DISSERTATION

OPTIMAL WEEKLY RELEASES FROM A MULTIRESERVOIR HYDROPOWER SYSTEM

Submitted by

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In partial fulfillment of the requirements

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION

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ABSTRACT OF DISSERTATION

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The operation of a multi-unit electric energy generating system is studied under certain and uncertain future inflow conditions. The generating units include thermoplants, hydroplants with regulating reservoir and run-of-river hydroplants. The objective is to minimize the expected cost of the operation of the system while meeting a previously defined energy demand. A case study is formulated based on the electric energy generating system of the South of Brazil. The system is composed of 6 hydroplants with regulating reservoirs, 2 run-of-river hydroplants, and 8 thermoplants.

In order to obtain a better insight into the nature and peculiarities of the system's operation it is initially studied considering the future to be deterministic. An aggregation-optimization-disaggregation procedure is proposed to identify a near optimal solution while reducing substantially the computational effort. This consists of the development of an aggregated representation of the system composed of a hypothetical and unique reservoir with overall energy inflows and releases. Optimal operation of the aggregated system is determined by a new and efficient optimization technique specifically developed for this problem. A disaggregation procedure defines the operation of each system's unit given the operation of the aggregated system. The procedure is based on a heuristic approach that has as a main objective to minimize water spills.

An aggregated representation of the system is again adopted for the definition of optimal strategy of operation when the future inflows are uncertain. The characteristics of operation of each reservoir are

introduced into the aggregated formulation utilizing the peculiarities of the optimal deterministic operation. A modification of Massé's Chain of Marginal Expectations is used in the computations.

The resultant strategy of operation can be presented as a function of aggregated values of energy storage and inflow. The strategy explicitly considers the autocorrelation of aggregated energy inflows. The strategy also implicitly accounts for the cross-correlations among the energy storages and inflows to each reservoir. Finally, a substantial part of the autocorrelation of the energy inflows and storages in each reservoir is indirectly considered in the strategy. Theoretical significance of the strategy is obtained without burdensome computational effort.

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Chapter 1

INTRODUCTION

Energy consumption is a significant indicator of economic activity in modern economies. Exhaustion of cheap, non renewable supplies of oil has significantly increased the demand for electric energy as an alternative. In Brazil 28 percent of the energy consumed in 1973 was electricity. By 1985 the estimated percentage is 35 (Ministério das Minas e Energia, 1976). The figures in the USA are 26 percent in 1973 and 42 percent by the year 2000 (U.S. Army Engineer Institute for Water Resources, 1975).

Increasing consumption of electricity causes the increase in number of units in electric energy generating systems. With the introduction of Extra High Voltage lines that allow transmission of electricity to large distances, the trend is toward interconnection of the expanded systems.

The operation of electric energy generating system becomes more complex when the system increases in number of units. The operational complexity is particularly notable when a large number of water reserves is used to regulate the water supply to hydroelectric power plants.

Mathematical Optimization techniques to define the optimal operation of water resources systems have been widely utilized since the beginning of the 70's. However, when the number of water reserves, users, and study time increases, the exponential increment in computation time severely limits the uses of such techniques. As a consequence, a great deal of simplification has been used in formulating these problems. This decreases the computation time but also reduces the significance of results.

Presumably the first researcher to address a general theory of reservoir operation was Massé (1946a and b). He presented an analytical approach to the problem based on the Calculus of Variations. In his work the optimal conditions of operation of a single reservoir are derived for the cases where the future inflows are either deterministic or uncertain. The formulation under uncertainty of the future inflows defined Massé's Theory of the Marginal Expectations which was the first explicit recognition of the Principle of Optimality in multistage decision processes (Arrow, 1957, p. 525; Sobel, 1975, p. 770; Morel-Seytoux, 1976, p. 9-33). Some years later Bellman (1953) noticed the generality of the principle and derived what he called the Theory of Dynamic Programming.

In its original formulation Dynamic Programming is an analytical approach from which the Theory of Marginal Expectations is a particularization. Bellman also presented a numerical approach for Dynamic Programming under the name of Discrete Dynamic Programming. The approach is based on substitution of the analytical formulation of the problem's functions by their discrete formulations.

Some pioneer applications of Dynamic Programming to the optimal operation of reservoirs were performed by Little (1955), Gessford (1959) and Fukao et al. (1959). Little derived the optimal operating strategy for a single reservoir with uncertain future inflows. As expected, his formulation is identical to Massé's. The solution procedure however was based on Discrete Dynamic Programming. This approach was later referred to as Explicit Stochastic Dynamic Programming (Roefs, 1968, p. 66). Gessford presented an analytical solution for the same kind of problem at the expense of a very simplified formulation. Apparently the first application of Dynamic Programming to the deterministic operation of a multireservoir system was accomplished by Fukao and his co-workers. Fukao noticed the computational difficulties caused by the dimensionality of the problem and presented some artifices to decrease the computational requirements. They are now known as the Successive Approximation and the State Increment artifices of Dynamic Programming (Larson, 1968).

The associated computational difficulties of the Explicit Stochastic Dynamic Programming when serial correlation is present in the inflow led Hall and Howell (1963) to suggest the identification of the optimal operation strategy through samples of numeric values of releases. The samples were obtained by computation of the optimal operation under several inflow series using Discrete Dynamic Programming. Identification of the optimal operation strategy through regression analysis was proposed by Young (1967). This approach was later called the Implicit Stochastic Approach of Dynamic Programming (Roefs, 1968, p. 66).

The Dynamic Programming procedure in its explicit or implicit stochastic approaches was the basis of a majority of the recent contributions to the state-of-the-art of optimal reservoir operation studies. It was observed however that the implicit stochastic approach tended to produce unreasonable results in extreme and hence rare hydrologic situations (Butcher, 1968; McKerchar, 1973, p. 219). To improve the results an extensive number of generated hydrologic series is required to increase the occurrence of rare events in the samples upon which the regression analysis is applied. This considerably increases the computational requirements of the approach. When the number of reservoirs considered in the problem increases not only the computational requirements become overwhelming but the operating strategies defined by regression analysis were reported to be unable to prescribe "reliable and acceptable overall operation... over an extended time horizon" (Divi et al., 1978, p. 14).

Application of most reliable explicit stochastic approaches includes vast computational requirements that tend to increase exponentially with the number of reservoirs in the system. This necessitates utilizing the approach under simplified assumptions and/or following extensive iterative procedures. The solutions are generally derived with no guarantee the results will be applicable to the real system (Schweig and Cole, 1968; Takeuchi and Moreau, 1974; Sen et al., 1977; Houcks and Cleland, 1977).

Sigvaldson (1976, p. 343) summarizing the state-of-the-art of multireservoir operating studies correctly stated: "For larger (reservoir) systems...say, more than about five or six reservoirs... the most successful modeling strategy still seems to be with the use of simulation models."

Some research effort developed parallel with the previous works has exploited the use of aggregated formulations of the multireservoir systems (Smith, 1979). The aggregated formulation represents the multireservoir system by a unique equivalent reservoir which is supposed to capture all relevant characteristics of the real system. Difficulty of application of such an artifice is related to the derivation of aggregated objective function and restrictions. Thus, applications are restricted to problems in which the sequence of the temporal overall releases is more important than the spatial distribution of the releases among reservoirs. This requirement restricted the application of aggregated formulation basically to hydropower systems (Morlat, 1951; Stage and Larsson, 1961; Lindqvist, 1962; Ahmed, 1967; Arvanitidis and Rosing, 1970 a and b; Hall, 1971; Sen et al., 1978 and Turgeon, 1979). Aggregated formulations were used with explicit stochastic approaches to define the operation of hydropower systems in Sweden, Brazil, Canada and Colombia (Millan and Mejia, 1979).

The apparent trend in the development of the state-of-the-art of multireservoir operational studies has been the use of increasingly sophisticated techniques of Mathematical Optimization (El-Hawary and Christensen, 1972, 1973, 1977; Delebecque and Quadrat, 1978 and Murray and Yakowitz, 1979). Not enough attention has been directed to the characteristics of the optimal operation and their exploitation in search of optimal strategies. In this aspect the Theory of Marginal Expectations is one of the few approaches where the primary objective is the analysis of optimality conditions of the operation as the first step in the procedure for its definition.

Only recently this approach was recognized as a particularly useful alternative to reservoir operation studies (Morel-Seytoux, 1976). It was successfully applied to define the optimal operation for a single seasonal reservoir under deterministic and uncertain future inflows (Laufer, 1977; Laufer and Morel-Seytoux, 1979).

The following study of multireservoir hydropower system operation has its methodological orientation based on the study of the characteristics of the optimal operation. The main source of inspiration is Massé's contributions to the theory of the optimal management of reserves of which the operation of water reservoirs is a particular aspect. Under the basic assumption that no problem in management is general, a system of 6 reservoirs and 8 hydropower plants is studied and methods of optimization of its operation under deterministic and uncertain future inflows are developed. These methods are tested and their results analyzed from theoretical consistency and computational efficiency aspects.

Chapter 2

THE RESERVES AND THEIR OPTIMAL OPERATION

In the following general discussion a complex problem that includes the design, management and operation of reserves is analyzed according to operational aspects. The analysis stresses the kind of decisions involved in the operation, their optimality conditions, the objectives to be sought and some particularities of the real world decision processes.

2.1 Objectives in Operation of Reserves

Two alternative problems may occur in the operation of reserves (Massé, 1953, p. 120). The first problem referred to as the output optimization problem occurs in cases in which the output derived from the operation of the reserves must be computed to maximize the resultant net benefit. The price factors that define the operating cost and revenues are known. In an electrical energy generating system this problem occurs when the level of energy output along a given time period is computed to maximize the net benefit derived from energy production, distribution and sale. In this case, the cost of operation refers to the cost of resources and labor needed to achieve a given level of energy output in the system. The revenue from the operation is obtained from the sale of the energy output.

A second type of problem occurs when the level of output derived from operation of the reserves is pre-defined. The price factors which compose the cost of operation are known. The objective is the definition of the output production process in order to minimize the resultant operating cost. This problem will occur in an electric energy generating system when the energy output is established by contracts, for instance. The operating cost derives from the cost of the energy generation and distribution. The optimal solution is obtained by selecting a given level of energy generation in each unit of the system such that the contracted level of energy production is achieved at the minimum cost. This problem is referred to as the process optimization problem.

2.2 Strategic and Tactical Decisions

In the task of operation of reserves two kinds of decisions may be identified: the strategic and the tactical decision. The strategic decisions refer to the decisions concerning the operation of the system in order to optimize some measure of satisfaction in an expanded time horizon and consequently under uncertain future. The tactical decisions refer to the short term decisions required to secure the strategic goals. The future in tactical decisions is known or assumed known with an acceptable degree of accuracy. In this study any set of decisions made under complete knowledge of the future is considered to be of the tactical type.

A basic distinction and a paradox exist between tactical and strategic decisions:

> "In the world of certainty we can (and we must) make our present and our future decisions today. In the world of uncertainty we are apparently faced with an unsurmountable dilemma. We can not today make our present and future decisions at once because the progressive influence of change upon the known factors will give us valuable information in making our future decisions. But we cannot take our present decisions today either, (if we) let our future decisions fall where they may because we could not make long-term optimum calculations" (Massé, 1963, p. 249).

The implications of this paradox are that inherent to any strategy there will be forecastings of the alternative futures that must be updated as long as new information about the future is known. This attitude of postponing the decisions for the sake of better forecastings is the main difference between tactical and strategic decisions.

In a strategic decision the present optimal decision is made taking into account what may occur in the future if this decision is made and then what the optimal decision would be. A tree of decisions may be abstractly defined with each node representing a possible state of the system in the future. State of the system in this case is defined as the collection of all relevant information available to define the instantaneous condition of the system and to forecast possible future conditions. It eventually includes among other informations quantification of the reserves and past inflows. Branches linking two nodes represent decisions made and the consequent transition between system's states. The optimization of the operation is performed by selecting in each node (or state) the best decision to be made for the best overall results.

Strategy definition can be overpowering when several decisions, alternative futures and informations about the system's state exist. A simplification used in order to decrease the computational effort is to consider only a limited set of states or decisions of the system. Sometimes this task is performed through the discretization of the continuous variables of the problem. Other simplification restricts the amount of information used in the definition of the system's states. Empirism to define the strategy is also used for simplification.

These kinds of simplifications define an incomplete strategy. In the first two examples, an incomplete definition is presented because not all states of the system or decisions are considered. In the last case an incomplete conceptualization of the tree of decisions is used.

It is clear that an incomplete strategy can be optimal only as an approximation. However, in the presence of a considerable body of information the definition of complete strategies quickly becomes impractical (Arrow, 1957, p. 529).

Hence, there are two hinderances to optimal operation of reserves that are increasing the cost one has to pay for the uncertainty. One, inherent to the uncertainty itself, is the insufficient knowledge of reality. This causes delays in decision making until part of the "uncertainty becomes known" (Massé, 1946a, p. 27), or until better predictions can be obtained. The second barrier is formed by the limitations in processing considerable amounts of information concerning the future in the strategy definition.

2.3 Optimal Operation of Reserves

The fundamental decision (tactical or strategic) in operation of reserves is to decide the quantity of goods to consume now and the amount of goods to leave in reserve for future usage. The reserve acts as a device through which the goods are transferred from present to future. This temporal allocation involves a well defined complexity.

> "The same good available in two different times must be considered as representative of two different goods from the economic point of view" (Massé. 1946a, p. 41).

If the good had the same worth (or implied same satisfaction) whether used now or in the future no problem of operation would exist. Any allocation would be optimal since it would imply the same overall satisfaction.

Suppose the revenue (or satisfaction) derived from the instantaneous usage of the goods in reserve is known and depends uniquely on the level of usage of the goods at each instant. Suppose also the revenue (or satisfaction) obtained from maintaining the goods in reserve for optimal use in the future is known. An optimal decision relating the quantity of goods to be used now and those to be maintained in reserve may be made through the following reasoning. Consider a given level of instantaneous usage of the goods. Suppose an additional infinitesimal quantity of goods is allotted to be used at the present instant, decreasing the remaining quantity of goods in storage. Suppose also it will increase the instantaneous revenues and decrease the revenues obtained with the future usage of the remaining goods in reserve. If the increase in the instantaneous revenue is greater than the decrease in the future revenue of the remaining storage, the new level of allocation of goods defines an improved solution. In this case, improved solutions may be further obtained by increasing the instantaneous usage of goods. Otherwise, if the increase in the instantaneous revenue is less than the decrease in the future revenue, the initial level of allocation of goods is better than the new level. In this case, improvement may be further attained by increasing the reserves. A situation may occur where the increase in the instantaneous revenue is equal to the decrease in the future revenue when an infinitesimal increase in the instantaneous usage of the goods is made. In this case no improvements will result by increasing infinitesimally the usage or the reserves of the goods at the given instant. In an optimal allocation this situation obviously occurs.

In mathematical terminology this necessary condition for the optimal decision may be stated as: the optimal decision occurs when the marginal value of revenues derived from instantaneous usage of goods equals the marginal value of revenues derived from future usage of the remaining goods in reserve. Hence, the problem of operation of reserves is solved through the definition of the functions of revenues from instantaneous and future usage of goods. The values of instantaneous usage of goods and the resultant remaining reserves are optimally obtained when the marginal values of the respective functions of revenue are equal.

The revenues derived from the use of goods at each instant of operation are generally known. Difficulties may exist in relation to the definition of revenues of future usage of goods in storage. When the future is deterministic, all information is available for computation. In Chapter 5 a method is proposed to accomplish this task in a particular problem of operation of reserves. When the future is uncertain however, some complexities are introduced. The reserves depend on the decisions made in the past. Reciprocally, the decisions made in the past must take into account the future revenue of the reserves. However, since the future is uncertain so is the optimal decision and the future revenue. In this case one is compelled to compare an instantaneous and certain revenue derived from the instantaneous usage of the goods with a future and uncertain revenue derived from the future usage of the remaining goods in reserve. In some circumstances the uncertain future revenue of the reserves may be considered through its mathematical expectation. It occurs in cases where the actuarial risk situation prevails (Laufer, 1977, p. 200). This situation arises whenever many decisions are concurrently made and are similar and independent of each other. Moreover, one is interested in the overall consequences of the decisions and not in their isolated consequences. In this case, the optimal decision is obtained when the marginal value of revenues from instantaneous usage of goods equals the expected marginal value of revenues from future usage of remaining goods in reserve.

In both problems it is implicitly assumed that the function of revenues from the instantaneous use of the goods is readily available. The problem may become more complex when the instantaneous revenues also depend on the future. This situation occurs when the instantaneous worth of the goods depends on its future availability. For instance, if the commodity is water and the usage is crop irrigation some future temporal pattern of water supply has to be available in order to establish the instantaneous worth of the water. For example, a given volume of water used for irrigation in the beginning of the crop season will have no value if the crop is lost later due to inadequate water supply. In this case, a guarantee of future water supply must exist for the water to be benefitially used in the present.

In some circumstances the instantaneous revenue function may be considered independent of the future availability of goods. In electric energy generating systems this may occur in the two basic optimization problems. In the first problem, output optimization, the guarantee or dependability of the energy supply does not affect the selling price in cases when the demand market is large compared with production capability of the system and several alternate systems can supply the energy. In the process optimization problem the independence comes naturally when the generating units are always able to provide the energy demand.

2.4 Elementary Time Interval of Operation

Decisions must be made anticipating the time interval of implementation. This permits the groups involved in the production (suppliers) and usage (users) of the goods to take the adequate measures to make the decision effective. For instance, suppose the decision refers to the quantification of the water release from a reservoir. Some anticipation may be needed to transmit the decision to the reservoir's operator and allow him enough time to set the proper gate opening. Some anticipation is also required to allow the users of the water to perform the necessary steps to receive and use the released water.

Generally a program of operation valid for an elementary time interval is also established. The objective is to prevent suppliers and users to continually adapt to a changing condition of operation. Taking the same example given before, releases from the reservoir are established for an elementary time interval equal to, say, one day, one week or one month.

The length of this elementary time interval defines two basic schemes of operation of water reserves (Massé, 1946b, p. 131-133). In the first scheme the water inflow during the elementary time interval can be accurately forecast when the decision is made. Therefore no problem exists in the determination of the remaining storage at the end of the elementary time interval. The decision about the optimal water release may be taken in order to enforce the necessary condition of optimality, the equality between the marginal revenue of the instantaneous release and the marginal expected future revenue of the remaining storage.

In the second scheme of operation the forecast of the water inflow during the elementary time interval is subject to some degree of uncertainty. Therefore, at the time of the decision, one cannot know for sure what will be the remaining storage of the reservoir at the end of the elementary time interval. In such a situation the necessary condition for the optimal operation cannot be enforced. Another source of uncertainty, the uncertainty in following the optimal strategy of operation, is then introduced into the problem.

Therefore in the choice of the elementary time interval two opposing considerations are present. It must be short enough to avoid inaccurate forecasts and risks involved in not following the optimal strategy. It must also have enough length to allow the suppliers and the users to take the necessary measures to adequately implement the decision.

A third consideration has been widely used in reservoir operation studies to define elementary time intervals. An increase of the elementary time interval is adopted as an artifice to decrease the computational effort in derivation of the optimal strategy. This artifice generally determines the utilization of the second scheme of operation.

2.5 The Water Reserves in Electric Energy Generating Systems

Electric energy presents some particularities that regulate production, transmission, and usage. It may feasibly be transformed into almost any kind of work; it is relatively easy to be transported large distances without time delay; but the large scale storage of electric energy is economically infeasible. As a consequence, the production of electric energy follows the demand fluctuation although the generating units may be located far from the users.

In the production of electric energy the most common primary energy supply used in the process is the kinectic or potential energy of the water or the thermal energy from fuel burning or nuclear fission. An important characteristic of the generating units is related to the availability and cost of this primary energy supply. In the case of the kinetic or potential energy of water the availability depends on the water inflow which is random in natural water streams. In the case of fuel burning or nuclear fission it may be considered deterministic. The variable operating cost of a single purpose, hydropower unit is practically null. In the other units it is directly related to the quantities of fuel required for the electrical generation. Thus, the generating units may be conveniently classified as hydroelectric or thermoelectric units.

In a hydroelectric generating unit without water reserves the water supply cannot be stored. If this unit is part of a hydrothermal system it is logical to use prioritarily all water supply to produce electricity. The remaining demand would then be supplied by the thermal units. If the primary energy supply to the thermal units is assumed unlimited no problem of allocation between present and future exists. In such a situation the operational problem is essentially tactical since the future is not relevant for the present decision. The decision will be defined by the best combination of production from each thermal unit to provide the remaining electricity demand at minimum production cost. If electricity output is fixed as in the process optimization problem (section 2.1), the solution is determined simply by first using the less expensive units until all demand is provided. Otherwise, in the output optimization problem, the optimal level of electric production is such that the marginal benefit of the electric supply equals the marginal cost at the most expensive thermal unit or the thermal units that are generating at the full capacity.

Introduction of water reserves in the system initiates the trade-off with the future. The breakdown between hydro and thermoenergy production is only part of the whole solution procedure. This part defines the variable cost of electricity production under several alternative repartitions of the production between hydro and thermal units. As the variable cost of hydrogenerating is null the variable cost of electricity production is determined by the thermogeneration.

