Federal do Rio Grande do Sul, sob a orientação dos professores Dr. Nathan Willig Lima e Dr. Cláudio José de Holanda Cavalcanti

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RESUMO

Na presente dissertação, com objetivo de contribuir com o Ensino de Física Quântica, buscamos caracterizar a gênese, a tradução e a estabilização do Princípio da Incerteza entre variáveis posição e momentum na teoria quântica não-relativística nos artigos originais e em livros didáticos de mecânica quântica do ensino superior. Para isso, partimos de uma fundamentação teórico-metodológica centrada nos Estudos das Ciências de Bruno Latour e recorremos a elementos históricos e fontes primárias para fornecer profundidade e contextualização ao Princípio da Incerteza. Nesse contexto, a dissertação foi estruturada em três estudos independentes. No primeiro estudo, buscamos determinar quais características uma narrativa histórica deve apresentar para estar consonante com as premissas teórico-metodológicas da Sociologia da Tradução e, dessa forma, reconstruímos historicamente a gênese do Princípio da Incerteza. No segundo estudo, a partir de uma proposta didática inspirada em Max Jammer, apresentamos o Princípio da Incerteza em três níveis: formalização matemática, fenomenologia e interpretações. Dessa forma, reconstruímos as principais derivações do Princípio da Incerteza para variáveis posição e momento, as principais discussões fenomenológicas e interpretações do princípio, de forma estruturada para que possam ser utilizadas no contexto pedagógico. Por fim, no terceiro e último estudo, a partir da análise da apresentação do Princípio da Incerteza trazida por 34 livros didáticos de Física Quântica do Ensino Superior, mapeamos os principais actantes da rede que constitui o Princípio da Incerteza no contexto pedagógico-científico e, com o suporte da Teoria Ator-Rede associada a Análise das Redes-Sociais e o auxílio do *software R* e do algoritmo *ForceAtlas2*, foi possível identificar que alguns elementos do desenvolvimento histórico do Princípio da Incerteza são mais estáveis do que outros. Além disso, nossos resultados apontam que muitos livros abordam o Princípio da Incerteza de uma maneira muito semelhante, sendo uma abordagem muito parecida com aquela proposta pelo próprio Werner Heisenberg em 1927, o que resulta no apagamento de discussões e desenvolvimentos posteriores.

Palavras-Chave: Princípio da Incerteza, Ensino de Física, Estudos da Ciência, Física Quântica, Sociologia da Tradução.

ABSTRACT

In the present dissertation, with the objective of contributing to the Teaching of Quantum Physics, we seek to characterize the genesis, translation and stabilization of the Uncertainty Principle between the position and momentum variables in non-relativistic quantum theory in original articles and in quantum mechanics textbooks of higher education. For this, we start from a theoretical-methodological foundation centered on Bruno Latour's Science Studies and resort to historical elements and primary sources to give depth and contextualization to the Uncertainty Principle. In this context, the dissertation was structured in three independent studies. In the first study, we sought to determine which characteristics a historical narrative must present to be in line with the theoretical-methodological assumptions of the Sociology of Translation and, thus, we historically reconstruct the genesis of the Uncertainty Principle. In the second study, based on a didactic proposal inspired by Max Jammer, we present the Uncertainty Principle at three levels: mathematical formalization, phenomenology and interpretations. Thus, we reconstruct the main derivations of the Uncertainty Principle for position and moment variables, the main discussions and phenomenological interpretations of the principle, in a structured way so that they can be used in the pedagogical context. Finally, in the third and final study, based on the analysis of the presentation of the Uncertainty Principle brought by 34 Quantum Physics textbooks in Higher Education, we mapped the main actors of the network that constitutes the Uncertainty Principle in the pedagogical-scientific context and, with the support of the Actor-Network Theory associated with the Analysis of Social Networks and the help of the R software and the ForceAtlas2 algorithm, it was possible to identify that some elements of the historical development of the Uncertainty Principle are more stable than others. In addition, our results indicate that many books approach the Uncertainty Principle in a very similar way, being an approach very similar to that proposed by Werner Heisenberg himself in 1927, which results in the erasure of discussions and subsequent developments.

Key words: Uncertainty Principle, Teaching of Physics, Quantum Physics, Science Studies, Sociology of Translation.

SUMÁRIO

1 INTRODUÇÃO

Pelo menos desde as últimas décadas do século XX os estudos na área de Ensino de Ciências vêm apontando para a necessidade de incluir tópicos de Física Moderna e Contemporânea (FMC) na Educação Básica. Mais especificamente, tais estudos apontam para a necessidade de elaboração do ensino de Física Quântica no Ensino Médio – os motivos são variados e, dentre eles, podemos destacar que a escola é, muitas vezes, o último contato de grande parte da população brasileira com a educação formal; existe uma grande necessidade em formar novos cientistas; diversos fenômenos presentes nos avanços tecnológicos podem ser explicados pela Física Quântica (DA SILVA; DE ALMEIDA, 2011; LIMA; OSTERMANN; DE HOLANDA CAVALCANTI, 2017; MOREIRA; OSTERMANN, 2000). Entretanto, para que se possa, de fato, discutir as possíveis estratégias de inserção da Teoria Quântica na educação escolar é necessário olhar para a Teoria Quântica que é ensinada nos cursos de graduação em licenciatura em Física, pois a Física que é levada à Educação Básica está diretamente relacionada à Física que é aprendida no Ensino Superior pelos futuros professores de ciências (ROCHA; MOREIRA; HERSCOVITZ, 2010).

As pesquisas que tratam sobre o ensino da Teoria Quântica nos cursos de graduação fornecem indícios de que esse é um ensino tecnicista, centrado na demasiadamente na resolução de problemas e no desenvolvimento do formalismo matemático (JOHANSSON et al., 2018; ROCHA; MOREIRA; HERSCOVITZ, 2010; SOUZA et al., 2021), partindo de uma proposta pedagógica que é fortemente apoiada na pedagogia científica desenvolvida no período da Guerra Fria (KAISER, 2005). Esse tipo de abordagem, quando desvinculado das demais dimensões da teoria, além de negligenciar parte de toda a riqueza da Teoria Quântica, pouco contribui para que essa seja, posteriormente, levada às salas de aula da Educação Básica.

Uma possível maneira de integrar os diferentes aspectos da Teoria Quântica, aliando elementos matemáticos, filosóficos, fenomenológicos e conceituais, é retornar aos artigos originais. Embora não seja uma tarefa simples, uma vez que necessita de grande planejamento e elaboração por parte dos professores universitários (KARAM, 2021), esse tipo de abordagem possui grande potencial para contribuir com discussões mais profundas e frutíferas ao Ensino de Física (BATISTA, DRUMMOND, FREITAS, 2015).

Dessa forma, na presente dissertação recorremos a elementos históricos e a fontes primárias, para contribuir com o ensino de um tópico relevante da Teoria Quântica: o Princípio da Incerteza. Entendemos que, ao olhar para a história, podemos resgatar ideias originais, concepções filosóficas e conceituais, diferentes formalizações matemáticas e diferentes fatores que foram relevantes na concepção do Princípio da Incerteza, fornecendo profundidade e contextualização ao mesmo tempo, o que ajuda a romper com o ensino tecnicista e a enfatizar, também, a complexidade do empreendimento científico.

A dissertação foi estruturada na forma de três artigos, sendo que os dois primeiros já estão publicados e o terceiro em processo de submissão. Devido ao formato escolhido para a dissertação, ao final de cada capítulo estão dispostas, respectivamente, suas referências.

Ao longo de todo o trabalho, a fundamentação epistemológica está centrada nas discussões sobre ensino de ciências a partir dos Estudos de Ciência (*Science Studies)* (LIMA et al., 2019; LIMA; OSTERMANN; CAVALCANTI, 2018; VAZATA et al., 2020), presente em maior ou menor grau em cada um dos artigos. O primeiro estudo discute, principalmente, contribuições da Sociologia da Ciência para a historiografia da Física, e faz um resgate histórico do contexto e dos acontecimentos de 1927, com o objetivo de caracterizar a gênese do Princípio da Incerteza. Esse artigo foi publicado na revista *Transversal: International Journal of History of Science* (LIMA; ROSA; BENTO, 2020). O segundo estudo possui um caráter didático, em que, inspirados por Max Jammer (JAMMER, 1974), apresentamos o Princípio da Incerteza em três níveis (formalismo matemático, fenomenologia e interpretações), resgatando algumas propostas de derivações históricas das relações de incerteza, dois experimentos mentais pensados por Werner Heisenberg no artigo de 1927 e algumas interpretações do Princípio da Incerteza apontadas pela literatura (JIJNASU, 2016). Esse artigo foi publicado na Revista Brasileira de Ensino de Física (ROSA; LIMA; CAVALCANTI, 2022). Por fim, o terceiro e último estudo busca analisar as representações do Princípio da Incerteza presente em 34[1] livros didáticos de Física Quântica do ensino superior a partir da teoria ator-rede, com o objetivo de identificar quais de seus aspectos foram mais ou menos estabilizados no contexto científico-pedagógico.

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[1] Escolhidos por seu protagonismo no plano de ensino de disciplinas introdutórias de Física Quântica de diversos cursos de graduação de instituições de ensino superior brasileiras.

De uma forma geral, podemos entender a estrutura dessa dissertação a partir da enunciação de um objetivo geral, quatro objetivos específicos, e de quatro questões de pesquisa. Esses elementos são apresentados na sequência.

1.1 OBJETIVOS

Podemos sintetizar os objetivos dessa pesquisa em um objetivo geral e três objetivos específicos

1.1.1 Objetivo Geral

Caracterizar a gênese, a tradução e a estabilização do Princípio da Incerteza entre variáveis posição e momentum na teoria quântica não-relativística nos artigos originais e em livros didáticos de mecânica quântica do ensino superior

1.1.2 Objetivos Específicos

I. Determinar quais características uma narrativa histórica deve ter para estar consonante com as premissas teórico-metodológicas da Sociologia da Tradução;
II. Reconstruir historicamente a gênese do Princípio da Incerteza à luz da Sociologia da Tradução;
III. Reconsturir as principais derivações do princípio da incerteza para variáveis posição e momento, as principais discussões fenomenológicas e interpretações do princípio de forma estruturada para serem apresentadas no contexto pedagógico.
IV. Mapear quais elementos do desenvolvimento histórico do Princípio da Incerteza foram mantidos, quais elementos foram apagados e quais elementos foram transformados nos livros didáticos de ensino superior.

1.2 QUESTÕES DE PESQUISA

As questões que nortearam esse estudo foram:

a) Quais significados foram atribuídos ao Princípio da Incerteza originalmente por Heisenberg em 1927? Como ele justifica sua articulação e os significados atribuídos a ele?
b) Quais são as disputas entre os atores envolvidos dentro da gênese do Princípio da Incerteza?
c) Como o Princípio da Incerteza foi estabilizado (ou não) no contexto científico? Quais elementos são trazidos em sua demonstração e justificação? Quais significados são atribuídos a ele?
d) Como os livros didáticos traduzem os diferentes elementos envolvidos no Princípio da Incerteza? Ou seja, quais elementos encontrados nos artigos originais são apagados, quais são adicionais e quais são transformados? O que tais processos de tradução nos informam sobre o contexto pedagógico-científico?

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2 TRAJETÓRIA DE PESQUISA

Uma mesma narrativa histórica pode ser contada a partir de diferentes tradições historiográficas que, por sua vez, possuem diferentes fundamentações filosóficas e, quando se trata da História da Ciência, cada tradição propagará, também, um determinado posicionamento epistemológico (VIDEIRA, 2007). Por exemplo, tradições historiográficas centradas em uma filosofia positivista tendem a favorecer visões lineares sobre o fazer científico, tratando a Ciência como algo pronto e acabado, e reforçando o mito dos gênios que descobrem as verdades da natureza. Esse tipo de narrativa, que foi predominante na História da Ciência por muito tempo, contribuiu imensamente para a propação de visões pouco realistas do empreendimento científico (CHALMERS, 1993). Tendo em vista que a presente dissertação utiliza de elementos históricos para contribuir com o Ensino de Física, iniciamos nossa trajetória de pesquisa determinando as principais características que uma narrativa histórica deve apresentar para ser condizente com as premissas teórico-metodológicas da Sociologia da Tradução – que subsidia os artigos desse trabalho. Dessa forma, em nosso primeiro estudo, reconstruímos historicamente a gênese do Princípio da Incerteza a partir do que chamamos de História Simétrica.

Partindo dos preceitos da Antropologia Simétrica de Bruno Latour (2001), seguimos uma narrativa em que humanos e não-humanos compartilham a ação, considerando ambos como atores da história. Tais os atores não existem de maneira independente, mas são resultado do estabelecimento de redes a partir da articulação de outros atores, de modo que, no empreendimento científico, cabe ao cientista o papel de manipular elementos para tornar real um novo ator. Nesse contexto, todos os enunciados são válidos em redes específicas. Entretanto, ao comparar as redes mobilizadas por cada um dos diferentes atores, é possível hierarquizá-las, resultando na existência de proposições mais reais que outras, conforme mobilizem mais atores. Ademais, na História Simétrica, elementos internos e externos à ciência se misturam, de modo que fenômenos naturais, políticos e sociais contribuem para a estabilização (ou não) de determinado ator ou fato científico.

A partir da História Simétrica pudemos olhar para a história e, por meio dela, identificar os diferentes atores que foram mobilizados por Werner Heisenberg em ao propor o Princípio da Incerteza, resgatando, dessa forma, suas ideias originais, concepções filosóficas e conceituais, as formalizações matemáticas e os demais fatores que foram relevantes para a concepção do Princípio da Incerteza.

Com o primeiro estudo, ficou evidente que, quando o Princípio da Incerteza foi proposto, em 1927, a Física passava por um período de intensa atividade científica e, mais especificamente, a Mecânica Matricial, inaugurada em 1925 por Werner Heisenberg, e a Mecânica Ondulatória, proposta em 1926 por Erwin Schrödinger, disputavam pela interpretação da Teoria Quântica (JAMMER, 1966, 1974). Embora os dois programas de pesquisa fossem matematicamente equivalentes, tal como demonstrado pelo próprio Schrödinger, possuíam interpretações físicas da realidade incompatíveis – enquanto a Mecânica Matricial estava essencialmente conectada com os saltos quânticos, a descontinuidade e uma visão puramente corpuscular da matéria, a Mecânica Ondulatória sustentava que, tal como a radiação eletromagnética, os elétrons também poderiam ser descritos por meio de fenômenos ondulatórios – de modo que suas divergências renderam profundos debates filosóficos e conceituais (HEISENBERG, 1996, 2000; JAMMER, 1966, 1974). Nesse contexto, o Princípio da Incerteza surge em 1927 quando Werner Heisenberg, com o objetivo de defender a Mecânica Matricial e sua visão de mundo puramente corpuscular, articula diversos actantes, incluindo formalismo matemático, experimentos mentais e interpretações, e se torna o porta-voz desse novo conceito físico. Ao longo do texto, abordamos em detalhes algumas das principais controvérsias relacionadas ao artigo de 1927.

Por outro lado, diferente do primeiro estudo que teve grande ênfase no quadro teórico da Sociologia da Tradução e na gênese do Princípio da Incerteza, no segundo artigo buscamos caracterizar didaticamente o Princípio da Incerteza sem explorar os conceitos de nossa fundamentação teórica – mas sempre partindo da premissa de que as derivações matemáticas, as interpretações e os experimentos são actantes na rede que compõe um conceito físico. Dessa forma, no segundo estudo aportamos nossa abordagem na proposta de Max Jammer (1974) sobre os níveis de uma teoria e sugerimos que um conceito físico seja tratado em três níveis: formalismo matemático, fenomenologia e interpretações.

No primeiro nível, utilizando uma abordagem didática em que cada operação e manipulação matemática são explicitadas, resgatamos quatro derivações do Princípio da Incerteza baseadas em textos históricos – contemplando as derivações de Heisenberg e Weyl para as variáveis posição e momentum, e as derivações de Robertson e Schrödinger para dois observáveis quaisquer A e B. No segundo nível discutimos em detalhes dois experimentos mentais propostos por Werner Heisenberg, conhecidos como “o microscópio de raios-gama” e “o experimento do elétron passando por uma fenda”, tal discussão é realizada com o intuito de demonstrar a fenomenologia associada ao Princípio da Incerteza. Por fim, no terceiro nível retomamos quatro possíveis interpretações propostas pela literatura para o Princípio da Incerteza nas variáveis posição e momento (JIJNASU, 2016), incluindo duas interpretações que definimos como “ônticas”, pois percebem a incerteza como uma propriedade inerente da matéria, e duas interpretações que definimos como “epistêmicas”, pois percebem a incerteza como uma limitação referente ao processo de medida, ao nosso conhecimento das coisas. De modo geral, o segundo artigo possui, principalmente, a função de ser uma ferramenta auxiliar ao ensino da Física Quântica em que buscamos explicitar a complexidade do Princípio da Incerteza nas variáveis posição e momentum, de modo que já nesse estudo é possível perceber a pluralidade de actantes presentes na rede do Princípio da Incerteza.

Por fim, o terceiro e último estudo possui o intuito de contribuir com a comunidade de pesquisa na área de Educação em Ciências investigando como os diferentes livros didáticos fazem a transposição de um mesmo tópico científico, nesse caso, o Princípio da Incerteza. De fato, os livros didáticos desempenham um papel central na educação científica, sendo grandes responsáveis pela iniciação de novos cientistas e professores de ciências no contexto de suas disciplinas (KUHN, 2017). Ademais, existe um senso comum de que os livros didáticos difundem apenas fatos científicos – ou seja, aquilo que já foi consolidado no contexto científico e que não é mais fruto de investigações ou disputas – constituindo, dessa forma, um corpo único de conhecimentos. Dessa forma, com o objetivo de investigar como o Princípio da Incerteza foi traduzido e estabilizado no contexto científico-pedagógico e como os livros didáticos podem atuar nesse processo de estabilização, buscamos mapear suas representações presentes em 34 livros didáticos de Física Quântica do Ensino Superior.

Ao analisar os trechos destinados ao Princípio da Incerteza em cada um dos livros didáticos escolhidos, identificamos 43 aspectos que foram dispostos em 6 diferentes categorias – incluindo posição na estrutura dos livros, variáveis físicas, derivação, formalismo matemático adotado, experimentos mentais utilizados para abordar o conceito, interpretação dada à incerteza nas variáveis posição e momentum e outros elementos (tal como discussão filosófica ou conexão com o Princípio da

Complementaridade). A partir desses dados, criamos uma tabela binária em que dispomos cada aspecto presente em cada um dos livros e, partindo da Teoria Ator-Rede associada à Análise das Redes Sociais, e com o auxílio do *software R* (R CORE TEAM, 2015) e do algoritmo *ForceAtlas2* (KLOCKIEWICZ; ALVAREZ, 2015), foi possível fazer a análise estatística e a interpretação dos resultados.

Primeiramente, devido à ampla variedade de atores mobilizados na rede do Princípio da Incerteza, tornou-se evidente a essência plural desse actante. Além disso, também foi possível constatar que alguns atores estão mais presentes nos livros didáticos do que outros, de modo que é possível percebê-los como mais estáveis – tais como a incerteza nas variáveis posição-momento e tempo-Energia e o experimento mental do microscópio de raios-gama. No que se refere ao papel dos livros didáticos, foi possível identificar que muitos livros compartilham conteúdos muito parecidos, remetendo à existência de certo consenso sobre o Princípio da Incerteza. Além de outras inferências feitas ao longo do estudo, percebemos, também, grande potencialidade em aliar a análise das redes à análise metalinguística de Bakhtin.

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3 PRIMEIRO ARTIGO ORIGINAL

TRANSLATIONS, BETRAYALS AND CONTROVERSIES IN THE ARTICULATION OF THE UNCERTAINTY PRINCIPLE: POTENTIALITIES AND CHALLENGES OF A SYMMETRICAL HISTORY OF PHYSICS

3.1 ABSTRACT

In this paper, we discuss the potentialities and challenges of historical approaches related to the Symmetrical Anthropology proposed by Bruno Latour and collaborators. To accomplish this goal, first, we provide a brief account about how Sociology and Anthropology of Science evolved, stressing how these different movements correlate with historiographical approaches. Second, we introduce the metaphysical scheme of Symmetrical Anthropology and discuss which characteristics a historical narrative should have to be consistent with this world vision. Third, we briefly describe the articulation of the Uncertainty Principle focusing on appropriating such characteristics. Based on this concrete historical account, we discuss the potentialities and challenges of this approach to History of Physics.

3.2 INTRODUCTION

As any other utterance, historical narratives cannot be understood in isolation, since they are committed to different values, world views, and, in the case of the history of science, to different conceptions about nature of science. Hence, we may say that the different historical approaches always correlate with the various disciplines of their time (such as sociology, philosophy, anthropology, and sciences themselves). In this sense, the historiographic work consists of making this correlation explicit. As Videira (2007, p.127) outlines, historiography should be a critical discourse that reveals, to the greatest extent, the epistemological, historical, political, and axiological roots on which historical discourses are built. In other words, historiography reveals the relation between the historical approach and different world views.

In the present work, we assess these relations, chiefly addressing that between History of Science (and, more specifically, History of Physics) and

Sociology/Anthropology of Science. It is important to mention that there are many studies that propose a Sociology of Physics, which, of course, has important implications to History of Physics (Reyes Galindo, 2011). Nonetheless, our main goal is to discuss the potentialities and the challenges of historical approaches that are, at some level, committed to the worldview that underlies what Bruno Latour (1993) calls Symmetrical Anthropology[2] – which has not been much explored in the field of History of Physics. We will call such historical approaches as Symmetrical History[3].

In order to follow Videira's recommendation, we briefly discuss different possibilities of History of Science according to its possible relations with Sociology and Anthropology of Science, following Latour's (1993) reasoning. In the sequence, we introduce Latour's world view (Latour 1993; 1999d; Latour et al. 2012; Latour 2016; 1988b; 1999e; 1988a; 2005), which claims to be rooted in a different metaphysical formulation when compared to previous sociological trends. Then, to make the potentialities and challenges of the approach clearer, we introduce a symmetrical history account of the articulation of the Uncertainty Principle[4]. And, finally, we present our final remarks.

3.3 THE HISTORY OF SYMMETRICAL HISTORY

The presentation of historical narratives about science and physics had taken place through their whole development process, even though History of Science was only constituted as an autonomous discipline in the 20th century (Kragh, 1987). In the beginning, the production of historical accounts had the purpose of contributing to the stabilization of science as a valid tradition in the pursue of truth, quarrelling with religion and philosophy (Videira, 2007). The positivist doctrine proposed by Augusto Comte in the 19th century, for instance, suggested a linear conception of science progress, which unavoidably runs into a final point, which is the contemporary knowledge (Comte 1830).

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[2] Latour uses this term in the essay We have never been modern (Latour, 1993).

[3] As we will discuss, the worldview defended in this essay is consistent with further propositions of the author such as the Actor-Network Theory (Latour, 2005).

[4] The articulation of the Uncertainty Principle, first presented in a paper written by Heisenberg in 1927, is addressed in several different works (Tanona, 2004; Camilleri, 2007; M. S. Longair, 1984; Jammer, 1966; Jijnasu, 2016). Thus, we do not aim to claim any historical novelty, but rather the opposite, that is to explore the potentiality of looking to a known historical event with a new metaphysical worldview.

According to Foucault (1979), this kind of history is concerned with the study of origins (Ursprung). In the origin resides the conception of the thing-in-itself (before any accident or distortion), the essence and the truth. In the Positivist History, to search for the origins is to search for the seed that ultimately and unavoidably will lead to our present knowledge; is to show the solid ground where contemporary conceptions stand upon.

So, this first era of historical approaches can be characterized by the narratives of scientists and epistemologists in defense of a specific conception of nature of science (Videira, 2007; Alfonso-Goldfarb, 1994). In this perspective, the progress of science is explained by some natural element, such as the discovery of truth or of an essence. We may call these descriptions as “Epistemological History of Science”[5]. They relate to Sociology and Anthropology by denying their role in characterizing scientific knowledge.

The structural changes that took place in the industrial revolution in the end of the 19th century corroborated the conception of science as the source of economic growth and social welfare, reinforcing the myth of linear progress (Auler and Delizoicov, 2001). It seems to be a consensus, however, that confidence on science and on its capability of promoting social well-being was drastically called into question after the World War II (Lopes, 2013), a scenario that allowed the rise of more critical accounts of the history of science, being pursued by historians, sociologists, and anthropologists (Lightman, 2016). Latour (1993) describes this new era of sociological and anthropological accounts on science as disputed by two different movements: the critique and the deconstruction[6].

Following Latour (1993), we may call “Critical History of Science” those historical accounts that explain science progress by mobilizing only elements from society (and no longer from nature). Shapin & Schaffer (1985) and Boris Hessen (2009)[7] exemplify

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[5] Bruno Latour (1993) discusses Bachelard’s description of science to characterize epistemology as the discipline that explains scientific knowledge using natural elements only. Although proposed in a different period, Lakatos’ (1978) rational reconstruction also could be included in this sort of historiography.

[6] Videira (2007) speaks about a post-positivist period in History of Science – from 1945 to 1970 – in which Thomas Kuhn’s Structure of Scientific Revolutions (Kuhn 1996) plays an important role – and a post-modernist period, from 1970 on. It is possible to make a parallel between the post-positivist history and what Latour calls “critique” and post-modernist history and “deconstruction”, although these categories do not fit completely.

[7] Originally delivered in 1931 at the Second International Congress of the History of Science in London, it is prior to the 1945 turning point of History of Science (Videira 2007). Despite of that, its proposal embodies the spirit of what Latour calls critique.

this sort of historiography. Some premises of such descriptions were synthetized in the Strong Programme of Sociology (SPS) (Bloor, 1991). Particularly, the program explores social causes for the "success" and "failures" of science instead of the asymmetrical description of the "Epistemological History", in which natural causes explain the success and social causes explain the failures.[8]

As Latour (1993) points out, however, the SPS deconstructs nature as the source of truth, but it still reifies social structures. If the natural essences are not objective and intrinsically real for the SPS, the social structures are.[9] In this sense, Latour (1993) claims that it is not possible to say that SPS is fully symmetrical.

On the other hand, the "deconstruction movement" and consequently what we may call "Deconstructive History" went further, dissolving not only nature but also society – reducing reality to games of language and power (Latour, 1993). Although Derrida (1997) is often mentioned as the leading figure of deconstructivism, a clear example of "Deconstructive History" can be found in Foucault's (1979a; 1979b) discussion on the relation between truth and power and his proposition of genealogical studies. More specifically, Foucault (1979a) proposes genealogy as a study opposed to the search of origins (Ursprung).

Instead of adopting the teleological perspective of the positivist history – the supra-historic standpoint from which is possible to analyze history, the genealogist is committed to highlighting the singularities and specificities of each event. Thus, genealogy is devoted to the accidents and not the essences.

Therefore, genealogists do not search for the Ursprung (the thing-in-itself, the essence and the truth): what they search for is the Herkunfut (provenance) and the Entestehung (emergence). While Herkunft is associated to singularities of events, as well as accidents and distortions, Entestehung is associated to the dispute, conflict, and shock of forces.

Emergence is only produced in a determined state of powers. In this sense, genealogy does not provide any certainty, neither it shows us that science was created on a solid foundation – on the contrary, it stresses the lack of any foundations, rationality or stability as the characteristic of events, which always are singular.

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[8] For instance, the "Epistemological History" explains geocentrism by saying that it was grounded in religious tradition while heliocentrism was allowed by the discovery of the true system. In other hand, "Critical History" would explain both movements through social causes, as the adoption of a specific religious view.

[9] Latour's critiques on SSP were challenged by Bloor (1999) and then, reaffirmed by Latour (1999b).

In summary, Epistemological History describes science as something independent of society. On the other hand, Critical History ascribes to social structures the source of scientific progress (its successes and failures) and Deconstructive History gives up any attempt to provide solid ground for scientific endeavors. Instead of deciding which claim is true, Latour (2016) proposes to take the controversy of the different approaches as the object of study and to explain how this was possible in the first place. The objective of Latour's (2016) historical accounts is to show how something that was politically disputed, that depended on the social affairs and that was constructed upon accidents and mistakes, in the end rises as objective and natural.

In order to do so, it is not possible to be committed with the metaphysical perspective of the previous historiographical trends (Latour, 1999d). It is necessary to adopt another posture about the relation between nature and society – what can be called fully symmetrical perspective (Latour, 1993). This new metaphysical perspective was built up by Latour and collaborators through decades and is still object of philosophical construction (Harman, 2009). It is chiefly grounded in Sartres's Existentialism (Sartre, 2007), Callon's Sociology of Translation (Callon, 1984), Whitehead's Philosophy of Propositions (Whitehead, 1978) and Tarde's Monadology (Tarde, 2007).

3.4 SOCIOLOGY OF TRANSLATION, PHILOSOPHY OF PROPOSITIONS AND ACTOR-NETWORK THEORY: A MONADOLOGICAL PERSPECTIVE TO DESCRIBE HISTORY OF SCIENCE SYMMETRICALLY

According to Latour (1993), the modernist period is an attempt to forge an absolute separation between nature and society, as what we observe in Kant's (2005) philosophy. The "Epistemology" is firmly grounded in this ontological scheme. Despite of that, when we look to laboratories and historical primary sources, what we find is the process of intense hybridization of natural and social elements in what Latour (1993) calls quasi-objects (or hybrids).

On the other hand, the Critique dissolves nature while sustaining society as an ontological pole, and Deconstruction dissolves everything. As we have pointed out, however, although the Positivist History seems not to resist to an accurate and deep analysis of the primary sources, it seems that scientific knowledge at some point resists to human volition and subjectivity. Otherwise, in the middle of a pandemic, should we consider scientific orientations only as an effect of discourse?