In the output optimization problem the concern is with the net benefit of the electricity output. The decisions in each time interval are 1) the total electricity production and 2) the breakdown between hydro and thermoenergy productions. Focusing attention on to one time interval only the best breakdown is obtained by using all hydroelectricity available since its cost is null. After that, thermoelectricity will be generated until thermal units are producing energy at their maximum capacities or the marginal cost of energy production equals the marginal benefit. However, when the trade-off with the future is introduced this solution usually becomes non optimal due to the limited and random supply of water. Even then a characteristic of the optimal unique time interval operation continues to be present. The marginal benefit of electricity supply has to be equal to marginal cost of the production. As a consequence, if the optimal level of hydroelectric energy production is known the optimal thermoelectric production may be defined trivially at the equality of the marginal values of cost and benefit. The definition of the optimal level of hydrogeneration may then be determined independently of thermogeneration.

When the future is deterministic the optimal instantaneous hydrogeneration is such that its marginal benefit is equal to the marginal benefit derived from optimal usage of the remaining water storage. Suppose mathematical expectations can be utilized when the future is uncertain. Then the optimal instantaneous hydrogeneration will be defined when its marginal benefit is equal to the marginal expected future benefit derived from optimal operation of remaining water storage. Consequently, the optimal trade-off with the future is in both cases accomplished in terms of hydrogeneration only. This conclusion is not surprising since no trade-off with the future is required for thermogeneration because its related primary energy supply is considered unlimited.

The process optimization problem is easier to analyze. In this problem electric output is previously defined and the objective is to minimize the cost of operation. The thermogeneration will be trivially defined as the remaining energy output to be achieved after the optimal hydrogeneration was determined. The cost of operation and more important, its marginal value is derived from the thermogeneration since the cost of hydrogeneration may be considered null. Consequently the objective is to minimize the cost of thermogeneration. The resultant optimal instantaneous level of hydrogeneration is such that the marginal instantaneous operating cost equals the marginal expected future operating costs with the optimal operation of remaining water reserves.

In conclusion, problems of output and process optimization in hydrothermal electric energy generating systems will be solved essentially by the same procedure of trade-off with the future. In both trade-offs attention is focused on the optimal operation of the hydroelectric units only.

A final consideration related to the definition of an elementary time interval of operation is needed. As previously stated, electric energy cannot be economically stored on a large scale. As a consequence the electricity is produced, transported and distributed according to the instantaneous demand. The use of elementary time interval of operation results in the consideration of average values of the variables in the time interval. This situation normally does not cause difficulties in water supply systems as long as secondary regulations are available to provide the demand according to its occurrence pattern inside the elementary time interval. However, electric energy systems have no secondary regulations. The problem of provision of the energy demand inside the elementary time interval must be considered in the operation. This problem is usually called dispatching.

Chapter 3

PROBLEM FORMULATION

A hypothetical hydrothermal electric energy generating system is defined in this chapter. It is based on a hydrothermal system in southern Brazil; the projected completion date is 1983. When available, real figures were adopted in the formulation of the problem; however, assumptions were used when they appeared to be sound or when information was not available.

3.1 Formulation

The configuration of the system is presented in Figure 1. The region includes three Brazilian states: Paraná, Santa Catarina and Rio Grande do Sul. The forecasted weekly energy demand for the region by the year 1983 is presented in Table 1. The energy unit adopted is the Equivalent Megawatt (Eq. MW) which is equal to the average power demand during a given time period. The conversion to Megawatt-hours (MWh) is made by multiplying Eq. MW by the number of hours in the period.

The hydroelectric generating system is composed of 8 hydropower plants with a total capacity of 6,588 MW when installed. The water reserves are formed by 6 reservoirs located on 4 rivers with a total storage capacity of 13,449 hm^3. Two sets of hydroplants with 3 units each are formed by serially linked hydroplants. The set at the Jacuí River is regulated by one reservoir only. The other set is located at the Iguaçú River and has one reservoir at each plant. A transbasin diversion is located at the Capivari - Cachoeira hydropower development. On the Capivari river a reservoir with a small regulating capacity was

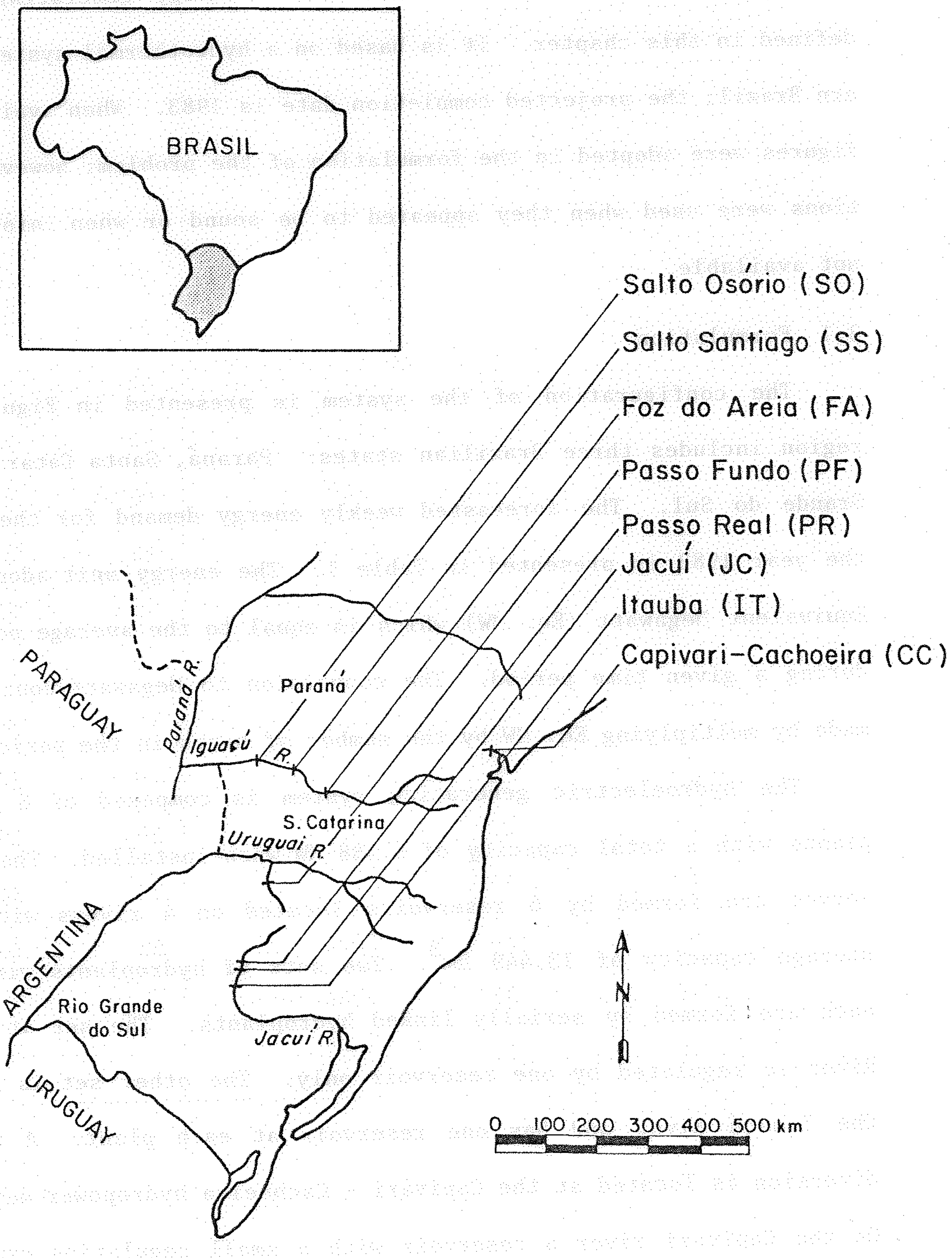


Figure 1. Hydropower Development in the Region

Table 1. Weekly energy demand.
(Eq. MW)

Week	Demand	Week	Demand	Week	Demand
1	2746	19	2811	37	2883
2	2758	20	2826	38	2890
3	2778	21	2839	39	2899
4	2806	22	2850	40	2909
5	2833	23	2859	41	2920
6	2855	24	2866	42	2928
7	2869	25	2865	43	2933
8	2862	26	2863	44	2939
9	2849	27	2859	45	2946
10	2831	28	2856	46	2956
11	2812	29	2856	47	2978
12	2797	30	2862	48	3002
13	2784	31	2868	49	3026
14	2775	32	2874	50	3051
15	2768	33	2879	51	3070
16	2769	34	2880	52	3087
17	2779	35	2880		
18	2795	36	2881		

Source: GCOI-SUL, 1978.

built and its water is diverted through a 13 km tunnel to the hydroplant located at the Cachoeira River. Table 2 presents the characteristics of each hydroplant.

The overall hydroplant efficiencies and hydraulic losses are considered constant in each hydroplant. The net hydraulic head is computed by considering the variation in the reservoir levels; the tail-water elevation is assumed constant. The transmission losses due to the electricity transportation are included in the overall efficiency.

Some small hydropower developments scattered through the region are considered as a block contribution to electric production in the system. Their contribution has been evaluated based on the report of GCOI-Sul (1978, Annex 6 and 9). Their maximum weekly contribution is 172 Eq. MW.

Two transfers of energy from the electric generating system in southeast Brazil are available with a constant total of 460 Eq. MW (Table 3).

The thermoelectric generating system is composed of 8 units with a total installed capacity of 963 MW. The characteristics of each one are presented in Table 3.

A source of pre-fixed energy generation is the minimum required generation of each thermoplant (GCOI-Sul, 1979, p. 11 and Annex 10). The minimum value of generation in thermoplants is adopted to avoid delay from starting a unit in an idle state and to bringing it up to the required production rate. It also eliminates the consideration of the cost to start up the unit. The minimum thermogeneration is equal to 562 Eq. MW during a week of maximum contribution.

Table 4 presents the minimum and maximum energy production provided by thermoplants, small hydroplants and energy transfers for the year 1983, including maintenance considerations. The minimum energy

Table 2. Characteristics of the hydroplants.

Site	turbine type	hydraulic heads max (m)	min (m)	rated (m)	rated flow (m^3/s)	efficiency and losses (%)	- (m)	load factor (%)	capacity (MW)
PR	propeller	46.0	32.0	40.9	415.	.90	0.4	1.	150.
JC	reaction	92.0	92.0	92.0	212.	.88	4.0	1.	168.
IT	reaction	87.7	87.7	87.7	660.	.88	.06	1.	500.
FA	reaction	150.3	110.5	125.6	2075.	.88	1.0	1.	2250.
SS	reaction	109.0	84.0	99.7	2324.	.88	1.0	1.	2000.
SO	reaction	72.0	64.0	68.3	1844.	.85	1.0	1.	1050.
PF	reaction	261.0	247.0	251.1	101.	.88	5.0	1.	220.
CC	impulse	755.4	730.9	716.3	40.	.88	30.	1.	250.

Site	PR	JC	IT	FA	SS	SO	PF	CC
Maximum Storage (hm^3)	3640	31	500	7565	6760	1270	1568	180
Minimum Storage (hm^3)	1280	31	500	2552	2670	830	180	22
Minimum Release (m^3/s)	10	10	10	78	114	160	15	6

Source: ELETROSUL, 1979.

Table 3. Characteristics of the thermoplants and energy transfers.

Plant	electricity production maximum (Eq.MW)	minimum (Eq.MW)	average cost ($/MWh)	fuel
# 1	369-425	208-241	12.5	coal
# 2	178-357	134-268	25.8	coal
# 3	46-61	14-29	28.7	coal
# 4	16-24	5-5	31.5	coal
# 5	9-14	0-0	34.4	coal
# 6	300-300	0-0	40.0	transfer I
# 7	28-56	16-16	45.0	oil
# 8	8-16	0-0	80.0	oil
# 9	10-10	0-0	87.0	oil
# 10	160-160	0-0	100.0	transfer II

Notes: 1) the variation of the maximum and minimum production values refers to the planned outages through the year,
2) transfer refers to hydroelectricity brough from outside of the region.

Sources: 1) maximum and minimum production from GCOI-Sul, 1978,
2) average costs are assumed based in personnal communications (ELETROSUL, 1979).

Table 4. Maximum and minimum energy production provided by thermoplants, small hydroplants and energy transfers.

(Eq. MW)

week	min	max	week	min	max	week	min	max
1	600	1416	19	603	1417	36	733	1579
2	600	1408	20	603	1417	37	733	1579
3	600	1408	21	603	1417	38	733	1579
4	600	1416	22	603	1432	39	733	1579
5	604	1408	23	609	1404	40	702	1540
6	604	1408	24	609	1405	41	702	1540
7	604	1408	25	609	1405	42	702	1532
8	604	1408	26	609	1405	43	702	1540
9	618	1432	27	612	1442	44	737	1597
10	618	1432	28	612	1442	45	737	1589
11	618	1432	29	612	1442	46	737	1589
12	618	1432	30	612	1442	47	737	1590
13	618	1432	31	682	1503	48	737	1598
14	590	1393	32	682	1503	49	693	1531
15	590	1389	33	682	1503	50	693	1531
16	590	1390	34	682	1503	51	693	1531
17	590	1394	35	733	1554	52	693	1531
18	603	1417						

Source: GCOI-Sul, 1978.

production will be subtracted from the total energy demand giving the net demand for energy.

Table 5 presents some statistical parameters of the inflows to each hydroplant during selected weeks of the year. In serially-linked hydroplants the value of the inflows refers solely to that part of the total observed inflow which originated after the section where the upstream hydroplant is located. For instance, the values of the series of the inflows in Salto Osório are given by the observed inflows in Salto Osório discounted by the observed inflows in Salto Santiago for each time interval. The evaporation in each reservoir is assumed to be equivalent to rainfall and secondary water inflows to the reservoir; hence, it is not accounted.

The travel times of the water between serially linked hydroplants are considered constant. Table 6 presents the distance between the hydroplants and the assumed travel times in number of weeks. The distances are always short enough to have travel times shorter than one week. In other words, the water being released from a hydroplant will reach the downstream hydroplant during the same time interval. To increase the generality of the case study the travel time between the hydroplants of Foz do Areia and Salto Santiago was considered 1 week. In this case, the release from Foz do Areia will reach Salto Santiago during the following week.

The operational objective of the system is to provide the electric energy demanded at minimum cost. Hence, the problem discussed here is one of the process optimization (see section 2.1). Estimate of the energy demand is assumed to have a sufficient order of accuracy to be considered deterministically as compared with the other sources of uncertainty in the problem.

Table 5. Statistical parameters of the inflows.

(m^3/s)

Site	Weeks:	1st	6th	11th	16th	21th	26th	31th	36th	41th	46th	50th
PR	aver.	131.4	102.7	87.8	129.4	191.7	215.4	214.4	281.7	307.8	166.0	147.9
	st.dv.	86.0	65.4	58.0	132.0	199.8	131.4	116.5	167.9	194.4	92.1	112.4
	skew	1.34	1.62	3.23	1.99	3.23	0.69	1.07	0.52	1.04	0.96	1.31
	corr.	0.94	0.98	0.91	0.96	0.97	0.98	0.97	0.99	0.99	0.97	0.99
IT	aver.	53.1	39.9	38.9	53.2	76.8	81.2	92.4	114.6	116.0	65.1	57.7
	st.dv.	34.4	25.3	29.6	49.4	72.8	52.2	51.1	64.7	63.5	34.5	41.0
	skew	1.28	1.45	2.51	1.63	2.28	0.72	0.86	0.40	0.97	0.84	1.01
	corr.	0.93	0.98	0.94	0.96	0.98	0.98	0.96	0.98	0.99	0.97	0.99
FA	aver.	433.9	512.4	547.0	400.5	453.6	535.2	489.2	547.0	712.6	586.0	489.8
	st.dv.	201.6	254.5	326.4	236.1	316.7	406.1	433.5	485.0	396.7	322.7	300.4
	skew	1.25	0.97	1.81	1.84	1.11	1.49	2.84	3.35	1.03	0.72	2.18
	corr.	0.92	0.99	0.98	0.94	0.97	0.98	0.97	0.99	0.98	0.97	0.99
SS	aver.	298.6	344.7	294.6	269.4	330.4	377.8	331.0	358.4	561.8	424.3	329.3
	st.dv.	198.8	292.7	169.1	179.5	310.2	344.3	308.2	303.8	539.1	282.5	250.9
	skew	1.56	2.99	0.80	1.80	2.58	2.15	2.22	2.21	4.22	1.29	2.76
	corr.	0.95	0.99	0.96	0.91	0.97	0.98	0.95	0.98	0.99	0.96	0.99
SO	aver.	43.3	59.0	39.9	35.5	41.5	49.6	46.0	62.5	77.7	55.5	44.2
	st.dv.	37.2	90.8	24.6	24.6	34.4	45.8	40.6	104.8	73.8	34.5	33.8
	skew	3.39	5.24	0.77	1.73	1.40	2.26	1.54	5.26	3.92	1.03	2.78
	corr.	0.96	0.99	0.94	0.94	0.97	0.98	0.93	0.99	0.96	0.96	0.95
PF	aver.	35.3	28.6	26.3	31.7	47.2	54.1	57.7	73.7	78.3	52.3	40.9
	st.dv.	23.4	16.8	21.5	25.1	32.7	34.0	29.1	51.0	49.2	36.3	31.0
	skew	1.92	1.27	1.85	1.34	1.28	1.65	0.78	1.36	0.98	1.42	1.40
	corr.	0.92	0.97	0.97	0.98	0.96	0.98	0.95	0.99	0.98	0.97	0.99
CC	aver.	20.1	24.4	22.3	16.5	14.9	13.9	12.3	14.0	17.9	18.2	18.9
	st.dv.	4.3	8.0	7.6	5.2	5.5	5.2	5.7	7.0	6.9	8.0	5.8
	skew	0.20	1.48	1.63	0.91	1.21	1.04	3.09	2.70	1.37	1.85	0.58
	corr.	0.91	0.99	0.98	0.95	0.93	0.96	0.98	0.98	0.98	0.98	0.98

Table 6. Distances and travel times between serially-linked plants.

Plant sites upstream - downstream	distance km	travel time weeks
P. Real - Jacuí	6	0
Jacuí - Itaúba	35	0
F. Areia - S. Santiago	185	1
S. Santiago - S. Osorio	72	0

The elementary time interval of operation is considered to be one week. The following reasons support this choice. A week is long enough to permit adequate forethought so that effective decisions can be made. It is short enough to allow accurate forecasts of the water entering in the system during the interval. Finally it has the amplitude of one of the fundamental cycles of the demand variation.

The average cost of thermoenergy production or transfers is presented in Table 3. These values were computed based on information from ELETROSUL (1979) or were assumed considering information about the economic efficiency of operations in each unit presented in GCOI-Sul (1978, Annex 11).

The water resources developments are single purpose developments. Their objective is solely the production of electric energy. The variable costs of their operation are considered null. Minimum release values allow for other possible uses of water. However, these are assumed to be secondary objectives met on the basis of water availability, except in the hydropower development of Capivari-Cachoeira.

The operating costs of the system are derived from the operating costs of its thermoplants. Energy shortages may eventually occur in the operation. Nevertheless, their magnitude is not considered a disaster-level occurrence. Therefore, the shortage is approached through a cost function according the practice adopted in the operation of the actual system (GCOI-Sul, 1978). This cost function reflects the cost of the mitigating measures used in case of shortage occurrences. It may include the purchase of energy from other regions, the adoption of energy conservation programs, etc. Eventually, the cost of the shortage may reflect the actual damages to the users. The report GCOI-Sul (1978, p. 17) does not mention the basis upon what such a cost function has been defined.

Figure 2 presents a typical function for the marginal cost of the thermoenergy generation plus shortages. The function was defined by ordering the thermoplants according to their average cost of generation. The marginal cost of shortages is attached to the portion of the energy demand which eventually will not be provided. Each bar in the histogram presented in Figure 2 represents a thermoplant. The abscissa shows the variable energy generation and the ordinate stands for the average cost of generation of each thermoplant. The maximum variable energy generation is the maximum value of energy generation less the minimum fixed energy generation of each unit (see Table 3). As the maximum and minimum generation vary along the year, a cost function is defined for each week.

The marginal cost of the shortages was defined based on the data presented in the report of GCOI-Sul (1978, p. 17). After a sudden linear increase in the marginal cost of the most expensive thermoplant

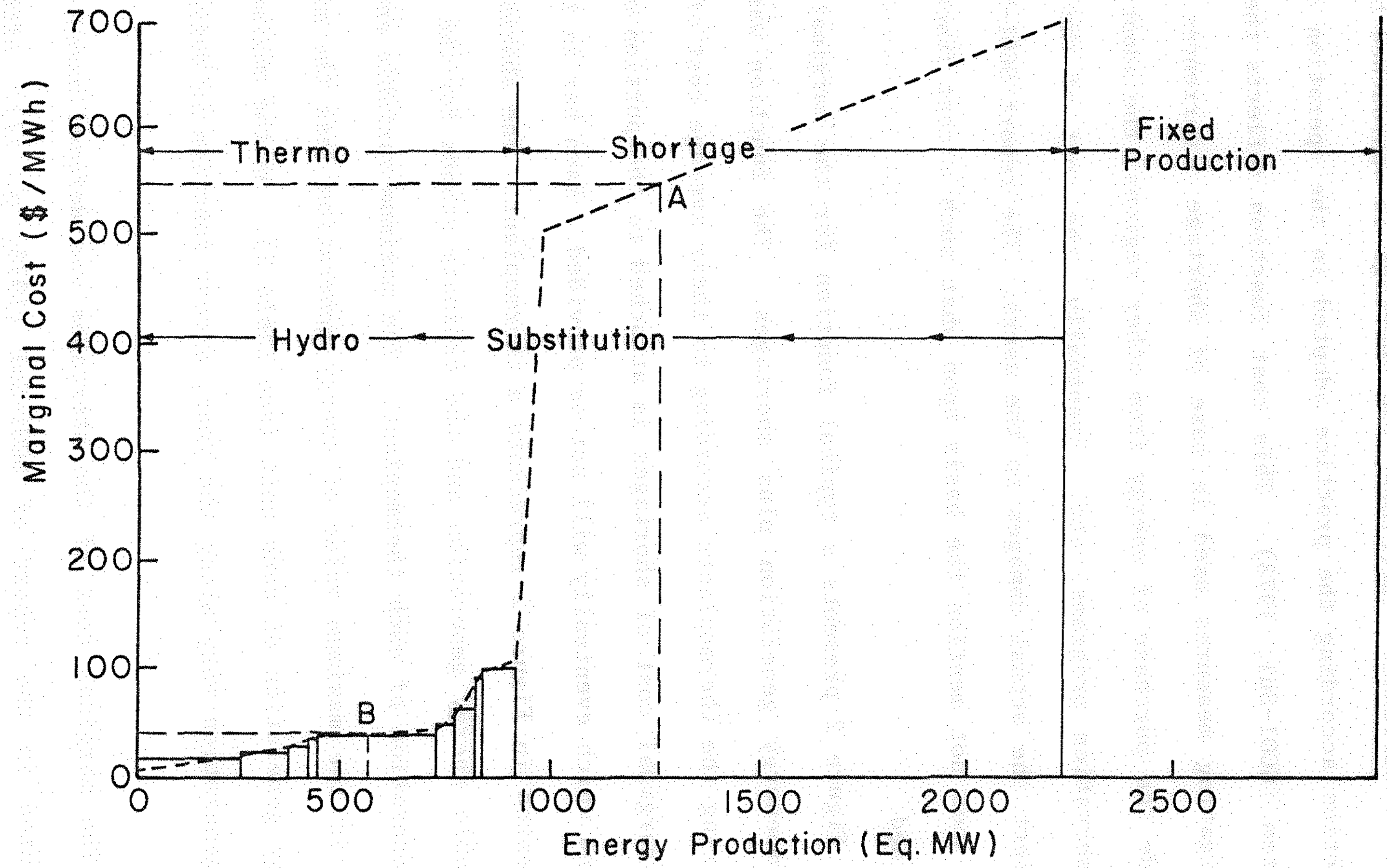


Figure 2. Marginal Cost of Energy Production

to \$500 per MWh, the marginal cost of shortages varies linearly up to the value \$700 per MWh. A piecewise linear function was also used to represent the marginal cost of the thermoenergy generation. It was adjusted following the trend represented by the histogram of the average costs of thermogeneration (Figure 2).

Figure 2 also depicts the way the system is operated. The energy demand, which has a total value of 3000 Eq. MW in the figure, will initially be discounted by the fixed energy production. The fixed energy production is defined by the minimum thermal energy generation plus the firm energy generation of the small hydroplants in the system (see Table 4). The net energy demand, about 2250 Eq. MW, will be supplied by thermo and hydroenergy. Initially, the hydroenergy will be used to avoid energy shortages. When no shortages occur, hydroenergy generation will substitute the most expensive thermal units. For instance, assume that hydroenergy is used to meet the demand except for a remaining demand of 1250 Eq. MW. This situation is shown by the point A in Figure 2. The remaining demand is represented by the value of the abscissa of A. It cannot be supplied entirely by thermoplants. Consequently, the thermoplants are generating at their maximum capacities. Moreover, a shortage, with a marginal cost of about \$540 per MWh (ordinate of A), will occur. Another example is shown by the point B in Figure 2. In this case the hydroenergy is used to meet the demand except for a remaining demand of 600 Eq. MW. This remaining demand can be entirely provided by the thermoplants. In this case, the four cheapest thermoplants are generating at their full capacities. The fifth thermoplant is loaded below its maximum capacity. The resultant marginal cost of operation is about \$40 per MWh. In both cases the cost of operation is given by the area below the marginal cost curve up to point A or B, respectively.

It has to be mentioned that despite the assumptions made in the definition of the marginal cost function, it has a very general shape. The related cost function obtained by integration would have a convex piecewise quadratic form which is well suited to almost any convex function. The marginal cost function is a non-decreasing function of the thermoenergy production. Therefore, the law of non-increasing marginal values applies to benefits from the substitution of thermoenergy by hydroenergy. It states that the most expensive sources of energy will be substituted first in the optimum operation.

3.2 Mathematical Modeling

The objective of the system's operation is the minimization of the expected operating cost to provide the energy demand in 1983.

The mathematical formulation of this objective is

$$\underset{\substack{\dot{x}(t),\\ t=1,52}}{\text{Minimize}} \quad E\left\{\sum_{t=1}^{52} F_t[\dot{x}(t)]\right\} = E\left\{\sum_{t=1}^{52} \int_{x1}^{x2} f\,[t,\ D(t) - x(t)]\ dx\right\} \qquad (3.1)$$

where $f[\cdot]$ is the marginal cost of operation of the system when hydroenergy generation is $x(t) = \dot{x}(t)$ (see Figure 2). The term $D(t) - x(t)$ represents the net energy demand less the hydrogeneration during the t-th time interval; hence, it refers to the thermoenergy generation and eventual shortage. The integral computes the operating cost of the system if the hydroenergy generation is $\dot{x}(t)$. Its lower limit is given by the point where no thermoenergy is used or the hydro generation equals the net demand $[x(t) = x1 = D(t)]$. The upper limit is defined by the point where the hydroenergy generation is $x(t) = x2 = \dot{x}(t)$.

The symbols are defined as:

$E\{\cdot\}$ is the mathematical expectation operator

t is the index of the t-th time interval of operation (week)

D(t) is the net electric energy demand during the t-th time interval, given by the total energy demand less the fixed energy production (Eq. MW)

x(t) is the hydroenergy generation (Eq. MW) during the t-th time interval, which is given by

$$x(t) = \sum_{j} x(j,t) \qquad (3.2)$$

where x(j,t) is the energy generation at the hydroplant of index j during the t-th time interval

The computation of the energy production in each hydroplant is made by

$$x(j,t) = alpha(j,t) * w(j,t) \qquad (3.3)$$

where w(j,t) is the water released through the turbines of the hydroplant j during the t-th time interval (hm^3)

alpha(j,t) is the energy conversion factor given by

$$alpha(j,t) = 9.81 * Eff(j) *\{h[j,\bar{s}(j,t)] - Hloss(j)\}/Transf \qquad (3.4)$$

where 9.81 is the gravitational acceleration (m/sec^2)

Eff(j) is the efficiency for the hydroplant of index j

$h[j,\bar{s}(j,t)]$ is the average hydraulic head at the hydroplant of index j during the t-th time interval (m)

$\bar{s}(j,t)$ is the average water storage in the reservoir of index j during the t-th time interval (hm^3)

Hloss(j) is the hydraulic losses in the hydroplant of index j (m)

Transf is the constant factor of transformation to Equivalent Megawatts

The optimization is subject to the following constraints:

1. Water Balance Equalities

$$s(j,t+1) = s(j,t) + q(j,t) + qu(j,t) - w(j,t) - spill(j,t) \quad (3.5)$$

for all j and t = 1,52

where $s(j,t+1)$ is the water content in the reservoir of index j at the beginning of the (t+1)-th time interval (hm^3)

$s(j,t)$ is the water content in the reservoir of index j at the beginning of the t-th time interval (hm^3)

$q(j,t)$ is the natural water inflow to reservoir j during time interval t (hm^3)

$qu(j,t)$ is the inflow to reservoir j during time interval t caused by previous releases from the upstream reservoir (hm^3)

$spill(j,t)$ is the water volume spilled from reservoir j during time interval t (hm^3)

2. Maximum Reservoir Content Inequalities

$$s(j,t) \leq Smax(j), \text{ for all j and } t = 1,52 \quad (3.6)$$

3. Minimum Reservoir Content Inequalities

$$s(j,t) \geq Smin(j), \text{ for all j and } t = 1,52 \quad (3.7)$$

4. Maximum Release Inequalities

$$w(j,t) \leq Wmax(j,t,s(j,t)), \text{ for all j and } t = 1,52 \quad (3.8)$$

5. Minimum Release Commitment

$$w(j,t) \geq Wmin(j), \text{ for all j and } t = 1,52 \quad (3.9)$$

where $Smax(j)$ is the maximum water content in reservoir j (hm^3)

$Smin(j)$ is the minimum water content in reservoir j (hm^3)

$Wmax(\cdot)$ is the maximum feasible release through the turbines of reservoir j during time interval t, of which the derivation is presented in section 3.3.3 (hm^3)

$Wmin(j)$ is the minimum water volume to be released from reservoir j during the t-th time interval when enough water is available. For the hydropower development of Capivari-Cachoeira the enforcement of such an inequality is always required (hm^3).

3.2.1 Hydraulic Head Function

The hydraulic head is given by the difference between the water elevation upstream of the plant and the water elevation in the tail-water. The upstream water elevation is given for the hydroplants with reservoirs by a polynomial function with the value of the water content in the reservoir. For run-of-river plants the hydraulic head is fixed.

The tailwater is assumed to have a constant elevation calculated as the average value of the variable elevation in each reservoir.

3.2.2 Maximum Release Computation

Two upper limits exist for the instantaneous release through a turbine-generator system; one is given by the turbine limitation, the other is given by the generator capacity (Pereira, 1976). For impulse and reaction turbines the limitation (xtur) is proportional to the square root of the quotient of the actual (h) and the rated head (hrated). The scale factor is the rated flow for the turbine (xrated)

$$xtur = (h/hrated)^{\frac{1}{2}} * xrated \tag{3.10}$$

For propeller turbines the limitation is the rated flow itself

$$xtur = xrated \tag{3.11}$$

The instantaneous flow limitation for generators (xgen) is a function of the installed capacity (cap)

$$xgen = cap/alpha \tag{3.12}$$

where alpha is the energy conversion factor defined in Equation (3.4).

The value of the maximum instantaneous release is given by the minimum of the two limitations

$$ximax = \min(xtur, xgen) \tag{3.13}$$

The maximum instantaneous release cannot be operated continuously in a power plant (Pereira, 1976). Problems related with heating of the generator and equipment outages decrease the maximum continuous release while in operation. The equipment outages are generally calculated through a percentage outage factor representing the time percentage of outages (off). The heating limitation is calculated through a maximum capacity factor (lfmax) i.e., a maximum quotient between the maximum capacity of the generator and the maximum feasible continuous load while in operation.

If the plant were in continuous operation with the maximum instantaneous release ximax, the maximum attainable capacity factor (cf) would be

$$cf = xgen/ximax \tag{3.14}$$

If the generator is limiting the maximum instantaneous water release or ximax = xgen, cf will equal 1 and heating problems will occur as long lfmax < 1. Therefore the maximum continuous release will be such that cf = lfmax and considering the outage factor

$$xmax = (1 - off) * ximax/lfmax \tag{3.15}$$

If the turbine limits the maximum instantaneous release, then cf < 1. If cf is also less than lfmax no heating problems will occur and

$$xmax = (1 - off) * ximax \tag{3.16}$$

If, however, lfmax < cf < 1 the maximum attainable capacity factor is still greater than the maximum feasible factor. The generator, however, will never be at its full capacity and a practical approximation for xmax is given by

$$xmax = (1 - off) * ximax/(lfmax/cf) \tag{3.17}$$

3.2.3 Run-of-river Plants

The existence of two run-of-river plants downstream of a reservoir plant creates a singular situation. Since they are run-of-river plants no reservoir controls their releases. However, part of the water which flows to the run-of-river plants has originated from releases made at an upstream reservoir plant. Hence, the release of water by the reservoir plant must consider what the consequences will be when this release reaches the downstream run-of-river plants.

Travel time from the reservoir plant to the farthest downstream run-of-river plant is less than one week. This allows the use of an aggregated conception for approaching the operation of these plants. The water release from the reservoir should be computed as being operative immediately in all downstream plants. The maximum (continuous) release should be computed in order to avoid spill in any plant. However, if spill occurs at the reservoir plant the water passing into the river will be utilized as much as possible by the run-of-river plants.

Chapter 4

OPTIMALITY CONDITIONS FOR DETERMINISTIC OPERATION

In this chapter the optimal operation of a multi-unit hydroelectric energy generating system is derived under deterministic future. An inductive approach is used; simpler situations are analyzed before the more complex ones. The assumption of a deterministic future is by itself a simplification of the real situation where the future is uncertain. Application of the results to the real situation of an uncertain future will be discussed later.

There are reasons for the study of the optimal operation conditions under deterministic future. One is the availability of theorems from Mathematical Programming for derivation of the optimality conditions. Hopefully, from the optimality conditions under deterministic future the optimality conditions for the operation when the future is uncertain may be extrapolated. Moreover, from the behavior of the system in its optimal deterministic operation some peculiarities that may be noted could be used in the study of the operation when the future is uncertain.

Initially, the operation of a single reservoir hydroplant with constant hydraulic head will be studied. Next, systems with multiple reservoir hydroplants will have their optimal operation studied. The assumption of invariant hydraulic head will then be relaxed.

4.1 Single Reservoir with Constant Hydraulic Head

The mathematical formulation is

$$\underset{\substack{\dot{x}(t) \\ t = 1,52}}{\text{Minimize}} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} = \left\{ \sum_{t=1}^{52} \int_{x1}^{x2} f[t, D(t) - x(t)]\, dx \right\} \qquad (4.1)$$

This integral defines the total cost of operation during each time interval t. The function f(·) represents the marginal cost of operation. The expression D(t) - x(t) computes the value of the energy demand less the hydroenergy generated during the t-th time interval. Its value defines the total thermoenergy generation plus eventual shortages during the time interval t. The limits of integration are given by

$$\left.\begin{aligned} x(t) &= x1 = D(t) \\ x(t) &= x2 = \dot{x}(t) \end{aligned}\right\} \qquad (4.2)$$

The limit x1 represents the marginal cost curve at the origin (see Figure 2) when no thermogeneration is used and the demand is supplied by hydrogeneration alone. The limit x2 represents the marginal cost curve when the hydrogeneration is $\dot{x}(t)$.

The hydroenergy generation is given by

$$x(t) = alpha * w(t) \qquad (4.3)$$

where w(t) is the water release through the turbines of the hydroplant during the time interval t and

alpha is the energy conversion factor, function of the fixed hydraulic head, losses and efficiency of the hydroplant's operation.

The constraints are:

1. Water Balance Equalities

$$s(t+1) = s(t) + q(t) - w(t) - spill(t), \quad t = 1,52 \qquad (4.4)$$

where s(t+1) is the water content in the reservoir at the beginning of the (t+1)-th time interval (hm^3)

s(t) is the water content in the reservoir at the beginning of the t-th time interval (hm^3)

q(t) is the inflow to the reservoir during the t-th time interval (hm^3)

spill(t) is the water spilled from the reservoir during the t-th time interval (hm^3)

2. Maximum Reservoir Content Inequalities

$$s(t) \leq Smax, \quad t = 1,52 \qquad (4.5)$$

where Smax is the maximum water content capacity of the reservoir (hm^3)

3. Minimum Reservoir Content Inequalities

$$s(t) \geq Smin, \quad t = 1,52 \qquad (4.6)$$

where Smin is the minimum water content in the reservoir (hm^3)

4. Maximum Release Inequalities

$$w(t) \leq Wmax, \quad t = 1,52 \qquad (4.7)$$

where Wmax is the maximum release capacity of the reservoir during a time interval (hm^3). Its computation is performed by the procedure presented in section 3.2.2

5. Minimum Release Inequalities

$$w(t) \geq Wmin, \quad t = 1,52 \qquad (4.8)$$

where Wmin is the minimum required release from the reservoir during a time interval (hm^3)

The problem is deterministic in nature and thus an initial value for the reservoir content has to be defined. This initial value will be S1. The final reservoir content will be S53, defined as a strategic goal.

The water balance equation may be written as

$$s(t+1) = S1 + \sum_{k=1}^{t} [q(k) - w(k) - spill\ (k)], \quad t = 1,51 \qquad (4.9)$$

$$s(53) = S53 \qquad (4.10)$$

Substituting the above equations in the constraints

$$\sum_{k=1}^{t} [q(k) - w(k) - spill(k)] + S1 - Smax \leq 0, \quad t = 1,51 \qquad (4.11)$$

$$-\sum_{k=1}^{t} [q(k) - w(k) - spill(k)] - S1 + Smin \leq 0, \quad t = 1,51 \qquad (4.12)$$

$$w(t) - Wmax \leq 0, \quad t = 1,52 \qquad (4.13)$$

$$- w(t) + Wmin \leq 0, \quad t = 1,52 \qquad (4.14)$$

The final reservoir content value defines what may be called Gross Water Balance Equality given by

$$\sum_{t=1}^{52} [q(t) - w(t) - spill(t)] + S1 - S53 = 0 \qquad (4.15)$$

4.1.1 Application of the Kuhn-Tucker Theorem

The total cost of operation is a convex function of the hydroenergy generation during a time interval and this is a linear function of the water release in the same time interval when the hydraulic head is constant (Equation 4.3). Hence the objective function is a convex non-linear function of the releases. The constraints are all linear, consequently the problem is a non-linear convex optimization problem. The necessary conditions for the optimum operation may be derived by application of the Kuhn-Tucker Theorem of Mathematical Programming.

Following the development presented by Morel-Seytoux (1972, lectures 19 and 20, p. 2) it comes

A. Lagrangian

$$L[w(t),spill(t),\alpha(t),\beta(t),\gamma(t),\lambda(t),\theta] =$$

$$= \sum_{t=1}^{52} F_t[\dot{x}(t)] + \sum_{t=1}^{51} \left\{ \alpha(t) * \left[\sum_{k=1}^{t} [q(k) - w(k) - spill(k)] + S1 - Smax \right] \right\} +$$

$$\sum_{t=1}^{51} \left\{ \beta(t) * \left[-\sum_{k=1}^{t} [q(k) - w(k) - spill(k)] - S1 + Smin \right] \right\} +$$

$$\sum_{t=1}^{52} \left\{ \gamma(t) * [w(t) - Wmax] \right\} + \sum_{t=1}^{52} \left\{ \lambda(t) * [-w(t) + Wmin] \right\} +$$

$$\theta * \left\{ \sum_{t=1}^{52} [q(t) - w(t) - spill(t)] + S1 - S53 \right\} \ldots . \qquad (4.16)$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $\lambda(t)$ and θ are the Lagrange multipliers related to the constraints (4.11) to (4.15), respectively.

B. Optimality Condition Equations

By derivation with respect to each variable the optimality conditions are obtained as

1. Either

$$\frac{\partial}{\partial w(k)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} - \sum_{t=1}^{k} \alpha(t) + \sum_{t=1}^{k} \beta(t) + \gamma(k) - \lambda(k) - \theta \leq 0$$

$$\text{if} \quad w(k) = 0 \tag{4.17}$$

or

$$\frac{\partial}{\partial w(k)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} - \sum_{t=1}^{k} \alpha(t) + \sum_{t=1}^{k} \beta(t) + \gamma(k) - \lambda(k) - \theta = 0$$

$$\text{if} \quad w(k) \geq 0 \tag{4.18}$$

for $k = 1,51$

2. Either

$$- \sum_{t=1}^{k} \alpha(t) + \sum_{t=1}^{k} \beta(t) - \theta \leq 0 \quad \text{if spill}(k) = 0 \tag{4.19}$$

or

$$- \sum_{t=1}^{k} \alpha(t) + \sum_{t=1}^{k} \beta(t) - \theta = 0 \quad \text{if spill}(k) \geq 0 \tag{4.20}$$

for $k = 1,51$

3. Either

$$\sum_{t=1}^{k} [q(t) - w(t) - \text{spill}(t)] + S1 - Smax \leq 0$$

$$\text{if} \quad \alpha(k) = 0 \tag{4.21}$$

or

$$\sum_{t=1}^{k} [q(t) - w(t) - \text{spill}(t)] + S1 - Smax = 0$$

$$\text{if} \quad \alpha(k) \geq 0 \tag{4.22}$$

for $k = 1,51$

4. Either

$$-\sum_{t=1}^{k} [q(t) - w(t) - spill(t)] - S1 + Smin \leq 0$$

$$\text{if} \quad \beta(k) = 0 \qquad (4.23)$$

or

$$-\sum_{t=1}^{k} [q(t) - w(t) - spill(t)] - S1 + Smin = 0$$

$$\text{if} \quad \beta(k) \geq 0 \qquad (4.24)$$

for $k = 1,51$

5. Either $w(t) - Wmax \leq 0$ if $\gamma(t) = 0$ (4.25)

or $w(t) - Wmax = 0$ if $\gamma(t) \geq 0$ (4.26)

for $k = 1,52$

6. Either $- w(t) + Wmin \leq 0$ if $\lambda(t) = 0$ (4.27)

or $- w(t) + Wmin = 0$ if $\lambda(t) \geq 0$ (4.28)

for $k = 1,52$

7.
$$\sum_{t=1}^{52} [q(t) - w(t) - spill(t)] + S1 - S53 = 0 \qquad (4.29)$$

8. All variables but θ are non negative.

The equation (4.18) may be presented as

$$\frac{\partial}{\partial w(k)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} = \sum_{t=1}^{k} \alpha(t) - \sum_{t=1}^{k} \beta(t) - \gamma(k) + \lambda(k) + \theta \qquad (4.30)$$

Developing the derivative of the left-hand side

$$\frac{\partial}{\partial w(k)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} = \frac{\partial}{\partial w(k)} \left\{ \sum_{k=1}^{52} \int_{x1}^{x2} f[t, D(t) - x(t)] \, dx \right\} =$$

$$= \frac{\partial}{\partial w(k)} \left\{ \sum_{k=1}^{52} \int_{x1}^{x2} f[t, D(t) - alpha * w(t)] * alpha * dw \right\} =$$

$$= - alpha * f[k, D(k) - alpha * w(k)]$$

Therefore

$$- \text{alpha} * f[k, D(k) - \text{alpha} * w(k)] =$$

$$= \sum_{t=1}^{k} \alpha(t) - \sum_{t=1}^{k} \beta(t) - \gamma(k) + \lambda(k) + \theta \qquad (4.31)$$

This condition states that in the optimum the marginal instantaneous benefit of the release is equal to the summation of the past and instantaneous (t = k) values of the Lagrangian multipliers associated with the maximum reservoir content constraint

1. less the summation of the past and instantaneous values of the Lagrangian multipliers associated with the minimum reservoir content constraint

2. less the instantaneous value of the Lagrangian multiplier associated with the maximum release constraint

3. plus the instantaneous value of the Lagrangian multiplier associated with the minimum release constraint

4. plus the value of the Lagrangian multiplier associated with the gross water balance constraint.