In order to provide an alternative description of scientific knowledge and progress, one that is epistemic but not only epistemic, sociological but not only sociological, and discursive but not only discursive, Latour starts from Sartre's (2007) existentialism, according to which the rejection of the conception of God in contemporary philosophy implies that human nature has no essence. Humans were not created to be something, so they do not have a pre-existing essence – they produce and stabilize their essence along their lives.

What Latour (1993) proposes with Symmetrical Anthropology is to extend Sartre's conception to all non-humans, to all quasi-objects: their essence is not something preexisting too, but something to be stabilized along time. In this sense, the scientific practice does not discover nature, but it creates nature and makes it stable. There is a particularly important but subtle element in this perspective: Symmetrical Anthropology is also non-essentialist as Deconstruction, but it emphasizes the capacity that different actors[10] have of creating and stabilizing essences. In this sense, its focus is on construction and association and not on destruction.

This conception has direct impact on how history is told: one should not look for objective pre-existing beings that meet with each other. On the other hand, what one seeks is the articulation of actors, whose essence is not objective and immutable. One aims to explain how these new articulations change their essence and create new actors. In the end, nature and society are created and stabilized by the practices of the actors and not the contrary (this is the key feature of Actor-Network Theory). One way of describing such articulations is through the terminology of the Sociology of Translation, which is based on three principles: agnosticism, generalized symmetry, and free association (the abandonment of all a priori distinctions between the natural and the social) (Callon, 1984, p. 196).

The generalized symmetry and the free association principles lead us to propose that humans and non-humans must share agency along history (Latour, 1999a). In this sense, nonhumans are not material objects waiting to be used by

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[10] Actor is defined in the following sense: "Instead of starting with entities that are already components of the world, science studies focus on the complex and controversial nature of what it is for an actor to come into existence. The key is to define the actor by what it does-its performances under laboratory trials" (Latour, 1999d, 303).

humans, as they also change the course of human actions. When scientists speak, they are proposing something that was constructed in the articulation of humans and non-humans. Every time this happens, the result is a process of “translation”, which is never the simple combination of the original programs of action, but rather something new: *“In place of a rigid opposition between context and content, chains of translation refer to the work through which actors modify, displace, and translate their various and contradictory interests”* (Latour, 1999d, p.311).

In the process of translations, the scientist, in particular, may assume the role of spokesman – representing all non-humans (which cannot speak) in the same way political representatives speak in the name of an assembly of humans that would not be listened if they were to speak all at the same time (Latour, 1993). This translation always involves uncertainties, and, sometimes, the spokesman may betray the group (Callon, 1984).

This non-essentialist and symmetrical perspective was adopted by Latour (1999c) to organize a historical account on Pasteur’s work on fermentation. In this case, Pasteur’s work resulted in the existence of a new actant – the yeast. This process of coming into existence by the mediation and translation is called by Latour as articulation, which derives from Whitehead’s Philosophy of Proposition (Whitehead, 1978). According to this perspective, each actor may be recognized as a proposition, which only exists by the articulation of other propositions. It is important to note, however, that when Pasteur mobilizes equipment, theories and samples in his laboratory trials, the microbes become articulated by all these propositions and they become independent of Pasteur himself (it is not subjective anymore). That is why Epistemology, Critique and Deconstruction are at some point right: all of them emphasize different dimensions of the same process.

Therefore, Symmetrical Anthropology (and, as a consequence, what we call Symmetrical History) attempts to explain how the whole collective of humans and nonhumans come into existence and how they change over time. All actors (humans and nonhumans) have agency, they transform reality and impact other actors’ agency as well as they are transformed and have their agency impacted by other actors. This metaphysical perspective, however, is not new in Sociology (Latour et al. 2012; Latour, 2001). Gabriel Tarde (1843-1904) proposed a monadological sociology, defending that the use of the concept of monad (minimum element, whose existence is actually sustained by the relations with other monads) was crucial to sociology (Tarde 2007).

According to Tarde (2007), science does not owe its progress to the adoption of a positivist perspective, but to the search of monads – such as atoms, molecules and cells.

3.5 A SYMMETRICAL HISTORY OF PHYSICS: METHODOLOGICAL CONSIDERATIONS

From the Symmetrical Anthropology and its metaphysical scheme, we propose six characteristics to Symmetrical History. These characteristics should not be understood as rules, but a translation of the metaphysical perspective. It is very important to highlight that for an account to be a Symmetrical History it does not have to use Latour's concepts explicitly but only to be consistent with the metaphysical perspective (Latour, 2005). In other words, it does not need to speak about quasi-objects, actors, and so on. Certainly, if during the process the necessity of mobilizing a specific concept rises, it is possible to use it, yet it is not necessary. Accordingly, Symmetrical History is the one that shows the following characteristics:

a) The history moves toward the stabilization of nature and society. Symmetrical History reveals how elements of nature and social structures were articulated and stabilized after a controversy. We may describe the provenance and emergence – which reveal the singularities of the event. However, we must highlight what makes nature and society stable.

b) Non-humans have agency: Instead of telling a history in which humans use objects to make history, Symmetrical History observes how humans and non-humans articulate, mediate, and translate each other. Of course, the scientist plays the role of spokesman, but again their will is affected by non-humans' agency at some point.

c) Actors do not exist independently – they are the articulation of other actors. Instead of considering "reality" a binary property, Symmetrical History acknowledges reality as a continuous spectrum. An actant exists according to the number and stability of associations of its network. In this sense, Symmetrical History is non-essentialist. In Symmetrical History, one shows the work of the scientist to mobilize elements to make a new actor real.

d) Knowledge and Belief are symmetrical: All statements are valid in a specific network, in a specific set of propositions – what Latour calls space-time envelope (Latour, 1999d).

e) It is possible to hierarchize propositions: Although there is not any essential difference between knowledge and belief, in a certain spacetime envelope, it is possible to compare the networks mobilized by different actors. In this sense, it is possible to hierarchize propositions. There are propositions that exist more than others, and as a consequence some statements are truer than others.

f) Interior and exterior of science are mixed: Instead of separating the social from the natural, Symmetrical History deals with collective of humans and non-humans. Thus, it is not possible to separate interior from exterior of science, ontology from epistemology and epistemology from politics (Latour, 1999d). This does not mean that all these aspects must be present all the time, but rather that they may be (Latour, 2005).

g) Actors must speak: Instead of projecting a priori categories onto the history or aiming to find the “real history”, one should focus on listening and reporting the actor’s own narratives. In this sense, the symmetrical history is always based on narratives of the own protagonists, with all subjectivities and controversies that this can bring about. We do not expect to mirror “historic reality”, but to articulate what Latour (2005) calls a risky account. By bringing more points of view into the account, one makes it more stable.

The six characteristics that we propose for Symmetrical History may also be found in two of Latour’s historical studies on Physics (Latour, 1988a; 1999f). When we think about Physics, however, some specificities that were not addresses by Latour may appear. Specially, there are theoretical works on Physics that do not deal with any laboratory experiment, which is a key element of Latour’s description. In this case, a different sort of actor seems to play an important role: the “mathematical actor”. Mathematical symbols in theoretical Physics may be considered actors in the same way as laboratory equipment is. And mathematical manipulations should be like laboratory experiment. In this sense, we claim that mathematical symbols should be treated as any other non-human for Symmetrical History. Although Latour does not discuss this issue, the reader can find a wide literature about the interplay between Physics and Mathematics (Ferreira and Silva, 2020; Lützen, 2013; Paty, 2003).

In the next section, we will propose a symmetrical discussion on the articulation of the Uncertainty Principle, chiefly discussing Heisenberg's (1927) paper.[11] We intend to discuss how it was possible to pass from a deterministic world to a world of indeterminacies in 1927.[12] In order to achieve such a goal, we will try to answer the following questions: what were the associations necessary for the Uncertainty Principle to come into existence? What were the translations? Which were the variations of meaning and agency? Were there betrayals? What was Heisenberg's program of action? Has he succeeded or failed?

3.6 THE ARTICULATION OF THE UNCERTAINTY PRINCIPLE: TRANSLATIONS AND BETRAYALS[13]

3.6.1 Emergence (Entestehung) of the Uncertainty Principle

The beginning of the 20th century was colored by the intense proliferation of new actants (such as the quantum, the atom, the wave function, and so on) and of new principles legislating the behavior of this new "nature", such as the Ehrenfest's Adiabatic Principle and Bohr's Correspondence Principle (Jammer 1966). Although many movements and actors can be mentioned, two special programs of action are important to be highlighted. Leaded by Albert Einstein, the first one promoted the study of radiation from the perspective of thermodynamics and Statistical Mechanics (Klein, 1967), leading to the articulation of a corpuscular radiation in 1905 and of a dual radiation in 1909, based on a strong realist perspective and a deep sense of unification.

In another direction, we may observe the studies about the structure of matter and, more specifically, the development of atomic models. In this scenario, Niels Bohr may be mentioned as one of the exponents (Kragh, 2012). The physicist developed

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[11] We will follow the English translation (Heisenberg, 1983a). We also address a comment made by Heisenberg (1983b) in 1967. It is important to stress that it is far known that, after World War II, Heisenberg aimed to present himself in a nice picture (Howard, 2004). Thus, this narrative should be considered a risky account – in which the interests and conceptions of Heisenberg (the spokesman) are already hybridized with the primary sources. If we were interested about discovering the consistencies of Heisenberg's narrative, we should search for other spokesmen and to confront their accounts – which could be object of another study.

[12] There is a distinction between uncertainty and indeterminacy. The first refers to fluctuations associated to a measurement while the second refers to something that is intrinsic in nature (Jammer, 1966). As we will discuss, Heisenberg's 'Uncertainty Principle' led to an indeterminate worldview.

[13] As we will discuss in the final remarks, along the text we overemphasized some categories to highlight the potentialities and challenges of this historiographic trend. In a usual historic presentation these categories would not need to be discussed explicitly.

another culture of approaching Physics, as he had a different philosophical background when compared to Einstein. He was much influenced by William James' pragmatism and by Harald Høffding's studies on Søren Kierkegaard (Jammer, 1966). Many of his ideas were directed toward the possibility of blurring the boundaries between the subject and the object – the distinct and non-accessible Kant's ontological poles –, a concept that plays an important role in the Copenhagen Interpretation (Heisenberg, 1958). The development of Bohr's 'astronomical' model for the atom motivated a revival of the interest for mathematical methods used in Astronomy (such as in the Celestial Mechanics written by Laplace in the beginning of the 19th century), which could be used in the description of matter.

In 1925, Werner Heisenberg produced a paper in which he adopted the mathematical formalism coming from the studies on celestial mechanics to inaugurate what would become Quantum Mechanics. Heisenberg was committed not only to a determined way of practicing Physics, but also to a specific Philosophy of Physics, firmly grounded in the positivist doctrine, as it can be seen from the abstract of his Umdeutung paper: *"The present paper seeks to establish a basis for theoretical quantum mechanics founded exclusively upon relationships between quantities which in principle are observable"* (Heisenberg, 1967, p. 261).

The development of Heisenberg's program in the subsequent years by Heisenberg himself, Born, Dirac, Jordan and Pauli would lead to Matrix Mechanics and to Transformation Theory. Furthermore, after 1924, Heisenberg started an intense collaboration with Niels Bohr, and in 1926 he became a lecturer in Copenhagen, at the same time Dirac and Jordan were also there. Matrix Mechanics, thus, can be considered the translation of the Mechanics of the Atom, atomic spectra and Niels Bohr's original program. The formulation is grounded in the discontinuity of atomic processes, matrix formalism, pragmatism, existentialism, and positivism.

However, the stabilization of this program would have been deeply impacted by the rise of a competitor, not only rooted in a different philosophy, but grounded in a different mathematical formalism and supported by other set of physical data. To be more precise, Erwin Schrödinger had long studied Statistical Mechanics, and for many years was searching a description of quantum phenomena that could be compatible not only with Special Relativity but also with General Relativity (Joas and Lehner, 2009). Schrödinger, as Einstein, was committed to a realistic perspective and, in the year of 1926, proposed Wave Mechanics, in which not only electromagnetic radiation was described as continuous waves but electrons too. In this way, Schrödinger's program was based on continuity. It also described atomic spectra, dealt only with differential equations (and not matrixes) and was grounded in realism.

In 1925, Werner Heisenberg attended one of Schrodinger's lectures in Munich (Heisenberg, 1983b), when he presented his undulatory interpretation of Quantum Mechanics. Heisenberg was disturbed by Schrödinger opposition to quantum jumps and discontinuities, but nobody seemed to agree with his objections – on the contrary, Schrödinger's interpretation seemed to just gain popularity among the theoretical physicists:[14] it offered a clear picture of what was happening in the quantum level and it used mathematical formalism with which physicists were acquainted (differential equations).

Not much later, Niels Bohr invited Schrödinger to go to Copenhagen to debate the interpretation of Quantum Theory. After long and exhaustive debates (which, according to Heisenberg (1983b) led Schrödinger to be physically sick), Schrödinger's continuous description and Bohr's quantum jumps could not be reconciled. After Schrodinger had left Copenhagen, the researchers of Bohr Institute centered their attention towards the problem of formalism and interpretation of Quantum Mechanics, analyzing the many paradoxes that the different interpretations could produce.

This was the scenario that allowed the "emergence" of the Uncertainty Principle, where there is the confrontation between two worldviews, with their own philosophies, mathematical structures, conceptual bases, phenomena and scientists. Each one of the opposite networks are in a struggle to stabilize itself and destabilize the other. This conflict is summarized in the introduction of Heisenberg's 1927 paper:

> The physical interpretation of quantum mechanics is still full of internal discrepancies, which show themselves in arguments about continuity versus discontinuity and particle versus wave. Already from this circumstance one might conclude that no interpretation of quantum mechanics is possible which uses ordinary kinematical and mechanical concepts (Heisenberg 1983a, p. 62).

3.6.2 HEISENBERG AS A SPOKESMAN

In 1927, Niels Bohr was still not convinced about how to solve such controversy – and it was only when he left Copenhagen on vacation that Heisenberg decided to

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[14] In 1927, for instance, Heisenberg refers to Schrödinger's interpretation as "popular" (Heisenberg 1983a).

take the lead and to assume the role of spokesman, proposing his own interpretation (Heisenberg, 1983b). In this sense, we may understand Heisenberg’s paper as the defense of the whole program that started in 1925.

Heisenberg had a cause to defend and an alternative narrative to deconstruct. Like a lawyer in the tribune, his objective was to defend a claim and make it plausible. To do so, he had to expose the claim, show proofs, find allies and witnesses, and disqualify the opposite interpretation. He had to convince that he had the most suitable way to describe and interpret Quantum kinematics and dynamics.

Some elements of his claim are explicitly expressed on the paper’s abstract, others are distributed throughout the paper and are defended only implicitly. In order to make the argument clearer, we synthetize Heisenberg’s claim in five utterances. Along our presentation, we will show how Heisenberg defend all of them:

a) Quantum objects are particles. Waves should not play any role in Quantum Mechanics.

b) One can only speak about observable quantities. Discontinuity, thus, is a key feature of quantum reality.

c) Dirac-Jordan theory is the correct formalism to mathematically describe quantum mechanics (Heisenberg 1983a, p. 62). Schrödinger’s formalism is therefore unnecessary[15].

d) Canonically conjugate quantities can be determined simultaneously only with a characteristic indeterminacy (Heisenberg 1983a, p. 62). Although the term used is indeterminacy, Heisenberg understands that the relation of uncertainty is caused by the measurement equipment.

e) This indeterminacy is the real basis for the occurrence of statistical relations in quantum mechanics (Heisenberg 1983a, p. 62). Since all we can speak about is what we measure, to say that there is uncertainty in the measure is equivalent to say that there is indeterminacy. This indeterminacy is the source of all statistical relations, which are described by the formalism as a sort of error propagation.

To convince the scientific community about his claim, Heisenberg needs to mobilize a set of witnesses and allies.

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[15] Although Jordan's theory of transformation needs Schrodinger's formalism.

3.6.3 Gedanken Laboratory Witnesses

To exemplify what he meant, Heisenberg proposed throughout the paper different Gedanken experiments involving the simultaneous measuring of non-commutable observables such as position and momentum, time and energy and action and angle variables. In each case, Heisenberg provided rough phenomenological descriptions and found a relation between the uncertainty associated to each pair of variables. For the case of the relation between momentum and position, he discussed the case of an electron observed in a gamma-ray microscope. For the case of the relation between time and energy, he discussed the split of a beam of atoms in a Stern-Gerlach experiment. Finally, for the case of the relation between action and angle variable, he discussed the Franck-Hertz experiment. In each Gedanken experiment involving non-commutable variables by which Heisenberg found an uncertainty relation, the experiment became a witness of his claim. Let us see how he presented this sort of reasoning:

> At the instant when the position is determined – therefore, at the moment when the photon is scattered by the electron – the electron undergoes a discontinuous change in momentum. This change is the grater the smaller the wavelength of the light employed – that is, the more exact the determination of the position. At the instant at which the position of the electron is known, its momentum therefore can be known up to magnitudes which correspond to that discontinuous change. Thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely. In this circumstance, we see a direct physical interpretation of the equation $pq - qp = i\hbar$. Let q_1 be the precision with which the value q is know (q_1 is, say, the mean error of q), therefore here the wavelength of the light. Let p_1 be the precision with which the value p is determinable; that is, here, the discontinuous change of p in the Compton effect. Then, according to the elementary laws of Compton effect p_1 and q_1 stand in the relation $p_1 q_1 \sim h$. (Heisenberg 1983a, p.64)

Since the microscope cannot speak, Heisenberg spoke for it and used its "speech" in his defense. We will discuss in more detail some aspects of this translation in the next sections. At this point, we just want to highlight the role that non-humans play in the articulation of Heisenberg's proposition.

3.6.4 Mathematical Witnesses

The non-human witnesses of the Gedanken laboratory truly corroborated the case and participated in the articulation of Heisenberg's proposition. However, Heisenberg’s presentation of such examples was always rough, and, in a sense, they give the impression that if, for instance, another experiment was chosen to determine position and momentum, it would be possible to overcome the indeterminacy limits. To provide a universal principle, it would be necessary to provide more than “laboratory trials”; it was necessary to provide something that would be more stable than the choice of a laboratory equipment. He needed a “mathematical trial”. When one uses the mathematical formalism to represent a quantity, one represents it regardless of the equipment used. Thus, all Heisenberg must do is to speak in the name of mathematics. To do so, Heisenberg proposed that when we represent a particle using Jordan’s formalism with a Gaussian function, the standard deviation should be pragmatically interpreted as the standard deviation (or mean error) of a measurement. This interpretation, however, is not something that is expressed in the equation, but added by the spokesman when he speaks in the name of the equations. By proposing this interpretation, Heisenberg wrote the gaussian function representing a particle in the q-space (position-space). Multiplying the function by its complex conjugate (what, nowadays, we recall as the probability density function), he obtained the expression

$$S\overline{S}' \, proportional \, to \exp\left[-\frac{(q-q')^2}{q_1^2}\right] \tag{1}$$

Also, using Dirac-Jordan Theory is possible to transform S from q-space to p-space and, again, compute the product with its complex conjugate. Heisenberg found that

$$S\overline{S}' \, proportional \, to \exp\left[-\frac{(p-p')^2}{p_1^2}\right] \tag{2}$$

Where p_1 and q_1 are related through

$$p_1 q_1 = \hbar \tag{3}$$

It should be stressed that Heisenberg proposed an equality (and not an inequality, like it is expressed nowadays).[16] Despite of that, in the same paper, Heisenberg showed that the product between the uncertainties can be larger than $\hbar$, but because of his pragmatic interpretation he considered that the equality is the only

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[16] According to what is known today, the equality only holds for gaussian packages at a specific time. For other wave packet shapes and as time runs, the product becomes larger than $\hbar/2$.

right expression. According to Heisenberg, the wave packet described in $t = 0$ establishes a region where the particle can be found around a mean value with some precision. After some time, the inaccuracy increases in the position of the particle, since the original indeterminacy is propagated, making the wave packet broadened. However, after a second measurement, the position of the particle turns to be determined in a specific region whose length is equal to the initial one (and which is determined by the precision of the equipment used). In other terms, the measurement reduces the extension of the wave packet – what nowadays could be called the collapse of the wave function.

Furthermore, Heisenberg computed the extension of the wave packet after a period t, and he showed that it was bigger than the original one. In this sense, Heisenberg had the chance to rewrite his uncertainty equation as an inequality ($q_1 p_1 \geq \hbar/2$), but as he interpreted the relation as something that spoke about the measurement situation, where the wave packet was reduced to the original length, he sustained the relation as an equality.

3.6.5 Philosophical Argumentation

To make the case plausible, Heisenberg must assign the phenomenological outputs to the mathematical formalism and provide a consistent worldview. This worldview is articulated by three philosophical standpoints. First, Heisenberg addressed the problem of how concepts coming from classical mechanics can be used in the atomic dimension. Influenced by an argument that often appears in Bohr's argumentation (Heisenberg, 1996), Heisenberg assumed that we cannot describe reality with concepts that we have not used classically, since our way of describing reality (our conception of space and time, for instance) are a priori conditions of knowledge itself, as it is expressed in the Kantian philosophy (Kant, 2005). Despite of not being able to provide new concepts to speak about reality, it does not mean that the a priori judgements are absolute and universal. The concepts that we use in the classical world can only be used in Quantum Mechanics considering that there is always some uncertainty related to the measurement of two conjugate quantities. In this sense, Heisenberg advocates for a revision of Kantism. Furthermore, it is possible to recognize some influence of Kierkegaard in Heisenberg's argument. Heisenberg claims that it is not possible to isolate a quantum object:

> In order to be able to follow the quantum-mechanical behavior of any object one has to know the mass of this object and its interaction with any fields and other objects. Only can then the Hamiltonian function be written down for the quantum mechanical system (...) About the “gestalt” (construction) of the object any further assumption is unnecessary; one most usefully employs the word “Gestalt” to designate the totality of these interactions. (Heisenberg, 1983a, p.64)

The “Gestalt” construction that Heisenberg described directly confronts the Kantian scheme in which there are object in themselves (independent of their surroundings) as it was explained by Heisenberg (1996) himself – and it holds a parallel with Kierkegaard’s philosophy. Moreover, this Gestalt worldview was hybridized with positivist and pragmatic postures. In order to call attention to the depth of what he was about to propose, Heisenberg, again, makes a parallel with the Special Theory of Relativity, stressing that to determine a position means to determine the experiment with which is possible to determine position:

> When one wants to be clear about what is to be understood by the words “position of the object”, for example of the electron (relative to a given frame of reference), then one must specify definite experiments with whose help one plans to measure the position of the electron; otherwise, this word has no meaning. (Heisenberg, 1983a, p. 64)

Finally, he mobilized a positivist interpretation in its highest expression in the following excerpt: “I believe that one can fruitfully formulate the origin of the classical “orbit” in this way: the “orbit” comes into being only when we observe it” (Heisenberg, 1983a, p. 73).

3.6.6 Attacks against Schrödinger’s Wave Mechanics

Besides defending his own claim, Heisenberg attacked Schrödinger’s Mechanics three times throughout the paper. In the first one, he mentioned that Dirac’s formulation is the truly invariant formalism, which was an important feature if one thinks in terms of the development of a relativistic quantum mechanics. Second, Heisenberg discussed the relation between Micro and Macromechanics (this is the title of one of Schrödinger’s papers), opposing to Schrödinger’s previous discussions: “The transition from micro- to macromechanics has already been treated by Schrödinger, but I do not believe that Schrödinger’s considerations get to the heart of the problem” (Heisenberg, 1983a, 73). Finally, in the third attack, Heisenberg stressed his disagreement with Schrödinger:

> Certainly, one cannot overestimate the value of the mathematical (and to that extent physical) mastery of the quantum-mechanical laws that Schrödinger's theory has made possible. However, as regards questions of physical interpretation and principle, the popular view of wave mechanics, as I see it, has actually deflected us from exactly those roads which were pointed out by the papers of Einstein and de Broglie on the one hand and by the papers of Bohr and by quantum mechanics on the other hand. (Heisenberg, 1983a, p. 82)

3.6.7 Heisenberg's Final Defense: A New Nature

The problem that Heisenberg addressed to close his defense is how it is possible to link the statistical nature of Quantum Mechanics with the existence of conservation of physical quantities. Heisenberg's answer is that the problem is not in the logical structure of causality, but in its premises, i.e, it is never possible to determine precisely the initial state of a system, so we cannot predict precisely its subsequent states:

> But what is wrong in the sharp formulation of the law of causality, "When we know the present precisely, we can predict the future," is not the conclusion but the assumption. Even in principle we cannot know the present in all detail. For that reason, everything observed is a selection from a plenitude of possibilities and a limitation on what is possible in the future. (Heisenberg, 1983a, p.83)

In this sense, it seems that Quantum Mechanics suggests that there is a real independent world where causality holds. However, this world is inaccessible to us because every measurement perturbs the original system. In this sense, all we have access to, all we can observe is not contemplated by this "causal world" and is subject of uncertainty relations. Taking the positivist position seriously, all we can talk about is the measurement results, and thus causality finally fails:

> As the statistical character of quantum theory is so closely linked to the inexactness of all perceptions, one might be led to the presumption that behind the perceived statistical world there still hides a "real" world in which causality holds. But such speculations seem to us, to say it explicitly, fruitless and senseless. Physics ought to describe only the correlation of observations. One can express the true state of affairs better in this way: Because all experiments are subject to the laws of quantum mechanics, and therefore to equation (1), it follows that quantum mechanics establishes the final failure of causality. (Heisenberg, 1983a, p.83)

The indeterminate nature was born.

3.6.8 Translations and Betrayals

We have passed through the main points of Heisenberg's defense: his claim, the mobilization of gedanken laboratory experiments, the mobilization of Dirac-Jordan mathematical formalism, the use of pragmatism, existentialism and positivism to interpret all the articulation and his final defense. The combination of all these elements corresponds to a translation that is conducted by Heisenberg, the spokesman. Nevertheless, in this process at least two characters were betrayed. The first was Bohr, who did not commit to the full rejection of wave description of quantum phenomenon. As we will discuss on the next section, when Bohr read the first version of the paper, already approved but not published, he convinced Heisenberg to add a note mentioning that the wave description of radiation was used in the analysis of the gamma ray microscope and that it was an essential part of Heisenberg's description! Actually, the defense of both pictures – corpuscular and undulatory – was performed by Bohr in 1928 in his proposition of the Complementarity Principle (Bohr, 1928). In this sense, Heisenberg's intention of rising as the spokesman of the Copenhagen program was frustrated by Bohr's opposition to a purely corpuscular description. Moreover, a central aspect of Heisenberg's proposition is the derivation of the uncertainty relation from the Dirac-Jordan's formalism. Despite of using their formalism, Heisenberg denied Jordan's interpretation of Quantum Mechanics. In 1927, Jordan had already proposed a probabilistic interpretation, which was not associated simply to the measurement process (Longair, 2013). Heisenberg thus mobilized Jordan's formalism but betrayed his interpretation:

> Of course, we would also like to be able to derive, if possible, the quantitative laws of quantum mechanics directly from the physical foundations—that is, essentially, from relation (1). On this account Jordan has sought to interpret the equation $S(q,q'') = \int S(q,q')S(q',q'')dq'$, as a probability relation. However, we cannot accept this interpretation (§2). We believe, rather, that for the time being the quantitative laws can be derived out of the physical foundations only by use of the principle of maximum simplicity. (Heisenberg, 1983a, p.82)

These two betrayals had concrete consequences to Heisenberg's program.

3.6.9 Provenance (Herkunfut) of the Uncertainty Principle

In the process of mobilizing allies, sometimes Heisenberg presented contradictory statements or, at least, not very rigorous reasonings, as pointed by Bohr. Some of them were addressed in the note added during the editing process. Let us examine some of the “unstable points” of the microscope *gedanken* experiment:

- i) $q_1$is called the mean error of q. The calculation of this parameter is based on the wave theory of light (it is caused by diffraction), while p_1 is computed with a corpuscular theory (it is caused by the Compton Effect), which makes Heisenberg sustains both pictures at the same time, the opposite of what we claimed to do, as pointed out by Bohr.
- ii) p_1 is not a mean error, but the maximum variance of momentum. So, while q_1 is an error, p_1 is a disturbance (Jijnasu, 2016).
- iii) Heisenberg is not considering the problem of the real microscope and ignores concrete factors such as the objective diameter.
- iv) Heisenberg does not show how the indeterminacy comes from the commutator relation; he just attaches them. Along the paper, Heisenberg claims that the commutator is the origin of the uncertainty relations, but he does not prove it.
- v) Heisenberg has stressed the necessity of expressing how a physical quantity is measured. In the *gedanken* experiment, he describes how q is measured and what q_1 means in the experiment, there isn’t any concern to measure p and to empirically define p_1.

Besides these internal instabilities, in the case of the uncertainty principle for time and energy, also there are some contradictions that would be pointed out by other scientists:

- a) Heisenberg claims that the uncertainty principle for time and energy comes from the commutation relation between these two quantities. However, in the subsequent years, it was shown that it is not possible to propose an operator of time that it is self-adjoint through all the spectrum (Bush, 2008).

b) In the Stern-Gerlach experiment, time is treated as the time necessary to perform a measurement. However, Heisenberg also uses time as the interval of a transition:

> the time transitions or ‘quantum jumps’ must be as concrete and determinable by measurements as, say, energies in stationary states. The spread within such an instant is specifiable is given according to equation (2) by $\frac{h}{\Delta \mathrm{E}}$, if ΔE designates the change of energy in the quantum jump (Heisenberg, 1983a, p.76).

While the first is an external measurement of time, the second is an intrinsic parameter (Busch, 2008). Finally, Heisenberg’s derivation using Dirac-Jordan’s theory used a gaussian-wave packet arbitrarily, without any justification. Despite of that, he claimed that the uncertainty relation is valid to all conjugate variables.