Suppose the optimal operation in a given period has the following characteristics

1. the reservoir will never be maximum or minimum

2. the releases will never be maximum or minimum.

If these conditions occur Equation 4.21, 4.23, 4.25 and 4.27 state that the Lagrange multipliers $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\lambda(t)$ are null for any time in the given period. Eliminating from Equation 4.31 the null variables, the optimality condition will be

$$- \text{alpha} * f[t, D(t) - \text{alpha} * w(t)] = \Omega \qquad (4.32)$$

where Ω stands for the summation of the Lagrange multipliers before the analyzed period.

Assume the reservoir reaches maximum storage at the beginning of time interval k within the given period. Equation 4.22 states that $\alpha(k) \geq 0$. The optimality condition will be

for $t < k$: - alpha * f[t, D(t) - alpha * w(t)] = Ω (4.33)

for $t \geq k$: - alpha * f[k, D(k) - alpha * w(k)] = $\alpha(k) + \Omega$ (4.34)

Therefore the marginal instantaneous benefit of the releases will still be constant before and after the maximum reservoir content occurs. After this instant, though, it will increase by $\alpha(k)$.

If the reservoir reaches the minimum content at the beginning of the time interval k, rather than the maximum the marginal instantaneous benefit of the releases will decrease by $\beta(k)$ after k (see Equation 4.24).

Suppose now the release is maximum during the time interval k. Equation 4.26 states that $\gamma(k) \geq 0$. Equation 4.33 will continue to state the optimal condition of operation except during the interval k where the optimal operation is given by

- alpha * f[k, D(k) - alpha * w(k)] = $-\gamma(k) + \Omega$ (4.35)

or the marginal instantaneous benefit of the release will decrease by $\gamma(k)$ during the time interval k. Otherwise if the release is minimum during interval k the marginal instantaneous benefit of the release will then increase by $\lambda(k)$ (see Equation 4.28).

The conclusion is that any time one of the constraints is tightened (or active) the related Lagrange multiplier will assume a non-null value. This value will be present on the right-hand side of the optimality condition given by Equation 4.31 for any instant thereafter if the constraint refers to an extreme value of the reservoir content (maximum or minimum).

The interpretation of the optimality condition formulated by the Equation 4.31 did not take into consideration the marginal economic meaning of the Lagrange multipliers. This marginal meaning states that in the optimum solution the Lagrange multiplier associated with a constraint represents the rate improvement in the optimal value of the objective function when the constraint is infinitesimally relaxed. An analysis of each Lagrange multiplier under this scope shows

1. $\alpha(t)$ refers to the maximum reservoir content constraint. The relaxation of this constraint means Smax is being increased. When this happens the rate of improvement in the value of the optimal objective function will be equal to $\alpha(t)$ for an infinitesimal increase in Smax. This improvement is derived from the possibility of additional water storage at the time of relaxation which will be released later.

2. $\beta(t)$ refers to the minimum reservoir content constraint. Relaxation of this constraint means Smin will decrease. The rate of improvement of the objective function for an infinitesimal relaxation will be equal to $\beta(t)$. This is derived from the possibility of releasing more water before the relaxation.

3. $\gamma(t)$ refers to the maximum release constraint. A rate of improvement in the optimal value of the objective function equal to $\gamma(t)$ will be derived from an infinitesimal relaxation of the referred constraint during the time interval t. This relaxation acts to increase Wmax. The improvement will be derived from the possibility of the release of more water during the time interval t.

4. $\lambda(t)$ refers to the minimum release constraint. A rate of improvement equal to $\lambda(t)$ in the optimal value of the objective function will be caused by an infinitesimal relaxation of the beforementioned constraint during time interval t. Relaxation will now act to decrease Wmin. The improvement here is caused by the possibility of the release of less water during time interval t.

5. θ refers to the gross hydrologic balance equality. One of the possible interpretations of θ is that it represents the rate of improvement of the optimal value of the objective function derived by an infinitesimal decrease in the final water content in the reservoir. The improvement will be caused by the possibility of an increase in the releases in the operation.

The economic interpretation of the Lagrange multipliers allows them to be classified into two groups. The first group includes the Lagrange multipliers $\alpha(t)$, $\beta(t)$ and θ. It refers to the rate of variation of the optimal value of the objective function which results from the infinitesimal variation of storage somewhere in the optimal operation. Therefore, these Lagrange multipliers refer to the marginal benefit of storage in the optimal operation. The second group includes the Lagrange multipliers $\gamma(t)$ and $\lambda(t)$. These refer to the rate of variation of the optimal value of the objective function which results from the infinitesimal variation of the releases somewhere in the optimal operation. Hence, these Lagrange multipliers refer to the marginal benefit of the releases during a given time interval or to the marginal instantaneous benefit of the releases in the optimal operation.

In section 2.3 it was shown that the optimal decision in the operation of reserves occurs when the marginal instantaneous benefit of the releases equals the marginal future benefit of the operation of the remaining storage. Equation 4.31 is the mathematical formulation of such a condition.

The terms which refer to the instantaneous benefit of the releases in Equation 4.31 are grouped at the left-hand side

$$- alpha * f[k, D(k) - alpha * w(k)] + \gamma(k) - \lambda(k) =$$

$$= \sum_{t=1}^{k} \alpha(t) - \sum_{t=1}^{k} \beta(k) - \theta \qquad (4.36)$$

Suppose during a given time interval k the optimal release is not limited by the maximum or the minimum release constraint. In this case, the Lagrange multipliers $\gamma(k)$ and $\lambda(k)$ are null. The left hand side of Equation 4.36 will be formed by the marginal instantaneous benefit of the release. The right hand side will represent the marginal future benefit of the remaining storage.

If the optimal release is limited by the maximum or minimum release constraint during the k-th time interval, $\gamma(k)$ or $\lambda(k)$ will be non-null. Nevertheless the marginal future benefit of the remaining storage does not change during the time interval k. The Lagrange multipliers $\gamma(k)$ and $\lambda(k)$ may be interpreted as a correction on the value of the marginal instantaneous benefit of the releases when the release is tightened by the maximum or minimum release constraint. The actual value of these Lagrange multipliers in the optimal operation have small significance. As long as they are null the optimal release is unconstrained by its maximum or minimum limitations. The corresponding value of the release will be defined for each time interval when the marginal instantaneous benefit of the release equals the marginal future benefit of the remaining storage. Otherwise, when $\gamma(k)$ or $\lambda(k)$ are non-null the solution is trivial; the optimal release during the time interval k will be maximum or minimum, respectively (Equations 4.26 and 4.28).

To summarize, the optimal operation of a single reservoir in a system with the characteristics of the case study has the following conditions:

Optimality Condition 1. In the optimal operation of a single reservoir the marginal instantaneous benefit of the releases is equal to the marginal future benefit of the remaining storage.

When the optimum release is constrained by the maximum and minimum releases a trivial solution occurs (Equations 4.26 and 4.28).

The values of the Lagrange multipliers $\alpha(t)$ and $\beta(t)$ will be always null unless the maximum or minimum storage constraints tighten the optimal solution during the time interval t (Equations 4.21 and 4.23). Or:

Optimality Condition 2. In the optimal operation of a single reservoir the marginal future benefit of its storage is constant as long as the optimal operation is not tightened by the extreme (maximum or minimum) storage constraints.

When the maximum storage constraint tightens the optimal operation at the beginning of a time interval the respective Lagrange multiplier $\alpha(t)$ is non-null (Equation 4.22). Or:

Optimality Condition 3. In the optimal operation of a single reservoir the marginal future benefit of its storage increases only when the maximum storage constraint is tightening the optimal operation.

When the minimum storage constraint tightens the optimal operation at the beginning of a time interval the correspondent Lagrange multiplier $\beta(t)$ is non-null (Equation 4.24). Or:

Optimality Condition 4. In the optimal operation of a single reservoir the marginal future benefit of its storage decreases only when the minimum storage constraint is tightening the optimal operation.

The optimal occurrence of spills is stated by Equations 4.19 and 4.20. The derivative of the objective function related to the spill is obviously null. A spill occurrence represents water being lost in the operation. Any release of this water through the turbines will improve the results of the operation. Consequently, spills may occur in the optimal operation only after the release reaches its maximum value. This maximum value is given by the minimum between the maximum release capacity (Wmax) and the water release which will generate the net energy demand during the time interval [D(t)].

Equations 4.19 and 4.20 state that the marginal future benefit of the remaining storage is non-negative. It will be null when a spill occurs during the time interval. But optimality condition 2 states that the marginal future benefit of the remaining storage will be constant as long as no maximum or minimum storage occurs. Therefore, when spill occurs in the optimal operation it will occur inside a sub-period of operation with null marginal future benefit of the remaining storage. This sub-period will start and end with maximum or minimum storage.

Assume a sub-period starts with the maximum storage. Optimality condition 3 states that the marginal future benefit of the remaining storage will increase. As the marginal future benefit of the remaining storage is null during the sub-period, it must have previously been negative. It violates the non-negativity condition for the marginal future benefit of the remaining storage stated by Equations 4.19 and 4.20. If the sub-period starts with the reservoir empty the marginal future benefit of the remaining storage will then decrease (optimality condition 4) and no violation will occur.

Assume now a sub-period with spilling occurrence ends with the reservoir empty. Optimality condition 4 states that the marginal future benefit of the remaining storage will decrease. If the marginal future benefit of the remaining storage is null within the sub-period, it should become negative after it; this cannot occur. If however the reservoir is full at the end of the sub-period the marginal future benefit of the remaining storage will increase and no violation will occur.

Concluding, the sub-period with spilling occurrences must start with minimum reservoir storage and must end with maximum reservoir storage.

Optimality condition 1 states that optimal releases will be made during this sub-period with the marginal instantaneous benefit equal to the marginal future benefit of the remaining storage, which is null. The marginal instantaneous benefit of the release is null when the net energy demand is being provided entirely by hydroenergy generation (section 2.1 and Figure 2). Eventually, the release will be tightened by the maximum release constraint. This situation results in the trivial solution which has a maximum release. Therefore, the optimal operation within the sub-period with spilling occurrences is defined by what is here called the maximum release tactic. In this tactic, the release is equal to the minimum between the release which will provide the energy demanded and the maximum release capacity.

These considerations define the fifth optimality condition of operation of the system.

Optimality Condition 5. In the optimal operation of a single reservoir spill may occur in a given period of operation only if the marginal future benefit of the storage is null during this period. In such a situation, the reservoir storage will be minimum at the beginning and maximum at the end of the period. The optimal releases in the period are given by the maximum release tactic.

4.1.2 An Alternative Approach

All optimality conditions derived by the application of the Kuhn-Tucker theorem may be derived through an alternative, non-mathematical approach. It is presented in this section since this permits more insight into the nature of the optimal operation of a reservoir.

A reserve is a device which allows the transfer of resources available in the present to the future. In the case of water reserves the water may be stored when it arrives at the reservoir and kept there for future use as long as space is available. Consequently when the reservoir is full no transfer of water for future use can be made. The water must be used as soon as it arrives or it is spilled.

The idea of "transfer to the past" can also be considered in optimization under deterministic future. With deterministic future the optimal sequence of releases is given by a tactic which is derived at the beginning of an operation. Assuming the optimal operation is obtained through iterations a transfer to the past occurs when given a solution for the operation an improved solution may be obtained by "transferring" releases in the future to be done in the past. The transfer to the past has also its limitations. One can not transfer more water before a given time than the total amount of water available at that time. This amount is given by the initial water in reserve plus inflows and less spillings from the beginning of the operation up to the present time. At this limit the reservoir will be at its minimum content.

The concept of water transfer is very useful in the analysis of this problem. In the optimal operation water will always be transferred from time t1 to time t2 when the marginal instantaneous benefit of the release at t2 is greater than at t1. The optimum marginal instantaneous benefit of the releases is constant for all times if no limitation in transfer occurs. This is what optimality conditions 1 and 2 state.

Consider now the case where the reservoir content achieves its maximum during a given time. Suppose also the releases before and after this time are made with constant marginal instantaneous benefits. If the marginal instantaneous benefit of releases decreases after the reservoir has its maximum content no limitation exists in the transfer of water to the past and the solution is not optimal. However, if the marginal instantaneous benefit of the releases increases after the maximum reservoir content no improvement can be obtained since transfer of releases to the future is infeasible; therefore, the operation is optimal. This is the optimality condition 3.

In the case where reservoir content is at a minimum at a given time an improved solution can always be achieved when the marginal instantaneous benefit of releases increases after that time. If the marginal instantaneous benefit of the releases decreases the only way to improve the solution is by transferring water to the past which is then infeasible. This is what the optimality condition 4 states.

In the occurrence of spillings the optimal operation is obviously given by the maximum release tactic since it is more beneficial to release water than to let it be spilled. Spilling means there is a surplus of water during a period which cannot be used, stored or transferred for other periods. This can only occur if the period of surplus starts with a minimum reservoir content and ends with maximum reservoir content, avoiding any transfers. This is the optimality condition 5.

4.2 Multiple Unlinked Reservoirs with Constant Hydraulic Heads

The mathematical formulation of the problem is

$$\underset{\substack{\dot{x}(t) \\ t = 1,52}}{\text{Minimize}} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} = \sum_{t=1}^{52} \left\{ \int_{x1}^{x2} f[t, D(t) - x(t)]\, dx \right\} \qquad (4.37)$$

The integral computes the total cost of operation during the time inverval t given its marginal cost by the function f(·). D(t) is the energy demand during the t-th time interval and x(t) is the total energy generation by the hydroplants in the system during the same time interval. The limits of integration are given by

$$\left. \begin{aligned} x(t) &= x1 = D(t) \\ x(t) &= x2 = \dot{x}(t) \end{aligned} \right\} \qquad (4.38)$$

The limit x1 represents the point where no thermogeneration is used and the energy demand is entirely supplied by the hydroplants. The limit x2 represents the situation where hydroenergy generation is $\dot{x}(t)$.

Hydroenergy generation is given by

$$x(t) = \sum_{j} \text{alpha}(j) * w(j,t) \qquad (4.39)$$

where w(j,t) is the water released through the turbines of hydroplant j during the time interval t (hm^3)

alpha(j) is the energy conversion factor for hydroplant j which is a function of the hydraulic head, hydraulic losses, and the efficiency at hydroplant j.

The constraints are

1. Water Balance Equalities

$$s(j,t+1) = s(j,t) + q(j,t) - w(j,t) - spill(j,t) \qquad (4.40)$$

for t = 1,52 and all j.

where s(j,t+1) is the water content in the reservoir j at the beginning of the time interval t+1 (hm^3)

s(j,t) is the water content in the reservoir j at the beginning of the time interval t (hm^3)

q(j,t) is the water inflow to the reservoir j during the time interval t (hm^3)

w(j,t) is the water release through the turbines of the reservoir j during the time interval t (hm^3)

spill(j,t) is the water spilled from the reservoir j during the time interval t (hm^3).

2. Maximum Reservoir Content Inequalities

$$s(j,t) \leq Smax(j) \quad \text{for} \quad t = 1,52 \text{ and all } j \qquad (4.41)$$

where Smax(j) is the maximum storage capacity of the reservoir j.

3. Minimum Reservoir Content Inequalities

$$s(j,t) \geq Smin(j) \quad \text{for} \quad t = 1,52 \text{ and all } j \qquad (4.42)$$

where Smin(j) is the minimum storage in the reservoir j.

4. Maximum Release Inequalities

$$w(j,t) \leq Wmax(j) \quad \text{for} \quad t = 1, 52 \text{ and all } j \qquad (4.43)$$

where Wmax(j) is the maximum release capacity for the hydroplant j during a time interval. Its computation is made by the procedure described in section 3.2.2.

5. Minimum Release Inequalities

$$w(j,t) \geq Wmin(j) \qquad (4.44)$$

where Wmin(j) is the minimum release in the reservoir j during a time interval.

The maximum release capacity in each reservoir is invariant since the hydraulic head is invariant. The problem is deterministic, therefore, an initial and a final reservoir content must be defined for each reservoir. These are defined as S1(j) and S53(j).

The water balance equalities may be written as

$$s(j,t+1) = S1(j) + \sum_{k=1}^{t} [q(j,k) - w(j,k) - spill(j,k)] \qquad (4.45)$$

for t = 1, 51 and all j

and

$$s(j,53) = S53(j), \text{ for all } j \qquad (4.46)$$

The substitution of the values of s(j,t) from Equation 4.45 in the constraints 2 to 5 defines for any j

$$\sum_{k=1}^{t} [q(j,k) - w(j,k) - spill(j,k)] + S1(j) - Smax(j) \leq 0 \qquad (4.47)$$

for t = 1,51

$$-\sum_{k=1}^{t} [q(j,k) - w(j,k) - spill(j,k)] - S1(j) + Smin(j) \leq 0, \qquad (4.48)$$

for t = 1,51

$$w(j,t) - Wmax(j,t) \leq 0, \quad t = 1,52 \qquad (4.49)$$

$$- w(j,t) - Wmin(j,t) \leq 0, \quad t = 1,52 \qquad (4.50)$$

and the gross water balance equation

$$\sum_{t=1}^{52} [q(j,t) - w(j,t) - spill(j,t)] + S1(j) - S53(j) = 0 \qquad (4.51)$$

The objective function is convex as in the single reservoir case. The application of the Kuhn-Tucker Theorem provides the following necessary and sufficient conditions for the optimum.

A. Lagrangian

$$L[w(j,t),spill(j,t),\alpha(j,t),\beta(j,t),\gamma(j,t),\lambda(j,t),\theta(j)] =$$

$$\sum_{t=1}^{52} F_t[\dot{x}(t)] +$$

$$\sum_j \sum_{t=1}^{51} \left\{ \alpha(j,t) * \left[\sum_{k=1}^{t} [q(j,k) - w(j,k) - spill(j,k)] + S1(j) - Smax(j) \right] \right. +$$

$$\sum_j \sum_{t=1}^{51} \left\{ \beta(j,t) * \left[\sum_{k=1}^{t} [q(j,k) - w(j,k) - spill(j,k)] - S1(j) + Smin(j) \right] \right. +$$

$$\sum_j \sum_{t=1}^{52} \left\{ \gamma(j,t) * [w(j,t) - Wmax(j)] \right\} + \sum_j \sum_{t=1}^{52} \left\{ \lambda(j,t) * [-w(j,t) + Wmin(j)] \right\} +$$

$$\sum_j \left\{ \theta(j) * \left[\sum_{t=1}^{52} [q(j,t) - w(j,t) - spill(j,t)] + S1(j) - S53(j) \right] \right\} \qquad (4.52)$$

where $\alpha(j,t)$, $\beta(j,t)$, $\gamma(j,t)$, $\lambda(j,t)$ and $\theta(j)$ are the Lagrange multipliers related to the constraints 4.47 to 4.51, respectively.

B. Optimality Conditions

1. Either

$$\frac{\partial}{\partial w(j,k)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} - \sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \beta(j,t) + \gamma(j,k)$$

$$- \lambda(j,k) - \theta(j) \leq 0$$

$$\text{if} \quad w(j,k) = 0 \qquad (4.53)$$

or

$$\frac{\partial}{\partial w(j,k)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(k)] \right\} - \sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \beta(j,t) + \gamma(j,k)$$

$$- \lambda(j,k) - \theta(j) = 0$$

$$\text{if} \quad w(j,k) \geq 0 \qquad (4.54)$$

for $k = 1,51$.

2. Either

$$-\sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \beta(j,t) - \theta(j) \leq 0 \text{ if } spill(j,k) = 0 \qquad (4.55)$$

or

$$-\sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \beta(j,t) - \theta(j) = 0 \text{ if spill } (j,k) \geq 0 \qquad (4.56)$$

for k = 1,51 and all j.

3. Either

$$\sum_{t=1}^{k} [q(j,t) - w(j,t) - spill(j,t)] + S1(j) - Smax(j) \leq 0$$

$$\text{if } \alpha(j,k) = 0 \qquad (4.57)$$

or

$$\sum_{t=1}^{k} [q(j,t) - w(j,t) - spill(j,t)] + S1(j) - Smax(j) = 0$$

$$\text{if } \alpha(j,k) \geq = 0 \qquad (4.58)$$

for k = 1,51 and all j.

4. Either

$$-\sum_{t=1}^{k} [q(j,t) - w(j,t) - spill(j,t)] - S1(j) + Smin(j) \leq 0$$

$$\text{if } \beta(j,k) = 0 \qquad (4.59)$$

or

$$-\sum_{t=1}^{k} [q(j,t) - w(j,t) - spill(j,t)] - S1(j) + Smin(j) = 0$$

$$\text{if } \beta(j,k) \geq 0 \qquad (4.60)$$

for k = 1,51 and all j.

5. Either $w(j,t) - Wmax(j) \leq 0$ if $\gamma(j,t) = 0$ (4.61)

or $w(j,t) - Wmax(j) = 0$ if $\gamma(j,t) \geq 0$ (4.62)

for k = 1,52 and all j.

6. Either $- w(j,t) + Wmin(j) \leq 0$ if $\lambda(j,t) = 0$ (4.63)

or $- w(j,t) + Wmin(j) = 0 \quad \text{if} \quad \lambda(j,t) \geq 0$ (4.64)

for k = 1,52 and all j.

7. $$\sum_{t=1}^{52} [q(j,t) - w(j,t) - spill(j,t)] + S1(j) - S53(j) = 0 \qquad (4.65)$$

for all j.

8. All variables but $\theta(j)$ are non negative.

If the derivative in condition 1 is developed such a condition can be presented as

$$-alpha(j) * f[k,D(t) - \sum_{j} alpha(j) * w(j,t)] + \gamma(j,t) - \lambda(j,t) =$$

$$= \sum_{t=1}^{k} \alpha(j,k) - \sum_{t=1}^{k} \beta(j,k) + \theta(j) \qquad (4.66)$$

The resultant conditions for the optimal operation of each reservoir in a system with multiple unlinked reservoirs are identical to the conditions derived previously for the optimal operation of a single reservoir system. Equation 4.66 states (as Equation 4.36 did) that in the optimal operation of each reservoir the marginal instantaneous benefit of its releases equals the marginal future benefit of its remaining storage. When the maximum or minimum release constraints are tightening the optimal operation, the Lagrange multipliers $\gamma(j,t)$ and $\lambda(j,t)$ will be non-null. In this case, the solution will be easily obtained with the release in the reservoir equal to the maximum or minimum release, respectively (Equations 4.62 and 4.64).

Equations 4.57 and 4.59 state that the marginal future benefit of the remaining storage in each reservoir will be constant along with the optimal operation as long as no extreme storage (maximum or minimum) occurs in the reservoir. Moreover, as in the single reservoir system, the marginal future benefit of the remaining storage in each reservoir may increase only after maximum storage is reached in the referred reservoir (Equation 4.58). The marginal future benefit of the remaining storage in the reservoir may decrease only after minimum storage level has occurred (Equation 4.60).

Some additional characteristics related to the joint operation of the system can be derived through further reasoning. In the case study the objective function is related to the summation of the energy produced in each hydroplant of the system. This means that benefits of hydro energy generation are independent on the hydroplant or hydroplants which provide this energy. Identically, the benefits derived from the additional future energy generation are independent of which reservoir or reservoirs have provided the additional storage. The result is that it will always be feasible to store additional water for energy generation later in the operation as long as at least one reservoir has the adequate storage space. It will also be feasible to release an additional volume of water to generate energy during a given time interval as long as at least one reservoir has sufficient water stored. Therefore, the concept of temporal transfer of water developed for a single reservoir operation in section 4.1.2 is somewhat modified in the case of multiple unlinked reservoirs. Now the restrictions of water transfer to the future or to the past refer to a situation dictated simultaneously by all of the reservoirs in the system.

The marginal economic interpretation of the Lagrange multipliers must be adapted to the situation of joint operation. Suppose a reservoir j reaches its maximum storage during a time interval t during a period without any spilling occurrences. If at least one reservoir in the system does not have its operation tightened by the maximum storage, additional water can still be stored in the system during the t-th time interval. Therefore the optimal allocation of releases or storages before and after the t-th time interval is not tightened by any constraint. If the maximum storage constraint in the reservoir j is infinitesimally relaxed no additional benefit will be obtained in the operation. This follows from the fact that this constraint is not actually tightening the optimal operation of the system as a whole. The result is that the Lagrange multiplier $\alpha(j,t)$, which refers to the maximum storage constraint in reservoir j during the t-th time interval, is null. Or, finally, the marginal future benefit of the remaining storage in reservoir j is identical before and after the t-th time interval.

The same reasoning can be applied to the case where a reservoir reaches its minimum storage at the beginning of a period without spills and there are still reservoirs in the system with storages above their minimums. In this case the marginal future benefit of the remaining storage in this reservoir will be identical before and after the time interval of minimum storage.

Finally, with the occurrence of spills in a reservoir the marginal future benefit of its remaining storage will be null (Equation 4.56). Together with the previous conditions, the marginal future benefit of the remaining storage will be null during a period which starts with the reservoir empty and ends with the reservoir full. This situation is implied by the non-negativity of the marginal future benefit of the remaining storage stated by Equation 4.55. The optimal operation of the reservoir will then be defined by the maximum release tactic. As defined before, this tactic refers to a case where the releases are made at the minimum between the reservoir's maximum release capacity and the release which will completely meet the energy demand.

Concluding, the optimal operation of a system of multiple unlinked reservoirs is defined by the following optimality conditions.

Optimality Condition 1. In the optimal operation of multiple unlinked reservoirs the marginal instantaneous benefit of the releases from each reservoir is equal to the marginal future benefit of its remaining storage.

Optimality Condition 2. In the optimal operation of multiple unlinked reservoirs the marginal future benefit of the storage in each reservoir is constant as long as the optimal operation of the system is not tightened by either extreme (maximum or minimum) storage constraint.

Optimality Condition 3. In the optimal operation of multiple unlinked reservoirs the marginal future benefit storage in each reservoir increases only when the reservoir's maximum storage constraint is tightening the optimal operation of the system.

Optimality Condition 4. In the optimal operation of multiple unlinked reservoirs the marginal future benefit of storage in each reservoir decreases only when the reservoir's minimum storage constraint is tightening the optimal operation of the system.

The storage constraints will tighten the optimal operation of the system in two cases. The first case occurs when a given reservoir is operated during periods of surplus water. Minimum storage in the reservoir at the beginning of this period precludes the possibility of additional transfers of water to the past. Maximum storage in the reservoir at the end of the same period precludes the possibility of additional transfers of water to the future.

The second case in which the storage constraints tighten the operation refers to the operating conditions of the system as a whole. It occurs when all reservoirs in the system simultaneously reach the maximum or minimum storage. As in the previous cases additional transfers of water to the future or to the past are restricted by the maximum and minimum storage constraints.

The occurrence of spills in the optimal operation of the system is considered in the following optimality condition.

Optimality Condition 5. In the optimal operation of multiple unlinked reservoirs spills can occur in a reservoir during a given period of operation when the marginal future benefit of its storage is null. In such a situation, its storage will be minimum at the beginning and maximum at the end of the period. The optimal releases from the reservoir during the period are given by maximum release tactic.

4.3 Multiple Serially Linked Reservoirs with Constant Hydraulic Heads

The mathematical formulation of the problem in this case is given by

$$\underset{\substack{\dot{x}(t)\\ t=1,52}}{\text{Minimize}} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} = \left\{ \sum_{t=1}^{52} \int_{x1}^{x2} f[t,D(t) - x(t)]\, dx \right\} \qquad (4.67)$$

The integral computes total cost of operation during the time interval t given the marginal cost of operation by the function $f(\cdot)$. $D(t)$ is the total energy demand and $x(t)$ refers to the total hydro energy generation during the time interval t. The limits of the integration are given by

$$\left. \begin{aligned} x(t) &= x1 = D(t) \\ x(t) &= x2 = \dot{x}(t) \end{aligned} \right\} \qquad (4.68)$$

The limit x1 refers to the marginal cost when energy demand is totally provided by hydro energy. The limit x2 stands for the hydro energy generation $\dot{x}(t)$.

The hydro energy generation is computed by

$$x(t) = alpha(j) * w(j,t) \qquad (4.69)$$

where w(j,t) is the water release through the turbines of the hydroplant j during the time interval t (hm^3)

alpha(j) is the energy conversion factor for the hydroplant j. It is function of the invariant hydraulic head, hydraulic losses and efficiency of the hydroplant j.

The operation's constraints are given by

1. Water Balance Equalities

$$s(j,t+1) = s(j,t) + q(j,t) + qu(j,t) - w(j,t) - spill(j,t) \qquad (4.70)$$

for t=1,52 and all j

where s(j,t+1) is the water content in the reservoir j at the beginning of the time interval t+1 (hm^3)

s(j,t) is the water content in the reservoir j at the beginning of the time interval t (hm^3)

q(j,t) is the water inflow to the reservoir j during the time interval t (hm^3)

qu(j,t) is the water inflow to the reservoir j during the time interval t caused by releases from the upstream reservoir (hm^3)

if ju is the index of the reservoir upstream of the reservoir j and the travel time of the water from the reservoir ju to the reservoir j is given by ℓ_{ju} time intervals, the value of qu is given by

$$qu(j,t) = w(ju,t-\ell_{ju}) + spill(ju,t-\ell_{ju}) \qquad (4.71)$$

where $w(ju,t-\ell_{ju})$ and $spill(ju,t-\ell_{ju})$ are respectively the release and spill from the reservoir ju occurred a travel time before the time interval t.

w(j,t) is the water release through the turbines of the hydroplant j during the time interval t (hm^3)

spill(j,t) is the spill from the reservoir j during the time interval t (hm^3).

2. Maximum Reservoir Content Inequalities

$$s(j,t) \leq Smax(j) , \quad \text{for} \quad t = 1,52 \quad \text{and all } j \tag{4.72}$$

where Smax(j) is the maximum storage capacity of the reservoir j.

3. Minimum Reservoir Content Inequalities

$$s(j,t) \geq Smin(j) \quad \text{for} \quad t = 1,52 \quad \text{and all } j \tag{4.73}$$

where Smin(j) is the minimum storage in the reservoir j.

4. Maximum Release Inequalities

$$w(j,t) \leq Wmax(j), \quad \text{for} \quad t = 1,52 \text{ and all } j \tag{4.74}$$

where Wmax(j) is the maximum release capacity through the turbines of the hydroplant j during a time interval. The computation is made by the procedure presented in section 3.2.2.

5. Minimum Release Inequalities

$$w(j,t) \geq Wmin(j) \quad \text{for} \quad t = 1,52 \text{ and all } j \tag{4.75}$$

where Wmin(j) is the minimum required release from the reservoir j during a time interval.

As the problem is deterministic initial reservoir contents are defined as S1(j). The final contents in the reservoirs are given by S53(j).

The water balance equation may be stated as

$$s(j,t+1) = S1(j) + \sum_{k=1}^{t} [q(j,k) + w(ju,k-\ell_{ju}) - w(j,k) + spill(ju,k-\ell_{ju}) - spill(j,k)] \tag{4.76}$$

for t = 1,51 and all j

$$\text{and} \quad s(53) = S53(j), \quad \text{for all } j \tag{4.77}$$

The constraints become after substituting the value of s(j,t)

$$\sum_{k=1}^{t} [q(j,k) + w(ju,k-\ell_{ju}) - w(j,k) + spill(ju,k-\ell_{ju}) - spill(j,k)]$$

$$+ S1(j) - Smax(j) \leq 0 \qquad (4.78)$$

$$-\sum_{k=1}^{t} [q(j,k) + w(ju,k-\ell_{ju}) - w(j,k) + spill(ju,k-\ell_{ju}) - spill(j,k)]$$

$$- S1(j) + Smin(j) \leq 0 \qquad (4.79)$$

for all j and t = 1,51

$$w(j,t) - Wmax(j) \leq 0 \qquad (4.80)$$

$$- w(j,t) + Wmin(j) \leq 0 \qquad (4.81)$$

for all j and t = 1,52

and the gross balance equation

$$\sum_{t=1}^{52} [q(j,t) + w(ju,t-\ell_{ju}) - w(j,t) + spill(ju,t-\ell_{ju}) - spill(j,t)]$$

$$+ S1(j) - S53(j) = 0 \qquad (4.82)$$

The application of the Kuhn-Tucker theorem provides

A. Lagrangian

$$L[w(j,t),spill(j,t),\alpha(j,t),\beta(j,t),\gamma(j,t),\lambda(j,t),\theta(j)] =$$

$$= \sum_{t=1}^{52} F_t[\dot{x}(t)] + \sum_j \sum_{t=1}^{51} \left\{ \alpha(j,t) * \left[\sum_{k=1}^{t} [q(j,k) + w(ju,k-\ell_{ju}) - w(j,k) \right.\right.$$

$$\left.\left. + spill(ju,k-\ell_{ju}) - spill(j,k)] + S1(j) - Smax(j) \right] \right\} +$$

$$\sum_j \sum_{t=1}^{51} \left\{ \beta(j,t) * \left[-\sum_{k=1}^{t} [q(j,k) + w(ju,k-\ell_{ju}) - w(j,k) \right.\right.$$

$$\left.\left. + spill(ju,k-\ell_{ju}) - spill(j,k)] - S1(j) + Smin(j) \right] \right\} +$$

$$+ \sum_j \sum_{t=1}^{52} \left\{ \gamma(j,t) * [w(j,t) - Wmax(j)] \right\} + \sum_j \sum_{t=1}^{52} \left\{ \lambda(j,t) * \right.$$

$$\left. * [- w(j,t) + Wmin(j)] \right\} +$$

$$\sum_{j} \left\{ \theta(j) * [\sum_{t=1}^{52} [q(j,t) + w(ju,t-\ell_{ju}) - w(j,t) + spill(ju,t-\ell_{ju}) - spill(j,t)] + S1(j) - S53(j)] \right\} \tag{4.83}$$

B. Optimality Conditions

1. Either

$$\frac{\partial}{\partial w(j,t)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} - \sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \alpha(jd,t+\ell_{jd}) + \sum_{t=1}^{k} \beta(j,t) - \sum_{t=1}^{k} \beta(jd,t+\ell_{jd}) + \gamma(j,k) - \lambda(j,k) + \theta(jd) - \theta(j) \leq 0$$

$$\text{if} \quad w(j,t) = 0 \tag{4.84}$$

or

$$\frac{\partial}{\partial w(j,t)} \left\{ \sum_{t=1}^{52} F[x(t)] \right\} - \sum_{t=1} \alpha(j,t) + \sum \alpha(jd,t+\ell_{jd}) + \sum_{t=1} \beta(j,t) - \sum_{t=1}^{k} \beta(jd,t+\ell_{jd}) + \gamma(j,k) - \lambda(j,k) + \theta(jd) - \theta(j) = 0$$

$$\text{if} \quad w(j,t) \geq 0 \tag{4.85}$$

for $k = 1,51$ and all j

where jd and ℓ_{jd} are respectively the index of the reservoir immediately downstream of the reservoir j and the travel time of the water between reservoirs j and jd.

2. Either

$$- \sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \alpha(jd,t+\ell_{jd}) + \sum_{t=1}^{k} \beta(j,t) - \sum_{t=1}^{k} \beta(jd,t+\ell_{jd}) + \theta(jd) - \theta(j) \leq 0 \quad \text{if} \quad spill(j,k) = 0 \tag{4.86}$$

or

$$- \sum_{t=1}^{k} \alpha(j,t) + \sum_{t=1}^{k} \alpha(jd,t+\ell_{jd}) + \sum_{t=1}^{k} \beta(j,t) - \sum_{t=1}^{k} \beta(jd,t+\ell_{jd})$$

$$+ \theta(jd) - \theta(j) = 0 \quad \text{if} \quad spill(j,k) \geq 0 \qquad (4.87)$$

for $k = 1,51$ and all j

3. Either

$$\sum_{t=1}^{k} [q(j,t) + w(ju,t-\ell_{ju}) - w(j,t) + spill(ju,t-\ell_{ju}) -$$

$$- spill(j,t)] + S1(j) - Smax(j) \leq 0 \quad \text{if} \quad \alpha(j,k) = 0 \qquad (4.88)$$

or

$$\sum_{t=1}^{k} [q(j,t) + w(ju,t-\ell_{ju}) - w(j,t) + spill(ju,t-\ell_{ju}) -$$

$$- spill(j,t)] + S1(j) - Smax(j) = 0 \quad \text{if} \quad \alpha(j,k) \geq 0 \qquad (4.89)$$

for $k = 1,51$ and all j

4. Either

$$- \sum_{t=1}^{k} [q(j,t) + w(ju,t-\ell_{ju}) - w(j,t) + spill(ju,t-\ell_{ju}) -$$

$$- spill(j,t)] - S1(j) + Smin(j) \leq 0 \quad \text{if} \quad \beta(j,k) = 0 \qquad (4.90)$$

or

$$- \sum_{t=1}^{k} [q(j,t) + w(ju,t-\ell_{ju}) - w(j,t) + spill(ju,t-\ell_{ju}) -$$

$$- spill(j,t)] - S1(j) + Smin(j) = 0 \quad \text{if} \quad \beta(j,k) \geq 0 \qquad (4.91)$$

for $k = 1,51$ and all j

5. Either $\quad w(j,t) - Wmax(j) \leq 0 \quad \text{if} \quad \gamma(j,t) = 0 \qquad (4.92)$

or $\quad w(j,t) - Wmax(j) = 0 \quad \text{if} \quad \gamma(j,t) \geq 0 \qquad (4.93)$

for $k = 1,52$ and all j

6. Either $-w(j,t) + Wmin(j) \leq 0$ if $\lambda(j,t) = 0$ (4.94)

or $-w(j,t) + Wmin(j) = 0$ if $\lambda(j,t) \geq 0$ (4.95)

for $k = 1,52$ and all j

7. $$\sum_{t=1}^{52} [q(j,t) + w(ju,t+\ell_{ju}) - w(j,t) + spill(ju,t+\ell_{ju}) -$$

$$- spill(j,t) + S1(j) - S53(j) = 0 \qquad (4.96)$$

for all j

8. all variables but $\theta(j)$ are non negative.

Derivation of conditions 1 and 2 presents some complexities concerning releases from an upstream reservoir. If a release $w(j,k)$ is made in reservoir j during time interval k, it is involved in the hydrologic balance of the reservoir j during the same time interval. After a period equal to the travel time between reservoir j and downstream reservoir jd, the same release should be included in the hydrologic balance of the downstream reservoir. This causes the Lagrange multipliers related to the maximum and minimum storage constraints of the downstream reservoir to be considered in the optimal operation conditions of the upstream reservoir.

When a serially linked reservoir releases water, the result is an instantaneous benefit from energy generation and a prospective benefit due to usage of the increased storage in the downstream reservoir. In this case it is still profitable to increase the releases from a reservoir as long as additional benefits from the releases are greater than the resultant decrease in future benefits of the remaining storage. When the additional benefits of the releases are less than the decrease in the future benefit the optimal decision is inverse. The point of equilibrium which provides the optimal decision in the time interval is obtained when the equality of the two terms occurs.

The identification of the Lagrange multipliers which refer to each kind of benefit may be obtained through analysis of their marginal economic meaning. Consider the equation 4.85 and the Lagrange multipliers.

The Lagrange multipliers $\gamma(j,k)$ and $\lambda(j,k)$ refer to the rate of improvements on the optimal value of the objective function when an infinitesimal relaxation on the maximum or minimum release constraints is made in the reservoir j during the k-th time interval. The improvement is from the possibility of releasing more or less water from reservoir j during the time interval k. Therefore, these Lagrange multipliers refer to the marginal instantaneous benefit of the releases from the reservoir j.

The Lagrange multipliers $\alpha(j,t)$ and $\beta(j,t)$ refer to the rate of improvements on the optimal value of the objective function when an infinitesimal relaxation of the maximum or minimum storage constraint is made in the reservoir j at the beginning of the time interval t. The improvement is from the possibility of increasing or decreasing storage quantities in reservoir j at the beginning of time interval t. Therefore, these Lagrange multipliers refer to the marginal future benefit of the remaining storage in j.

Finally, the Lagrange multipliers $\alpha(jd,t+\ell_{jd})$ and $\beta(jd,+t\ell_{jd})$ establish a linkage between operation of the upstream and downstream reservoir. They refer to the marginal future benefit of the variation of storage in the downstream reservoir caused by operation of the upstream reservoir or in other words, the marginal future benefit of the water transferred from the upstream to the downstream reservoir.