3.6.10 Stabilizing Nature

Heisenberg’s analysis of the gamma microscope contradicted his own claim. He also forgot many important features of the experiment. He claimed to speak about measurement errors, but he referred to position error and to momentum perturbation (this one, only estimated and not measured). He changed the concept of uncertainty along the paper, sometimes referring simply to an interval (as in the case of time). He claimed that the uncertainty relation came from the commutation relation, but he did not show it. He assumed a commutation relation for time and energy that cannot be written uncritically. His derivation was for a specific case and he claimed it to be universal. We have all these reasons to agree with Foucault that in the base of science there is only error and accident.

Despite of that, nature became somehow indeterminate after 1927. Somehow an Uncertainty Principle emerged and remains alive until today. The way we can explain that is by recognizing that the uncertainty relation was not only articulated by Heisenberg. He was the spokesman. But the proposition was also articulated by non-humans. And non-humans have their own agency. In the case of the uncertainty relation, the equations mobilized by Heisenberg turned out to speak what Heisenberg could not speak himself.

Heisenberg’s interpretation, his main claim and defense, the one based on the betrayal of Bohr’s conception and Jordan’s interpretation, was not successful at all. Today, we learn Quantum Mechanics through Schrödinger’s Equation and not Dirac-Jordan’s formalism. And we speak about probability waves and not error propagation.

However, the mathematical structure of the uncertainty principle (without Heisenberg's interpretation) was more solid than his intention, and its association resisted. Following Jammer's (1966) reconstruction, just after Heisenberg published the paper, in the same year, Kennard (1927) has shown that the Gaussian case was the minimum bound to the indeterminacy relation. Also, in 1928, Hermann Weyl (1950) demonstrated the Uncertainty relation as an inequality (which is valid for gaussian and non-gaussian packages). In the same year, C. G. Darwin (1928) has pointed out that the transformation between the position and momentum representations can be understood in terms of a Fourier transform, Ruark (1928) called the uncertainty relation as The Uncertainty Principle, and Kennard (1928) computed the uncertainty in position and momentum for an electron passing through a shutter, showing that it corresponds to the Fourier resolution of a train of waves passing through the same shutter. The mathematical proof that the uncertainty relation holds to all canonical conjugate variables only came in 1929 (Robertson, 1929). After Robertson's publication, Schrödinger (1930), whose interpretation of Quantum Physics was competing with Heisenberg's conception, found a new and more general expression for the Uncertainty Principle. So, what was successful in Heisenberg's proposition was not his interpretation, his claim, but the objective mathematical structure that was independent of him, and that resisted to his interpretation. This was the stable association that allowed the Principle of Uncertainty to survive along history. An important question is why, then, this mathematical association was so stable? If we take Gabriel Tarde's claim seriously, that science progresses every time it adopts a monadological perspective, the success of Heisenberg was to introduce the monad of determinacy, what can be thought – as Heisenberg himself mentions – the monad of "volume in the phase space" (Heisenberg, 1983a, p.65)[17]. As Tarde claims, this monad ought to be an essential part of reality, dependent- of course- of other monads- but still stable enough to resist Heisenberg's translations and betrayals.

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[17] According to the intrinsic interpretation of the Uncertainty Principle (Jijnasu 2016) one may think that, differently of Classical Mechanics, according to which a particle may occupy a single point in the phase space, quantum objects are fuzzy, that they are distributed over a region of phase space, whose minimum volume is given by the Uncertainty Principle. In the same way, Tarde interpreted the atom as the monad of matter, so we may interpret the Uncertainty Principle as an expression of the monad of volume in phase space.

3.7 FINAL REMARKS

We discussed possible characteristics of a historical account that relate with Symmetrical Anthropology. We understand that, for a history to be symmetrical, it does not have necessarily to use specific concepts, but it must agree with the presented metaphysical scheme. We exemplified that with the discussion on the articulation of Uncertainty Principle. The main challenge to produce such a narrative was to decide about when to explicitly use the concepts of Symmetrical Anthropology. Not using them at all would make difficult for the reader to connect the narrative with the theoretical discussion; using them too much could make it obscure. So, there is an equilibrium point that is not easy to achieve and that must be pursued in every narrative. On the other hand, our narrative allowed us to speak about theoretical physics without using a sectarian language – we reframed Heisenberg’s work as the work of a lawyer in the tribune (characteristic “f”). We discussed the necessity of mobilizing different actors for the Uncertainty Principle to exist (characteristic “c”). These actors were laboratory equipment, mathematical formalism, and philosophical doctrines (characteristic “f”). What explains Heisenberg’s success and failures were not beliefs nor knowledge, but his ability in articulating propositions (characteristic “d”). His betrayal of Bohr’s and Jordan’s interpretation may have played an important role in the failure of his interpretation, while the further articulation of the mathematical expression allowed its survival. The fact that the mathematical expression survived with a different meaning exemplifies the fact that non-humans have their own agency independently of human volition (characteristic “b”). In the end, we speculate that the stability of the Uncertainty Principle is due to the fact that Heisenberg introduced, without knowing, a monad of determinacy, agreeing with Gabriel Tarde’s claim. In this sense, we understand that Symmetrical History has the potentiality of allowing us to speak about Physics without having to commit to an internalist or to an externalist approach and without having to assume an absolutist or a relativist perspective. Also, it may allow us not only to describe Physics, but also to provide explanations for its progress according to the stability of associations.

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4 SEGUNDO ARTIGO ORIGINAL

DIFERENTES PROPOSIÇÕES DO PRINCÍPIO DA INCERTEZA PARA POSIÇÃO E MOMENTUM: INTEGRANDO FORMALISMO MATEMÁTICO, FENOMENOLOGIA E INTERPRETAÇÕES NO ENSINO DA TEORIA QUÂNTICA

4.1 RESUMO

O Princípio da Incerteza, proposto pelo físico Werner Heisenberg em 1927, possui centralidade no desenvolvimento Teoria Quântica e desde sua formulação assume uma pluralidade de interpretações, derivações matemáticas e experimentos mentais que buscam explorá-lo em profundidade. Entretanto, os livros didáticos, que possuem papel importante na formação de cientistas e professores de ciência, usualmente optam por privilegiar somente alguns de seus principais aspectos, limitando a apresentação do Princípio da Incerteza nos cursos de Teoria Quântica do ensino superior. Com o objetivo de ampliar as discussões sobre o Princípio da Incerteza para as variáveis posição e momentum e contribuir com o Ensino da Teoria Quântica, ao longo deste texto apresentamos o Princípio da Incerteza em três diferentes níveis (formalismo matemático, fenomenologia e interpretação). Começamos resgatando de forma didática quatro diferentes derivações históricas do Princípio da Incerteza: as derivações de Heisenberg e Weyl para as variáveis posição e momentum, e as derivações de Robertson e Schrödinger para dois observáveis quaisquer A e B. Em seguida, retomamos dois experimentos mentais propostos por Heisenberg: o microscópio de raios-gama e o elétron passando por uma fenda. Por fim, apresentamos quatro principais escolas de interpretação do Princípio da Incerteza.

Palavras-chave: Princípio da Incerteza, Teoria Quântica, Ensino de Física.

4.2 INTRODUÇÃO

O Princípio da Incerteza foi, primeiramente, formulado por Werner Heisenberg [1] em 1927[18]. No ano seguinte, Niels Bohr [2] incorporou o Princípio da Incerteza como um dos elementos principais do que ficou conhecimento como Interpretação da Complementaridade – tornando-se um elemento fundamental da Teoria Quântica. Apenas para fornecer alguns exemplos da centralidade do Princípio da Incerteza na Teoria Quântica, em 1933, em seu famoso livro didático sobre Mecânica Quântica, Pauli [3] a introduz por meio do Princípio da Incerteza e da Complementaridade. Em 1935, no artigo de Einstein, Podolski e Rosen [4], que ficou conhecido como paradoxo EPR, os autores apresentam a interpretação usual da Teoria Quântica ressaltando o fato de que variáveis que não comutam não podem ser conhecidas simultaneamente com qualquer precisão. Em 1952, no seu artigo sobre variáveis ocultas, David Bohm menciona "*A interpretação física usual da teoria quântica se centra em torno do Princípio da Incerteza*" (1952, p. 167, tradução livre) [5]. Apesar de tal centralidade, os livros didáticos contemporâneos de Mecânica Quântica, muitas vezes, tratam o tema de forma periférica ou, pelo menos, não discutem em detalhes as diferentes formas de se derivar as relações de incerteza, os diferentes experimentos mentais que podem ser mobilizados para entendê-lo e, sobretudo, as diferentes interpretações que ele enseja. Ademais, em um estudo preliminar [6], mostramos que os livros didáticos usualmente fazem uma escolha: (1) privilegiam a apresentação do formalismo matemático em profundidade ou (2) enfatizam os aspectos históricos e interpretativos da teoria.

Embora exista uma ampla literatura voltada para discussões acerca do Princípio da Incerteza, incluindo diferentes derivações e generalizações do princípio [7]–[17], estudos voltados para discussões fenomenológicas, centrados no experimento mental do microscópio de raios-gama proposto por Heisenberg em 1927 [18]–[21] e, ainda, trabalhos que buscam discutir conceitualmente o Princípio da Incerteza, suas implicações teóricas e possíveis interpretações, [22]–[25], a maioria dos trabalhos costuma ser voltada a pesquisadores e estudiosos da área, tornando pouco provável sua utilização como instrumento auxiliar para o ensino da Teoria

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[18] O termo "Princípio da Incerteza" só foi proposto ao final de 1927 por A. E. Ruark [2] e, embora seja amplamente utilizado, não condiz com o fenômeno em si. O que conhecemos e ensinamos como Princípio da Incerteza não se trata de um princípio de fato, mas de uma relação de incerteza que se estende a qualquer sistema descrito por ondas e, dessa forma, se aplica também a outras variáveis.

Quântica. Na literatura nacional já existem estudos voltados para a discussão de aspectos matemáticos [26] e filosófico-interpretativos do Princípio da Incerteza [27].

Neste contexto, com o presente trabalho, buscamos contribuir para o ensino desse tópico central da Teoria Quântica ao fornecer um panorama geral e uma proposta de exploração didática. Reconhecendo a dificuldade dos alunos em compreender o formalismo da Teoria Quântica [28], apresentamos aqui um texto de caráter didático que, ao abordar diferentes aspectos do Princípio da Incerteza, é organizado em três categorias: formalismo matemático, fenomenologia e interpretação, com o intuito de que tal abordagem favoreça sua aprendizagem. Em especial, apresentamos diferentes derivações inspiradas em textos históricos do Princípio da Incerteza (explicando passos que foram omitidos nos textos originais e organizando a apresentação de forma detalhada), retomamos a explicação de dois experimentos mentais imaginados por Heisenberg para explicar o princípio da incerteza [1] e apresentamos quatro principais escolas de interpretação do princípio [23].

Essa apresentação é feita a partir dos próprios trabalhos originais da Teoria Quântica (fontes históricas primárias) e de comentários já existentes (fontes históricas secundárias). Tal texto pode ser introduzido em disciplinas de Mecânica Quântica, após a apresentação tradicional, contribuindo para complementar o que foi estudado no livro didático e para enriquecer as discussões e compreensões sobre tal princípio. Essa proposta, nesse sentido, se alinha aos trabalhos que propõem trazer fontes primárias para o Ensino de Física como uma forma de enriquecer a atividade didática e adotar um pluralismo metodológico e didático [29]–[32]. A escolha de apresentar o texto didático dividido em três categorias (formalismo matemático, fenomenologia, e interpretação) é inspirada na discussão feita por Max Jammer [33] sobre os elementos de uma teoria física e dialoga com trabalhos da área de Ensino de Física.

O texto está organizado da seguinte forma: na seção 2, apresentamos uma breve discussão sobre a proposta didática de discutir conceitos físicos em três categorias, explicamos cada uma das categorias e justificamos a importância de apresentá-las de forma clara no contexto pedagógico. Na seção 3, fazemos um breve resgate histórico da gênese do Princípio da Incerteza, para situar o leitor no momento histórico do desenvolvimento das discussões que aparecem na sequência. Na seção 4, introduzimos o primeiro nível, o formalismo matemático, resgatando de forma didática quatro derivações históricas do Princípio da Incerteza. Na sequência, na seção 5, explicamos dois experimentos mentais mencionados no artigo de 1927 de Heisenberg, explicando detalhes que não aparecem no artigo original. Na seção 6, apresentamos quatro formas distintas de se interpretar o princípio da incerteza. Por fim, na seção 7, apresentamos nossas considerações finais, fazendo algumas sugestões para o uso didático desse texto.

4.3 PROPOSTA DIDÁTICA: APRESENTANDO CONCEITOS FÍSICOS EM TRÊS CATEGORIAS (FORMALISMO MATEMÁTICO, FENOMENOLOGIA E INTERPRETAÇÕES)

A proposta didática de discutir conceitos físicos em três categorias é inspirada na discussão levantada por Max Jammer [33] no livro "*The Philosophy of Quantum Physics"* em que o autor propõe que teorias físicas são constituídas por, pelo menos, três componentes distintos: formalismo abstrato, regras de correspondência e interpretações. O formalismo abstrato é essencialmente a estrutura lógica de uma teoria, seus postulados desprovidos de significado empírico. Por meio das regras de correspondência, o formalismo abstrato deixa de ser um conjunto de símbolos e passa a representar um sistema físico. Na teoria quântica, por exemplo, o que usualmente é chamada de interpretação probabilística de Born pode ser considerada como a regra de correspondência, pois conecta um elemento abstrato da teoria, a função de onda, com uma grandeza mensurável, a probabilidade de se encontrar uma partícula em uma certa região. Por fim, Jammer [33] aponta que as teorias físicas ainda possuem um terceiro elemento, sua interpretação, a qual contempla o conjunto de enunciados da teoria que descrevem a realidade (como modelos utilizados) mas que não alteram as previsões experimentais da teoria [34].

Dessa forma, propomos que tal estrutura não se restrinja ao estudo de teorias físicas, mas que possa ser ampliada para o estudo de conceitos e princípios físicos em um contexto mais amplo, tal como o Princípio da Incerteza, e de maneira análoga a Max Jammer, sugerimos que sejam apresentados em três diferentes categorias: formalismo matemático, fenomenologia e interpretações[19].

Os conceitos apresentados na Teoria Quântica não são simples e, muitas vezes, contrariam nossas intuições mais fundamentais, de modo que os estudantes

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[19] Proposta semelhante, mas subsidiada por diferentes razões teóricas, foi apresentada para a dualidade onda-partícula por Cheong & Song [35].

em geral costumam ter dificuldade para compreender sua estrutura [28]. Por esse motivo, apresentar tais conceitos em três categorias pode contribuir para uma melhor organização da apresentação didática[20]. Utilizando dessa apresentação, primeiramente, o aluno entende como o Princípio se desenvolve a partir do formalismo matemático, aprendendo como que ele se conecta com a estrutura geral da Teoria. Na sequência, são discutidos aspectos fenomenológicos, ou seja, resultados empíricos obtidos em experimentos com sistemas quânticos. Nesse momento não é discutido o que o princípio significa, mas como que os resultados experimentais traduzem tal princípio. Somente então, nas situações em que o professor achar necessário e tiver interesse, discute-se a interpretação desse formalismo e seus resultados. Ressaltamos que a abordagem do terceiro nível deve ser considerada como uma escolha para professores e professoras que considerem as discussões filosóficas como frutíferas ao Ensino de Física e que encontrem espaço para elas em suas aulas.

4.4 UM BREVE HISTÓRICO DA GÊNESE DO PRINCÍPIO DA INCERTEZA

No início do século XX, os estudos sobre matéria e radiação conduziram a uma série de resultados que contrariavam concepções fundamentais da Física Clássica. Em especial, a radiação, usualmente tratada por campos contínuos, passou também a ser representada por uma caracterização corpuscular, como nos trabalhos de Einstein[21], e a matéria, usualmente tratada de forma corpuscular também ganhou um caráter ondulatório, como nos trabalhos de Louis de Broglie[22]. Essas representações contraintuitivas, embora fossem capazes de explicar uma série de resultados experimentais, não conseguiam formar um quadro teórico completo e consistente. Na metade da segunda década do século XX, dois programas de pesquisa surgiram justamente com o objetivo de articular tal quadro completo e coerente [2].

Um desses programas foi inaugurado em 1925 pelo físico alemão Werner Heisenberg por meio do desenvolvimento da Mecânica Matricial que, inspirada por

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[20] Embora tal separação seja artificial e não reflita toda a complexidade envolvida na estabilização de um conceito ou uma teoria física.

[21] A. Einstein, Ann. de Phisik., 17, 132 (1905); A. Einstein, Ret. Jan., 29 (1925).

[22] L. de Broglie, J. Phys. Radium, 3, 22 (1922); L. de Broglie, Comptes Rendus, 177, 507 (1923); L. de Broglie, Philos. Mag., 47, 446 (1924).

ideias consistentes com a teoria da relatividade de Einstein [2], com sua essência centrada na concepção corpuscular da matéria e na descontinuidade dos processos atômicos, propôs um formalismo completo e totalmente novo para a física quântica. Em seu texto, Heisenberg rejeitou as noções clássicas de posição e velocidade (ou momentum) para elétrons em átomos, trabalhando apenas com grandezas observáveis, e empregou o princípio da correspondência de Bohr no cerne de seu esquema matemático [36].

Apesar do sucesso de Heisenberg em descrever o comportamento dos fenômenos quânticos utilizando da Mecânica Matricial, a comunidade científica, em geral, relutou em aderir a sua formulação, tanto pela estranheza ao formalismo matemático quanto pela abstração teórica do artigo de 1925 [2], [37]. Entretanto, tal abstração chamou a atenção de alguns físicos teóricos da época, resultando em um maior desenvolvimento da Mecânica Matricial pelos físicos Born e Jordan [38], [39], Pauli [40] e Dirac [41] de maneira que, finalmente, foi formulada a teoria de Dirac-Jordan[23] [42]–[45].

No ano seguinte, partindo do teorema de Broglie sobre a equivalência entre o Princípio de Maupertuis e o Princípio de Fermat, o físico austríaco Erwin Schrödinger desenvolveu a Mecânica Ondulatória em uma série de quatro artigos [46] provando, nos três primeiros, que a Mecânica Ondulatória era matematicamente equivalente à Mecânica Matricial. Entretanto, apesar da equivalência formal entre os dois programas, a interpretação da teoria e a maneira como cada uma explicava os fenômenos quânticos levantou um dilema irreconciliável, uma vez que Heisenberg e Schrödinger utilizavam de diferentes estruturas para descrever a realidade, reforçando os paradoxos entre a representação corpuscular e ondulatória da mecânica quântica. Enquanto Heisenberg e a Mecânica Matricial representavam os objetos quânticos por meio de fenômenos corpusculares, Schrödinger e a Mecânica Ondulatória centravam-se na ideia metafísica de que qualquer partícula é representada por um conjunto de ondas, de modo que a descrição da dinâmica da partícula seria feita por meio da descrição do transporte dessas ondas.

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[23] Embora não esteja presente no estudo contemporâneo da Teoria Quântica, e sua contribuição dificilmente será encontrada em algum livro didático, destacamos aqui sua importância pois, segundo o próprio Heisenberg no artigo de 1927, o Princípio da Incerteza foi primeiramente formulado utilizando da teoria de Dirac-Jordan [1].

Principalmente por apresentar um formalismo matemático com o qual os físicos já estavam adaptados [2], [37], baseado na resolução de equações diferenciais parciais e compatível com a Teoria de Hamilton-Jacobi, a formulação matemática da Mecânica Ondulatória de Schrödinger rapidamente recebeu a aprovação da comunidade científica [2], [33], sendo adotada como formalismo hegemônico da teoria quântica, presente majoritariamente, até hoje, nos livros didáticos e nos cursos de formação em física quântica.

Por outro lado, a interpretação física de tal formalismo não foi tão bem aceita, principalmente no que se refere a sua interpretação para a função de onda, que trazia uma série de problemas [2], sendo a principal contradição encontrada no pacote localizado, uma vez que a redução do pacote de onda por si só já estaria associada a um efeito não-local [34].

Nesse período, ademais, a natureza estatística das transições atômicas passou a chamar a atenção dos físicos para a necessidade de um rompimento com características importantes da Física Clássica. Em 1925, por exemplo, Einstein [47] deu a primeira interpretação estatística às ondas de de Broglie, uma vez que, ao tentar tornar a dualidade onda-partícula mais compreensível, interpretou o quadrado das amplitudes das ondas ópticas como densidade de probabilidade para detectar um fóton em uma dada região [37]. A descrição probabilística da realidade foi posteriormente desenvolvida nas obras de Born, Pauli e Jordan no período de 1925 a 1927. Apropriando-se do formalismo ondulatório de Schrödinger e associando-o a uma interpretação corpuscular da matéria, e tendo como base o estudo dos processos de colisões atômicas, Max Born propôs que a função de onda Ψ deveria representar a densidade de probabilidade para elétrons (ou outras partículas) [24] [37], de modo que a probabilidade de detectar um único quantum em determinada região do espaço seria proporcional ao quadrado da amplitude Ψ da onda associada à região.

Conforme Heisenberg [49], foi no início de 1926 que ele tomou conhecimento das ideias de Schrödinger e, ao final do período letivo o mesmo ano, teve a oportunidade de assistir uma palestra do físico austríaco em Munique. Em seu livro [49], o físico alemão aponta que ficou inconformado com o fato de a Mecânica Ondulatória negar a descontinuidade dos processos atômicos, e que, após a palestra, levantou uma série de objeções à teoria. Entretanto, conforme Heisenberg [49]

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[24] Embora em 1954 Max Born alegue que tenha se inspirado na proposta de Einstein para a probabilidade de ocorrência do fóton, Mara Beller [48] traz argumentos que contradizem tal afirmação.

destaca, suas contestações foram isoladas e, de modo geral, a Mecânica Ondulatória foi bem aceita pelo público de físicos presentes. Em setembro do mesmo ano Niels Bohr convidou Schrödinger para visitar Copenhague com o objetivo de discutir, em detalhes, as duas diferentes interpretações. Segundo Heisenberg [49], [50], foram dias de intermináveis discussões, começando nas primeiras horas da manhã e estendendo-se até tarde da noite. Tais discussões não ajudaram a resolver o impasse entre as duas teorias concorrentes, e cada lado saiu convencido de estar trilhando pelo caminho certo [49], [50]. Nos meses subsequentes, o foco de Bohr e Heisenberg foi dirigido para a busca por uma interpretação física da Mecânica Quântica por meio da discussão de diversos experimentos mentais [50].

Ainda segundo Heisenberg [49], em fevereiro de 1927, enquanto Bohr passava por um período de férias esquiando na Noruega, ao tentar descrever matematicamente a trajetória percorrida pelo elétron no experimento da câmara de nuvem, o físico lembrou de uma discussão que havia tido em Gottigen sobre a possibilidade de determinar a posição de uma partícula com um microscópio de raios-gama. Nesse momento, o físico alemão se questionou se a Teoria Quântica poderia representar, com alguma imprecisão, o fato de um elétron se encontrar em determinada posição e se mover com determinada velocidade e, ainda, se existiria a possibilidade de tornar tais imprecisões tão pequenas de modo que não produzissem obstáculos experimentais. Dessa forma, com o objetivo de legitimar a Mecânica Matricial, os saltos quânticos e sua interpretação corpuscular da matéria, Heisenberg publicou o artigo intitulado "*The Physical Content of Quantum Kinematics and Mechanics*" que, segundo ele, levaria a uma interpretação da mecânica quântica livre de inconsistências e onde definiu, pela primeira vez, o Princípio da Incerteza: *"quantidades canonicamente conjugadas podem ser determinadas simultaneamente apenas com uma indeterminação característica"* (1983, p. 1, tradução livre) [1].

Sendo fortemente influenciado pela Teoria da Relatividade Especial, Heisenberg propôs que para especificar o que deve ser entendido por "posição de uma partícula" seria necessário determinar o aparato experimental que permite a medida da sua posição, caso contrário este termo não possui significado[25]. Usando tal metáfora, Heisenberg apresentou alguns experimentos mentais que, de acordo com ele, revelariam a impossibilidade de dois parâmetros conjugados serem medidos

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[25] Tal como na Relatividade que, ao falar de eventos simultâneos, é necessário que se especifique quais relógios serão utilizados para a realização das medidas de tempo.

simultaneamente com precisão arbitrária e uma demonstração matemática que, conforme ele aponta, foi baseada na teoria de Dirac-Jordan.

4.5 PRIMEIRO NÍVEL: O FORMALISMO MATEMÁTICO ASSOCIADO AO PRINCÍPIO DA INCERTEZA

Ao longo dessa seção, apresentaremos uma reconstrução didática de quatro derivações inspiradas em textos históricos do Princípio da Incerteza para posição e momentum. Para acompanhar algumas das premissas aqui utilizadas, é recomendável que o leitor já tenha conhecimento prévio dos Postulados da Teoria Quântica, que podem ser retomados no Material Suplementar – Apêndice A. Também será necessária certa familiaridade com conceitos da Teoria Estatística, que podem ser consultados no Material Suplementar – Apêndice B.

4.5.1 Incerteza entre as variáveis posição e momentum para um pacote de ondas gaussiano – reconstrução didática da derivação presente no trabalho de Heisenberg (1927)

Para formalizar matematicamente a relação de incerteza para posição e momentum, no texto de 1927 Werner Heisenberg [1] aponta que utilizou a teoria da Transformação Matricial de Dirac-Jordan (designação do próprio autor), assumindo seu formalismo matemático, mas discordando de sua interpretação original[26]. Heisenberg propôs que, ao representar uma partícula utilizando o formalismo de Jordan, com uma função gaussiana, por exemplo, o desvio padrão deveria ser pragmaticamente interpretado como o desvio padrão de uma medição.

Ao longo dessa apresentação utilizaremos um raciocínio similar, mas com uma abordagem contemporânea[27] [51]. Desta forma, temos que, uma partícula quântica pode ser descrita por meio de uma função de onda $\Psi(x,t)$ (Postulado 1 do apêndice

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[26] Na formulação da teoria de Dirac-Jordan, Jordan propôs uma interpretação probabilística para a mesma.

[27] Para a presente derivação, utilizamos de propriedades ondulatórias da Teoria Quântica que, embora contrarie a ideia original de Heisenbeg (que utilizou somente de argumentos corpusculares), ajudam a oferecer uma abordagem mais didática e alinhada ao que é ensinado nos cursos de física atualmente.

A), cuja frequência angular e o número de onda podem ser descritos pelas expressões de Einstein e de Broglie

$$p = \hbar k \,, \tag{1}$$

onde $k = 2\pi/\lambda$ é o número de onda. E, para a energia,

$$E = \hbar\omega \,. \tag{2}$$

No caso de uma partícula livre, toda a sua energia estará associada a seu movimento, o que significa que será puramente cinética

$$E = \frac{p^2}{2m} \,. \tag{3}$$

Portanto,

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} \,. \tag{4}$$

Dessa forma, podemos começar expressando uma função de onda qualquer como uma “soma” de ondas planas. Uma vez que k apresenta um espectro contínuo, podemos expressar a função de onda como uma integral em relação a k

$$\Psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(k) e^{i(kx-\omega t)} dk \,. \tag{5}$$

Em que o termo $\frac{1}{2\pi}$ será explicado mais adiante. Nesse caso, a função $g(k)$ representa o coeficiente de expansão da função de onda em relação aos autoestados.

Ademais, Heisenberg propõe preparar um arranjo experimental em que, em um específico instante, pode-se dizer $t = 0$, a partícula estará em torno de uma posição média $\langle x \rangle$ com precisão $\sigma/\sqrt{2}$. Podemos traduzir tal arranjo experimental em termos da função $\Psi(x, 0)$. Por exemplo, neste caso, podemos assumir que a função de onda é caracterizada como uma distribuição gaussiana em torno de $\langle x \rangle$ com desvio padrão σ (ver expressão B6 do apêndice B).

$$\Psi(x,0) = \sigma^{-\frac{1}{2}} \pi^{-\frac{1}{4}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\langle x \rangle}{\sigma}\right)^2} . \tag{6}$$

Se considerarmos que o x médio da medida é a origem, então $\langle x\rangle = 0$. Desta forma, (conforme Postulado 5 do apêndice A) a função densidade de probabilidade pode ser escrita como

$$|\Psi(x,0)|^2 = \frac{1}{\sigma\sqrt{\pi}} e^{-\left(\frac{x}{\sigma}\right)^2} . \tag{7}$$

e tem um desvio padrão $\sigma_x = \sigma/\sqrt{2}$. As constantes da equação (7) e (6) são determinadas de forma que a integral de $|\Psi(x,0)|^2$ sobre todo espaço seja igual a 1 (propriedades da integral gaussiana estão no apêndice B). Podemos, agora, descrever $\Psi(x,0)$ em termos da função de coeficiente $g(k)$

$$\Psi(x,0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(k) e^{ikx} dk . \tag{8}$$

É possível reconhecer que $g(k)$ é a Transformada de Fourier de $\Psi(x,0)$ – por isso o termo $1/2\pi$ foi introduzido em (5) (comparar, por exemplo, com as expressões B7 e B8 do apêndice B). Além disso, também é possível interpretar a equação acima como a decomposição espectral de $\Psi(x,0)$ em termos dos autoestados do operador $\hat{\mathbf{k}}$. Podemos ainda reescrever a integral, com o objetivo de tornar o momentum da partícula explícito, por meio da seguinte mudança de variáveis

$$k = \frac{p}{\hbar} . \tag{9}$$

De modo que

$$g(k) = g\left(\frac{p}{\hbar}\right) = \sqrt{\hbar}\,\psi(p) . \tag{10}$$

E, portanto, é possível reescrever a expressão (8) em termos da variável p

$$\Psi(x,0) = \frac{1}{2\pi\sqrt{\hbar}}\int_{-\infty}^{+\infty} \psi(p) e^{\frac{ipx}{\hbar}} dp . \tag{11}$$

Calculando a transformada de Fourier (apêndice B), obtemos

$$\psi(p) = \frac{1}{\sqrt{\hbar}}\int_{-\infty}^{+\infty} \Psi(x,0)e^{-\frac{ipx}{\hbar}}dx\,, \tag{12}$$

que é a função de onda na representação de momentum, escrita como uma integração sobre os autoestados do operador $\hat{\mathbf{x}}$. Usando $\Psi(x,0)$ de (6) e substituindo em (12), temos

$$\psi(p) = \frac{1}{\sqrt{\hbar}}\int_{-\infty}^{+\infty} \sigma^{-\frac{1}{2}}\pi^{-\frac{1}{4}}e^{-\frac{1}{2}\left(\frac{x-\langle x\rangle}{\sigma}\right)^2} e^{-\frac{ipx}{\hbar}}dx\ . \tag{13}$$

Resolvendo a integral com $\langle x\rangle = 0$, (ver apêndice B) obtemos:

$$\psi(p) = \left(\frac{\sigma^2\pi}{\hbar^2}\right)^{\frac{1}{4}} e^{-\frac{\sigma^2p^2}{2\hbar^2}}\,. \tag{14}$$

Esta é uma nova função gaussiana, mas cujo desvio padrão é igual a $\hbar/\sigma$. Com isso podemos determinar a função densidade de probabilidade na representação de momentum:

$$|\psi(p)|^2 = \frac{\sigma\sqrt{\pi}}{\hbar}e^{-\frac{\sigma^2p^2}{\hbar^2}}\,. \tag{15}$$

Esta equação expressa a densidade de probabilidade para medir certo valor de momentum para a partícula. Neste caso, é possível determinar que o desvio padrão da função de densidade é $\sigma_p = \hbar/(\sigma\sqrt{2})$ (conforme apêndice B).