Developing the derivative and regrouping the terms of the equation 4.85 in order to gather the Lagrange multipliers related to the benefits of the releases in the left-hand side it becomes

$$\left\{- \text{alpha}(j) * f[k,D(k) - \sum_{j} \text{alpha}(j) * w(j,k)]\right\} + \gamma(j,k) - \lambda(j,k) +$$

$$+ \left\{ \sum_{t=1}^{k} \alpha(jd,t+\ell_{jd}) - \sum_{t=1}^{k} \beta(jd,t+\ell_{jd}) + \theta(jd) \right\} = \sum_{t=1}^{k} \alpha(j,t)$$

$$- \sum_{t=1}^{k} \beta(j,t) + \theta(j) \qquad (4.97)$$

where the terms in braces on the left-hand side refer respectively to the marginal instantaneous benefit of releases from reservoir j and to the marginal future benefit of water transferred from reservoir j into the immediately downstream reservoir, jd. The right-hand side term refers to the marginal future benefit of the remaining storage in reservoir j. Optimal release per time interval is determined by the equality of Equation 4.97. A trivial solution occurs when the Lagrange multipliers $\gamma(j,t)$ or $\lambda(j,t)$ are non-null. In this case the optimal release is respectively maximum and minimum.

Variation of marginal future benefit of the remaining storage is identical to the case of multiple unlinked reservoirs. Lagrange multipliers $\alpha(j,t)$ and $\beta(j,t)$ which refer respectively to the maximum and minimum storage constraints will be null as long as such a constraints are not tightening the optimal operation. Moreover the marginal future benefit of remaining storage may increase in a reservoir only when the storage is maximum (relation 4.89). Otherwise, the marginal future benefit of the remaining storage decreases only when the storage is minimum (relation 4.91).

Relation 4.87 states the condition for spilling occurrences in the operation of a reservoir. Spill may occur in a reservoir when the marginal future benefit of water transferred to the downstream reservoir equals the marginal future benefit of the remaining reservoir's storage. Water spill in the operation of the system means loss of water. In such an occurrence the water must be stored or released through the turbines of the reservoir's hydroplant rather than lose it. Increase in releases are limited by energy demand per time interval and by the reservoir's maximum release capacity. The optimal release during this period will then be defined by the minimum value of these limitations. When the release from the reservoir provides the entire energy demand, its marginal instantaneous benefit is null. This is due to the fact that since the energy demand is satisfied any increment of release has null benefit. Therefore, the additional benefit of an increment of releases can only be derived from the water transferred to the downstream reservoir. Relation 4.87 states this exactly. Hence in serially-linked reservoirs, differently of single and multiple unlinked reservoirs, the marginal future benefit of the remaining storage may not be null in the optimal operation when spills occur. However, as in the other cases, the period when spill occurs must start with the reservoir empty and end with the reservoir full. This occurs as a consequence of the minimization of the spillings from the reservoir. If the period in consideration has a surplus of water to the degree it must be spilled the period of operation immediately anterior to it will be operated in such a way to transfer no water to the future. The transition of the operation between periods is made with the reservoir empty. On the other hand, the transition requires a maximum temporal transfer of water between the period of spilling and the next period of operation. This maximum transfer will occur when the transition of the operation between both periods is made with the reservoir full.

The concept of temporal water transfer logically explains the optimal operation of a single reservoir without utilizing mathematical analysis. In serially-linked reservoirs the water may also be transferred between reservoirs. In this case, extensions of the concept of temporal water transfer permits further identification of the characteristics of optimal operation of serially-linked reservoirs.

When the maximum storage constraint in a reservoir is tightening the optimal operation of the system, optimal transfer of water to the future is limited in this reservoir. In a system with multiple unlinked reservoirs this limitation occurs when all reservoirs simultaneously reach maximum storage. When serial linkages between reservoirs are present, another possibility of water transfer to the future may be exploited. Consider a system with two serially-linked reservoirs. Suppose the reservoirs reach maximal storage simultaneously at the beginning of time interval t and the travel time of the water between the reservoirs is ℓ_j. In other words, water released from the upstream reservoir will reach the downstream reservoir after ℓ_j time intervals. Hence, water stored in the upstream reservoir after the beginning of the time interval $(t-\ell_j)$ will reach the downstream reservoir after the time interval t when transfer of water to the future is restricted by maximum storage constraints. This results in the upstream reservoir being full at the beginning of the time interval $(t-\ell_j)$ to optimize transfer of water to the future.

The operation is tightened by the minimum storage constraints when all reservoirs are simultaneously empty. The limitation in this case prevents an increase of the releases before the time interval of simultaneous minimum storage. Let it be the t-th time interval of operation. The travel time of water between the upstream and downstream reservoirs is still ℓ_j time intervals. If the upstream reservoir has the minimum storage at the beginning of the time interval $(t-\ell_j)$ all the stored water is transferred to the downstream reservoir to be released there prior to simultaneous minimum storage. Therefore, it promotes an optimal transfer of water.

Using the previous reasoning through optimal operation the upstream reservoir must reach the maximum or minimum storage before it will be again at this condition simultaneously with the downstream reservoir. The time between extreme storage occurrences equals the travel time of the water between the upstream and downstream reservoirs. The analysis may be extended to a system with any number of serially-linked reservoirs. Assume such a system has J serially-linked reservoirs and the upstream reservoir is labeled reservoir Number 1 and the remaining reservoirs are numbered in the downstream direction to Number J. The travel time of the water between reservoirs Number j and k is ℓ_{jk}. For instance, the travel time between reservoirs 1 and 2 is $\ell_{1,2}$; between reservoirs J-1 and J is $\ell_{J-1,J}$. Suppose the system reaches an extreme (maximum or minimum) storage at the beginning of the t-th time interval of operation. The optimization of the water transfers during the operation results in all reservoirs but the reservoir J being at the same extreme storage at the beginning of the time interval $(t-\ell_{J-1,J})$. Again, all reservoirs except the reservoirs J-1 and J will be at the same extreme storage at the beginning of the time interval $(t-\ell_{J-2,J-1} - \ell_{J-1,J})$. The extreme storages will successively occur backwards until only reservoir Number 1 is at the extreme storage at the beginning of the time interval given by

$$(t - \sum_{j=2}^{J} \ell_{j-1,j})$$

This characteristic of the optimal operation of serially-linked reservoirs is referred to as the Chain of the Extreme Storages. The chain of the extreme storages presents a condition in which occurrence of maximum or minimum storages non-simultaneously in all reservoirs tightens the optimal operation of the system. Mathematics analyzing consider a system with two serially-linked reservoirs where the reservoirs ju and jd are respectively the upstream and downstream reservoirs. Suppose the travel time between the reservoirs is ℓ_j time intervals and they reach the maximum storage simultaneously at the beginning of the k-th time interval. The optimal transfer of water from the upstream to the downstream reservoir will occur if the upstream reservoir is full at the beginning of $k-\ell_j$ or the time interval that occurs a travel time of water between the reservoirs prior to the simultaneous maximum storage. The optimal operation of the reservoirs before the time interval $(k-\ell_j)$ is given by

1. upstream reservoir (ju):

$$-\text{alpha}(ju) * f[t,D(t) - \sum_j \text{alpha}(j) * w(j,t)] + \gamma(ju,t) - \lambda(ju,t) + \\ + \Omega_t(jd) = \Omega_t(ju) \qquad (4.98)$$

2. downstream reservoir (jd):

$$-\text{alpha}(jd) * f[t,D(t) - \sum_j \text{alpha}(j) * w(j,t)] + \gamma(jd,t) - \lambda(jd,t) = \\ = \Omega_t(jd) \qquad (4.99)$$

where t is an arbitrary time interval such that $t < k - \ell_j$.

$\Omega_t(j)$ is the marginal future benefit of the remaining storage in the reservoir index j (j = ju or jd) at the beginning of the time interval t. This value is given by the right-hand side of the Equation 4.97.

At the beginning of the time interval $(k-\ell_j)$ the upstream reservoir reaches maximum storage. At this point two non-null Lagrange multipliers enter in the equation that defines the optimal operation of the upstream reservoir. The Lagrange multiplier $\alpha(ju,k-\ell_j)$ is related to the maximum storage in the upstream reservoir ju at the beginning of the time interval $(k-\ell_j)$ and the Lagrange multiplier $\alpha(jd,k)$ is related to the maximum storage that occurs in the downstream reservoir jd at the beginning of the time interval k. Equation 4.99 continues to define the optimal operation of the downstream reservoir since no constraint is tightening its operation. Therefore, in the period starting at the beginning of the time interval $(k-\ell_j)$ and ending at the beginning of the time interval k the optimal operation of the reservoirs is given by

1. upstream reservoir (ju):

$$-alpha(ju) * f[t,D(t) - \sum_j alpha(j) * w(j,t)] + \gamma(ju,t) - \lambda(ju,t) +$$

$$+ \Omega_t(jd) + \alpha(jd,k) = \Omega_t(ju) + \alpha(ju,k-\ell_j) \qquad (4.100)$$

2. downstream reservoir (jd):

$$-alpha(jd) * f[t,D(t) - \sum_j alpha(j) * w(j,t)] + \gamma(jd,t) - \lambda(jd,t) =$$

$$= \Omega_t(jd) \qquad (4.101)$$

Finally, at the time interval k both reservoirs reach their maximum storage simultaneously. One non-null Lagrange multiplier is introduced into the operating equation of each reservoir. The Lagrange multiplier $\alpha(ju,k)$ is introduced into the upstream reservoir's equation and the Lagrange multiplier $\alpha(jd,k)$ in the downstream reservoir's equation. Their operation is given by

1. upstream reservoir (ju):

$$-alpha(ju) * f[t,D(t) - \sum_j alpha(j) * w(j,t)] + \gamma(ju,t) - \lambda(ju,t) +$$

$$+ \Omega_t(jd) + \alpha(jd,k) = \Omega_t(ju) + \alpha(ju,k-\ell_j) + \alpha(ju,k) \quad (4.102)$$

2. downstream reservoir (jd):

$$-alpha(jd) * f[t,D(t) - \sum_j alpha(j) * w(j,t)] + \gamma(jd,t) - \lambda(jd,t) =$$

$$= \Omega_t(jd) + \alpha(jd,k) \quad (4.103)$$

The marginal economic interpretation of some of the non-null Lagrange multipliers that enter the operating equations after the time interval $k-\ell_j$ are listed below

$\alpha(jd,k)$ is the Lagrange multiplier that enters the equations related to the operation of both reservoirs. At the upstream reservoir (Equations 4.100 and 4.102) it represents the rate of additional benefits of operation for an infinitesimal increase in the transfer of water to the downstream reservoir after the beginning of the $(k-\ell_j)$-th time interval. At the downstream reservoir (Equation 4.103) it represents the rate of additional benefits of operation for an infinitesimal increase in the storage in the downstream reservoir at the beginning of the k-th time interval.

$\alpha(ju,k-\ell_j)$ is the Lagrange multiplier that enters the equation related to the operation of the upstream reservoir only (Equations 4.100 and 4.102). It represents the rate of additional benefits of operation for an infinitesimal increase in the storage of the upstream reservoir at the beginning of the time interval $(k-\ell_j)$. This additional benefit of operation is derived from the possibility of maintaining more water in the upstream reservoir to be optimally transferred to the downstream reservoir. In this case, optimal transfer occurs when the water reaches the downstream reservoir after the beginning of the time interval k when the downstream storage is maximum. Therefore, this Lagrange multiplier represents the rate of additional benefit of operation for an infinitesimal increase on the water transfer from the upstream to the downstream reservoir after the beginning of the $(k-\ell_j)$-th time interval. Comparing this statement with the anterior interpretation of the Lagrange multiplier $\alpha(jd,k)$ in the operation of the upstream reservoir it follows

$$\alpha(ju,k-\ell_j) = \alpha(jd,k) \tag{4.104}$$

This conclusion is significant in the operation of serially-linked reservoirs. When the upstream reservoir reaches the maximum storage at the beginning of the time interval $(k-\ell_j)$, the optimal operation changes from the condition stated in Equation 4.98 to the condition stated in Equation 4.100. However, the Lagrange multipliers that make this change are identically valued. Therefore, the marginal instantaneous benefit of the releases from the upstream reservoir does not alter its optimal value. This is not surprising since the marginal instantaneous benefit of the releases in each reservoir is function of the overall energy production in the system weighted by the respective energy conversion factor. Therefore, any change in the marginal instantaneous benefit of the releases from the upstream reservoir will necessarily be simultaneously followed by the change of the marginal instantaneous benefit of the releases from the downstream reservoir. But this change in the operation of the downstream reservoir cannot be justified by Equations 4.99 and 4.101 that define its operation before and after the isolated maximum storage in the upstream reservoir.

The previously presented reasoning can be extended to any number of serially-linked reservoirs and any case of extreme (maximum or minimum) storage occurrence. The general conclusion is that during the occurrence of the chain of extreme (maximum or minimum) storages the marginal instantaneous benefit of the releases from each reservoir is invariant. The exception may occur only in the case of trivial solutions when the releases are maximum or minimum.

The resultant optimality conditions for the operation of serially-linked reservoirs are

Optimality Condition 1. In the optimal operation of serially-linked reservoirs the marginal instantaneous benefit of the releases from each reservoir plus the marginal benefit of its water transfer is equal to the marginal future benefit of its remaining storage.

Optimality Conditions 2. In the optimal operation of serially-linked reservoirs the marginal future benefit of the storage in each reservoir is constant as long as the optimal operation of the system is not tightened by extreme (maximum or minimum) storage constraints.

Optimality Condition 3. In the optimal operation of serially-linked reservoirs the marginal future benefit of storage in each reservoir increases only when the reservoir's maximum storage constraint is tightening the optimal operation of the system.

Optimality Condition 4. In the optimal operation of serially-linked reservoirs the marginal future benefit of the storage in each reservoir decreases only when the reservoir's minimum storage constraint is tightening the optimal operation of the system.

The storage constraints will tighten the optimal operation of serially-linked reservoirs in three cases. The first case refers to the condition of surplus of water occurring in the operation of one reservoir. Minimum storage at the beginning of the period and maximum storage at its end prevents transfer of the excedent water to the past or to the future, respectively.

The second case occurs when all reservoirs of the system reach maximum or minimum storage simultaneously. Under these conditions additional transfer of water to the future or past is prohibited by the maximum and minimum storage constraints of all reservoirs, respectively.

The third case occurs during the chain of extreme storages. Assuming reservoir is full during the occurrence of the chain of maximum storage, additional transfer of water to the downstream reservoir after the time when both reservoirs are simultaneously full is prohibited by the maximum storage constraint of the upstream reservoir. The opposite occurs if the reservoir is empty during the chain of minimum storages. In this case additional water can not be transferred to the downstream reservoir before the time interval when both reservoirs are empty.

The optimal occurrence of spills in serially-linked reservoirs is given by the following optimality condition.

Optimality Condition 5. In the optimal operation of serially-linked reservoirs spills can occur in a reservoir during a given period of operation when the marginal future benefit of its storage equals the marginal benefit of transfer of water to the downstream reservoir. In such a situation, the reservoir's storage is minimum at the beginning and maximum at the end of the period. The optimal releases from the reservoir during the period are given by the maximum release tactic.

4.4 Mix Configuration with Constant Hydraulic Head

The mix configuration of a system with multiple reservoirs refers to the situation where unlinked and serially-linked reservoirs coexist. Multiple serial linkages may also be included when reservoirs are located in the main stream and in tributaries.

The unlinked reservoir system may be considered as a particularization of a system of serially-linked reservoirs in which no transfer of water occurs between reservoirs. In this case, the variable qu(j,t), the inflow to a downstream reservoir caused by releases from the upstream reservoir, is null. It may be observed that the optimality conditions derived for serially-linked reservoirs will then apply to the unlinked reservoirs as well. However, in this case, there is no need for the chain of extreme storages to be utilized as an instrument to optimize water transfers.

Assuming a reservoir index j has several upstream reservoirs serially-linked with it, the total inflow to the reservoir j includes the sum of releases from the upstream, serially-linked reservoirs. The contribution of releases from the immediately upstream reservoirs is introduced into the water balance of reservoir j. Therefore, in the optimal operation of the upstream reservoirs the optimal transfer of water to the downstream reservoir must be determined. Although the transfers are now multiple, the same characteristics of the operation derived for serially-linked reservoirs apply to systems with multiple serial linkages. This may be demonstrated by following the same reasoning used in the study of serially-linked reservoirs. However, it is not presented here since this situations is outside the case study.

4.5 Systems with Variable Hydraulic Head

When the hydraulic head cannot be assumed invariant the problem of operation can not be assumed convex without further analysis. Consequently the optimality conditions may not be sufficient for the (global) optimum.

Problems related to the efficiency of energy generation restrict the variation of the hydraulic heads in hydropower developments to 30 to 40 percent of the gross hydraulic head (Federal Power Commission, 1968, p. 7). In the analyzed system this does not exceed 30 percent and is generally about 15 percent. Therefore it is unlikely that in this case a local optimum too far from the global optimum will be computed.

The main complexity introduced by the variation of the hydraulic head will be the computation of the maximum release capacity in each reservoir. The procedure of computation can not apparently be formulated in terms of a formal optimization problem (section 3.2.2).

It suggests the use of an iterative procedure where the optimality conditions are enforced for an assumed hydraulic head variation in each reservoir. The marginal instantaneous benefit of the releases may now be given with the value of the energy conversion factor [alpha(j)] varying in each reservoir along the time. Nevertheless, the optimality conditions do not change. With the defined optimal operation in the first iteration a new set of values of the hydraulic heads is obtained. Optimal operation is updated for this new set. Iterations are repeated successively until the computed and assumed hydraulic heads are roughly identical. For a single reservoir study, Laufer (1977, p. 166) reported that the convergence occurred during the second iteration using a similar approach. Although no proof of the convergence of the iterations is given it is expected that in a multireservoir problem it occurs in slightly more than two iterations.

Chapter 5

AN ALGORITHMIC APPROACH TO DETERMINISTIC OPERATION

A method of optimization of the reservoir system operation under study is presented in this chapter. It is based on the enforcement of optimality conditions of operation as derived by the Kuhn-Tucker Theorem of Mathematical Programming. Based on this characteristic it can be classified as an Indirect Method of Optimization (Wilde and Beightler, 1967, p. 18). However direct solution of the equations related to optimality conditions will usually be cumbersome. Therefore the method relies on an iterative procedure based on starting with a totally relaxed solution (no constraints considered in storage) and then introducing the violated constraints in each iteration. This method is akin to the Constrained Calculus of the Variations (Massé, 1964a) and is a generalization of the method proposed by Laufer (1977) and Laufer and Morel-Seytoux (1979).

The presented method yields an exact solution for the formulated problem of operation of a single reservoir for a pre-defined level of accuracy. The solution for a multireservoir operation is obtained by an aggregation-optimization-disaggregation procedure. This procedure successfully avoided the problems of dimensionality in the multireservoir system. Additionally, it handled the operational aspects of the problem which usually cannot be modeled by a standard mathematical programming formulation.

For hydropower systems the hydraulic head of each reservoir must be known in order to compute the energy generated. Hence in variable head power plants the iterative procedure described in the section 4.5 is applied. In the following section it is assumed that the hydraulic head variation with time of each reservoir is known or has been previously estimated.

An inductive approach is used in presenting this method. Derivation for the simpler problem of the operation of a single reservoir is done first and later it is extended to the multireservoir problem.

5.1 Single Reservoir Operation Optimization

In this section two algorithms are developed for the optimization of a single reservoir operation. The first algorithm is referred to as the Cumulative Minimum Surplus Analysis. Its purpose is to identify the periods of operation where spills occur. During such periods the optimal tactic of release is trivial. The second algorithm is referred as Constrained Marginal Analysis. It computes the optimal operation of the reservoir in periods when spills do not occur. It is based on the characteristics of the curves of variation of reservoir storage when releases are made with constant marginal instantaneous benefit.

5.1.1 Cumulative Minimum Surplus Analysis

When the optimality conditions of operation were derived the theoretical possibility of the existence of operating periods where the maximum release tactic is optimal was realized. In this tactic of operation the optimal release is equal to the minimum between a release which will entirely meet the energy demand and the reservoir's maximum release capacity. This trivial solution results from a surplus water inflow which occurs in the period. A maximum transfer of surplus water within the period requires that storage at the beginning of the period be at a minimum and at a maximum at the end of the period.

An observation about the period of surplus water inflow is that no optimization of its operation is needed. Its initial and final storage and its optimal releases have been previously established. The real problem refers to the identification of the periods of operation which have a surplus water inflow.

This identification can be accomplished through a procedure called the Cumulative Minimum Surplus Analysis. The minimum surplus of water is defined as the excess of water inflow into a reservoir after the maximum feasible release has been subtracted from it. If the minimum surpluses are computed and accumulated each time interval through the entire period of operation the cumulative minimum surplus curve is then defined. This curve has a shape like the one in Figure 3.

The sub-periods where the curve has positive tangents are sub-periods where the water surpluses are positive. There the reservoir will always be accumulating or spilling water since the release is already at a maximum during each time interval. The sub-periods where the tangents are negative are simply sub-periods where the maximum release tactic may only be adopted at the expense of the water reserves. The reserves, however, are limited by the reservoir capacity. Hence, a period of exclusively positive tangents in which the curve rises by more than the reservoir capacity value is a period when spill will occur no mater what the initial reservoir storage was (segments 1-2, Figure 3). Otherwise, in a period of exclusively negative tangents, if the curve falls by more than the reservoir capacity the maximum release tactic are infeasible no matter what the initial storage is (segments 3-4, Figure 3).

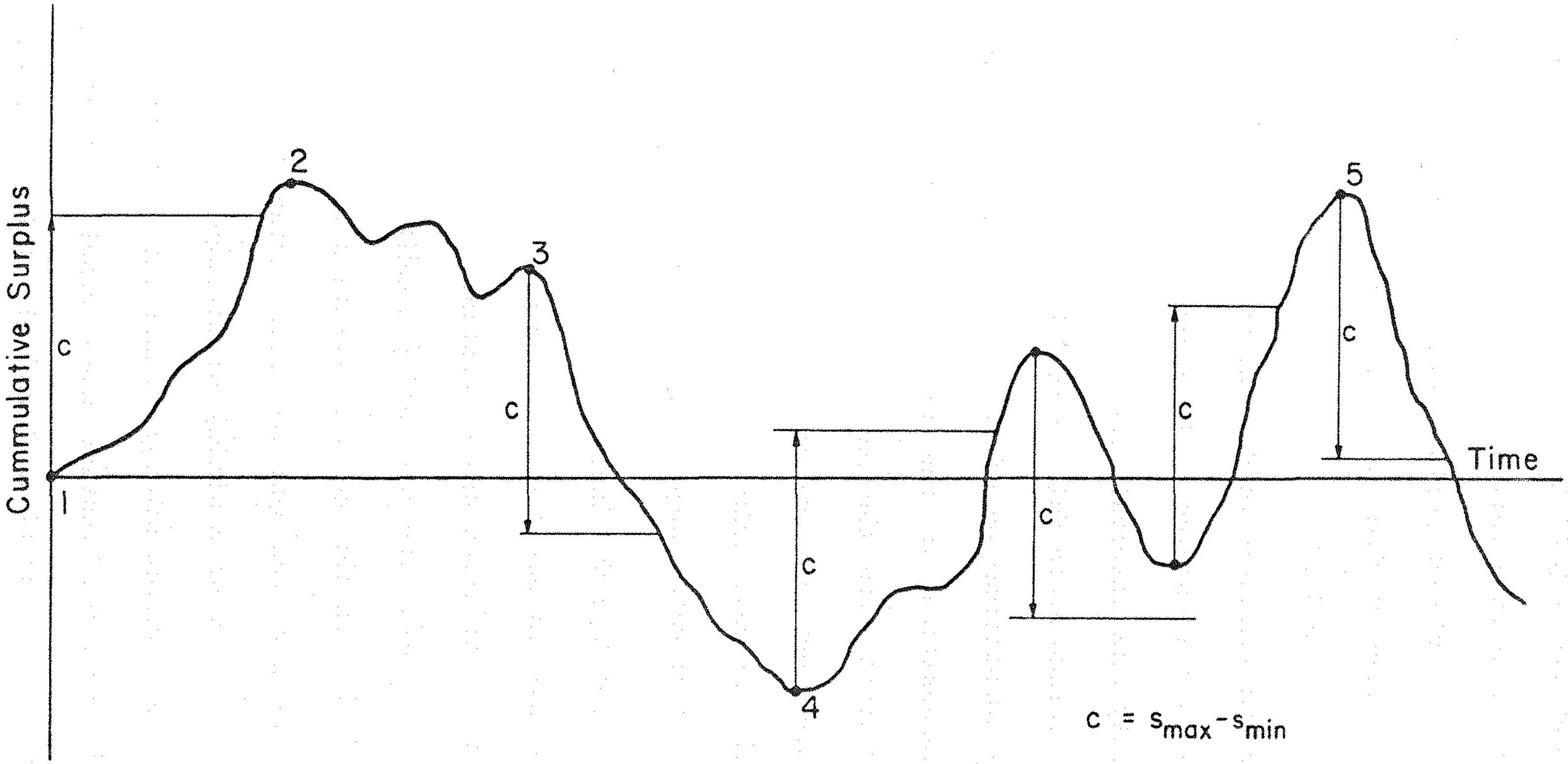


Figure 3. Cumulative Minimum Surplus Analysis

Suppose a segment of curve has positive and negative tangents. If it rises by the capacity of the reservoir between its extremities without falling by more than this same value, the maximum release tactic is feasible in the related period regardless of the initial reservoir storage. Consequently this property defines a water surplus period (segments 4-5, Figure 3).

An algorithm adaptable to digital computers was derived to identify the periods of water surplus. Its application to a period of operation with 1872 weekly time intervals took less than 0.3 seconds of processing time on the Colorado State University Cyber 172.

With the identification of the periods of water surplus a kind of temporal decomposition of the problem is obtained. The remaining periods will start with the maximum reservoir storage (or with the initial storage) and will end with the minimum reservoir storage (or with the final storage). Additionally, they will be separated from each other by periods of water surplus where the operating tactic is known.

5.1.2 Constrained Marginal Analysis

This analysis is based on the exploitation of characteristics of reservoir storage variation under constant marginal instantaneous benefits of the releases. The optimality conditions derived in the last chapter stated that in the optimal operation of a single reservoir the optimal releases have constant marginal instantaneous benefit between the extremes (maximum or minimum) of storage (optimality conditions 1 and 2). When the maximum storage constraint is tightening the optimal operation, the marginal instantaneous benefit of the releases will increase (optimality conditions 1 and 3). Otherwise, when the minimum storage constraint is tightening the optimal operation, the marginal instantaneous benefit of the releases will decrease (optimality conditions 1 and 4). Consequently the optimal operation of a single reservoir may be defined by one or more curves which represent the reservoir's storage variation under constant marginal instantaneous benefit of the releases. The characteristics of these curves are presented before the development of the technique of optimization.

Figure 4 presents some curves of variation of the storage in a hypothetical reservoir without maximum or minimum storage constraints. They start with identical storages and have their releases made with constant marginal instantaneous benefit. If the law of decreasing marginal benefits applies in this case the value of the releases decreases when the corresponding marginal instantaneous benefit increases. It implies that the resultant storage in the hypothetical reservoir increases when the marginal instantaneous benefit of the releases increases. Consequently, the bottom curve stands for the minimum marginal instantaneous benefit of the releases. In the reservoir system under observation this value is zero. In this case, the storage variation of the bottom curve refers to the operation with maximum release tactic. Let these curves of storage variation in the hypothetical reservoir be called trajectories.

The general equation for trajectories defined with initial reservoir storage equal to S1 is

$$s(\Omega,k) = S1 + \sum_{t=1}^{k-1} [q(t) - w(\Omega,t)] \qquad (5.1)$$

where $s(\Omega,k)$ is the hypothetical reservoir's storage at the beginning of time interval k under marginal instantaneous benefit of the release equal to Ω.

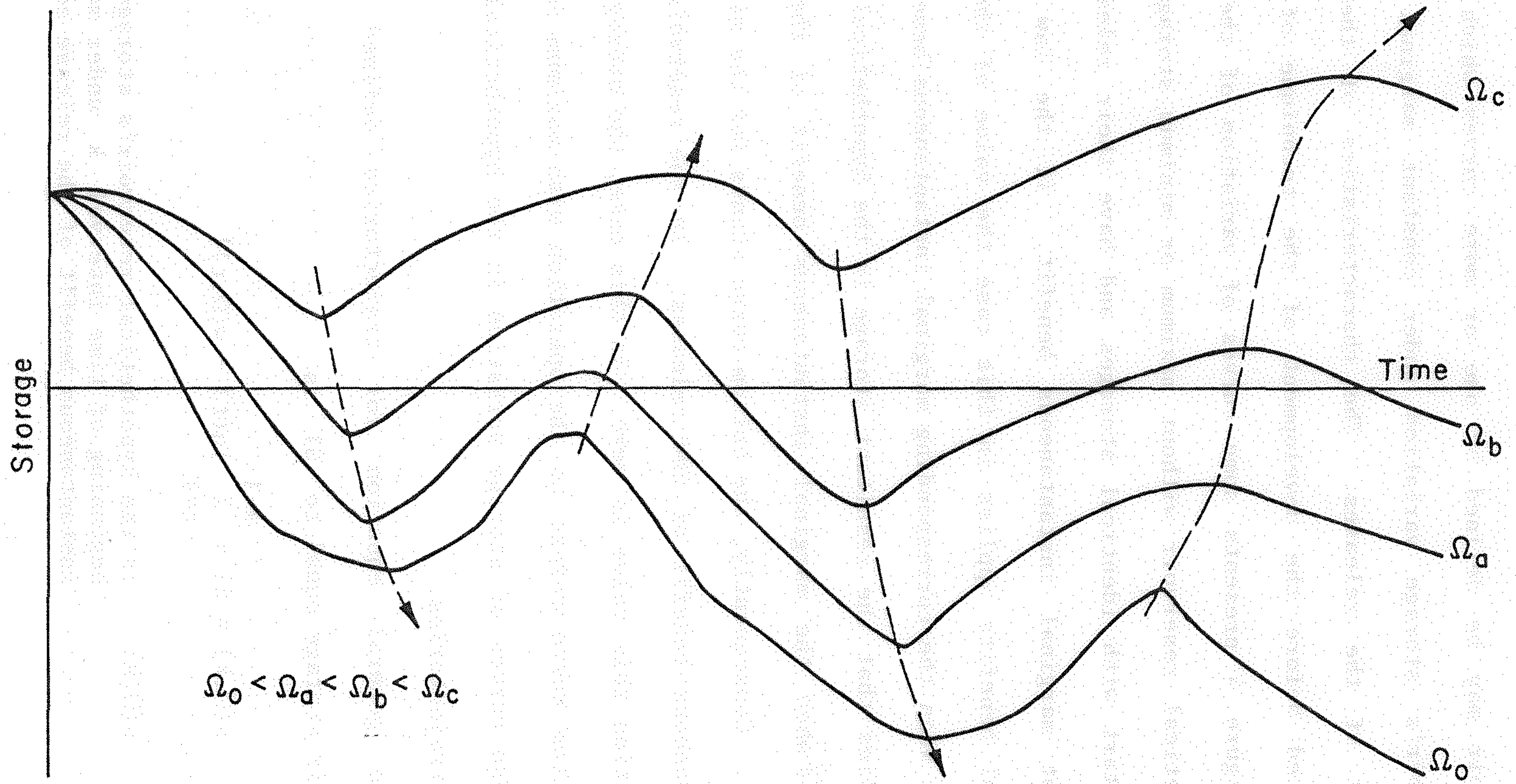


Figure 4. Trajectories with Identical Initial Storage

$q(t)$ is the inflow during time interval t.

$w(\Omega,t)$ is the release during time interval t with marginal benefit equal to Ω.

The minimum and maximum release constraints are considered in the definition of $w(\Omega,t)$. If the maximum and minimum releases during the time interval t are respectively equal to $Wmax(t)$ and $Wmin(t)$ the following expression computes the final value of $w(\Omega,t)$ in the trajectory's construction

$$w(\Omega,t) = \min\{\max[w(\Omega,t),\ Wmin(t)],\ Wmax(t)\} \tag{5.2}$$

The marginal instantaneous benefit is by definition a non-increasing function of the releases or

$$w(\Omega_a,t) \geq w(\Omega_b,t) \quad \text{for any } t \text{ and } \Omega_a < \Omega_b \tag{5.3}$$

Therefore, the hypothetical reservoir's storage is proportional to Ω in trajectories with identical initial storage. Then

$$s(\Omega_a,k) \leq s(\Omega_b,k) \quad \text{for any } k \text{ and } \Omega_a < \Omega_b \tag{5.4}$$

The trajectories with identical final storage also have theoretical interest. Figure 5 presents examples of such trajectories. The general equation for trajectories defined with identical final storage at the end of time interval n is

$$s(\Omega,k) = s(n+1) - \sum_{t=n}^{k} [q(t) - w(\Omega,t)] \tag{5.5}$$

where $s(n+1)$ is the storage at the end of the time interval n.

The releases are inversely proportional to their marginal instantaneous benefit (relation 5.3). Observing that the releases in Equation 5.5 will be added it follows that the hypothetical reservoir's storage is inversely proportional to the marginal instantaneous benefit of the releases in trajectories with identical final storage. Or

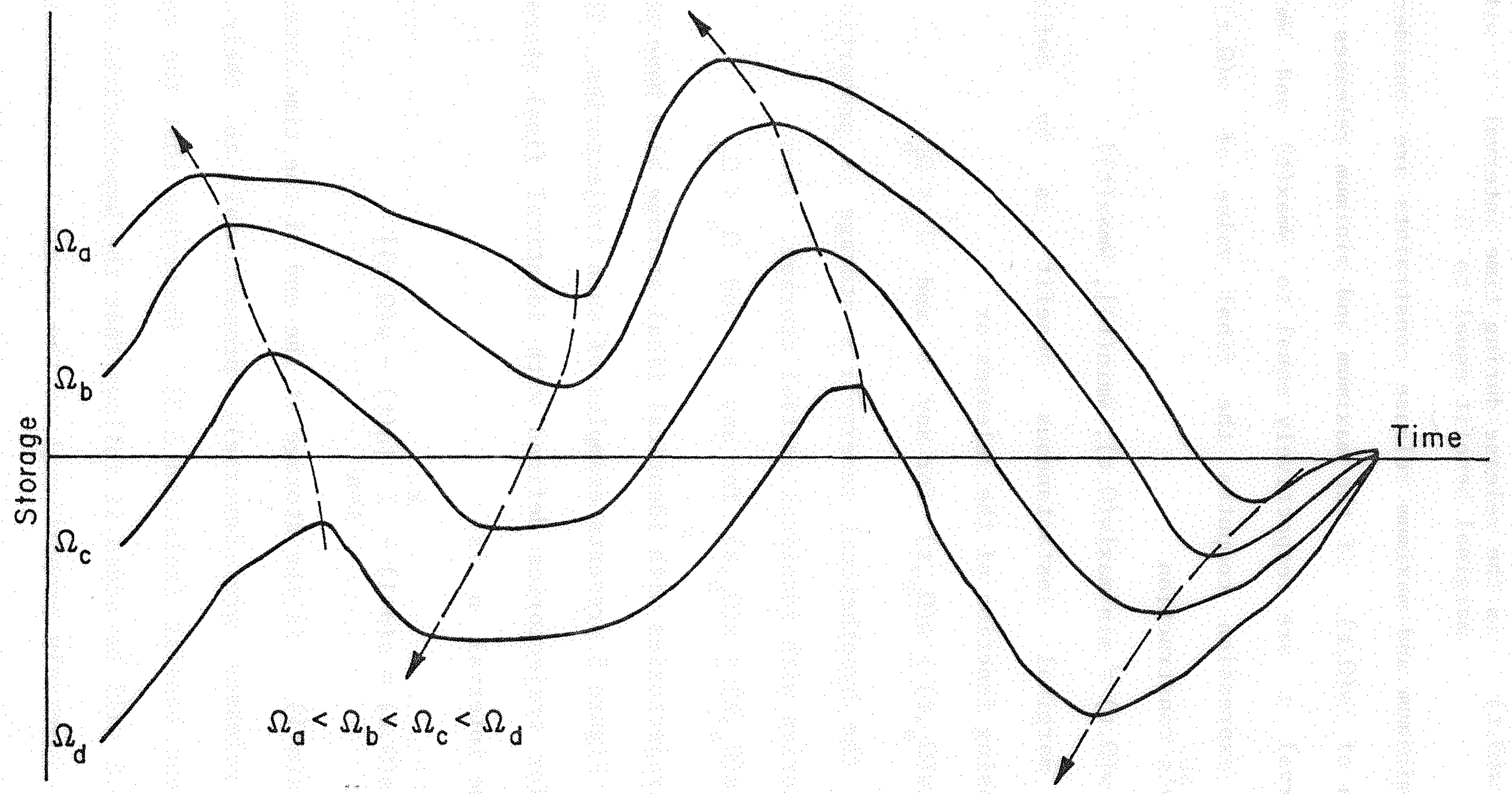


Figure 5. Trajectories with Identical Final Storage

$$s(\Omega_a,k) \geq s(\Omega_b,k) \quad \text{for all} \quad k \quad \text{and} \quad \Omega_a < \Omega_b \tag{5.6}$$

Suppose a trajectory with arbitrary initial and final storage has a local maximum at the beginning of the k-th time interval. A local maximum is defined by the following inequalities

$$s(\Omega,k-1) < s(\Omega,k) \geq s(\Omega,k+1) \tag{5.7}$$

where Ω is an arbitrary value of the marginal instantaneous benefit of the releases.

Substituting the value of storage into the inequalities by their expressions given in Equation 5.1 and cancelling the common term $S(1)$, it yields

$$\sum_{t=1}^{k-2} [q(t) - w(\Omega,t)] < \sum_{t=1}^{k-1} [q(t) - w(\Omega,t)] \geq \sum_{t=1}^{k} [q(t) - w(\Omega,t)] \tag{5.8}$$

Cancelling again common values of inflows and releases

$$q(k-1) > w(\Omega,k-1) \tag{5.9}$$

and
$$q(k) \leq w(\Omega,k) \tag{5.10}$$

It states that if a maximum occurs at the beginning of time interval k the inflow is greater than the release during time interval $(k-1)$ and the inflow is less than (or equal to) the release during time interval k. The result is logical. The storage must increase before the maximum and must not increase after it. When the storage increases the inflow is necessarily greater than the release. Otherwise, when the storage decreases (or is maximum) the release is necessarily greater than (or equal to) the inflow. The inequalities 5.9 and 5.10 state these conclusions mathematically.

If at the beginning of time interval k an arbitrary trajectory with marginal instantaneous benefit of the releases equal to Ω has a local minimum it occurs

$$s(\Omega,k-1) > s(\Omega,k) \leq s(\Omega,k+1) \tag{5.11}$$

Substituting the variables by their expressions given in Equation 5.1, after eliminating common terms it yields

$$q(k-1) < w(\Omega,k-1) \tag{5.12}$$

and

$$q(k) \geq w(\Omega,k) \tag{5.13}$$

The results are again obvious. When the storage is minimum it must decrease before the minimum and must no decrease after it. In this case, the inflow will be less than (or equal to) the release before the minimum and greater than (or equal to) the release after it.

An important observation is that the occurrence of the maximum and minimum storages may be defined in terms of inflows and releases alone. It follows that the time of occurrence of maximum and minimum storages is independent of the initial or final storages in the reservoir. Equations 5.1 and 5.5 which define the trajectories with identical initial and final storages respectively, confirm this observation. The variation of the initial and final storage in each case will cause the vertical translation of the trajectory. Its form (or shape) will not be modified, though.

The continuity of the storage variable in reservoir operation allows a trivial but important observation in the analysis. It is related to the sequence of local extreme (maximum or minimum) values in a trajectory; there is no possibility of two similar local extreme values to occur adjacently in a trajectory. In other words, if a local maximum occurs at the beginning of a given time interval the following local extreme value in the trajectory must be a minimum. Moreover, if the storage in a trajectory is increasing (or decreasing) in a given time interval the next local extreme value must be a maximum (or minimum).

These observations are useful in the following analysis. Consider two trajectories with marginal instantaneous benefit of releases equal to Ω_a and Ω_b , with Ω_a less than Ω_b. If a local maximum occurs in the first trajectory (Ω_a) at the beginning of the k-th time interval, Equations (5.9) and (5.10) state

$$q(k-1) > w(\Omega_a,k-1) \tag{5.14}$$

$$q(k) \leq w(\Omega_a,k) \tag{5.15}$$

The marginal instantaneous benefit is by definition a non-increasing function of the releases. Then, as $\Omega_a < \Omega_b$

$$w(\Omega_a,k-1) \geq w(\Omega_b,k-1) \tag{5.16}$$

$$w(\Omega_a,k) \geq w(\Omega_b,k) \tag{5.17}$$

Inequalities 5.14 and 5.16 imply

$$q(k-1) > w(\Omega_b,k-1) \tag{5.18}$$

Or, during (k-1)-th time interval the inflow is greater than the release with marginal instantaneous benefit equal to Ω_b. Hence, the storage is increasing during time interval k-1 in trajectory Ω_b. No conclusions can be made relating to the storage variation in the same trajectory during time interval k (see inequalities 5.15 and 5.17). Therefore, the first local extreme value in trajectory Ω_b occurring before time interval k is a minimum. And consequently, the first local extreme occurring at the beginning or after time interval k in the trajectory Ω_b is a maximum. Or the local maximum in the trajectory Ω_b occurs after or at the same time a local maximum occurred in trajectory Ω_a. This conclusion can be generalized: the time interval of occurrence of local maximum values in a trajectory is a non-decreasing function of the marginal instantaneous benefit of its releases.

Suppose now that instead of a local maximum a local minimum occurs at the beginning of time interval k in trajectory Ω_a. The inequalities are in the case

$$q(k-1) < w(\Omega_a, k-1) \tag{5.19}$$

$$q(k) \geq w(\Omega_a, k) \tag{5.20}$$

As Ω_a is still less than Ω_b the inequalities 5.16 and 5.17 still hold. Therefore, 5.20 and 5.17 imply

$$q(k) \geq w(\Omega_b, k) \tag{5.21}$$

It implies that the storage is increasing (or is constant) during time interval k in trajectory Ω_b. Or, the first local extreme value at the beginning or before time interval k in the trajectory Ω_b is a minimum. And, the first local extreme value after beginning of time interval k in the same trajectory must be a maximum. Therefore, the minimum in the trajectory Ω_b occurs before or at the time interval a minimum occurred in trajectory Ω_a. This conclusion can be generalized: the time interval of occurrence of a local minimum in a trajectory is a non-increasing function of the marginal instantaneous benefit of its releases.

In the construction of the trajectories on Figures 4 and 5 the relative position of maximum and minimum values followed the derived characteristics.