Finalmente, podemos calcular que o produto entre o desvio padrão da função de densidade de probabilidade nas representações posição e momentum é

$$\sigma_x.\sigma_p = \frac{\sigma}{\sqrt{2}}.\frac{\hbar}{\sigma\sqrt{2}} = \frac{\hbar}{2}\,. \tag{16}$$

Podemos observar que Heisenberg encontra uma igualdade e não uma desigualdade. Isso acontece porque calculamos a relação de incerteza para o caso de uma função gaussiana, o que corresponde justamente ao pacote de menor incerteza possível [52]. Assim, a derivação feita por Heisenberg caracteriza apenas um caso particular e não é uma demonstração geral da relação de incerteza entre posição e momentum. Ademais, ressaltamos que Heisenberg não relaciona diretamente a relação de incerteza com o comutador dos operadores conforme ele mesmo anuncia em seu artigo. Isso somente é feito em derivações futuras – como discutiremos mais adiante.

4.5.2 Incerteza entre as variáveis posição e momentum para um pacote de ondas generalizado – Derivação de Weyl (1928)

Uma outra maneira de demonstrar o Princípio da Incerteza é por meio da derivação proposta por Herman Weyl em 1928 [53]. Ao contrário da demonstração anterior, proposta por Heisenberg e exclusivamente válida para funções gaussianas, a estratégia utilizada por Weyl independe do formato da função de onda, e, portanto, podemos dizer que é para um pacote generalizado. Para isso, Weyl utiliza o fato de que observáveis são operadores Hermitianos e, portanto, (conforme apêndice B) para um observável arbitrário $\hat{\mathbf{A}}$, podemos escrever sua variância como o valor médio

$$\sigma_A^2 = \langle\left(\hat{\mathbf{A}} - \langle\hat{\mathbf{A}}\rangle\right)^2\rangle = \langle\hat{\mathbf{A}}^2\rangle - \langle\hat{\mathbf{A}}\rangle^2. \tag{17}$$

Que pode ser calculado como

$$\sigma_A^2 = \int \Psi^*\left(\hat{\mathbf{A}}^2 - \langle\hat{\mathbf{A}}\rangle^2\right)\Psi dx \ . \tag{18}$$

E, além disso, Weyl utiliza o fato de que o operador momentum, especificamente, pode ser escrito como

$$p = -i\hbar\frac{\partial}{\partial x} \ . \tag{19}$$

Consideramos, então, uma partícula na posição *x*. De modo que, por simplicidade, o valor médio da posição é 0. Então

$$\sigma_x^2 = \int \Psi^*(x^2 - \langle 0\rangle^2)\Psi dx \ . \tag{20}$$

Ou seja:

$$\sigma_x^2 = \int \Psi^* x^2 \Psi dx \ . \tag{21}$$

É possível proceder da mesma maneira para o momentum da partícula. Dessa forma, considerando, por simplicidade, $\langle p\rangle = 0$, podemos escrever

$$\sigma_p^2 = \int \Psi^* p^2 \Psi dx \ . \tag{22}$$

Lembrando, ainda, que é possível expressar o operador momentum como $p = -i\hbar\,\partial/\partial x$, temos

$$\begin{aligned}\sigma_p^2 = \int \Psi^*\left(-i\hbar\frac{\partial}{\partial x}\right)^2 \Psi dx \ &= \int \left(\frac{i\hbar\partial}{\partial x}\ \Psi\right)^* \left(\frac{-i\hbar\partial}{\partial x}\Psi\right) dx \\ &= -\hbar^2 \int \frac{\partial\Psi^*}{\partial x}\frac{\partial\Psi}{\partial x} dx\end{aligned} \ . \tag{23}$$

Também é possível utilizar o fato de que p é um operador Hermitiano. Tomando a desigualdade de Schwarz (conforme apêndice C)

$$\left\{\left[\int (f_1 f_1^* + f_2 f_2^*)\, dx\right] . \left[\int (g_1 g_1^* + g_2 g_2^*)\, dx\right]\right\} \geq \left|\int f_1 g_1 dx + \int f_2 g_2 dx\right|^2 \tag{24}$$

e substituindo $f_1 = x\Psi = f_2^*$ e $g_1 = \frac{\partial\Psi^*}{\partial x} = g_2^*$ no lado esquerdo da expressão (24), temos

$$\left[\int (x\Psi x\Psi^* + x\Psi^* x\Psi)\, dx\right] . \left[\int \left(\frac{\partial\Psi^*}{\partial x}\frac{\partial\Psi}{\partial x} + \frac{\partial\Psi}{\partial x}\frac{\partial\Psi^*}{\partial x}\right) dx\right] , \tag{25}$$

o que resulta em

$$4\left[\int \Psi x^2 \Psi^* \, dx\right] . \left[\int \frac{\partial \Psi}{\partial x}\frac{\partial \Psi^*}{\partial x} dx\right] . \tag{26}$$

Substituindo as expressões (21) e (22) em (26), obtemos que o lado esquerdo da expressão (24) pode ser escrito como

$$\frac{4\sigma_x^2 \sigma_p^2}{\hbar^2} \tag{27}$$

Já no lado direito da expressão (24), temos que

$$\left|\int f_1 g_1 dx + \int f_2 g_2 dx\right|^2 = \left|\int x\Psi \frac{\partial \Psi^*}{\partial x} dx + \int x\Psi^* \frac{\partial \Psi}{\partial x} dx\right|^2 = \left|\int \frac{x\partial}{\partial x}(\Psi^*\Psi) dx\right|^2 . \tag{28}$$

Fazendo a integração por partes, $\int u dv = u.v - \int v \, du$, escolhendo $u = x$ e $dv = \ \partial/\partial x \, (\Psi^*\Psi) dx$, obtemos

$$\int x \frac{\partial}{\partial x}(\Psi^*\Psi) dx = x \, . \, \Psi^*\Psi|_{-\infty}^{+\infty} - \int \Psi^*\Psi \, dx = -1 \, . \tag{29}$$

Assim, para que possa ter significado físico, a expressão $(x \, . \, \Psi^*\Psi)$ deve tender a zero para $|x| \to \infty$. Portanto,

$$\left|\int x \frac{\partial}{\partial x}(\Psi^*\Psi) dx\right|^2 = |-1|^2 \, . \tag{30}$$

Obtemos então a desigualdade

$$\frac{4\sigma_x^2 \sigma_p^2}{\hbar^2} \geq |-1|^2 \, . \tag{31}$$

E, portanto,

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \, . \tag{32}$$

Diferentemente da derivação de Heisenberg, que determina a relação de incerteza entre posição e momentum apenas para o caso gaussiano, com a derivação de Weyl, temos a relação para quaisquer formatos de pacote de onda.

4.5.3 Incerteza entre dois observáveis quaisquer A e B – Derivação de Robertson (1929)

Até esse momento, apenas a expressão do Princípio da Incerteza para as variáveis posição e momentum havia se mostrado consistente com o formalismo da Mecânica Quântica. Entretanto, como foi discutido anteriormente, Heisenberg havia afirmado que todas as variáveis canonicamente conjugadas deveriam obedecer a tal relação. A prova matemática para isso só veio em 1929 no artigo publicado por Robertson [54]. Neste artigo, Robertson utiliza a expressão para a desigualdade de Schwarz, assim como proposto por Weyl, para relacionar o quadrado do desvio padrão de dois observáveis representados por operadores Hermitianos. Portanto, (conforme apêndice B) para um observável arbitrário $\hat{\mathbf{A}}$, podemos escrever sua variância como

$$\sigma_A^2 = \langle \left(\hat{\mathbf{A}} - \langle \hat{\mathbf{A}} \rangle \hat{\mathbf{I}}\ \right)^2 \rangle = \langle \hat{\mathbf{A}}^2 \rangle - \langle \hat{\mathbf{A}} \rangle^2 = \langle \delta \hat{\mathbf{A}}^2 \rangle, \tag{33}$$

sendo $\delta\hat{\mathbf{A}} = \hat{\mathbf{A}} - \langle \hat{\mathbf{A}} \rangle \hat{\mathbf{I}}$, sendo $\hat{\mathbf{I}}$ o operador identidade (evidentemente $\langle \hat{\mathbf{I}} \rangle = 1$). A variância pode ser calculada como

$$\sigma_A^2 = \int \Psi^* \left(\hat{\mathbf{A}}^2 - \langle \hat{\mathbf{A}} \rangle^2\right) \Psi dx\ . \tag{34}$$

Considerando que para dois observáveis quaisquer ($\hat{\mathbf{A}}$ e $\hat{\mathbf{B}}$), podemos escrever suas variâncias, respectivamente, como

$$\sigma_A^2 = \langle \delta \hat{\mathbf{A}}^2 \rangle\ = \int \Psi^* \left(\hat{\mathbf{A}}^2 - \langle \hat{\mathbf{A}} \rangle^2\right) \Psi dx, \tag{35}$$

$$\sigma_B^2 = \langle \delta \hat{\mathbf{B}}^2 \rangle = \int \Psi^* \left(\hat{\mathbf{B}}^2 - \langle \hat{\mathbf{B}} \rangle^2\right) \Psi dx\ . \tag{36}$$

Agora, tomando as expressões $f_1 = (\hat{\mathbf{A}} - \langle\hat{\mathbf{A}}\rangle)\Psi^*$, $f_2 = (\hat{\mathbf{A}} - \langle\hat{\mathbf{A}}\rangle)\Psi$, $g_1 = (\hat{\mathbf{B}} - \langle\hat{\mathbf{B}}\rangle)\Psi$ e $g_2 = -(\hat{\mathbf{B}} - \langle\hat{\mathbf{B}}\rangle)\Psi^*$ e substituindo na desigualdade de Schwarz (24), no lado direito da inequação temos

$$\left[2\int \Psi^*(\hat{\mathbf{A}}^2 - \langle\hat{\mathbf{A}}\rangle^2)\Psi\, dx\right].\left[2\int \Psi^*(\hat{\mathbf{B}}^2 - \langle\hat{\mathbf{B}}\rangle^2)\Psi\, dx\right]\ . \tag{37}$$

De modo que, levando em conta as expressões para as variâncias $\Delta\hat{\mathbf{A}}$ e $\Delta\hat{\mathbf{B}}$ dadas pelas expressões (35) e (36), obtemos:

$$\begin{aligned} 4\left[\int \Psi^*\left(\hat{\mathbf{A}}^2 - \langle\hat{\mathbf{A}}\rangle^2\right)\Psi\, dx\right].\left[\int \Psi^*\left(\hat{\mathbf{B}}^2 - \langle\hat{\mathbf{B}}\rangle^2\right)\Psi\, dx\right] &= 4\sigma_A^2\sigma_B^2 \\ &= 4\langle\delta\hat{\mathbf{A}}^2\rangle\langle\delta\hat{\mathbf{B}}^2\rangle. \end{aligned} \tag{39}$$

Substituindo as expressões para *f* e *g* no lado esquerdo da desigualdade de Schwarz (24), temos

$$\left|\int (\hat{\mathbf{A}} - \langle\hat{\mathbf{A}}\rangle)\Psi^*(\hat{\mathbf{B}} - \langle\hat{\mathbf{B}}\rangle)\Psi dx + \int (\hat{\mathbf{A}} - \langle\hat{\mathbf{A}}\rangle)\Psi(\hat{\mathbf{B}} - \langle\hat{\mathbf{B}}\rangle)\Psi^* dx\right|^2 . \tag{40}$$

Abrindo os termos da expressão (40), e utilizando o fato de que $\int \Psi^*\hat{\mathbf{A}}\Psi dx = \int \Psi\hat{\mathbf{A}}^\dagger\Psi^* dx$ temos que o lado direito da expressão (24) resulta em

$$\left|\int \Psi^*(\hat{\mathbf{A}}\hat{\mathbf{B}} - \hat{\mathbf{B}}\hat{\mathbf{A}})\Psi dx\right|^2 . \tag{41}$$

A relação $\hat{\mathbf{A}}\hat{\mathbf{B}} - \hat{\mathbf{B}}\hat{\mathbf{A}}$ pode ser expressa como $[\hat{\mathbf{A}}, \hat{\mathbf{B}}]$ (operação de comutação entre os observáveis $\hat{\mathbf{A}}$ e $\hat{\mathbf{B}}$). Unindo as expressões (39) e (41) na expressão (24), obtemos

$$4\sigma_A^2\sigma_B^2 = 4\langle(\delta\hat{\mathbf{A}})^2\rangle\langle(\delta\hat{\mathbf{B}})^2\rangle \geq \left|\langle[\hat{\mathbf{A}}, \hat{\mathbf{B}}]\rangle\right|^2 . \tag{42}$$

Então

$$\sigma_A\sigma_B = \sqrt{\langle(\delta\hat{\mathbf{A}})^2\rangle\langle(\delta\hat{\mathbf{B}})^2\rangle} = \langle\Delta\hat{\mathbf{A}}\rangle\langle\Delta\hat{\mathbf{B}}\rangle \geq \frac{1}{2}\left|\langle[\hat{\mathbf{A}},\hat{\mathbf{B}}]\rangle\right|. \tag{43}$$

Na expressão acima, definimos as dispersões médias dos observáveis $\hat{\mathbf{A}}$ e $\hat{\mathbf{B}}$ respectivamente como $\langle\Delta\hat{\mathbf{A}}\rangle = \sqrt{\langle(\delta\hat{\mathbf{A}})^2\rangle}$ e $\langle\Delta\hat{\mathbf{B}}\rangle = \sqrt{\langle(\delta\hat{\mathbf{B}})^2\rangle}$. Assim, a afirmação de Heisenberg sobre a conexão entre as relações de incerteza e a propriedade de comutação foi finalmente provada. Assim, podemos concluir que para dois observáveis que não comutam, as suas respectivas variâncias são relacionadas por meio de uma desigualdade que depende do comutador entre esses mesmos observáveis. A relação (43) mostra que um sistema *não* pode ser preparado em um estado quântico tal que duas grandezas físicas cujos observáveis que as representam não comutam entre si sejam livres de dispersão. Uma dispersão baixa em uma das grandezas (baixa incerteza) acarreta uma dispersão alta na outra (incerteza alta). Se os observáveis comutam, essa limitação não existe.

4.5.4 A Derivação do Princípio da Incerteza por Schrödinger (1930)

Erwin Schrödinger [55] também parte da desigualdade de Schwarz (apêndice C), mas neste caso, escrita por meio da expressão simplificada

$$\int ff^*\,dx\,.\int gg^*\,dx \geq \left|\int fgdx\right|^2 \tag{44}$$

Entretanto, o passo que diferencia a derivação de Schrödinger da derivação de Robertson é o fato de que Schrödinger propõe escrever o produto de dois operadores (supostos hermitianos) como uma soma simétrica

$$\hat{\mathbf{A}}\hat{\mathbf{B}} = \frac{\hat{\mathbf{A}}\hat{\mathbf{B}}+\hat{\mathbf{B}}\hat{\mathbf{A}}}{2} + \frac{\hat{\mathbf{A}}\hat{\mathbf{B}}-\hat{\mathbf{B}}\hat{\mathbf{A}}}{2} \tag{41}$$

Agora, tomando $f = \hat{\mathbf{B}}\Psi$ e $g = \hat{\mathbf{A}}^\dagger\Psi^*$, podemos escrever

$$\int \hat{\mathbf{B}}\Psi\hat{\mathbf{B}}^\dagger\Psi^*\,dx\,.\int \hat{\mathbf{A}}^\dagger\Psi^*\hat{\mathbf{A}}\Psi\,dx \geq \left|\int \hat{\mathbf{B}}\Psi\hat{\mathbf{A}}^\dagger\Psi^* dx\right|^2. \tag{42}$$

De modo que, seguindo [56], temos

$$\begin{aligned}\int \Psi\hat{\mathbf{B}}^2\Psi^*\,dx\,.\int \Psi^*\hat{\mathbf{A}}^2\Psi\,dx\ &\geq \left|\int \Psi\hat{\mathbf{A}}\hat{\mathbf{B}}\Psi^* dx\right|^2 \\ &= \left(\int \Psi\hat{\mathbf{A}}\hat{\mathbf{B}}\Psi^* dx\right)\left(\int \Psi^*\hat{\mathbf{B}}\hat{\mathbf{A}}\Psi dx\right).\end{aligned} \tag{43}$$

No lado direito de (43) foi usada a propriedade $\left(\hat{\mathbf{A}}\hat{\mathbf{B}}\right)^\dagger = \hat{\mathbf{B}}^\dagger\hat{\mathbf{A}}^\dagger = \hat{\mathbf{B}}\hat{\mathbf{A}}$. Supondo, sem perda de generalidade[28], que $\langle\hat{\mathbf{A}}\rangle = \langle\hat{\mathbf{B}}\rangle = 0$, temos

$$\sigma_A^2 = \langle\hat{\mathbf{A}}^2\rangle = \int \Psi^*\hat{\mathbf{A}}^2\Psi dx \tag{44}$$

e

$$\sigma_B^2 = \langle\hat{\mathbf{B}}^2\rangle = \int \Psi^*\hat{\mathbf{B}}^2\Psi dx \tag{45}$$

Então,

$$\langle\hat{\mathbf{A}}\hat{\mathbf{B}}\rangle\langle\hat{\mathbf{B}}\hat{\mathbf{A}}\rangle = \left|\int \Psi AB\Psi^* dx\right|^2. \tag{46}$$

Temos

$$\langle\hat{\mathbf{A}}^2\rangle\langle\hat{\mathbf{B}}^2\rangle \geq \langle\hat{\mathbf{A}}\hat{\mathbf{B}}\rangle\langle\hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\,. \tag{47}$$

Escrevendo $\langle\hat{\mathbf{A}}\hat{\mathbf{B}}\rangle\langle\hat{\mathbf{B}}\hat{\mathbf{A}}\rangle$ em termos da soma simétrica dada por (41), obtemos

$$\langle\hat{\mathbf{A}}^2\rangle\langle\hat{\mathbf{B}}^2\rangle \geq \frac{1}{4}\left(\langle\hat{\mathbf{A}}\hat{\mathbf{B}} + \hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\right)^2 - \frac{1}{4}\left(\langle\hat{\mathbf{A}}\hat{\mathbf{B}} - \hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\right)^2 \tag{48}$$

Usando a propriedade $\left|\langle\hat{\mathbf{A}}\hat{\mathbf{B}} - \hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\right|^2 = -\left(\langle\hat{\mathbf{A}}\hat{\mathbf{B}} - \hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\right)^2$, a expressão (48) pode ser reescrita como

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[28] Qualquer operador $\hat{\mathbf{O}}$ pode ser reescrito em outro operador $\hat{\mathbf{O}}'$ cujo valor esperado é nulo. Basta simplesmente fazer $\hat{\mathbf{O}}' = \hat{\mathbf{O}} - \langle\hat{\mathbf{O}}\rangle\hat{\mathbf{I}}$. Nesse caso, por construção, $\langle\hat{\mathbf{O}}'\rangle = 0$.

$$\langle\hat{\mathbf{A}}^2\rangle\langle\hat{\mathbf{B}}^2\rangle \geq \frac{1}{4}\left(\langle\hat{\mathbf{A}}\hat{\mathbf{B}}+\hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\right)^2+\frac{1}{4}\left|\langle\hat{\mathbf{A}}\hat{\mathbf{B}}-\hat{\mathbf{B}}\hat{\mathbf{A}}\rangle\right|^2, \tag{49}$$

Definindo o anticomutador $\{\hat{\mathbf{A}},\hat{\mathbf{B}}\}=\hat{\mathbf{A}}\hat{\mathbf{B}}+\hat{\mathbf{B}}\hat{\mathbf{A}}$, e lembrando que $[\hat{\mathbf{A}},\hat{\mathbf{B}}\,]=\hat{\mathbf{A}}\hat{\mathbf{B}}-\hat{\mathbf{B}}\hat{\mathbf{A}}$, substituindo as expressões (44) e (45) em (49), obtemos

$$\sigma_A^2\sigma_B^2 \geq \frac{1}{4}\langle\{\hat{\mathbf{A}},\hat{\mathbf{B}}\}\rangle^2+\frac{1}{4}\left|\langle[\hat{\mathbf{A}},\hat{\mathbf{B}}\,]\rangle\right|^2=\sigma_{AB}^2+\frac{1}{4}\left|\langle[\hat{\mathbf{A}},\hat{\mathbf{B}}\,]\rangle\right|^2. \tag{51}$$

O segundo termo no lado direito da equação é igual à expressão de Robertson e o primeiro termo no lado direito pode ser reconhecido como a covariância $\sigma_{AB}=(1/2)\langle\{\delta\hat{\mathbf{A}},\delta\hat{\mathbf{B}}\}\rangle=(1/2)\langle\{\hat{\mathbf{A}},\hat{\mathbf{B}}\}\rangle-\langle\hat{\mathbf{A}}\rangle\langle\hat{\mathbf{B}}\rangle=(1/2)\langle\{\hat{\mathbf{A}},\hat{\mathbf{B}}\}\rangle$ entre os observáveis $\hat{\mathbf{A}}$ e $\hat{\mathbf{B}}$ (lembrando que seus respectivos valores esperados são nulos por hipótese) [57], [58]. No caso em que os valores esperados dos observáveis $\hat{\mathbf{A}}$ e $\hat{\mathbf{B}}$ não são nulos, é fácil mostrar que basta inserir em (51) a expressão completa para σ_{AB}[29]. Nesse sentido, pode-se entender que a Incerteza de Heisenberg é um caso especial de Incerteza de Schrödinger, no qual não se leva em conta a covariância entre as duas observáveis.

Por fim, é importante notar que o primeiro termo do lado direito da equação (51) é estritamente positivo (limite inferior), e isso, por si só, já é suficiente para provar a relação de incerteza conhecida quando os operadores envolvidos não comutam (quantidade positiva bem como na equação (51) - o primeiro termo só pode aumentar a contribuição do segundo termo). Como a propriedade importante é, se o produto de $\Delta x\Delta p$ der um número positivo, diferente de zero, temos o que precisamos: se a precisão no conhecimento de um aumenta, a precisão do outro deve diminuir, e vice-versa.

4.6 SEGUNDO NÍVEL DO PRINCÍPIO DA INCERTEZA: FENOMENOLOGIA

Na Teoria Quântica o processo de medição está diretamente associado ao colapso da função de onda (Apêndice A). Se, por exemplo, sabemos que uma partícula passou por uma fenda então, naquele instante, o pacote de ondas deve ter

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[29] A variância σ_A^2 nada mais é do que um caso particular da covariância, podendo ser entendida como a covariância do observável $\hat{\mathbf{A}}$ consigo mesmo. Basta fazer $\sigma_A^2\equiv\sigma_{AA}=(1/2)\langle\{\delta\hat{\mathbf{A}},\delta\hat{\mathbf{A}}\}\rangle=\langle\delta\hat{\mathbf{A}}^2\rangle=\langle\hat{\mathbf{A}}^2\rangle-\langle\hat{\mathbf{A}}\rangle^2$.

colapsado para o tamanho da fenda e, portanto, de acordo com a transformada de Fourier, a distribuição do momentum não poderá ser representada por um único valor (devendo ser, então, uma distribuição de valores). Entretanto, propomos que nesta seção o estudo dos experimentos propostos por Werner Heisenberg permaneça apenas a nível fenomenológico, de modo que algumas das possíveis interpretações de tais fenômenos subatômicos serão discutidas no próximo nível.

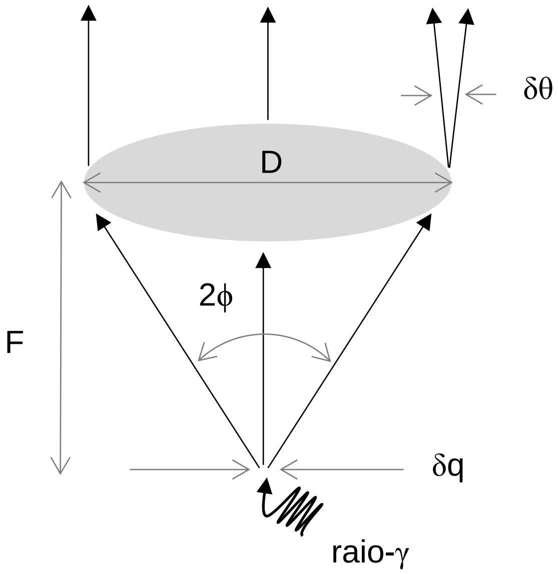
4.6.1 O Microscópio de raios Gamma

No artigo de 1927 o primeiro argumento utilizado para justificar o Princípio da Incerteza também é o primeiro experimento mental proposto por Heisenberg, que ficou conhecido como o Experimento do Microscópio. Nessa análise fenomenológica, Heisenberg discute a possibilidade de determinar a posição de um elétron utilizando um microscópio [1].

> Por exemplo, vamos iluminar o elétron e observá-lo ao microscópio. Então, a maior precisão alcançável na medição da posição é governada pelo comprimento de onda da luz. No entanto, em princípio, pode-se construir, digamos, um microscópio de raios gama e, com ele, realizar a determinação da posição com a precisão desejada. Nesta medição, há um recurso importante, o efeito Compton. Toda observação de luz espalhada proveniente do elétron pressupõe um efeito fotoelétrico (no olho, um no plano fotográfico, na fotocélula) e, portanto, também pode ser interpretada de forma que um quantum de luz atinge o elétron, é refletido ou espalhado, e então, mais uma vez dobrado pela lente do microscópio, produz o efeito fotoelétrico. No instante em que as posições são determinadas - portanto, no momento em que o fóton é espalhado pelo elétron - o elétron sofre uma mudança descontínua no momento. Essa mudança é tanto maior quanto menor for o comprimento de onda da luz empregada - ou seja, mais exata será a determinação da posição. No instante em que a posição do elétron é conhecida, seu momento, portanto, pode ser conhecido em magnitudes que correspondem a essa mudança descontínua. Assim, quanto mais precisamente a posição é determinada, menos precisamente o momento é conhecido e vice-versa. Nesta circunstância, vemos uma interpretação física direta da equação $pq - qp = -i\hbar$. Seja q_1 a precisão com a qual o valor q é conhecido (q_1 é, digamos, o erro médio de q), portanto aqui o comprimento de onda da luz. Seja p_1 a precisão com a qual o valor p é determinável; isto é, aqui, a mudança descontínua de p no efeito Compton. Então, de acordo com as leis elementares do efeito Compton, p_1 e q_1 estão na relação $p_1\, q_1 \sim h$ (1983, p. 64, tradução livre) [1].

Uma simplificação corrigida do experimento de Heisenberg é proposta por [18] e está representada na Figura 1.

Figura 1 – Representação esquematizada do microscópio de raios gamma.



Suponhamos que um feixe de fótons de raios gamma está inicialmente ao longo do eixo da lente do microscópio. Supondo que a lente possui um diâmetro D, a resolução angular $\delta\theta$ pode ser descrita como

$$\delta\theta \approx \frac{\lambda}{D} \tag{52}$$

Tomando F como a distância focal da lente, temos que o objeto está a uma distância δq descrita por

$$\delta q \approx F\delta\theta \approx \frac{\lambda F}{D} \tag{53}$$

O fóton originalmente possui um momentum p_γ que, após ser elasticamente espalhado pela lente, adquire um componente transversal no intervalo compreendido entre $-p_\gamma$ e $+p_\gamma$. Além disso, para ângulos pequenos, temos que

$$\operatorname{sen}\phi \approx \tan\phi = \frac{D}{2F} \tag{54}$$

Dessa forma, a incerteza no componente transversal do momentum do fóton será

$$p_{\gamma} \approx \left(\frac{D}{F}\right) p_{\gamma} \approx \frac{hD}{\lambda F} \tag{55}$$

e pela conservação do momentum, a incerteza do momentum do elétron torna-se

$$\delta p \approx \frac{hD}{\lambda F} \tag{56}$$

De modo que, multiplicando as equações (53) e (56), temos a relação de incerteza de Heisenberg

$$\delta q \delta p \sim h \tag{57}$$

4.6.2 Um elétron passando por uma fenda

Outro experimento mental a qual Heisenberg faz referência ao longo do texto de 1927 é o de um elétron atravessando uma fenda. O fenômeno consiste em fazer com que um elétron atravesse uma pequena abertura, seja ela um orifício ou uma fenda, de maneira que a onda associada ao movimento do elétron sofrerá o efeito de difração, resultando na alteração de sua trajetória. Dessa forma, a partir do momento em que o elétron atravessa a fenda, torna-se impossível prever exatamente o local em que o mesmo será detectado na chapa fotográfica [59].

> "Esta deflexão deve ser de pelo menos a mesma ordem de magnitude que o alargamento natural do feixe provocado pela difração pelas fendas, se qualquer medição for possível. O ângulo de difração é aproximadamente λ/d se λ denota o comprimento de onda de Broglie; portanto, $\lambda/d \sim E_1 t_1\, dp$, ou como $\lambda = h/p$, $E_1 t_1 \sim h$. " (1983, p. 67, tradução livre) [1].

Para calcular a incerteza associada ao experimento é preciso ter em vista que, antes de atravessar o diafragma, o estado de movimento da partícula pode ser representado por uma série de ondas planas de comprimento de onda λ, e seu momentum p é associado a λ segundo a relação de de Broglie:

$$p = \frac{h}{\lambda} \tag{58}$$

Entretanto, após a passagem pelo orifício, a onda associada ao movimento da partícula sofre o efeito de difração e, além disso, é possível interpretar que a partícula adquire um momentum em sua componente paralela à extensão da fenda (p_y). Tendo em vista que o padrão de distribuição gerado na chapa fotográfica por um conjunto de partículas incidindo sucessivamente por uma fenda é muito semelhante a um padrão de interferência, tal padrão pode ser determinado por meio de uma descrição ondulatória. Quando se incide luz coerente, a franja de máximo central concentra quase toda a energia luminosa, de modo que podemos considerar analogamente que a probabilidade de detecção da partícula se concentra quase que totalmente nessa região. Para determinar a largura dessa região basta calcular a posição da primeira franja de mínimo, dada pela equação (59) [60]:

$$arcsen(\theta) = \lambda \tag{59}$$

em que a é a largura da fenda e θ é a posição angular do mínimo. Expandindo a função $sen(\theta)$ em séries de Taylor podemos considerar $sen(\theta) \approx \theta$. Usando tal aproximação na equação (59) obtemos

$$\theta \approx \frac{\lambda}{a} \tag{60}$$

A incerteza associada ao momentum adquirido pela partícula Δp está, também, associada à largura do máximo central, de modo que quanto maior for o momentum p adquirido, maior será o desvio da sua trajetória e, portanto, mais próxima às extremidades dessa região a partícula será detectada. Dessa forma, a incerteza Δp corresponde ao maior valor que pode ser adquirido por p em sua componente p_y. Relacionando o módulo de ambos os vetores através de uma análise trigonométrica, temos

$$\mathrm{sen}\theta = \frac{p_y}{p} \tag{61}$$

de modo que

$$p_y \approx \theta p \tag{62}$$

e podemos descrever Δp como

$$\Delta p = p_{y\,máx} = \theta p \tag{63}$$

substituindo as equações (58) e (60) na equação (63):

$$\Delta p \approx \frac{h}{a} \tag{64}$$

Além da incerteza no momentum, existe também uma incerteza Δq na medida da posição da partícula no instante em que atravessa a fenda. Sabemos que a partícula passa pela fenda, mas não sabemos em que ponto exato da extensão da fenda ela está localizada. Já que os limites em que é possível encontrar a partícula correspondem ao espaço no interior da fenda, podemos definir Δq como sendo a própria largura a da fenda, de modo que

$$\Delta p \approx \frac{h}{\Delta q} \tag{65}$$

Reorganizando a expressão (65), obtemos a equação (66) nos mostra que as incertezas associadas estão de acordo com a relação de indeterminação.

$$\Delta p \Delta q \approx h \tag{66}$$

4.7 TERCEIRO NÍVEL DO PRINCÍPIO DA INCERTEZA: INTERPRETAÇÕES DO PRINCÍPIO DA INCERTEZA PARA AS VARIÁVEIS POSIÇÃO E MOMENTUM

O terceiro nível, que diz respeito interpretação de uma teoria ou conceito físico, está sempre na fronteira entre a Física e a Filosofia. É possível que, em cursos de graduação, possa-se evitar o terceiro nível, quando o foco da disciplina está mais voltado para o desenvolvimento do domínio do formalismo matemático. Entretanto, cursos voltados a abordagens históricas e conceituais podem ser mais favoráveis à sua incorporação de maneira integral à teoria, tal como Pessoa Jr. [34].

Apresentamos aqui quatro interpretações que a literatura aponta para o Princípio da Incerteza, que podem ser discutidas por professores que tenham interesse em trazer a dimensão interpretativa da Teoria para o ensino. Por outro lado, aqueles e aquelas que perceberem nessa seção aspectos que fujam do escopo da Física podem optar por não a utilizar.

Depois de mais de 90 anos da primeira apresentação do Princípio da Incerteza de Heisenberg uma ampla literatura foi desenvolvida com o objetivo de abordar seus diferentes significados e implicações [61]. Ao longo desta seção, iremos seguir a discussão e classificação proposta por Jijnasu [23]. Rigorosamente, as interpretações utilizadas para a incerteza nas variáveis posição e momentum podem ser divididas entre versões ônticas e versões epistêmicas. As versões do tipo ônticas são aquelas que propõe que a incerteza é uma característica inerente da matéria, independentemente de qualquer medida, enquanto as versões do tipo epistêmicas propõem que a incerteza é uma característica do procedimento de medida, embora, ainda assim, não possa ser evitada. Cabe destacar que a formulação original de Heisenberg mistura ambas concepções. Ao longo do artigo de 1927, na apresentação do experimento do microscópio de raios-gama e durante a derivação do Princípio da Incerteza utilizando o formalismo de Dirac-Jordan, Heisenberg assume uma postura epistêmica - atando as incertezas a medição. Entretanto, ao final do artigo, Heisenberg assume uma inclinação positivista e interpreta todos os resultados como se a indeterminação fosse uma característica intrínseca da matéria.

Definimos como perspectiva ôntica aquilo que Jijnasu [23] chama de perspectiva "incerteza intrínseca" (e designa pela letra grega Δ). A visão de uma “realidade não definida” foi posteriormente recuperada por Heisenberg [50] com o conceito de potencialidade, que, por sua vez, deu origem à Física Aristotélica[30]. Além disso, em 1935 Popper [62] discutiu o fato de que as relações de Heisenberg poderiam ser interpretadas estatisticamente, isto é, o Princípio da Incerteza não trataria de eventos singulares, mas de dispersões de conjuntos. Nesta concepção, não podemos falar de qualquer limitação da precisão em uma única medição. Em princípio, poderíamos medir a posição e o momentum com qualquer precisão arbitrária. O que não podemos fazer é preparar um conjunto tão nitidamente definido em posição e momentum quanto desejamos. Tal visão também foi endossada por interpretações mais contemporâneas da Mecânica Quântica [55] e pode ser considerada, também, como uma perspectiva ôntica, uma vez que reconhece as relações de indeterminação como uma característica natural da matéria (independentemente das medidas), e difere do primeiro caso, porém, por não considerar que elas possam ser aplicadas a uma única medição. Jijnasu [23] a chama de perspectiva “estatística” (e ele a designa

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[30] Este conceito também foi proposto recentemente por Busch e Jaeger [64] no âmbito das Medidas de Operador Positivo (valorizado) (POMs).

pela letra grega σ). Os adeptos da interpretação da indeterminação intrínseca entendem que a “interpretação do conjunto” apenas descreve como a realidade “não afiada” pode se manifestar nas medições, sendo a manifestação pragmática da incerteza intrínseca [63].

Além disso, a interpretação estatística não impõe qualquer restrição de precisão ou exatidão em medições simultâneas. No entanto, os experimentos mentais de Heisenberg indicam que algum tipo de restrição pode acontecer (embora o formalismo que ele desenvolveu não tenha sido capaz de demonstrar isso). Motivados por esse problema, muitos estudos têm sido desenvolvidos a fim de determinar a restrição em medidas simultâneas - que são classificadas como versões epistêmicas. Jijnasu [23] divide essas versões em dois grupos: o primeiro trata da relação entre a precisão de uma medida e a perturbação que essa medida causa na variável conjugada, como faz Heisenberg no caso do microscópio de raios gama [59]. Nesse caso, o momentum do elétron não foi medido - apenas a posição foi medida - e sobre o momentum foi apenas inferida sua faixa de possível perturbação. Jijnasu [23] chama essa perspectiva de incerteza de distúrbio de erro (e ele a designa por $\epsilon - \eta$), sendo possível encontrar muitas críticas à versão perturbação [30], [60][31]. Ademais, em vez de relacionar o resultado de uma única medição com o distúrbio que ela causa, também é possível realizar uma segunda medição para determinar o momentum do elétron. Nesse caso, também é possível provar que as incertezas entre o erro de posição e o erro de momentum obedecem a uma relação de incerteza [7], [61] – [63], Jijnasu [23] denomina tal perspectiva de incerteza como erro-erro (e ele a designa por $\epsilon - \epsilon$).

As quatro versões do Princípio da Incerteza/Indeterminação (duas ônticas e duas epistêmicas) estão resumidas abaixo na Tabela 1.

Tabela 1 – Interpretações do Princípio da Incerteza.

Interpretações do Princípio da Incerteza		
	Versão	Principais Aspectos
Ônticas	Intrínseca (Δ)	Refere-se a uma indeterminação intrínseca na estrutura da matéria. Quando o momentum de uma

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[31] Recentemente, Ozawa [65], [66], desafiou a apresentação original de Heisenberg do Princípio da Incerteza, introduzindo "uma generalização da relação de Heisenberg comprovada como válida para medições arbitrárias é proposta e revela dois tipos distintos de possíveis violações da relação de Heisenberg". No entanto, Busch, Lahti & Werner [67] mostraram que a relação de Ozawa não era universalmente válida.

		partícula é conhecido, suas coordenadas não têm significado físico. A discussão de Heisenberg no início de 1927 sobre a descontinuidade e a impossibilidade de determinar a velocidade é um exemplo dessa abordagem.
	Estatística (σ)	As indeterminações não se referem a um único quantum, mas a um conjunto. A indeterminação deve ser entendida como uma divergência estatística.
Epistêmicas	Erro-perturbação ($\epsilon - \eta$)	No caso do microscópio de raios gama, a posição é realmente medida e q_1 é o erro de medição, p, entretanto, não é medido e, portanto, p_1 é a perturbação máxima do sistema.
	Erro-erro ($\epsilon - \epsilon$)	Refere-se à medição real da posição (e seu erro) e momentum (e seu erro). Nesse caso, duas medições são realizadas concretamente.

Fonte: Elaboração realizada pelos autores com base no texto de Jijisanu [23].

4.8 CONSIDERAÇÕES FINAIS

Cabe ressaltar que, embora percebamos que a distinção entre as diferentes categorias de um conceito pode ser considerada uma maneira simplista de apresentar a Física, tendo em vista que tais aspectos não são desenvolvidos de maneira isolada, identificamos, também, seu valor didático, pois, sabendo do nível de abstração do Princípio da Incerteza, entendemos que estudar cada aspecto isoladamente pode ajudar os estudantes a entender melhor o conceito.

Nesse contexto, tendo em vista o papel central do livro didático na formação de cientistas e professores de ciências, o presente trabalho pode ser compreendido como uma ferramenta de apoio para o Ensino de Física, uma vez que os livros didáticos não costumam distinguir entre as diferentes interpretações do Princípio da Incerteza e, geralmente, apresentam apenas algumas das possíveis formas de demonstração do mesmo. Sendo assim, como material complementar, o presente trabalho pode enriquecer o estudo do conceito e ajudar em sua compreensão.

Como pôde ser percebido, existem várias formas de demonstrar o Princípio da Incerteza e investir mais tempo na dedução, ao invés de fornecer os alunos as fórmulas prontas, pode contribuir para que os estudantes adquiram mais familiaridade com as equações e possam desenvolver seu raciocínio matemático. Além disso conectar o Princípio da Incerteza com sua fenomenologia pode tornar a compreensão, tendo em vista a aplicação do conceito, mais nítida, além de ressaltar a importância da experimentação, mesmo que mental, em uma teoria Física. Demonstrar a pluralidade interna do Princípio da Incerteza implica em demonstrar, também, a riqueza da Teoria Quântica e contribui para passar aos estudantes uma noção de sua grande complexidade. Ademais, muito se discute sobre abordagens de Física Moderna na Ensino Médio, mas só podemos falar em ensinar Física Quântica na educação básica se discutirmos, primeiro, a formação de professores.

Uma possível maneira de trabalhar o presente material é através de uma abordagem didática que, complementar ao estudo do livro, permita dividir os alunos segundo qual das três diferentes categorias mais se identifiquem. Tal abordagem, apesar de possuir suas limitações, respeitaria a pluralidade existente em sala de aula, a diversidade dos alunos, e permitiria, assim, atingir mais estudantes e em níveis de abstração mais profundos.

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4.10 MATERIAL SUPLEMENTAR

4.10.1 Apêndice A: Postulados da Mecânica Quântica

Na Mecânica Clássica, o movimento de qualquer sistema físico pode ser descrito se sua posição e velocidade são conhecidas como uma função do tempo, de modo que o estado do sistema no tempo fixo t_0 será definido pela especificação de suas coordenadas generalizadas[32] $q_i\ (t_0)$ e seus momentos conjugados $p_i\ (t_0)$. Desta forma, conhecendo o estado inicial de um sistema, pode-se prever com os resultados de qualquer medição realizada em tempo futuro. Ademais, na Mecânica Clássica a evolução temporal do estado de um sistema pode ser descrita pelas equações de Hamilton-Jacobi, de modo que, por serem equações diferenciais de primeira ordem, sua solução será única se o valor dessas funções em um determinado momento for fixo, desta forma podemos dizer que na Mecânica Clássica, conhecendo o estado inicial de um sistema, o estado de um sistema poderá ser conhecido em qualquer outro instante [1].

Entretanto, descrever um sistema quântico é essencialmente diferente de descrever um sistema clássico e o objetivo deste apêndice é, seguindo Osvaldo Pessoa Jr. [2] fornecer, através de uma simplificação[33] dos seis postulados da Mecânica Quântica, os fundamentos básicos para que o leitor possa compreender a descrição de um sistema quântico. Tais postulados fornecem como descrever matematicamente o estado de um sistema quântico e, conhecendo-o, como prever seus estados futuros.

Postulados de Definição

Postulado 1. Em um determinado instante fixo t_0, o estado de um sistema físico isolado é definido por uma função de onda $\Psi(x, t_0)$.

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[32] As variáveis $q_i\ (t)$ e $p_i\ (t)$ $(i = 1,2, \ldots N)$ são chamadas de variáveis dinâmicas fundamentais, pois todas as grandezas físicas associadas a um determinado sistema (energia, momento angular, etc.) podem ser expressas em termos de $q_i\ (t)$ e $p_i\ (t)$.

[33] Uma vez que os postulados da Mecânica Quântica, propriamente, são muito mais gerais e tratam de representações no espaço de Hilbert.

Postulado 2. Toda a grandeza física mensurável (ou observável) Q é descrita por um operador[34] auto adjunto $\hat{Q}$.

Postulados de Evolução Linear

Postulado 3. A evolução temporal de uma função de estado $\Psi(x, t_0)$ é linear e determinista, sendo governada pela equação de Schrödinger dependente do tempo.

$$\left[-\frac{\hbar}{2m}\nabla^2 + V(r,t)\right]\Psi(r,t) = i\hbar\frac{\partial\Psi}{\partial t} \qquad (A1)$$

Postulados do Algoritmo Estatístico

Postulado 4. O único resultado possível para a medição de um observável Q é um dos autovalores q_i do operador correspondente $\hat{Q}$.

Postulado 5. Quando o observável Q é medido em um sistema no estado normalizado, a probabilidade $Prob(q_i)$ de se obter o autovalor do espectro discreto q_i do operador correspondente $\hat{Q}$ é

$$Pro(q_i) = \int_a^b \Psi.\Psi^* dx$$

Onde Ψ é a função de onda que descreve o estado associado ao autovalor q_i de $\hat{Q}$.

Postulado da Projeção

Postulado 6. Se a medição de um observável Q em um sistema descrito pela função de onda $\Psi(x,t)$ fornece o resultado q_i, então o estado do sistema imediatamente após a medição é a projeção normalizada de $\Psi(x,t)$ no auto subespaço associado a q_i.

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[34] Na teoria quântica, o estado de um sistema é sempre representado por um vetor enquanto que uma quantidade física é representada por um operador.

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4.10.2 Apêndice B: Conceitos da Teoria Estatística

Valores Médios e Desvio Padrão

Seguindo Salinas [1], consideremos uma variável aleatória u que possa assumir M valores discretos, tal que u_j ocorra com probabilidade $Pj = P\,(u_j)$, de modo que $0 \leq P_j \leq 1$, para qualquer j (considerando que $\sum_{j=1}^{M} P(u_j) = 1$, ou seja, que a distribuição já está normalizada). Temos que o **valor médio** da variável *u* é definido pela expressão

$$\bar{u} = \langle u \rangle = \sum_{j=1}^{M} u_j P(u_j) \qquad (B1)$$

A partir de então, podemos definir matematicamente outros conceitos, como o desvio da média

$$\Delta u = u - \langle u \rangle \qquad (B2)$$

a dispersão, ou variância

$$\langle (\Delta u)^2 \rangle = \langle (u - \langle u \rangle)^2 \rangle = \langle u^2 \rangle - \langle u \rangle^2 \qquad (B3)$$

Tendo em vista que $\langle (\Delta u)^2 \rangle \geq 0$, temos que $\langle u^2 \rangle \geq \langle u \rangle^2$. E, por fim, o desvio padrão

$$\sqrt{\langle (\Delta u)^2 \rangle} = \sqrt{\langle (u - \langle u \rangle)^2 \rangle} = \sqrt{\langle u^2 \rangle - \langle u \rangle^2} \qquad (B4)$$

Que, se comparado com o valor médio, fornece uma noção sobre a largura de distribuição de probabilidade [1].

Propriedades da Função Gaussiana

A integral gaussiana pode ser calculada conforme a seguinte equação

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\pi/a} \qquad (B5)$$

De modo que, se reescrevermos ela de tal maneira

$$f(x) = e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad (B6)$$

Temos que, na teoria estatística, tal função também é conhecida como distribuição normal de probabilidades e, a partir da própria equação, é possível identificar facilmente algumas propriedades importantes de determinada distribuição estatística, tal como a média, dada pela constante μ, a variância, que é fornecida por σ^2 e o desvio padrão, que é dado pela constante σ [2].

Transformada de Fourier

Seguindo Sauter e Azevedo [3], temos que dada uma função $f(t)$, seja ela real ou complexa, a transformada de Fourier $F(w)$ de $f(t)$ é definida como

$$F(w) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{+\infty} f(t) e^{-iwt} dt \qquad (B7)$$

E, dada uma função $F(w)$, seja ela real ou complexa, a transformada inversa de Fourier $f(t)$ de $F(w)$ é definida como

$$f(t) = \mathcal{F}^{-1}\{F(w)\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(w) e^{iwt} dw \qquad (B8)$$

No caso de uma função do tipo

$$f(t) = e^{-\frac{a}{2}t^2} \qquad (B9)$$

para $a > 0$, temos que a transformada de Fourier será

$$\mathcal{F}\{f(t)\} = F(w) = \frac{\sqrt{\pi}}{\sqrt{a}} e^{-\frac{w^2}{2a}} \qquad (B10)$$

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4.10.3 Apêndice C: Desigualdade de Cauchy-Schwarz

Seguindo a demonstração de Brito [1], consideremos uma função quadrática do formato

$$f(x) = (a_1x - b_1)^2 + (a_2x - b_2)^2 + (a_3x - b_3)^2 + \cdots + (a_nx - b_n)^2 \qquad (C1)$$

Desenvolvendo os quadrados das diferenças temos que

$$f(x) = (a_1^2x^2 - 2a_1b_1x + b_1^2) + (a_2^2x^2 - 2a_2b_2x + b_2^2) + \cdots + \ (a_n^2x^2 - 2a_nb_nx + b_n^2) \ (C2)$$

E organizando os semelhantes

$$f(x) = (a_1^2 + a_2^2 + \cdots + a_n^2)x^2 - 2(a_1b_1 + a_2b_2 + \cdots + a_nb_n)x + (b_1^2 + b_2^2 + \cdots + b_n^2) (C3)$$

Sabemos, por definição, que $f(x) \geq 0$ para qualquer valor de *x*. Tendo em vista o formato de uma função quadrática

$$f(x) = ax^2 + bx + c \qquad (C4)$$

E sabendo que neste caso temos que

$$\Delta = b^2 - 4ac \qquad (C5)$$

Podemos escrever

$$\Delta = [2(a_1b_1 + a_2b_2 + \cdots + a_nb_n)]^2 - 4[(a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)] \quad (C6)$$

Além disso, tendo em vista que para que as raízes da função sejam valores reais $\Delta \leq 0$, então

$$4(a_1b_1 + a_2b_2 + \cdots + a_nb_n)^2 - 4(a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2) \leq 0 \qquad (C7)$$

Somando o termo $4(a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)$ nos dois lados da expressão (C7) e dividindo por 4, temos

$$(a_1b_1 + a_2b_2 + \cdots + a_nb_n)^2 \leq (a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2) \qquad (C8)$$

De modo que a expressão (C8) também pode ser reescrita como

$$(a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2) \geq (a_1b_1 + a_2b_2 + \cdots + a_nb_n)^2 \qquad (C9)$$

Por fim, tomando $\int f_1f_1^* = a_1^2$, $\int f_2f_2^* = a_2^2$, $\int g_1g_1^* = b_1^2$ e $\int g_2g_2^* = b_2^2$, podemos reescrever a expressão (C9) como

$$\left[\int (f_1f_1^* + f_2f_2^*)\, dx\right] . \left[\int (g_1g_1^* + g_2g_2^*)\, dx\right] \geq \left|\int f_1g_1dx + \int f_2g_2dx\right|^2 \qquad (C10)$$

Referências

[1] F. W. M. Brito, “Otimização: uma aplicação para desigualdade das médias e para desigualdade de Chauchy-Schwarz,” (Universidade Federal da Paraíba, 2016).

5 TERCEIRO ARTIGO ORIGINAL

THE ROLE OF TEXTBOOKS IN THE STABILIZATION OF ACTANTS: A CASE STUDY ON THE STABILIZATION OF THE UNCERTAINTY PRINCIPLE.

5.1 ABSTRACT:

Textbooks are fundamental tools in Science Teaching and, in this study, we analyze their role in stabilizing an important topic in Quantum Physics, the Uncertainty Principle. Our analysis was performed on 34 Quantum Physics textbooks used in courses at university level and was carried out in the light of the Actor-Network Theory and the Social Network Analysis. For this, we defined 43 different aspects involved in the Uncertainty Principle, nested in six categories: structure (how the Uncertainty Principle is organized in the textbook), physical quantities (which physical quantities are presented), derivation, *gedanken* experiments used (eg gamma ray microscope), the class of mathematical formalism adopted (eg Dirac formalism) and other elements (general issues not addressed in the previous categories). Our results show that, in fact, the Uncertainty Principle is a plural actant with many aspects involved, and that some of these aspects are more stable and well established in the literature and scientific context than others. Furthermore, textbooks as a whole do not present a homogeneous discourse, but there are groups of traditional textbooks that tend to present very similar discussions.

Keywords: Textbooks; Actor-Network Theory; Uncertainty Principle.

5.2 INTRODUCTION

The textbook plays a fundamental role in general Education and in science teaching more specifically, being widely used as a tool in the pedagogic context from the early years of basic education to graduate courses. Among other aspects, textbooks differ from other literary genres mainly because of their authority, both pedagogical and scientific (SIMON, 2016), and their characteristic of systematizing the knowledge of a specific field, so that, it plays the role of introducing new students to the different subjects of study (FUSELIER; JACKSON; STOIKO, 2016).

Although it does not have much prestige in academic circles, mainly due to the supposed dichotomy between teaching and research (BADINO; NAVARRO, 2013; SIMON, 2016), at least since the 19th century, science teaching has widely used textbooks as an important tool for typical practices in this context (SIMON, 2016; STINNER, 1992), so that several generations of scientists and science teachers, in addition to the general population, were initiated into scientific disciplines through such a tool, providing to the textbooks an invaluable influence on scientific practice.

In view of the central role played by the textbook in the teaching-learning process, research in the science education field in the first half of the 20th century was already concerned with such teaching tool. Such studies focused mainly on the analysis of content and levels of difficulty explored by the books (BRAY, 1934; CURTIS, 1943; HERRIOTT, 1933; HULL, 1937; OVERN et al., 1928; SMITH, 1946; WYMAN, 1931). However, from the second half of the 20th century, the textbook also became an object of research based on a historical conception (SIMON, 2016). A landmark in the history of textbook studies is the systematization provided by Thomas Kuhn in 1962 in the book *The Structure of Scientific Revolutions*, in which the physicist and philosopher of science emphasizes that the textbook has the role of perpetuating normal science to new students and scientists of an area, giving a broad, homogeneous and linear view of the state of knowledge and the history of science, erasing any traces of previous problems and past paradigms (KUHN, 2017).

From that moment on, a whole new area of studies on the textbook was consolidated, since it started to be perceived as a relevant tool in the history of science and in scientific practice - at least in this first systematization, the textbook stops being understood only as a neutral collection of information about a given area and is now perceived as an instrument that contributes to the perpetuation of a specific vision of science. In this perspective, studies of science education suggest, for example, that textbooks are responsible for reproducing sexist and heteronormative discourses (METOYER; RUST, 2011; SNYDER; BROADWAY, 2004), besides reproduce a naive view of science (CHIAPPETTA; SETHNA; FILLMAN, 1993; FUSELIER; JACKSON; STOIKO, 2016), significantly affecting the way of students, scientists and society face such themes (WALTZ, 2006). Furthermore, recent studies indicate that the role of the textbook in the construction of science is not limited to a mere tool for transmitting knowledge and the current paradigm, but that the textbook can also play an active role as an agent in the process of scientific development (BADINO; NAVARRO, 2013; KAISER, 2005).

In the present work we intend to contribute to the studies on textbooks, by analyzing their role on the consolidation of specific scientific contexts. More specifically, we start from Actor-Network Theory, according to which every actor is defined by a heterogeneous network and a set of performances that it may provide. In this sense, we discuss how textbooks mobilizes different networks to define the essence of a specific actor, in this case, the Uncertainty Principle - a central topic in the theoretical framework of Quantum Theory (BOHM, 1952; GREENSTEIN; ZAJONC, 1997) and one of the pillars of the hegemonic interpretation of quantum physics (JAMMER, 1966, 1974), highlighting the different and common features and elements that compose such networks. In order to do so, we analyzed 34 college textbooks, which were selected for their strong presence as references in the teaching plans of Quantum Theory courses in Brazilian universities, some taught in introductory courses and others in advanced ones, mapping which elements are shared or not among them. Although there is a certain consensus that textbooks tend to present a stable and homogeneous discourse and that they reflect fixed and consensual knowledge, reproducing only what is scientifically well established and which can be considered indispensable for a given area, our results indicate that this instrument can be understood in more complex ways.

5.3 PRIOR CONSIDERATIONS: ACTOR-NETWORK THEORY AND TEXTBOOKS.

To construct a network of interaction between university Quantum Physics textbooks, focusing on Uncertainty Principle, we are not thinking of a network that is formed in a well-defined context, with textbooks playing a role of definite and static material inanimate things. Even agreeing that these textbooks are adopted and read in the universe of physics education in university level (undergraduate or graduate), they are not pure non-human static objects perfectly localized in some restricted space and time. On the contrary, their roots exceed much far from it.

In the light of Actor-Network Theory, the notion of “defined context” is inconsistent. Consider a network of Quantum Physics textbooks and a specific topic, Uncertainty Principle, which is what we propose here. Physics textbooks are important form of discourse and their purpose is not restricted uniquely to teaching scientific concepts. Limiting our analysis to the Uncertainty Principle, it is common that textbook authors make choices privileging specific interpretations of that topic over others, proposing selections from what will appear in the textbook. They also are responsive to commercial and academic issues, performing actions that potentially promote changes in other textbooks in the network. Textbooks bring to stage several forms of conceptions related to specific scientific concepts, forms of distributing them through the text, interpretations of outcomes of *gedankenexperiments*, all of them articulated to symbols, texts or visual graphics and to other texts in a dialogical interaction (e.g., other textbooks, scientific articles, all of which can be distributed in a large time period). Physics textbooks vehiculate a particular genre of scientific discourse, in a communication process that vehiculates a large set of characteristic signs of multisemiotic nature (O'Halloran, 1998, 2005), like graphs, figures, mathematical symbols, visualizations and so on (Kinard & Kozulin, 2008). In this multisemiotic discursive process, the authors highlight some conceptual and/or mathematical features of Uncertainty Principle, producing discursive stances that is dialogically founded in a series of external sources and, at the same time, also dialogically interact with discourse vehiculated in other textbooks of the network. In this sense, the textbooks can be considered as *actants*, a “specific” active and interacting element of the network. These conceptual and mathematical features can be framed as emerging from a network of explanations. Like Latour (1996) proposes, explanations can be tough as

> [The] attachment of a set of practices that control or interfere in one another. No explanation is stronger or more powerful than providing connections among unrelated elements or showing how one element holds many others. This is not a property that is distinct from networks but one of their essential properties (Latour, 1996, pp. 375-376).

In a broader perspective, since textbooks is a discourse that interacts with a bunch of external scientific discursive sources and also are grounded in technical and commercial issues (e.g., computers, graphic design, commercial management), they can be also considered as networks. In a similar way, the actions of these textbooks do no restrict in their interactions in this smaller environment, since it can impact larger educational environments by “through their engagement with a long chain of elements containing both humans and non-humans” (Vicsek, Király, & Kónya, 2016, p. 81). Thus, this “specific” network of Quantum Physics textbooks dealing with Uncertainty Principle can also be understood as an actant of a larger network (e.g., all university science textbooks). In this sense, the idea of “defined context” is usually problematic In Actor-Network Theory. These networks constitute not only from these actants, but also from the *interactions* between them. These interactions do not vanish in the spatial and temporal “limits” of that network.

An educational textbook is a technical tool that is engaged jointly with the humans to produce and spread knowledge. In other words, a textbook is a non-human entity that plays its role on educational settings not by itself, but with humans. A “thing” like a textbook does not take part on actions alone, but “might authorize, allow, afford, encourage, permit, suggest, influence, block, render possible, forbid, and so on” (Latour, 2005, p. 72). Thus, this process is not teleological, i.e., actants do not possess any self-purpose to take actions. Instead, they inevitably take part on a network with others interacting actants engaged on human agencies.

Textbooks’ actions in the network are mediated by discourses populated with physical concepts. They are semiotic basic entities in scientific discourse, forged by dialogical processes. In this sense they can be considered as actants. They are theoretical constructs with strong multisemiotic internal structure, constructed along history – that can be also understood as (semiotic) networks. These concepts are highlighted in different ways in each textbook. Along time, some interpretations or concepts can acquire more or less consensus through those textbooks, i.e., they appear in all textbooks or at least are cited, exhibiting some stabilization. We will try to infer which of these concepts or actants are more or less stabilized in the network and if there are differentiated roles of the textbooks in this process. The quantum physics textbooks themselves can be considered non-human actants, as they are, in essence, multisemiotic mediated forms of delivering scientific concepts. It is important to remember that actants are not defined *a priori*, they only develop a form of identity in-action, from their connections and performances represented in a network.

By means of the concept of actant *performance*, the process by which an actant are defined and stabilized (or not) through the interactions and/or relationships with other actants in the network and the responses they provoke in these interactions, is reasonable to suppose that the most expected response from other textbook authors is introduce the same or other similar innovation in their texts, changing the state of their textbooks in the network. If an author ignores this innovation, a naïve interpretation could lead to conclude that his textbook *per se* does not change. However, its state in the network will change – changes must be analyzed in relation to other actants, is not an absolute but a relative process. Even stabilization depends of how far we consider the chain of interactions of a given actant. This chain of interactions defines what Latour calls *translation*, a concept that must be understood not as a simple translation from one language to another.

In scientific endeavor, translations occur as a process that “moves” a given actant in relation to others in the network. In each step, the actant functions as a sign from the previous step and as raw matter for the next step, a hybrid of matter and sign. Such hybrid, at each stage, is associated with other actants, resulting in the translation of a new hybrid with new performances in the network, a “successive chains of translation” which “involve, at one end, exoteric resources (which are more like what we read about in the daily papers), and at the other end, esoteric resources (which are more like what we read in university textbooks)” (Latour, 2000, p. 91). Latour refers as translations also as a process that “consists of combining two hitherto different interests to form a single composite goal” (Latour, 2000, p. 88) or “to mean displacement, drift, invention, mediation, the creation of a link that did not exist before and that to some degree modifies the original two” (p. 179). In this sense, stabilization requires a series of negotiated translations to occur. This is why we cannot suppose that these networks are surrounded by a context. This “context” is a result of the network itself, through a series of translations which transforms meanings and connections (Mützel, 2009, p. 877). In this sense, the “educational context”, in each these textbooks and concepts are “immersed”, is resultant from these actants and their performances in the network. In a broader view, this network interacts with a broader network involving scientific activity and knowledge production, but with goals focused on educational issues.

This process of stabilization is also relative. A given actant that may be temporarily well stabilized in the broader world of academic research in Quantum Physics, considering the network of literature related to this area as primary sources, will not necessarily be stabilized in the narrower world of university textbooks. This can happen intentionally in a justified way, since the academic community has as a fundamental pillar the construction of new knowledge, while the educational community is more focused on teaching scientific concepts to students, who will likely become future researchers. Some physical concepts can be considered not well fashioned for teaching, or, in other perspective, they are yet not well articulated to the networks formed by the agents who or which are engaged in formative university courses of quantum physics. But this can be done also by simply omitting certain forms of translation, artificially stabilizing or privileging one given perspective, even if this consensus is not achieved in scientific community.

In other words, actants which have recently started to interact with other actants in a broader scientific network, whether they have or not a considerable degree of stability, they do not necessarily appear or are not necessarily stable in another network articulated in educational spaces. Conversely, an actant can show up a false stability in university textbooks, as is it was an ended debate. As an example, we can cite the Uncertainty Principle for relativistic quantum objects, which has only recently been articulated in the academic body of knowledge (Bialynicki-Birula & Bialynicka-Birula, 2012, 2019), but is not yet articulated in the network formed by introductory or advanced Quantum Physics textbooks analyzed here. In other words, although it is not yet a well-established actant in the network of relationships articulated in the academic scientific research, it has a certain degree of “existence” there. However, the same usually does not occur in the network articulated on the university undergraduate educational spaces, which in beginning end middle years are committed most on forming scientists or teachers than in production of cutting-edge research. If we access most of these textbooks, the debate around Uncertainty Principle concerning relativistic quantum objects seems to be inexistent, which is misleading. It’s more a matter of omitted translation that, in turn, lead to a poor-articulated concept in the networks having these textbooks and concepts/perspectives as main actants.

Before to perform the network analysis, some thoughts about possible approximation and differences involving Actor-Network Theory must be considered. This step is fundamental to turn possible interpret results in the light of Actor-Network Theory by use of some topological properties provided by the current knowledge of Graph Theory.

5.4 HOW CAN WE ARTICULATE ACTOR-NETWORK THEORY TO SOCIAL NETWORK ANALYSIS AND PRODUCE MEANINGFUL VISUALIZATIONS?

As cited in previous section, actor-networks must be concerned not only in their objects, nodes (or vertices) and edges (or connections), but mainly in the interactions between them. The nature of nodes can be considered as actants, but in a broader perspective they are also networks which involve humans and non-human agents. So, the limits of a network must be traced for methodological feasibility, but we must have in mind that it is artificial in some sense. In the specific case of building a network of textbooks, focusing in Uncertainty Principle, when first quantum physics textbooks came to light, it was expected that a somewhat messy and unstable network could represent their interaction.

Different views and interpretation of key aspects of Uncertainty Principle were object of debate and this instability was transferred to the networks. When some degree of consensus about an interpretation and/or physical concept was achieved, turning almost unnecessary to argument in favor of its “existence”, the concept and/or interpretations can become a black box, i.e., this means “that the reasons for its credibility are no longer being discussed” (Lima, Nascimento, Cavalcanti, & Ostermann, 2020, p. e2019013-3). The dynamics of knowledge refinement or improvement may consist in using previous black boxes to build new knowledge, in a process called chain of reference (Latour, 2000). When some concepts and/or interpretations stabilize in a network concerning textbooks, we are faced to a potential problem of opaque black boxing, which is problematic.

> […] introducing scientific facts as totally opaque black boxes may be misleading for pedagogical purposes. It may seem that “truths” appear as magic, or that only genial ideas have place in Science – what is recognized as the myth of the genius (Lima et al., 2020, p. e2019013-3).

For details on the conceptualization of *the myth of the genius* we recommend the paper of Allchin (2004).

The analytic techniques derived from modern Social Network Analysis, usually founded in Graph Theory, cannot be directly and seamlessly applied to Actor-Network Theory, at least without discussion of several limitations imposed by fundamental differences between usual social networks and actor-networks. Actor-networks are not directly resembled by graph theory, there are important disparities between these two types of networks.

However, in a more positive way, it is also possible circumvent some of these differences and find some convergences, as discussed in the works of Vicsek et al. (2016) and Venturini, Munk, and Jacomy (2019). These authors provide an excellent overview of difficulties in traditional Social Network Analysis with Actor-Network Theory, additionally discussing possible equivalences of two approaches. Based on some ideas of Mützel (2009), who argues that the "cultural turn"[35] in Social Network Analysis and Actor-Network Theory share many conceptual similarities, "although important differences remain" (p. 871), trying to connect "two current, typically separate strands in network thinking that treat *culture* and *structure* as intermingled rather than as autonomous entities of a duality" (p. 871). The central idea in the cultural turn of Social Network Analysis (or Relational Sociology) is "to identify social structures on the basis of social relations instead of cultural categories or individual attributes" (p. 873).

This, by itself, constitutes in an important approximation, since in Actor-Network Theory we must shift our focus from individuals to interactions, as cited earlier. In essence, a social network is not a collection of inert, static individuals whose connections between them have negligible meanings. On the contrary, social networks are "relational webs of meaning, discursively constituted in processes and essentially cultural products" (p. 873). When we think about a network of textbooks connected by each other through physical concepts about Uncertainty Principle that they vehiculate, the meaning of each connection cannot be regarded in a reductionism like "these textbooks share between them those physical concepts", there is much more. This is a discursive stance, rooted in a long cultural tradition of physicists' community. Physical concepts are successively translated until acquire some degree of stability, being published on papers and (usually) right after in textbooks.

This translations process affects the network as a whole, redefining all actants. Thus, the connections between textbooks are not single ties, but a result of a long chain of discursive exchanges which redefine all actants of the network. In the same way, physics textbooks and physical concepts are not individuals, but articulated forms of discourse. If a physics concept about Uncertainty Principle is privileged over others, if it is more stabilized in a network, this is not arbitrary. It is a reflect of relational interactions of human and non-human actors, its dynamics along time. It is important to stress that *time* is considered by Venturini et al. (2019) the most serious divergence between Actor-Network Theory and Social Network Analysis. Among other considerations, "movement in graph theory is usually movement through networks and not movement of networks" (p. 517).

This differentiation is a central issue, because the dynamics of actor-networks are constituted in *translactions*, not in literal or mechanic movements through the

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[35] See Mützel (2009, pp. 873-876).

network[36]. It is about the “movement” of the network as a whole, i.e., the evolution of the network in time. We will try to deal with this problem focusing our analysis in inferring which actants are more stabilized in the network and which possible issues account for these stabilizations. In this sense, the analysis is like consider “echoes” of time that could explain or clarify about the current state of the network, whose ties (or edges) between actors carries remnants of translactions, as if a set of stories (or histories) that were not been explicitly told, but produced marks.

Concerning the visualization of networks, Venturini et al. (2019) consider that “of all the techniques associated with graph analysis, the ones developed to visualize networks are those that most closely resonate with Actor-Network Theory”, since it has a potential power to, allied with theory, unveil some invisible structures whose nature can be help to explain or infer reasons that account for attractions or repulsions in some types of groups of actants interacting in networks. Those structures can appear in spatial distribution of nodes, depending on which algorithm we use to produce final layout. There are several techniques of spatializing networks, i.e., algorithms that produce different types of spatial distribution of its nodes and edges. It is intuitive to conclude that nodes that appear close together are strongest related than nodes that are far apart of each other (Jones, Mair, & McNally, 2018, p. 1742-1). Generally speaking, this is not the case.

A special case of spatialization can be obtained by force-driven layouts, which produce appealing visual results. However, the most commons force-driven algorithms produce spatial distributions that are good for visualizations purposes, but distances between nodes not necessarily are directly interpretable. Nevertheless, it is possible to use more sophisticated spatialization techniques (e.g., from Principal Component Analysis or Multidimensional Scaling), but we choose a different approach, a special force-driven layout algorithm called *ForceAtlas2* (Jacomy, Venturini, Heymann, & Bastian, 2014). In this algorithm, nodes repulse each other (as charged particles), while edges attract their nodes, like springs. These forces govern a dynamics on the network that converges to an equilibrium as a final state. The final spatial distribution of the nodes can be interpretable. We performed tests using at least three types of Multidimensional Scaling, all of which produced similar spatializations (with little/unimportant differences). The ForceAatlas2 algorithm produced clearer layouts.

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[36] As well spotted by Mützel (2009, p. 877), in Actor-Network Theory a “network is a metaphor for the flows of translations that actants go through in making connections”.

With this spatialization technique, centrality, betweenness, structural separation, and other conceptual metrics of Graph Theory “found a graphical equivalent”, as “they can be not only calculated, but also seen” (Venturini et al., 2019, p. 520). Following Venturini et al. (2019), we also believe that “this is also where the deepest bond between Social Network Analysis and Actor-Network Theory is to be found” (p. 520).

Posed the justifications above, our goal in the next sections is explain which aspects about Uncertainty Principle will be considered in each textbook and how the network will be constructed from data.

5.5 DEFINITION OF VARIABLES THAT SUMMARIZE CHARACTERISTICS OF UNCERTAINTY PRINCIPLE IN EACH TEXTBOOK

Starting from a preliminary reading of the textbooks, we initially defined 43 different aspects involved in Uncertainty Principle, nested in six categories: structure (how UP is organized in the textbook), physical quantities (which physical quantities are presented), derivation (how UP, in traditional or modified forms, is obtained), gedankenexperiments used (e.g. gamma ray microscope), the class of mathematical formalism adopted (e.g. Dirac’s formalism), the interpretation adopted for the uncertainty in the variables x-p and other elements (general issues not covered in previous categories). The categories and their details are shown in Table 1.

Table 1 – Categories considered from each textbook. These categories will be named as *objects* or *actant-objects*.

	OBJECTS (ACTANT-OBJECTS)	**OBJECT VARIABLE NAME**
STRUCTURE	**How the Uncertainty Principle is organized in the textbook?**	**Correspondent variable name**
	More than one part	STR_mtop
	One chapter	STR_1chap
	First Chapter	STR_1stchap
	One section	STR_1sct
	One section in the first chapter	STR_1sect1stchap
	Just mentioned along the text	STR_justment
	Not mentioned	STR_notment

	Which are the physical quantities presented?	**Correspondent variable name**
PHYSICAL QUANTITIES	$x - p$ (position and momentum)	PQ_xp
	$t - E$ (time and energy)	PQ_tE
	$t - \nu$ (time and frequency)	PQ_tf
	$J - \omega$ (action and angle variable)	PQ_Jtheta
	$L_z - \phi$ (z-component of Angular Momentum and Angle)	PQ_lzphi
	$P(\pi/4)$ e $P(\pi/2)$ (photon polarization)	PQ_phtpol
	$N - \phi$ (number of photons and phase)	PQ_nphi
	$A - B$ (commutation of two any physical quantities)	PQ_AB
DERIVATION	**Which form of derivation is developed?**	**Correspondent variable name**
	Gaussian package with Fourier Transform - Heisenberg $x - p$	DRV_gausspckg
	Generalized Package with schwarz inequality - Weyl $x - p$	DRV_genpckg
	Schwarz Inequality (for two any operators) – Robertson	DRV_schwrzinq
	Schrodinger's generalization	DRV_schrdg
	Direct analogy with the classical wave	DRV_anclasswav
	Phenomenological (gedaken experiments)	DRV_phenom
	No derivation at all	DRV_none
APHENOMENOLOGICAL	**Which are the used *gedankenexperiments*?**	**Correspondent variable name**
	Gamma-Ray Microscope	GED_gamraymicr
	Slit-experiments	GED_slitexp
	Metaphor with a classical wave	GED_mtphrclasswav
	Retrodiction experiments	GED_retrodexp
	Deflection in a magnetic field	GED_deflmgnfield
	Collision with a photon	GED_colphot
	Interaction with a electromagnetic field	GED_inteletmgnfld
	Einstein's box	GED_einstbox
FORMALISM	**Which mathematical formalism is adopted?**	**Correspondent variable name**
	Semi-classical notation	MATH_semicl
	Schrodinger's formalism with operators	MATH_schrod
	Brackets formalism	MATH_brckt

	Which meaning of the x-p uncertainty is presented?	Correspondent variable name
INTERPRETATION	Δ (intrinsic)	INT_intrsc
	σ (statistical ensemble)	INT_statens
	$\varepsilon - \eta$ (error-perturbation)	INT_ep
	$\varepsilon - \varepsilon$ (error-error)	INT_ee
	It says ε- ε but shows $\varepsilon - \eta$	INT_ee_ep
OTHER ELEMENTS	**Other elements**	**Correspondent variable name**
	It is made a metaphor with relativity	OTH_mtphrrel
	It is mentioned the Transformation Formalism	OTH_transform
	It is made a philosophical discussion	OTH_phildisc
	It is made a connection with the particle-wave duality	OTH_conpartwavedual
	It is made a connection with the Complementarity Principle	OTH_concomplm

The structure part of the table is considered a discursive stance, i.e., the distribution of the topic on the text is not something that comes merely from technical issues. It comes from author’s and/or editors’ position, which allows us to infer how they give prevalence to this topic. It falls on the compositional structure of discourse vehiculated by the textbook, an interactive/responsive characteristic of discourse, which can be also be considered as actants. We will name these textbooks’ components as *actant-objects* and the textbooks as *actant-textbooks*.

Regarding the other aspects of the table, it is also important to highlight some characterizations, as the formalism, which refers to the mathematical language that was used by the textbooks to describe or derive the Uncertainty Principle - the semi-classical formalism is the simplest, while the Schrodinger's formalism with operators is more elaborated and brackets formalism is the most abstract. Furthermore, the classification of the forms of derivation developed by the textbooks is in according to Rosa, Lima & Cavalcanti (2022), in which these derivations are didactically discussed and the forms of interpretation was based on Jijnasu (2016) propose of classification.

5.6 DATA COLLECTION AND CONFIGURATION

As previously cited, the analysis was performed on 34 Quantum Physics textbooks, all of them used in courses at university level. The defined 43 actant-objects defined in table 1 formed the guidelines of comparison between all different textbooks.

The data was structured as a matrix (or table) with the actant-textbooks disposed along the lines (one per line) and actant-objects disposed along the columns, defining a cross table actant-textbooks *versus* actant-objects. In this configuration, actant-objects play a role of variables. The books are classified as *introductory* (usually adopted in courses of general physics) and *advanced* (usually adopted in courses of quantum physics).

5.6.1 The complete bipartite textbook-object network

We define a binary vector (value 1 or 0), where 1 indicates that a given actant-textbook vehiculate a discursive construct[37] of a given actant-object and 0 indicates the opposite, resulting in a binary matrix of dimension 34×43. Speaking strictly in terms of network properties, a number 1 indicates that there is a connection (edge) between an actant-textbook (a node, on the lines) and an actant-object (a node, on the column). The number 0 indicates absence of connection (no edge). We call the matrix $\mathbf{M}_{\mathrm{BA}}$ (*B* means actant-*textbooks*, *A* means actant-objects). Our goal is focused on constructing a specific network in which actant-textbooks relates to each other indirectly, mediated by the actant-objects. In this network, suppose that a given actant-textbook B_1 acts on it mediated by the same twenty common actant-objects as a textbook B_2 does. Suppose also that B_1 acts on this network mediated by only five same actant-objects as a textbook B_3 does. In this case, the similarity between B_1 and B_2 is greater than the similarity between B_1 and B_3, since B_1 an B_2 puts in action a greater set of shared actant-objects than the dyad B_1 and B_3 does. In other words, actant-textbooks B_1 and B_2 interact in a stronger way than B_1 and B_3 do.

The sum of 1's on each column results in the number of adjacent edges to each actant-object (node) and their sums on each line results in the number of adjacent edges to each actant-textbook. The number of adjacent edges to a given node is the total number of edges *connected* to this node, as known as *node degree* or *vertex degree* (Kolaczyk & Csárdi, 2020, p. 44). The sum of lines results in the number of adjacent edges to each actant-object node, i.e., the degree of each of these nodes. Actant-objects that were not vehiculated by any actant-textbook were not included in analysis (STR_justment, STR_notment and INT_ee_ep). They would be represented

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[37] We will avoid here the utterance "the actant-textbook T *has* a given actant-object A", because an actant, as already said, is not something defined *a priori*. Here it is a discursive construct that comes into existence when it dialogically interacts in a network, i.e., it is defined only in the light of its actions on the network.

by isolated unconnected nodes, that is, they would not provide any relevant information beside the very fact of not have been vehiculated.

Table 2 – Small part of the complete matrix $\mathbf{M}_{\mathrm{BA}}$, which we call $\mathbf{m}_{\mathrm{BA}}$.

		OBJECTS								
		PQ_xp	**PQ_tE**	**PQ_tf**	**PQ_Jtheta**	**PQ_Izphi**	**PQ_phtpol**	**PQ_nphi**	**PQ_AB**	→
TEXTBOOKS	**ALONSO_1968**	1	1	0	0	0	0	0	0	→
	NUSSENZVEIG_1998	1	0	0	0	0	1	0	1	→
	TIPLER_2012	1	1	0	0	1	0	0	0	→
	HALLIDAY_2007	1	1	0	0	0	0	0	0	→
	YOUNG_2007	1	1	0	0	0	0	0	0	→
	SERWAY_2010	1	1	0	0	0	0	0	0	→
	↓	↓	↓	↓	↓	↓	↓	↓	↓	→

The actant-textbook degree indicates the number of different issues of uncertainty principle it discursively vehiculates, i.e., quantifies its actions on the network. The actant-object degree shows how many distinct actant-textbooks vehiculate it, quantifying the consensus that this actant-object must be privileged in educational settings. In other words, this can be assumed as a measure of *stabilization* of an actant-object in a network. Important to stress that what we consider *consensus* here in a very narrow definition, i.e., we consider that an actant-object achieves a consensual *status* when several actant-textbooks have vehiculated it, providing many connections on the network in which this actant-object takes part. A wider latourian analysis of consensus between physics textbooks about a specific topic would demand detailed analysis of discursive practices and other actions along a broad period of time, a complicated task that we will not develop here, although there are feasible alternatives to carry out studies of this type (see Shwed & Bearman, 2010, who develop an analysis on how scientific consensus evolves in citation networks).

In table 2 we show a small part of the complete table (which we call $\mathbf{m}_{\mathrm{BA}}$, a reduced version of complete matrix $\mathbf{M}_{\mathrm{BA}}$), containing only category *Physical Quantities* defined in table 1 and the introductory actant-textbooks. For example, the actant-textbook HALLIDAY_2007 share two vehiculated actant-objects with the remaining actant-textbooks, except with NUSSENZVEIG_1998. The actant-object PQ_xp is vehiculated by all introductory actant-textbooks (and all remaining ones that were included in our analysis, not shown in table 2). A network can be created from data shown in table 1, in which the actant-textbooks and actant-objects are nodes (or vertices). Each actant-textbook is connected by a line (or edge) to all actant-objects where its value in the table is 1 and not linked if its value is 0. As an example, the actant-textbook SERWAY_2010 is connected to PQ_xp and PQ_tE and not connected to remaining actant-objects. If we define two distinct sets, A and B, each containing only actant-objects and actant-textbooks nodes, respectively, the resultant network will exhibit only connections between nodes from different sets. In other words, only connections between actant-objects and actant-textbooks are present. There are no connections between nodes that belong to the same set. This class of network is called *bipartite network*. The matrix above is one of possible mathematical representations of a graph and it is called *incidence matrix*.

We can obtain symmetric matrices from $\mathbf{m}_{\mathrm{BA}}$ containing only the actant-textbooks and actant-objects shown in table 2. These matrices are useful to investigate relationships between different actant-textbooks and between actant-objects. This is not possible to do from $\mathbf{m}_{\mathrm{BA}}$, because in this matrix actant-textbooks are related to actant-objects and *vice-versa*. There is no edges connecting actant-textbooks to actant-objects, i.e., actants with distinct nature. It represents a *textbook-object* network. To obtain a *textbook-textbook* or a *object-object* network we can perform a *bipartite projection*. This can be done by means of simple operations from $\mathbf{m}_{\mathrm{BA}}$. The object-object matrix is obtained by

$$\mathbf{o} = \mathbf{m}_{\mathrm{BA}}^{T}\mathbf{m}_{\mathrm{BA}}, \tag{1}$$

where $\mathbf{m}_{\mathrm{BA}}^{T}$ is the transpose matrix of $\mathbf{m}_{\mathrm{BA}}$. The textbook-textbook matrix is obtained in a similar form:

$$\mathbf{t} = \mathbf{m}_{\mathrm{BA}}\mathbf{m}_{\mathrm{BA}}^{T} \tag{2}$$

5.6.2 The object-object network

The resulting matrices can be easily interpreted, since $\mathbf{m}_{\mathrm{BA}}$ is a binary matrix. Table 3 shows the matrix $\mathbf{a}$, in which we can directly see that diagonal elements (shaded in light gray) are the number of times that one given actant-object is vehiculated by all actant-textbooks. For example, the actant-object PQ_xp (position and momentum uncertainty) are vehiculated in all six actant-textbooks (this is the sum of column PQ_xp in table 2), so the result is 6. On the other hand, PQ_photpol (Uncertainty Principle discussed by means of photon polarization) is vehiculated only by one actant-textbook, so the result is 1.

Thus, the quantities on the diagonal are the degree of consensus about the respective actant-object, its node degree *on textbook-object network* (table 2). As already said, the greater the quantity of distinct actant-textbooks that vehiculates a given actant-object, the greater its stabilization in the network is.

The off-diagonal elements are the number of times that two distinct actant-objects are vehiculated together by all the actant-textbooks, i.e., the number of coincidences of such actant-objects. For example, the actant-objects PQ_xp and PQ_tE (time and energy uncertainty) are jointly vehiculated 5 times (number of coincidences of the number 1 on the columns PQ_xp and PQ_tE in table 2). The off-diagonal elements will be considered as *weights* of the edges in the object-object network, so that the more times two distinct actant-object appear together in textbooks as a vehiculated discourse, the strongest their interaction in the network and also more *stabilized* they will be considered as actants, considered the particular set of textbooks analyzed.

Considering these actant-objects essentially as vehiculated discursive constructs, the more times they are vehiculated together by two distinct textbooks, the greater is the degree of agreement between these two actant-textbooks in privileging it and the more consensual these two actant-objects are. If we sum up the off-diagonal elements of a column or a line of a specific actant-object, we obtain the *strength* of the node representing this actant-object in the object-object network. The strength is a generalization of vertex degree for weighted networks (Kolaczyk & Csárdi, 2020, p. 44).

Table 3 – Object-object matrix $\mathbf{o} = \mathbf{m}_{\mathrm{BA}}^{T}\mathbf{m}_{\mathrm{BA}}$.

OBJECTS	**PQ_xp**	**PQ_tE**	**PQ_tf**	**PQ_Jtheta**	**PQ_lzphi**	**PQ_phtpol**	**PQ_nphi**	**PQ_AB**
PQ_xp	**6**	5	0	0	1	1	0	1
PQ_tE	5	**5**	0	0	1	0	0	0
PQ_tf	0	0	**0**	0	0	0	0	0

OBJECTS	**PQ_xp**	**PQ_tE**	**PQ_tf**	**PQ_Jtheta**	**PQ_Izphi**	**PQ_phtpol**	**PQ_nphi**	**PQ_AB**
PQ_Jtheta	0	0	0	**0**	0	0	0	0
PQ_Izphi	1	1	0	0	**1**	0	0	0
PQ_phtpol	1	0	0	0	0	**1**	0	1
PQ_nphi	0	0	0	0	0	0	**0**	0
PQ_AB	1	0	0	0	0	1	0	**1**

For example, considering the actant-object PQ_xp, the number of adjacent edges to the node which represents it in the reduced object-object network of table 3 is 4 (one weighted 5, three weighted 1). However, the strength is the sum of weights, resulting 8. The strength can be interpreted as a measure of the intensity of *interaction* of this node with the others and can also be interpreted as a degree of *consensus* or *stabilization* of the actant-object in the network. The matrix represented in table.

5.6.3 The textbook-textbook network

The textbook-textbook matrix **t** is shown in table 4. Similar to object-object matrix, each element is readily interpretable. The diagonal elements are now the total number of distinct actant-objects vehiculated by each actant-textbook, i.e, the number of edges adjacent to each actant-textbook in the textbook-object network (its degree of centrality in this network, the sum of each line of table 2). The off-diagonal terms now indicate the number of actant-objects jointly vehiculated by two distinct actant-books.

For example, ALONSO_1968 and TIPLER_2012 vehiculate 2 common actant-objects considering this reduced universe of actants. In the table 2, this number is the number of coincidences of 1’s in the lines correspondent to ALONSO_1968 and TIPLER_2012. It is clear from this table that in this reduced universe ALONSO_1968 and TIPLER_2012 share the PQ_xp and PQ_tE actant-objects. As in the case of table 3, the off-diagonal elements of table 4 is the weights of the edges connecting two distinct actant-textbooks, their degree of interaction in this network. The strength of each node that represents the actant-textbooks is obtained summing up the off-diagonal elements of correspondent column or lines of table 4. The strength can be also interpreted as a degree of consensus between two different actant-textbooks, i.e., the grater the number of shared actant-objects two distinct actant-textbooks vehiculate, the more the degree of similarity between them, since they are acting in favor of stabilization of the same actant-objects on the network. In the textbook-textbook network, this consensus can be interpreted in terms of stabilization of the actant-textbook itself. If two actant-textbooks share many stabilized actant-objects in the network, they contribute to stabilize itself in the network. It is a *follow the majority* stance. Bring new actants to the network, i.e., bring innovations, ties the respective actant-textbook to an ill-stabilized actant-object and this can weak the stabilization of this actant-textbook in the network. This is not a bad issue, since good innovations are needed in most situations. However, some ill-stabilized actant-objects not necessarily are innovations, as we will explain later.

Table 4 – Textbook-textbook matrix $\mathbf{t} = \mathbf{m}_{\mathrm{BA}}\mathbf{m}_{\mathrm{BA}}^{T}$.

TEXTBOOKS	ALONSO_1968	NUSSENZVEIG_1998	TIPLER_2012	HALLIDAY_2007	YOUNG_2007	SERWAY_2010
ALONSO_1968	2	1	2	2	2	2
NUSSENZVEIG_1998	1	3	1	1	1	1
TIPLER_2012	2	1	3	2	2	2
HALLIDAY_2007	2	1	2	2	2	2
YOUNG_2007	2	1	2	2	2	2
SERWAY_2010	2	1	2	2	2	2

For example, considering only the introductory actant-textbooks and the physical quantities category of actant-objects, NUSSENZVEIG_1998 has the lowest strength on this reduced network (is equal to 5). This happens because it was the only textbook that bring to the reduced textbook-object network the actant-object PQ_phtpol (Uncertainty Principle discussed by means of photon polarization) and PQ_AB (Uncertainty Principle from commutation of two distinct observables, more akin to advanced books). We will see that these types of discursive actions contribute to destabilize NUSSENZVEIG_1998 in the textbook-textbook network of introductory actant-textbooks, bringing it closer to the advanced actant-textbooks.

In the next section we will present some visualizations of these networks, introducing some details of their topology which help us to emphasize subtle issues resulting from interaction between all actants.

5.7 VISUALIZATION AND INTERPRETATION OF THE NETWORKS

5.7.1 Analysis of bipartite network

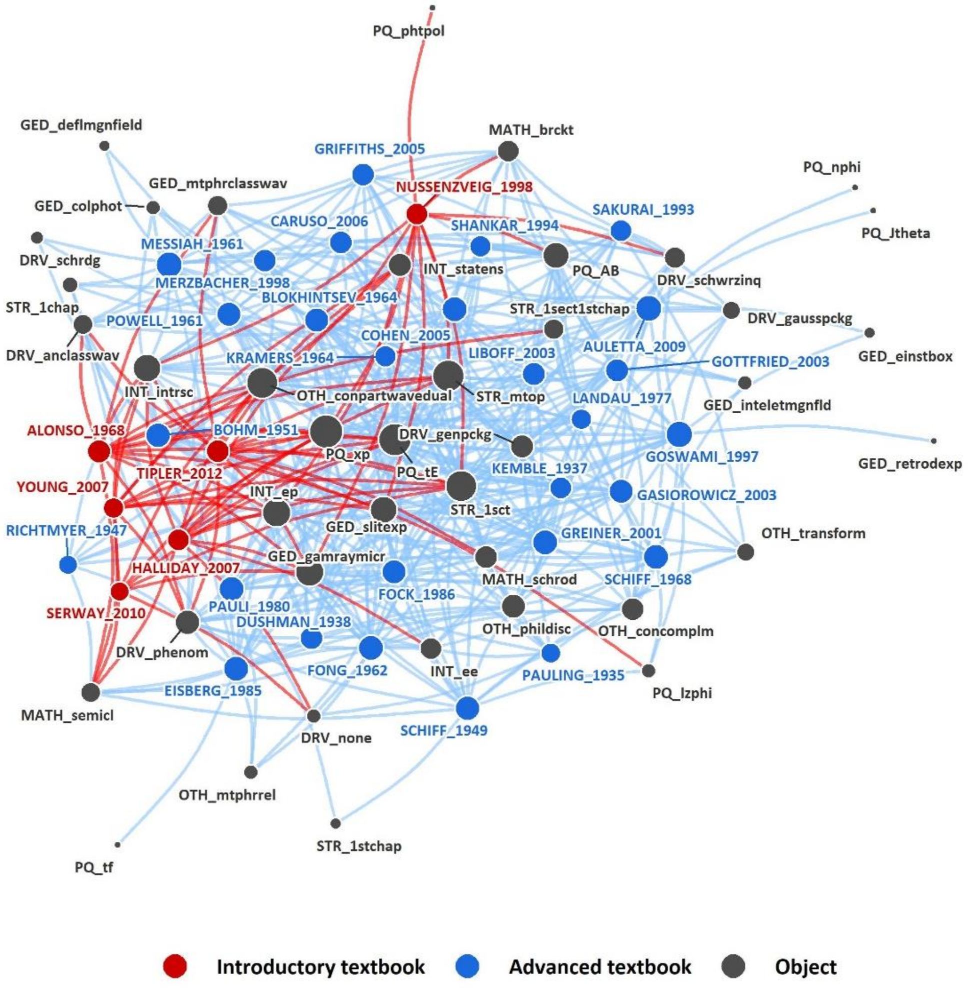
Network visualizations is large adopted to study relational data like what is compiled in this work. Although analyses grounded on networks are more descriptive and not aimed to modeling or inference (Kolaczyk & Csárdi, 2020), we plan to not limit ourselves to pure descriptions. The spatialization algorithm, ForceAtlas2, generates internode distances that are interpretable at some extent.

All analysis presented in this work was performed in R software (R Core Team, 2015), with the package *igraph* for network building (Csardi & Nepusz, 2005; Kolaczyk & Csárdi, 2020), *tidygraph* for network data maniputation (Pedersen, 2020) and *ggraph*, for visualizations (Pedersen, 2021). Moreover, we used a R-based development of ForceAtlas2 algorithm (Klockiewicz & Alvarez 2015). We will first present the complete textbook-object bipartite network, since from it we can develop interpretations and insights of the most important aspects involving the various actant-textbooks and actant-objects. In the discussion we will make use of some metrics characteristics of graph theory, namely the node degree, (edge) weights, node and edge betweenness, to identify some features of the network that would be hidden if these metrics were not considered. This network is shown in figure 1. The nodes are sized by their degree (number of edges adjacent to them) and colored by actant category, namely introductory textbook (red), advanced textbook (blue) or object (dark gray). The edges have all unitary weight and are so colored by textbook category. Color red indicates connections from introductory books and blue from advanced books.

An interesting result spatial pattern, generated by ForceAtlas2 algorithm, can be recognized: nodes with high degree tend to reach equilibrium in the central region of the network, while those with low degree reach this equilibrium in peripheral regions. This algorithm, by default, object to nodes an attraction “force” to the center of the spatialization space, called *gravity* (Jacomy et al., 2014), that linearly increases with the node degree – thus, the node degree plays the role similar of *mass* in the physical world and nodes with low degree are weakly attracted to the center of spatialization space. The repulsive force between nodes depends on the product of the degree of the nodes, similar to the case of a Coulomb force between two charges of equal sign. Finally, there is an attractive force between the nodes which depends linearly on the internode distance, multiplied by edge weights – a different mathematical model from the most known elastic force (that is quadratic on distance). The action of these forces produces an outcome with a spatial distribution that can be interpreted in terms of relative distances between nodes (not in terms of its absolute positions).

Since in the light of our theoretical assumptions it is supposed that the interaction between two actant-textbooks is mediated by the common actant-objects shared by them (or, conversely, the interaction between two actant-objects is mediated by actant-textbooks that jointly share them), the ForceAtlas2 algorithm is very well suited for our purposes. For example, the actant-textbook NUSSENZVEIG_1998, although classified as introductory, results far away from its peers (red color), localized all in bottom left of the network, after reach the equilibrium. It is visually clear from the network that it occurs because of the interactions of NUSSENZVEIG_1998 with many advanced books, by sharing similar actant-objects with them.

Figure 1 - Complete bipartite network.

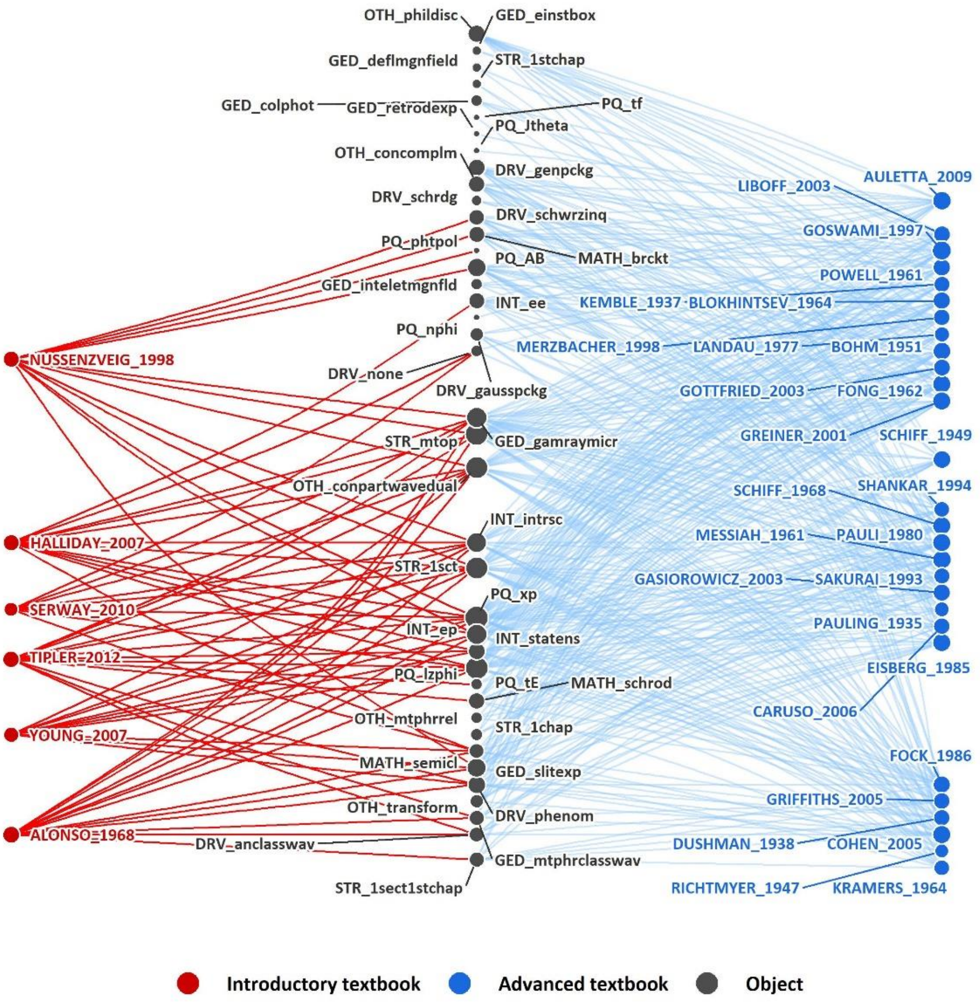


Nodes are sized by their degree and colored by actant category (introductory textbook, advanced textbook or object). Edges are colored by textbook category: red indicates connections from introductory books and blue from advanced books.

This actant-textbook shares actant-objects also vehiculated by actants-textbooks classified as advanced (dark gray nodes with many adjacent blue edges), that contribute to attract NUSSENZVEIG_1998 to a region populated with other advanced books. Thus, this algorithm is efficient in gathering similar actants into regions where one ends up close to the other. This allow to infer that NUSSENZVEIG_1998 is a hybrid advanced-introductory, which can be advantageous in many ways. Further, it brings to network an original actant-object, because it discusses the Uncertainty Principle by means of photon polarization (PQ_phtpol). As explained later, it potentially increases its connectivity on the network.

Another important nodes’ behavior that results from ForceAtlas2 algorithm is the central position of highest-degree actant-objects in the network (remember, gravity depends on node degree). This happens because the nodes representing actant-objects have on average more edges attached to them, i.e., on average have higher degrees than nodes representing actant-textbooks. The actant-textbook with highest degree is GOSWAMI_1997 (19), followed by AULETTA_2009 (18), whereas the actant-object with the highest degree is PQ_xp (34, vehiculated by all actant-textbooks analyzed), followed by PQ_tE (32). There is no single actant-textbook that vehiculates all 40 actant-objects included in this analysis. Also, on average, actant-textbooks classified as advanced have higher degree that their introductory peers, vehiculating more actant-objects. This is expected, as one of the main characteristics of advanced books is that they usually cover the contents in greater depth, mobilizing more complex translations and, as consequence, need to stablish more connections on the network vehiculating a larger number of more complex actant-objects. Not surprisingly, introductory actant-textbooks vehiculate only 23 distinct actant-objects, much less than the 40 vehiculated by advanced actant-textbooks. This is why introductory actant-textbooks are more peripheral than advanced ones.

Figure 2 – Stratified version of network obtained in figure 1.



Stratified version of network obtained in figure 1, with three layers, namely Introductory actant-textbooks (left), actant-objects (center) and advanced actant-textbooks (right).

The ForceAtlas2 makes that the most stabilized actant-objects, those with highest degrees, in most cases tend to achieve equilibrium in the center of the network (there are some deviations from this behavior, as the equilibrium also depends on other interactions between nodes). This can be verified on figure 1, as the actant-objects represented by larger sized nodes are the most central ones (remembering that the nodes were sized according with their degree). These actant-objects are jointly vehiculated by several actant-textbooks, i.e., are well articulated in the network as a result of consensus that they must be privileged on teaching Uncertainty Principle. The six larger degrees comes from actant-objects PQ_xp, PQ_tE, STR_mtop, STR_1sct, OTH_conpartwavedual and GED_gamraymicr.

Note that these six actant-objects are the most vehiculated by actant-textbooks classified as introductory. In figure 2, we show a modified layout of original network, a *stratified* layout in which actant categories are distributed into three different layers. This spatialization, even with internode distances that are not readily interpretable, can be useful to visualize which classes of actant-objects each category of actant-textbook vehiculates most. It is clear from this figure that, compared to the advanced actant-textbooks, introductory actant-textbooks are most bonded to the six most stabilized actant-objects cited above. It is also clear that advanced actant-textbooks vehiculates a wider class of actant-objects, since one of their goals is to articulate actants in the network that allow teach the Uncertainty Principle in greater depth.

As in the network shown in figure 1, we can visualize which actant-objects are most characteristic in introductory or advanced actant-textbooks. The majority of actant-objects from the bottom of stratified network are more associated to the introductory actant-textbooks.

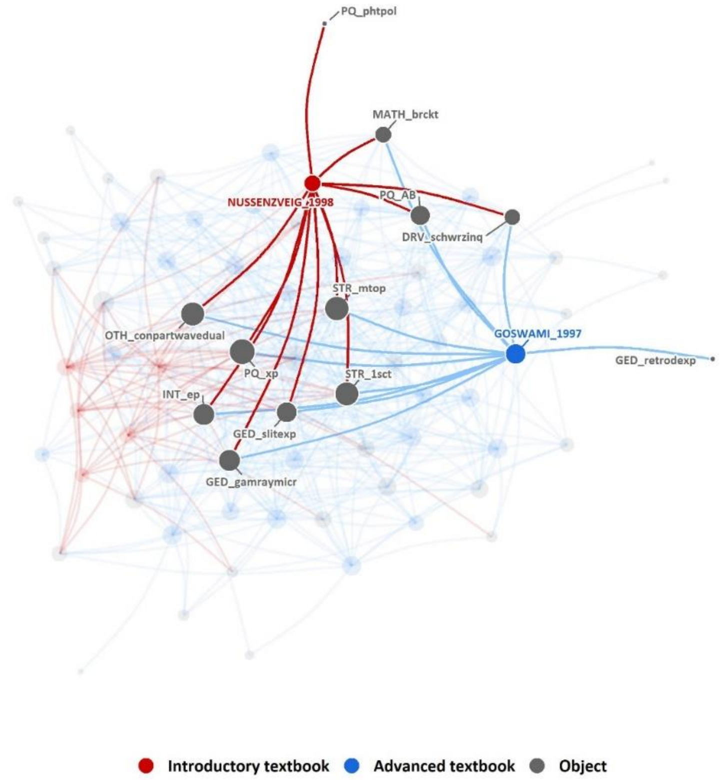
The higher ones are most usually related to advanced actant-textbooks. Thus, actant-textbooks positioned on higher regions of figure 2 are most similar to the advanced category, contrary to those positioned on lowest regions. For example, as we can see in the original network, NUSSENZVEIG_1998 is displaced from the lower region since it vehiculates four actant-objects mostly articulated by advanced actant-textbooks. This discursive action “moves” it to upper region, “moving” away from the other introductory actant-textbooks. This is what we call a movement *of* the network, in opposition to a movement *in* the network, i.e., an imaginary path that allows a flow between nodes of the network. Movement of the network refers to a “movement” of an actant, not *between* static actants by means of their connections. The discursive actions of actants provides connections and results in transformation on the topology and overall properties of the whole network, leading to reconfigurations. Similar “movement” also occurs in the network shown in figure 1.

We can also investigate possible outcomes of the actions of actant-textbooks or actant-objects other than their stabilization *status* in the network. A good measure of node importance in a network is called *betweenness*. Given any two different connected nodes in the network, generally is possible to find many possible paths that can unite these nodes. However, there is a particular class of paths, those that pass through the minimum possible number of nodes, called *shortest paths* or *geodesics*. Let V be a set of all nodes (or vertices) of the network. The betweenness of node $v \in V$ is given by (Kolaczyk & Csárdi, 2020, p. 48)

$$B(v) = \sum_{s \neq t \neq v \in V} \frac{\sigma(s, t|v)}{\sigma(s, t)}, \tag{3}$$

where $\sigma(s, t|v)$ is the total number of shortest paths between node s and t which pass through node v and $\sigma(s, t)$ is the total number of *all* shortest paths between s and t (not only the ones that pass through v). When the above sum is big (high betweenness) the node is candidate to be considered an *obligatory passage point* (Callon, 1986; Aka, 2019) or a bridge connection, i.e., is the main node that can unite (even potentially) several different nodes of the network, usually groups of nodes or clusters (Bail, 2016). First, we must clarify how a path can be interpreted in this network. By construction, there are no edges directly connecting two actant-textbooks or two actant-objects. An actant-textbook T_1 can only be connected to another actant-textbook T_2 by paths that pass through some of shared actant-objects between T_1 and T_2. Similarly, an actant-object A_1 can only be connected to another actant-object A_2 indirectly, by paths that pass by another actant-textbook. If both A_1 and A_2 are vehiculated by the same actant-textbook, then this unique actant-textbook connects the referred actant-objects. In this case the geodesic distance between A_1 and A_2 is 2, which is the number of edges of the shortest path between them. For example, the geodesic distance between the actant-objects MATH_brckt and PQ_AB is 2, passing through the actant-textbook NUSSENZVEIG_1998 or GOSWAMI_1997, as shown in figure 3.

Figure 3 – The geodesic distance between the actant-objects MATH_brckt and PQ_AB.

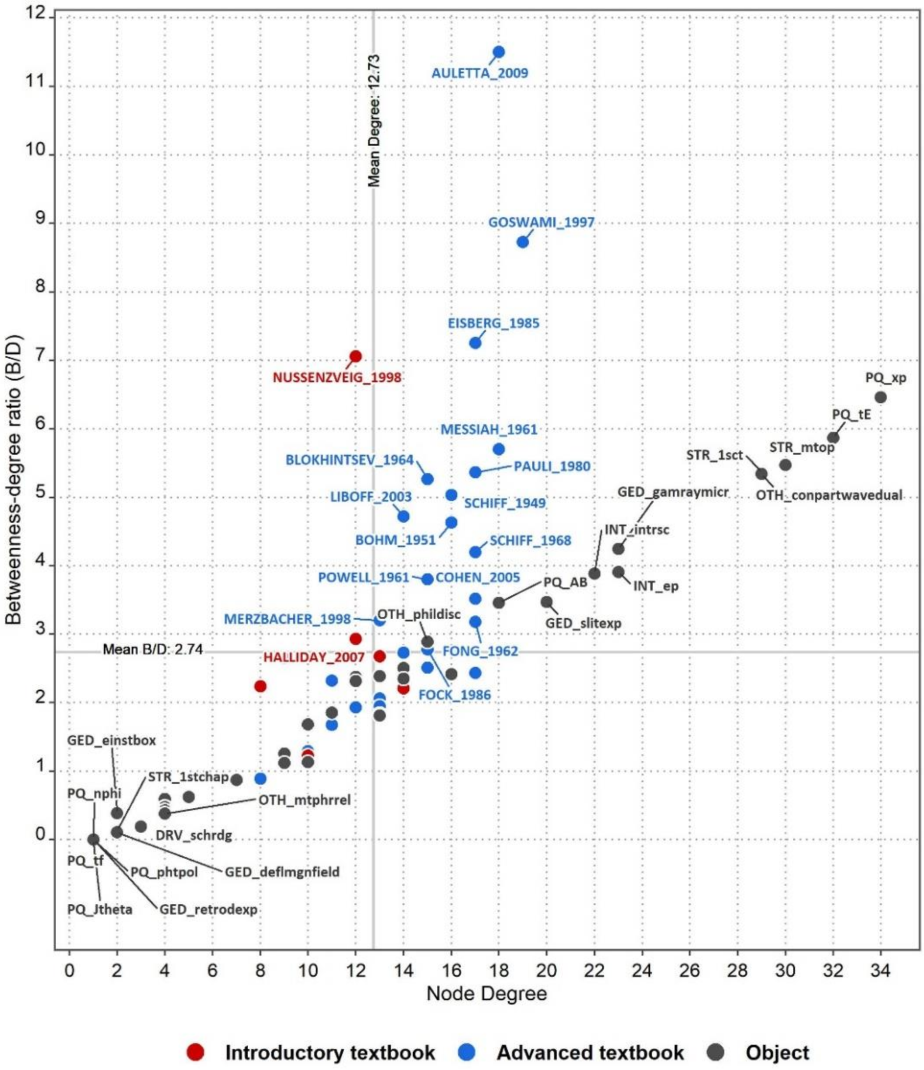


All shortest paths that connect actant-objects PQ_phtpol to GED_retrodexp, original contributions from NUSSENZVEIG_1998 and GOSWAMI_1997, respectively. All of them pass through four nodes, the minimum possible “distance” between these actant-objects. There are ten possible shortest paths between these actant-objects, all of them passing through ten mutually shared actant-objects.

However, more than one actant-textbook may be necessary to connect two given actant-objects. In figure 3 we can see this situation. Consider two actant-objects that are each one vehiculated only by one specific book, being original contributions of this actant-textbook to the network. In the figure we show PQ_phtpol (vehiculated only by NUSSENZVEIG_1998) and GED_retrodexp (vehiculated only by GOSWAMI_1997). The originality in vehiculating these two actant-objects results in at least these two actant-textbooks to connect them. This is why the most original actant-objects, those vehiculated by one (or very few) actant-textbooks, are “distant” to each other. In the example shown in figure 3, the geodesic distance is 4 (the number of edges on the shortest paths).

When we speak about *paths* between two given nodes in the network this is equivalent to speak in “*movement” in the network*, but this “motion” cannot be understood in its strict sense, as a material flow or something similar. Every actant-textbook in the network vehiculates a set of actant-objects, interacting with others by mediation of these vehiculated actant-objects. Thus, there is no direct edges between actant-textbooks or between actant-objects, only between actant-textbooks and actant-objects.

Figure 4 – Betweenness-degree ratio $B(v)/D(v)$ of actant-textbooks and actant-objects of the network.



The vertical and horizontal gray lines respectively show the mean node degree and mean betweenness-degree ratio.

In these terms, the “movement” in the network can be interpreted as resulting from a collaborative disposal of physical concepts embedded on several actant-objects that are vehiculated in the network by the actant-textbooks, symbolizing the repertoire of conceptual routes offered by the actant-textbooks in the network. This collective deployment of conceptual possibilities builds several bridges between actant-textbooks, organically constituted by mutually shared actant-objects. For example, in planning of a large (institutional) or small scaled (teachers or students) didactical program on quantum physics, especially on selecting topics related to Uncertainty Principle restricted to actant-textbooks of the network, a bibliography selection is usually performed to cover those topics. This action can be represented by the *potential* “movements” in the network, along the edges that connect the nodes, actants on the network. For example, when choosing an approach of Uncertainty Principle which includes non-usual approaches to Uncertainty Principle contextualizing the discussion on photon polarization (PQ_phtpol) and retrodiction experiments (GED_retrodexp), this potential route in the network is provided by actant-textbooks NUSSENZVEIG_1998 and GOSWAMI_1997.

Because they are the only ones that vehiculate these two actant-objects and share other ten actant-objects between each other (thus providing ten shortest paths between PQ_phtpol and GED), these two actant-textbooks are potential candidates to form *obligatory passage points* in the network and are big potential allies in approaching Uncertainty Principle mediated by these unique physics concepts. The obligatory passage points can be thought as “any material or immaterial devices by which each ally should inevitably go through if they want to solve their obstacles-problems and achieve their own interests”, and “the device of interessement and a part of final characteristics of the innovation” (Aka, 2019, p. 528).

In terms of topological properties of the network, these two nodes tend to have high *betweenness* values. When an actant-textbook vehiculates an innovation, in our case an original form of approach to Uncertainty Principle or a new related concept to feed the debate on this topic, it creates a potentially strong bridge on the network and gains importance as actant, since it potentially attracts allies. Defining this importance in terms of the most usual definition of betweenness given by (3), based on the number of shortest paths between two given nodes, two is the minimum number of actant-textbooks needed to cover Uncertainty Principle mediated by actant-objects PQ_phtpol and GED_retrodexp. There are more complex forms of betweenness centrality that consider more paths other than the geodesics between two nodes (see Estrada, Higham, & Hatano, 2009), but here we restrict our analysis to the simplest case.

The stabilization of actant-textbooks takes place in the network mainly when it connects to more stabilized actant-objects like PQ_xp, PQ_tE, GED_gamraymicr or INT_ep, but their action is not restricted to achieve stabilization, since there are institutional, commercial, academic and other forces driving actions in broader networks, with which the network analyzed here interacts. Innovations vehiculated in form of actant-objects can be considered as a substantive action that impacts this narrower network, providing a power of connectivity to the actant-textbook that brings these innovations to the network. Even if the innovation does not necessarily lead to preference of the actant-textbook among their peers, the potential *topological importance* achieved is undeniable.

Because there is a positive correlation between node degree $D(v)$ and node betweenness $B(v)$, the former increases as increases the latter (Oldham et al., 2019). In other words, nodes with high number of connections (adjacent edges) tend to have greatest betweenness. To account for this issue, we show in figure 4 not the betweenness, but the ratio $B(v)/D(v)$, which we will call *betweenness-degree ratio*, that give the efficiency of each actant in the network to perform a role of a mediator between two other actants. Two actants with distinct degrees and the same betweenness value does not have the same betweenness-degree ratio (the one with the lower degree, or less adjacent edges, has greater betweenness-degree ratio). Figure 4 shows that there is an approximately linear trend between betweenness-degree ratio and node degree for actant-objects. The more the degree of the actant-object, i.e., the greater is the stabilizations of actant-object in the network, the greater is its betweenness-degree ratio, i.e., the stabilization is not only a matter of number of connections, but it also leads to increase the potential power of actant-object in mediating interactions between all actant-textbooks.

The same is note true if we look at actant-textbooks. In this case, the relation between betweenness-degree ratio and node degree departs from linear relation. The actant-textbook AULETTA_2009 has greater betweenness-degree ratio than GOSWAMI_1997, even if AULETTA_2009 has a node degree 18 while GOSWAMI_1997 has a slightly higher degree (19). Inspecting figure 1, we can see that AULETTA_2009 vehiculates two unique actant-objects in the network (PQ_nphi and PQ_Jtheta), establishing an efficient potential connectiveness in the network. It is interesting to note that bringing unique (and therefore destabilized) actant-objects to the network causes gain of importance to the actant-textbook, due to potential achieve of mediating power, but not necessarily stabilization in the network. This is why AULETTA_2009, GOSWAMI_1997, EIBERG_1985 or NUSSENZVEIG_1998 (the higher betweenness-degree ratio among introductory actant-textbooks) occupy peripheric positions in the network. We will discuss actant-textbooks' stabilization in more detail in the following sections.

5.7.2 Analysis of bipartite projections: actants' stabilization

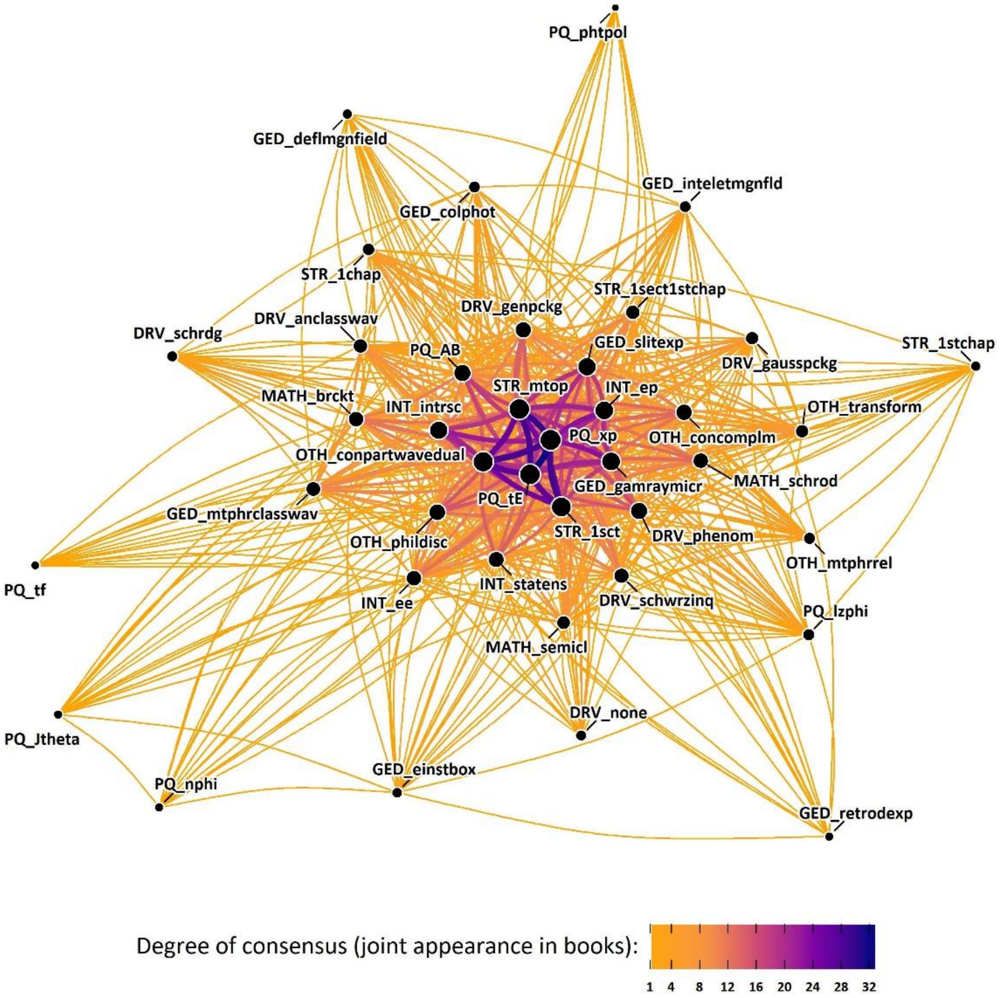
In sections 5.6.2 and 5.6.3 it was explained how we can obtain the adjacency matrices shown in tables 3 and 4, respectively representing the object-object and textbook-textbook networks.

In figure 5 we show the network obtained from the adjacency matrix shown in table 3. Each edge connects two different actant-objects and its weight is given by the number of times that these two actant-objects are jointly vehiculated by the actant-textbooks. Each node size was calibrated by the node strength, obtained by the sum of the weights of edges incident to the node (Kolaczyk & Csárdi, 2020, p. 44). It is reasonable to adopt these weights as a measure of the *degree of consensus* between actant-textbooks about the vehiculated actant-objects. Because the node strength is the sum of the weights of all edges connected to that node, the size of each node reflects the total degree of consensus that the correspondent node (actant-objects) experiment in the network.

The color and width of the edges are calibrated to reflect the weight differences in the network. Two actant-objects connected by orange and thin edges are vehiculated together by few actant-textbooks, those connected by thick and purple edges are vehiculated together by many actant-textbooks, evincing some degree of consensus in the sense the these actant-textbooks consider these actant-objects as essential to their approach to teach Uncertainty Principle. Thus, the more two given actant-objects are shared together by actant-textbooks, the greater is their mutual interaction in the network. The ForceAtlas2 algorithm reflects this and converge to a spatial distribution in which the strongest interacting actant-objects (thick and purple edges, greater nodes) are in the center of the network, while the weakest interacting actant-objects (thin and orange edges) end up in peripheral regions.

Among the most usual conceptual driven (remembering, considered multisemiotic objects) actant-objects, there are *structural* ones indicating how Uncertainty Principle is distributed on actant-textbooks. Actant-objects STR_mtop (more than one part) and STR_1sct (one section), PQ_xp (position and momentum uncertainty), PQ_tE (time and energy uncertainty), GED_slitexp (double-slit experiment), GED_gamraymicr (gamma-ray microscope experiment), INT_ep (error-perturbation interpretation, see Auletta, 2001, pp. 124-127) and INT_intrc (intrinsic interpretation, the idea that uncertainty is an intrinsic issue in the structure of matter, i.e., if the momentum of a quantum object is known at some precision, its position is dispersed and not well known) and OTH_conpartwavedual (connection with the wave-particle duality). It is clear a nuclear structure, with about nine actant-objects highly stabilized in the center of the network if we consider actant-objects with node strengths above the minimum threshold of 60 percent of maximum node strength (PQ_xp, strength 437). In figure 6 we show a bar plot of all strengths of each of the nodes representing the actant-objects in this network. It is possible to see that the stronger stabilization of nine concepts give support to the notion that introductory or advanced actant-textbooks jointly vehiculates just core concepts, most of them the ones which are more accessible to students (specially GED_slitexp and GED_gamraymicr), vehiculated in more than one part of the textbook (STR_mtop). This is expected, since the main proposal is obviously to teach Uncertainty Principle and its conceptual issues.

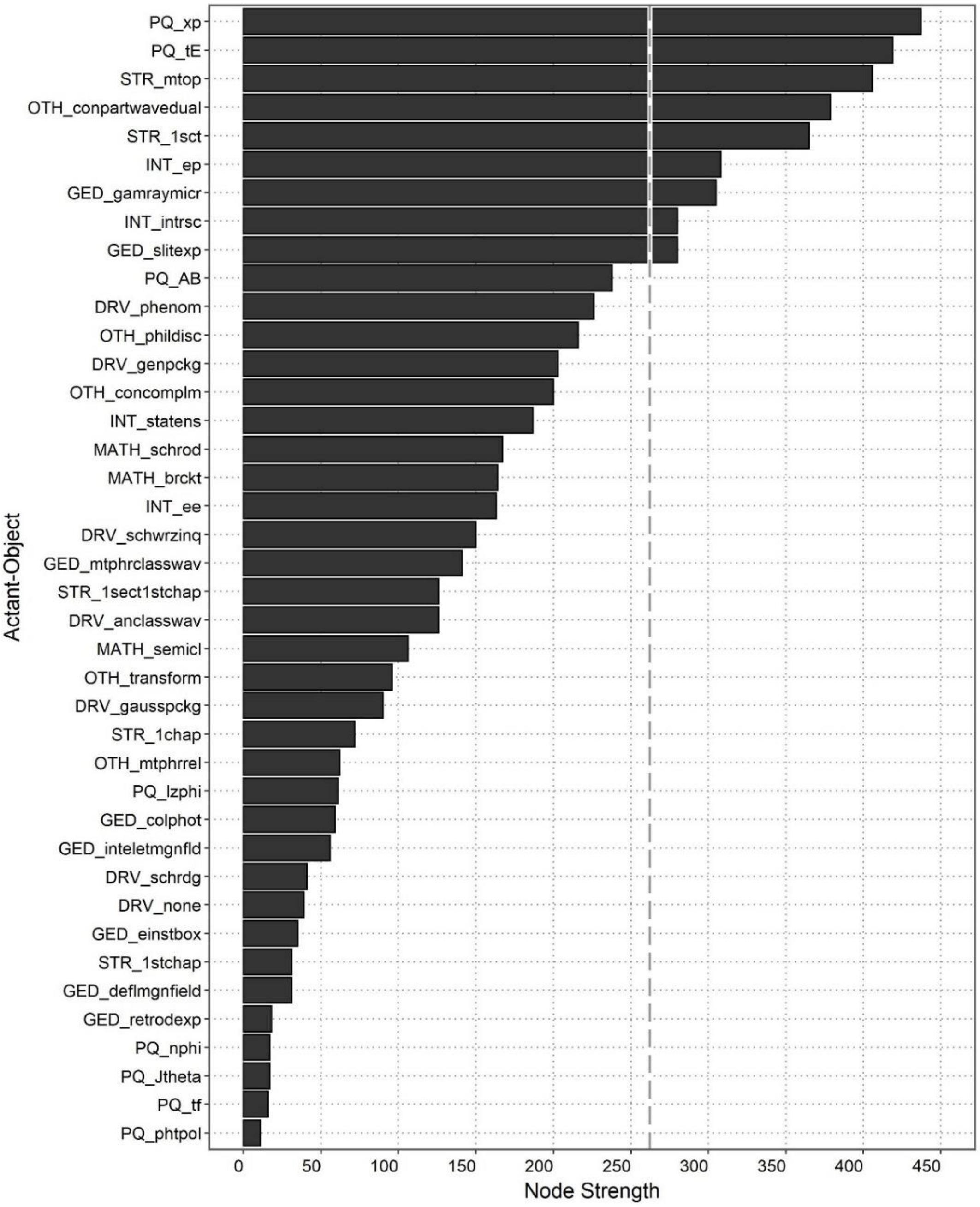
Figure 5 - Object-object network.



The object-object network was obtained from adjacency matrix shown on table 3.The edges' width and color are defined according to the edge weight, the number of actant-textbooks in which two actant-objects appear together. The size of the node is defined by the node strength, generalized version of node degree. The node strength is the sum of the weights of all the edges to which this node is connected (i.e., the *total degree of consensus*).

All these actant-objects are mutually and strongly connected and exhibit great stabilization in the network. In the case of actant-objects GED_gamraymicr and INT_ep, it would worth to investigate to what extent this may be a sign of black boxing. The idea of the gamma-ray microscope is clearly semiclassical. This conception considers the electron as a classical free particle moving with a certain velocity, the act of measuring its position being affected by the “collision” (actually, scattering) of a photon (measurement object) with this electron. Furthermore, Heisenberg's interpretation of the Uncertainty Principle – based on the microscope experiment – places the measurement process as fundamentally imbricated with the Uncertainty Principle (Auletta, 2001). This class of interpretation has been subject to criticism (Auletta, 2001, pp. 124-127; Brown & Redhead, 1981; Redhead, 1981, pp. 67-69).

Figure 6 - Strength of each actant-object.

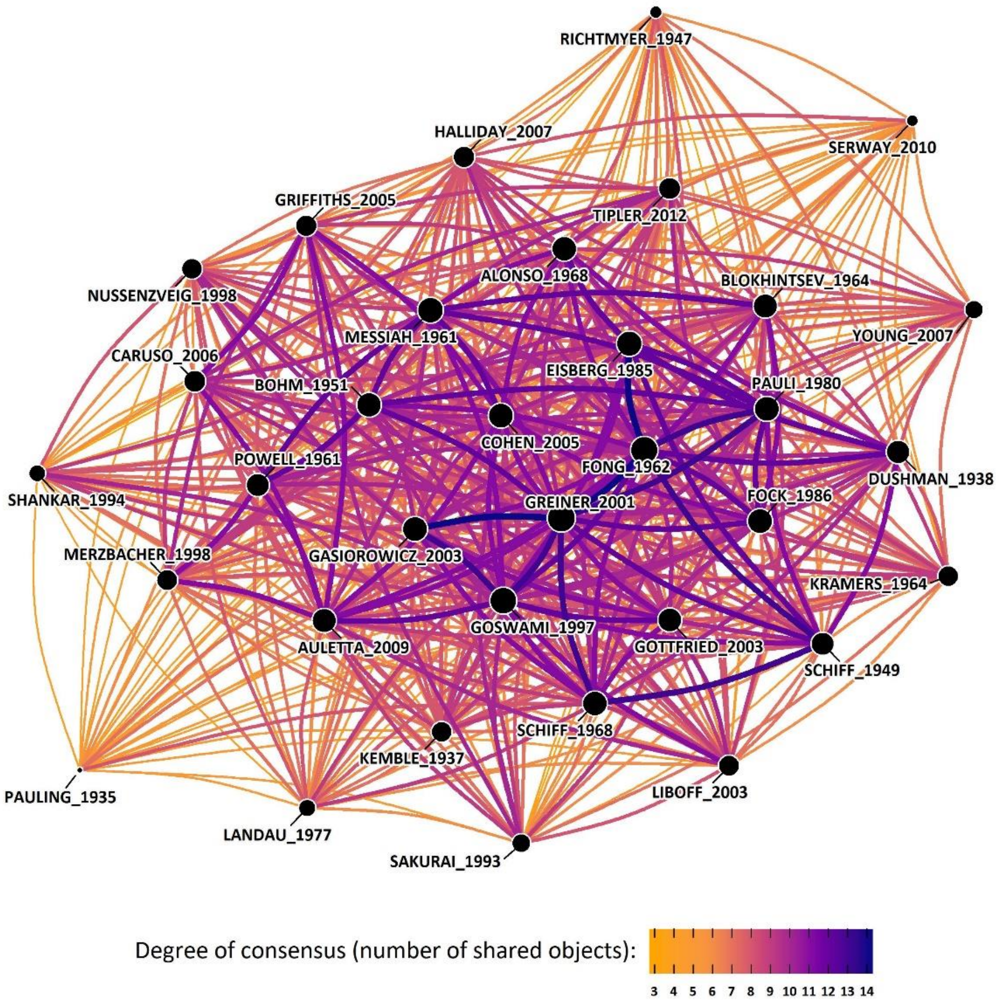


Strength of each actant-object shown in figure 5, ordered by node strength. The most stabilized actant-objects are represented by nodes with larger strengths. The vertical line corresponds to 60 percent of maximum node strength (PQ_xp).

However, it is quite intriguing in the literature, for example, to see that proposed approaches based on discrete events, processes compatible with corpuscular behavior, have been successfully applied to simulate results of a wide variety of experiments involving single photons and single neutrons, ranging from interference to entanglement. For a review of this interesting and thought-provoking approach see the work of H. De Raedt and Michielsen (2012). In relation to the Heisenberg Uncertainty Principle, this approach has also been successfully applied (Hans De Raedt & Michielsen, 2014).

Discussions about the foundations of Heisenberg's idea regarding the relationship between the measurement accuracy and the disturbance induced by the measurement (a central aspect in the error-perturbation interpretation) are still in progress, but is not common to appear in textbooks. Recently a group of researchers claimed to have found a violation of this relationship (Rozema et al., 2012), but a later work reinterprets the same foundations and claimed that the relationship is correct (Busch, Lahti, & Werner, 2013), claiming that they are redeeming Heisenberg's interpretation. It is interesting how this topic, treated in textbooks as an established concept, is still, in the literature, subject of intense debates related to the foundations of Quantum Physics. Therefore, the stabilization of this group of actant-objects must be viewed with caution. In some circumstances, stabilization can be a sign of black boxing processes.

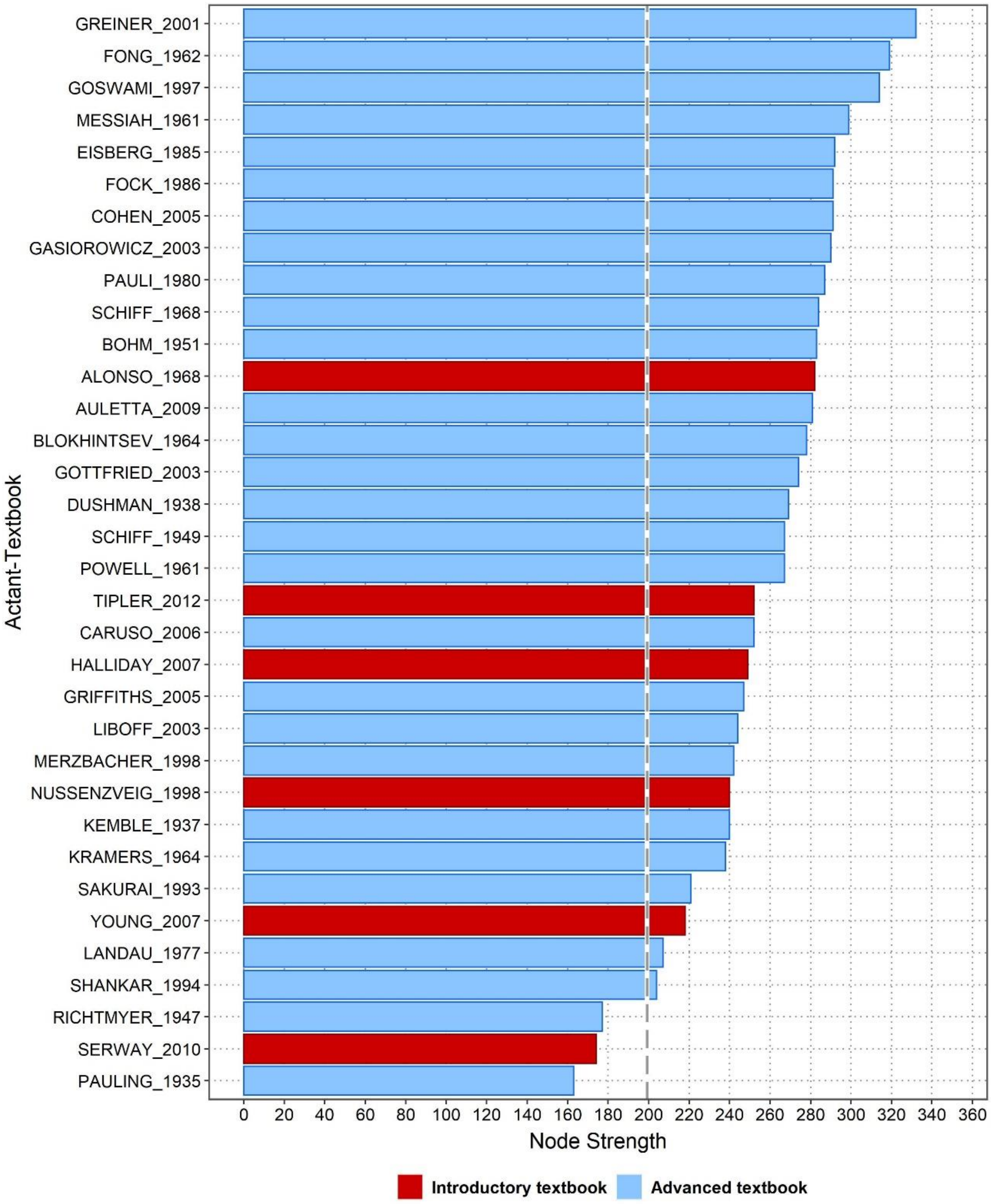
Figure 7 - Textbook-textbook network.



The textbook-textbook network was obtained from adjacency matrix shown on table 4. The edges' width and color are defined according to the edge weight (the number of actant-objects shared by two connected actant-textbooks). The node size is defined by the node strength (here it is also the total degree of consensus).

In figure 6 we show the textbook-textbook network, obtained from adjacency matrix shown on table 4. As in object-object network, the edges' width and color are defined according to the edge weight. Now, the edge weight is the number of actant-objects shared by two connected actant-textbooks. As in figure 5, the nodes' size is defined by the node strength. Unlike the object-object network, in which only nine actant-objects are strongly connected forming a small cohesive and stabilized group in the center of network, this group is clearly bigger in the case of textbook-textbook network.

Figure 8 - Strength of each actant-textbook.



The most stabilized actant-textbooks shown in figure 7. As in the figure 6, the actant-textbooks were ordered by its node strength. The vertical line again corresponds to 60 percent of maximum node strength (GREINER_2001).

These groups mutually share more actant-objects and, therefore, are the most consensual among themselves. As in the case of object-object network, node size reflects the degree of consensus each node experiments in the network – bigger nodes are result of bigger edge weights, which in turn indicates more stability.

There are some destabilized actant-textbooks in the network, in peripheral regions *and* connected mostly by orange edges, indicating that they share few actant-objects with all others in the network. This is the case of PAULING_1935, RICHTMEYER_1947, SERWAY_2010 and, in minor degree (confirmed by its larger node size), LANDAU_1977 and SHANKAR_1994. Most actant-textbooks are stabilized and experience strong connections.

Figure 8 shows the bar plot of node strengths obtained from the textbook-textbook network, shown in figure 7. It is noticeable that the nuclear structure of the object-object network was not reproduced in the textbook-textbook network. Only three actant-textbooks fall below the minimum threshold (vertical line in the bar plot), as already cited: RICHTMEYER_1947, SERWAY_2010 and PAULING_1935. This allows us to show a somewhat hidden feature in the bipartite network in figure 1: almost all actant-textbooks (31) are stabilized in the network jointly vehiculating a smaller set of actant-objects (9). In other words, is not possible to understand the relationships present in the complete bipartite network without inspecting relationships in the bipartite projections (and *vice-versa*). Although this can be verified inspecting horizontal axis of figure 1, it is much more evident when we look at the bipartite projections.

Another interesting aspect is related to time. Inclusion of actant-textbooks which come to light several years back (considering the first edition year as a measure of its age) allows us to infer about stabilization along time – the edition year ranges from 1935 (PAULING_1935) to 2010 (SERWAY_2010), a wide period. In the stabilized group is possible to identify actant-textbooks from 1949 (SHIFF_1949) to 2009 (AULETTA_2009), which shows that oldest actant-textbooks are yet well articulated to newest ones in topics related to Uncertainty Principle. This lack of significative mobility of the network as a whole that lead to a tendency of actant-textbooks to form this whole is compatible to the central idea asserting that “a black box contains that which no longer needs to be reconsidered, those things whose contents have become a matter of indifference” (Callon & Latour, 1981, p. 285). It would be worthwhile to implement additional studies that could confirm this potential problem in some actant-textbooks. One of the actant-textbooks appears two times, with two different edition years (SCHIFF_1949 and SCHIFF1968). As in the bipartite network shown in figure 1, the movement of this actant-textbook, although very perceptible, is not a signature of big changes in its perspective of Uncertainty Principle. This actant-textbook moves between two regions that are near to each other (movement *of* the network), indicating that even if new connections were built, they are not sufficient to imprint significative changes in the network as a whole. The node representing actant-textbook SCHIFF_1968 has a strength 284, slightly greater than the strength of the node representing SCHIFF_1947, equal to 267. In other words, SCHIFF_1968 is slightly better articulated to the network as a whole than its older one.

Concerning to introductory actant-textbooks, almost all of them seem to follow advanced actant-textbooks, since in most cases it is clear the presence of some purple and thick edges connecting its respective nodes to the other actant-textbooks (i.e., they share significative amount of the same actant-objects with the others). The exception is SERWAY_2010, represented by a small node. This is not a surprising or necessarily a negative feature. It can be a tendency of some introductory actant-textbooks to implement in-depth immersion in some topics, getting closer to how advanced actant-textbooks vehiculate their shared actant-objects.

5.8 FINAL REMARKS

In this paper, our main objective was study how the textbooks can act in the stabilization process of the Uncertainty Principle. First, we can identify the existence of a wide variety of aspects associated with the Uncertainty Principle among textbooks, so that we can affirm that there is a plurality, therefore, in the characterization of the essence of this actant. However, with our analysis we can go further and rank the characteristics of this actant – some are more stable than others, and the methods we employ allow us to identify what is most "essential," or stable, in the Uncertainty Principle. For example, we can see from Figures 1 and 5 that the aspects of the Uncertainty Principle most shared by textbooks, and thus more stable, are those that are at the center of the network, such as the physical quantities position-momentum (PQ_xp) and time-energy (PQ_tE), the Gamma-Ray Microscope (GED_gamaraymicr) and Slit-experiments (GED_slitexp) gedankenexperiments, the connection with the particle-wave duality (OTH_conpartwavedual) and the error-perturbation (INT_ep) and intrinsic (INT_intrsc) interpretations.

Furthermore, from the topics covered by the textbooks, we can also identify that the textbooks are not equal in terms of stability within the analyzed group. There are more stable books and those that become more peripheral, which indicates a plurality in the didactic genre itself.

The most stable textbooks are those that cover the most similar contents and are closer and in the most central regions of the network, while textbooks that are found in more peripheral regions of the network are not so stable, as they adopt a more original approach to the Uncertainty Principle, including aspects that most traditional textbooks ignore, or they are older textbooks, which have stopped in time and, therefore, differ from the others. From the methods used here, we can identify that there is a lot of consensus around few characteristics of the Uncertainty Principle, so that the most stable textbooks approach the concepts in a very similar way, using the same experiments and explanations, without much criterion or reflection. However, this stability is not necessarily a positive aspect and may indicate the existence of a black-box around the Uncertainty Principle – when the credibility of a scientific fact is accepted by the scientific community and there is no longer a need to explain all the “phenomenotechnique” involved in (LIMA et al., 2020). According to Lima et al. (2020) point out, although the blackboxing process is very important in the development of science, it can also be harmful to Science Teaching, as it ends up agreeing with a simplistic and naive view of scientific practice.

In general, our results seem to go against the existing common sense around the textbook. Contrary to what is expected, not all textbooks share a homogeneous discourse and deal with the same elements, this only occurs with more traditional textbooks, which tend to present a similar and simplistic discourse on the Uncertainty Principle, ignoring many aspects important and scientifically well-established about the subject.

Finally, although many books seem to say almost the same thing, the way they position themselves in the networks and their relationships with others differ (although little in some cases). Our analysis was restricted to the identification of the presence or not of certain perspectives on the Uncertainty Principle, but we perceive great potential in combining this study with a Bakhtinian analysis, so that, certainly, our network would gain more details and other fundamental differences would appear, because, however authorial and monological a textbook may seem, it always dialogues with other authors, with the past and the future, encompassing a series of voices in addition to the voice of its author. In this regard, every choice regarding the book (content that will be included, omitted, how it will be exposed, which philosophical strand the text is aligned with, and other aspects) is in itself a discursive act that can be deeply explored.

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6 CONSIDERAÇÕES FINAIS

Ao longo desta dissertação, apresentamos um estudo sobre a gênese e estabilização do Princípio da Incerteza no contexto científico-pedagógico. Para isso, partindo dos Estudos de Ciências de Bruno Latour e de suas premissas teórico-metodológicas, primeiramente investigamos o contexto histórico da gênese do Princípio da Incerteza e estudamos a fundo o artigo original de 1927 em que o físico Werner Heisenberg o propõe. Em seguida, com o auxílio de literatura secundária, caracterizamos alguns dos principais actantes que compoem a rede do Princípio da Incerteza e, por fim, investigamos como os livros didáticos do Ensino Superior traduzem o conceito.

Ao analisar o contexto de 1927 a partir do que definimos como História Simétrica, identificamos que a principal disputa entre os atores envolvidos na gênese do Princípio da Incerteza estava associada à formalização da Física Quântica – nesse contexto, o Princípio da Incerteza parece ser proposto como uma tentativa de advogar em favor do programa de pesquisa inaugurado em 1925 por Werner Heisenberg. Constatamos, também, que a proposta do Princípio da Incerteza traz consigo uma série de significados atribuídos por Heisenberg à própria Teoria Quântica: quando admite a existência de incerteza entre variáveis canonicamente conjugadas, percebe-a primeiramente como uma propriedade intrínseca aos fenômenos quânticos, pois seria uma limitação imposta pela descrição clássica da realidade; quando se propõe a analisar o microscópio de raios-gama, Heisenberg segue uma visão de mundo particular, em que objetos quânticos seriam essencialmente partículas e, ao descrever o processo de medição envolvido no experimento, interpreta a incerteza na posição como uma incerteza da medida e a incerteza no momentum como resultado de uma perturbação no experimento; quando utiliza da Teoria de Dirac-Jordan para formalizar matematicamente o Princípio da Incerteza, interpreta o desvio padrão pragmaticamente como o erro médio de uma medida e, ao analisar o comportamento estatístico da Teoria Quântica, assume-o como resultado das relações de incerteza.

Com o estudo do desenvolvimento do princípio, pudemos perceber que, embora nem todos os aspectos e interpretações propostos Heisenberg tenham se sustentado, o Princípio da Incerteza tornou-se um elemento de grande importância no quadro teórico da Física Quântica e, no decorrer do tempo, outros actantes foram se unindo a sua rede e ampliando-a, incluindo outras formalizações matemáticas, outros experimentos mentais e, inclusive, outras interpretações – nesse processo outros cientistas também foram responsáveis pela articulação de novos actantes a rede, tais como Erwin Schrödinger e Niels Bohr. Dessa forma, percebemos que a principal contribuição do segundo artigo está em explicitar uma pequena parcela da complexidade associada a rede do Princípio da Incerteza – principalmente no que se remete as variáveis posição e momentum – em que o objetivo foi contribuir para o ensino desse tópico em cursos de graduação.

Por fim, com o terceiro e último artigo pudemos identificar que, de fato, o Princípio da Incerteza está fortemente presente nos cursos de graduação em Física, uma vez que grande parte dos livros analisados dedicam ao menos uma subseção para discutir o tópico e todos os livros analisados trabalham o conceito. Entretanto, o que podemos perceber é que, no processo de tradução, consolidou-se muito consenso sobre poucos aspectos do Princípio da Incerteza, de modo que existe um grande grupo de livros didáticos que trabalham somente os mesmos actantes: muitos livros trabalham somente um tipo de demonstração matemática, uma ou duas interpretações para a incerteza posição-momento (sem explicitar quando trocam de uma para a outra), o experimento de raios-gama e experimentos com fendas. Esse processo de tradução dos livros considerados como mais tradicionais resulta no apagamento da complexidade envolvida na rede do Princípio da Incerteza.

Ademais, dessa forma foi possível identificar que alguns elementos são mais estáveis no contexto científico-pedagógico do que outros, pois estão presentes em grande parte dos livros – tais como a incerteza nas variáveis posição e momento e nas variáveis tempo e energia; as interpretações intrínseca e erro-perturbação; os experimentos do microscópio de raios-gama e de elétrons passando por fendas; e a conexão com a dualidade onda-partícula.

Dessa forma, com o terceiro estudo podemos concluir que a tradução realizada pelos livros didáticos está muito próxima do que Werner Heisenberg propõe no artigo de 1927, mas, em sua maioria, desconsidera grande parte das discussões e dos desenvolvimentos posteriores. Esse comportamento demonstra, também, a tendência dos livros didáticos em perpetuar uma visão ingênua do empreendimento científico, tratando os conceitos físicos como coisas a serem descobertas por um cientista e que, depois, permanecem cristalizadas no tempo e na história, apagando qualquer traço de controvérsia, debate ou inconsistência.

A pandemia de COVID-19, além de causar muito sofrimento e instabilidade nas mais diversas esferas da sociedade, tornou ainda mais evidente o abismo entre Ciência e Sociedade e fez emergir, mais fortemente, a necessidade de repensar o Ensino de Ciências. No contexto social em que vivemos, não basta que os estudantes decorem fórmulas e conceitos, tampouco que saibam apenas manipular equações matemáticas, é preciso que compreendam a Ciência dentro de sua complexidade, como um empreendimento – acima de tudo – humano, como um espaço para debates e questionamentos, que envolve rigorosidade, metodologia, empiria e filosofia, política, religião, crenças pessoais e visões de mundo.

Dessa forma, ao apresentar o Princípio da Incerteza como o resultado de uma rede, que foi articulada pela primeira vez em 1927 e, desde então, segue conectando novos atores, com o presente trabalho pretendemos romper com a tendência de ensino tecnicista e contribuir com um Ensino de Ciências que crítico e compromissado com uma visão complexa de Natureza da Ciência.

Por fim, o tempo disponível para a conclusão de uma dissertação de mestrado limita o presente estudo, mas percebemos potencialidades em dar seguimento ao mesmo por diferentes caminhos. Uma possibilidade de continuidade seria aliar a Teoria Ator-Rede, a Análise das Redes Sociais e a Metalinguística de Bakhtin, para investigar mais afundo as responsividades dos discursos repercutidos pelos livros didáticos, ou ainda poderia ser realizada uma investigação de como o Princípio da Incerteza para as variáveis energia e tempo se consolidou no contexto científico-pedagógico, como a literatura secundária o define e quais actantes são mais estáveis em sua rede.