The sequence of occurrence of maximum and minimum values in the trajectories is the basis for development of the algorithm. In the optimal operation the reservoir's storage variation may be represented by a unique trajectory provided no storage constraint tightens the operation. Otherwise, when a storage constraint tightens the operation the trajectories representing the storage variation must change. When the tightening constraint is the maximum storage constraint, the marginal instantaneous benefit of releases increases. Therefore, trajectories with increasing marginal values will represent the storage variation. When the tightening constraint is the minimum storage constraint the marginal instantaneous benefit of the releases decreases. Consequently, the storage variation is represented by trajectories with decreasing marginal values. When a reservoir becomes either full or empty the water inflow equals the water release. When this equality occurs inside a time interval k in a trajectory with marginal value equal to Ω the following inequalities hold.

1. When the reservoir is full

$$s(\Omega,k-1) < s(\Omega,k) \geq s(\Omega,k+1)$$

2. When the reservoir is empty

$$s(\Omega,k-1) > s(\Omega,k) \leq s(\Omega,k+1)$$

These inequalities define the occurrence of a local maximum or minimum in the trajectory at the beginning of the time interval k (see inequalities 5.7 and 5.11). It follows that the transition between trajectories in the optimal operation is made through local extreme values of successive trajectories.

It is interesting but not surprising to note that the sequence of maximum or minimum values in trajectories agrees with the requirement of optimal operation. The time interval of occurrence of maximum values of trajectories is a non-decreasing function of the marginal instantaneous benefit of the respective trajectory's releases. And the time interval of occurrence of the minimum values of trajectories is a non-increasing function of the marginal instantaneous benefit of the respective trajectory's releases. Therefore it will always be possible to perform a transition with increasing marginal values of trajectories through their maximum values. Otherwise, it will always be possible to perform a transition with decreasing marginal values of the trajectories through their minimum values (Figure 4 and 5).

At this point all the elements needed to present the algorithm have been derived. The algorithm seeks the identification of the trajectories which make up the reservoir's storage variation in the optimal operation. Each trajectory is identified by a marginal value equal to the marginal instantaneous benefit of its releases. Therefore, the optimal sequence of releases can be easily retrieved by their marginal instantaneous benefit.

Two basic routines exist in the algorithm. One is directed toward computation of the constant value of the marginal instantaneous benefit of the releases that defines a trajectory linking two time intervals with given storages. The computation is made through an iterative procedure which exploits the unimodality of the marginal instantaneous benefit function. Figure 6 illustrates the procedure. The trajectory which links the time interval 1 to k is to be defined such that the storage at the beginning of time intervals 1 and k must be S(1) and S(k) respectively. The convergence of the computed final storage to the value S(k) is accepted with a tolerance equal to TOL. Suppose two arbitrary trajectories Ω_1 and Ω_2 are initially defined. The trajectory Ω_1 defines a final storage below the required final storage S(k). The trajectory Ω_2 defines a final storage above it. Consequently, $\Omega_1 < \Omega_2$. The required trajectory will have its marginal value defined between Ω_1 and Ω_2. Suppose it is obtained by linear

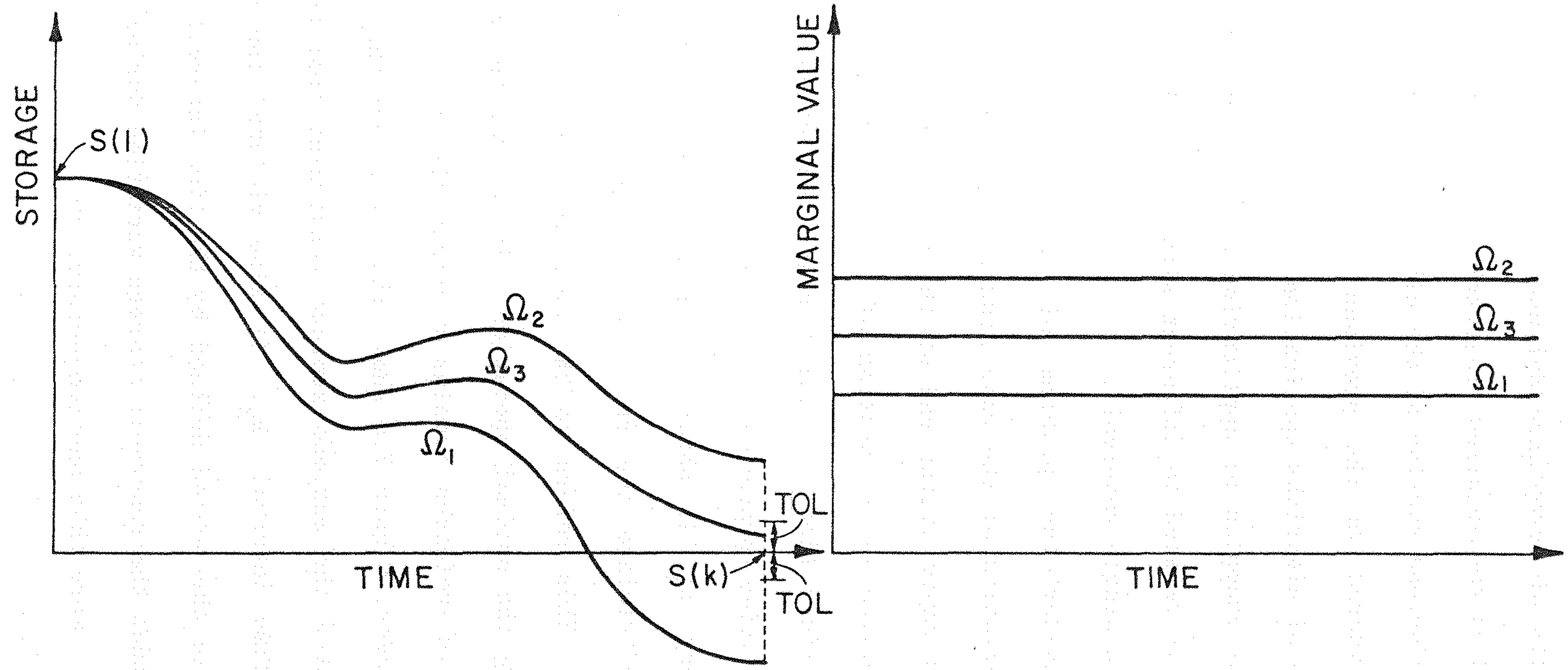


Figure 6. Routine for Trajectory Computation

interpolation. The new trajectory Ω_3 computes a final storage within the limits of acceptance defined by the tolerance value. Hence, it is accepted as the required trajectory.

The second routine of the algorithm defines the operating period to be considered in the previous routine. Its logic is presented as examples of algorithm application.

Four examples of application of the algorithm are presented. Each example is related to different situations likely to occur in the operation of a single reservoir.

Consider the problem of operation of a reservoir with the characteristics of the case study. The reservoir is full at the beginning and empty at the end of the operating period. The first step in the algorithm is computation of the trajectory which links the initial to the final time interval through respective storages (Figure 7.a). The iterative procedure explained before is applied. Assuming the trajectory Ω_1 is obtained (Figures 7.a and 8.a), if this trajectory does not violate the reservoir storage constraints, the solution is optimal. This refers to the case where no storage constraint is restricting the optimal operation. Therefore, the marginal instantaneous benefit of releases remains constant the entire period of operation.

However, Figure 7.a shows that it is not the case. The trajectory Ω_1 violated the maximum storage constraint from the time interval k_1 to the time interval k_3. It also violated the minimum storage constraint in the period defined by the time intervals k_4 and k_6. This implies that the reservoir storage variation during optimal operation is defined by more than one trajectory.

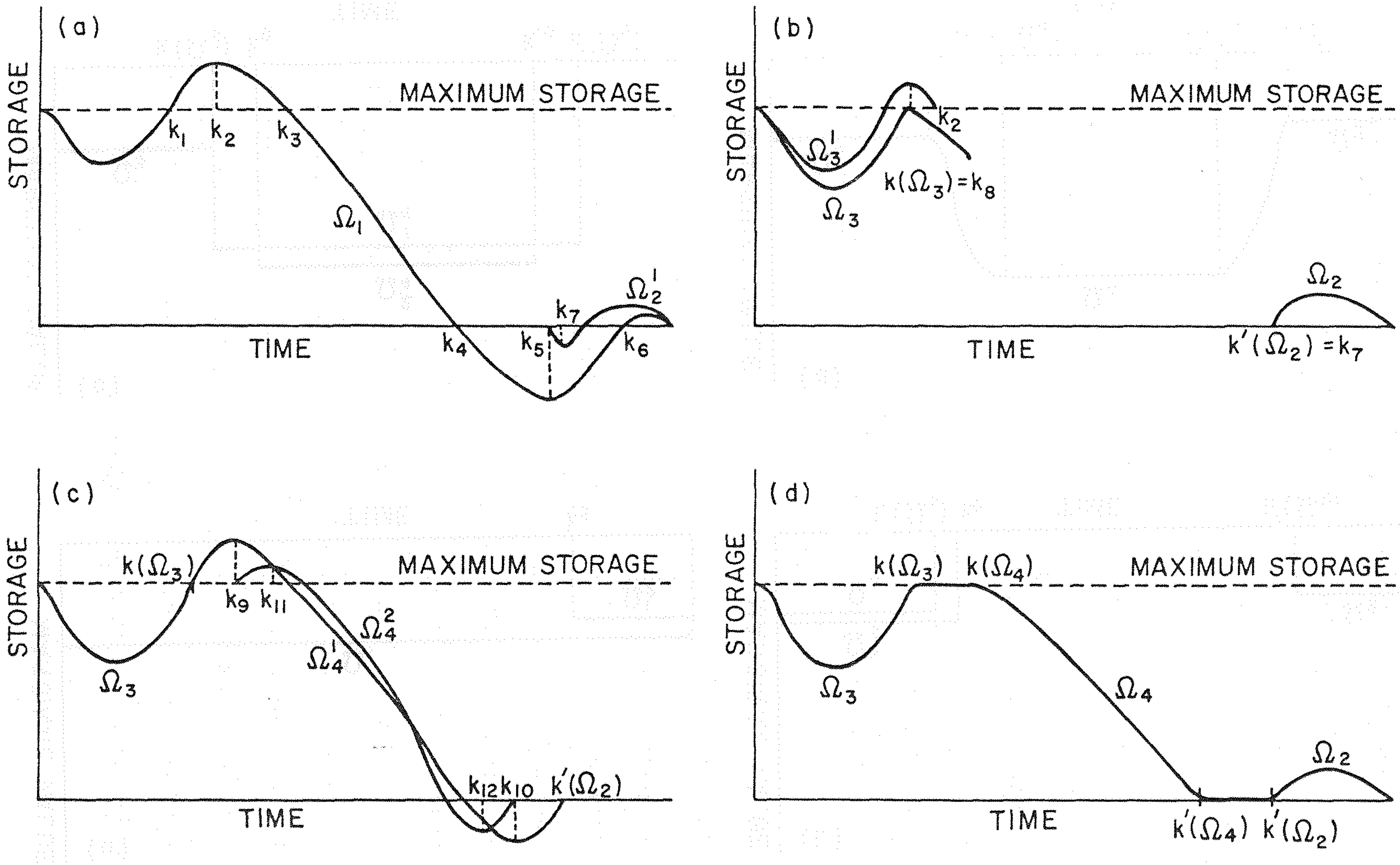


Figure 7. Algorithm Application, Example 1: Storage Variation

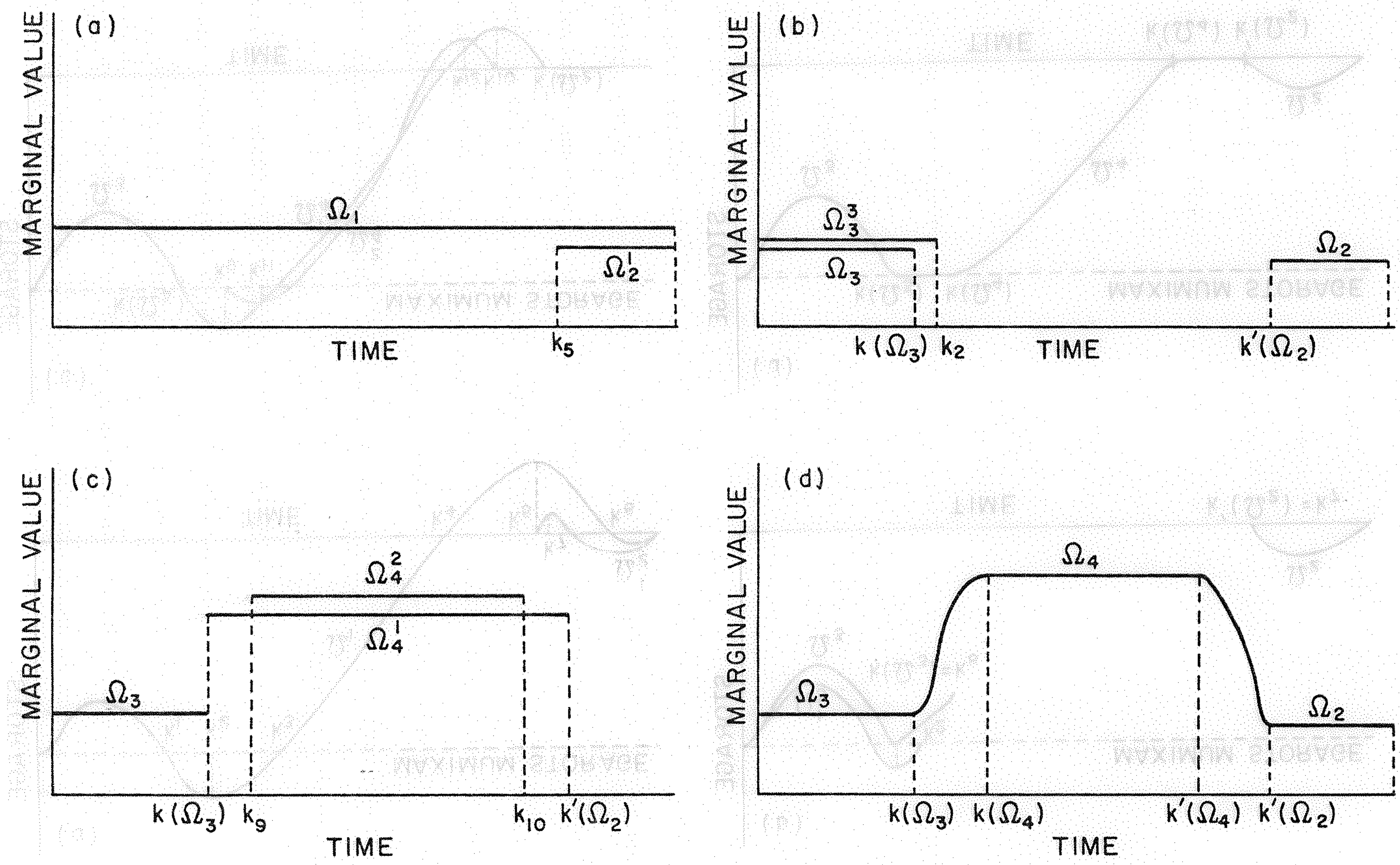


Figure 8. Algorithm Application, Example 1: Marginal Value Variation

At this point in analysis it may be helpful to recall the concept of temporal water transfer in single reservoir operation previously presented in section 4.1.2. The transfer of water to the future occurs when the storage increases. In this case the water entering in the reservoir is maintained for future use. No additional volume of water may be stored and consequently transferred to the future if the reservoir is full. Water may also be transferred to the past when given an initial solution for the operation it is modified by increasing the releases before a given time interval arbitrarily defined as the present. The limitation in this case developes when the reservoir is empty at the beginning of such a time interval.

The concept of water transfer may be used in the analysis of the violations of the trajectory Ω_1. This trajectory promotes the best transfer of water within the operating period if no storage constraint existed. Violation of the maximum storage constraint implies that less water can be transferred from the initial time intervals of operation to the future. Conversely, violation of the minimum storage constraints imply that less water can be transferred from the final time intervals of operation to the past. These restrictions cause an increase of the releases at the initial and final time intervals of operation and the resultant marginal instantaneous benefit of the releases during these time intervals will decrease.

At this point all elements required in the definition of optimal operation at the initial and final time intervals are available. Operation in the final time intervals is defined first. The objective in this case is to transfer the maximum quantity of water to the past without violating minimum storage constraints. This maximum transfer occurs when the reservoir is empty before the end of the operating period. Computation of the time interval of emptiness is made through an iterative procedure that uses the characteristics of the trajectories and the optimality conditions of operation.

Assuming the optimal operation in the period is given by a trajectory Ω_2, the following reasoning is used in the computation

1. As previously demonstrated, Ω_1 is greater than Ω_2 because releases in trajectory Ω_2 are greater than releases in trajectory Ω_1. The minimum value of trajectory Ω_2 must be equal to the minimum storage.

2. A previously mentioned characteristic of the trajectories refers to the time interval of local minimum values occurrence. It was shown that this time interval is a non-increasing function of the marginal instantaneous benefit of releases in the trajectory. Consequently, if Ω_1 is greater than Ω_2, the time interval k_5 (Figure 7.a) where local minimum of the trajectory Ω_1 occurs is a lower bound for the time interval when the local minimum value of trajectory Ω_2 occurs.

It is convenient to start the iterations with a lower bound for the time of occurrence of the minimum value of the trajectory Ω_2. A trajectory is computed linking the time interval k_5 to the final time interval. It is represented in Figure 7.a as the trajectory Ω_2^1. The minimum value of this trajectory occurs below the minimum storage. This indicates that the marginal instantaneous benefit of releases in trajectory Ω_2^1 is still greater than in trajectory Ω_2. However, trajectory Ω_2^1 defines a new lower bound for the minimum value of trajectory Ω_2. This is time interval k_7 when the minimum value of trajectory Ω_2^1

occurs. The iterative procedure continues defining a trajectory to link time interval k_7 to the final time interval. The resultant trajectory does not violate the minimum storage constraint (Figure 7.b). Any trajectory with marginal value less than that obtained in the last trajectory will have its minimum value above the minimum storage. In this case, it will not promote a maximum transfer of water to the past. Therefore the trajectory defined in the last iteration is the feasible trajectory that promotes the maximum transfer of water to the past intervals of operation. Therefore it is the optimal trajectory Ω_2. It may be noted that the iterative procedure proposed for computation of Ω_2 converges asymptotically to the optimal value through trajectories with marginal value greater than Ω_2. In other words, it converges through infeasible solutions to the trajectory Ω_2.

The objective in the initial time intervals of operation is to transfer the maximum quantity of water to the future intervals. This requires the reservoir be full sometime following the initial time interval of operation. Let the trajectory that defines the storage variation in these time intervals be trajectory Ω_3. The following reasoning is used in its computation

1. The releases in trajectory Ω_3 are greater than in trajectory Ω_1. Therefore Ω_3 is less than Ω_1. The maximum value of the trajectory Ω_3 is equal to the maximum storage.

2. The time interval of occurrence of local maximum values in a trajectory is a non-decreasing function of the marginal instantaneous benefit of the releases. Therefore, as Ω_1 is greater than Ω_3, the time interval k_2 (Figure 7.a and 7.b) is an upper bound for the time interval of occurrence of the maximum value in trajectory Ω_3.

Let the time interval when the maximum value of trajectory Ω_3 occurs be time interval $k(\Omega_3)$. The first estimate for this time interval is made on its upper bound, time interval k_2. A trajectory is computed linking the beginning of the initial time interval of operation to the beginning of time interval k_2. This trajectory is represented in Figure 7.b by trajectory Ω_3^1. It violates the maximum storage constraint. Therefore, Ω_3 is still less than Ω_3^1. However, trajectory Ω_3^1 defines another upper bound for $k(\Omega_3)$ at the beginning of time interval k_8 when a maximum value occurs. The new iteration computes a trajectory linking the initial time interval to time interval k_8 with maximum storage occurring at the beginning of both time intervals. The new trajectory does not violate any storage constraint (Figure 7b) Any trajectory with the marginal value less than this trajectory's marginal value will not have its final storage equal to the maximum storage. Otherwise, any trajectory with the marginal value greater than this trajectory's marginal value will violate the maximum storage constraint. Therefore trajectory Ω_3 is the feasible trajectory to promote the maximum transfer of water to the future time intervals.

The remaining period of operation starts at the beginning of time interval $k(\Omega_3)$ and ends at the beginning of time interval $k'(\Omega_2)$ (Figures 7.b and 7.c). The reservoir is full at its beginning and empty at its end. According to optimality conditions, the marginal instantaneous benefit of releases will increase during time interval $k(\Omega_3)$ and decrease during the time interval immediately preceding $k'(\Omega_2)$. Consequently, during these time intervals a transition occurs between the trajectories that define the storage variation of the reservoir. This transition follows local extreme (maximum or minimum) values of the trajectories. Under these conditions the inflows to the reservoir equal the releases.

The algorithm continues by computing a trajectory to link time interval $k(\Omega_3)$ to time interval $k'(\Omega_2)$ with their respective storages. The resultant trajectory is represented by trajectory Ω_4^1 in the Figure 7.c. This trajectory violates both storage constraints. Therefore it defines a situation similar to that which occurred with the trajectory Ω_1 (Figure 7.a). Trajectory Ω_4^1 would promote the most efficient transfer of water within the period if no storage constraint existed. Violation of the storage constraints indicates that less water can be transferred to a period inside the remaining period of operation. Consider the period of operation defined by the time intervals $k(\Omega_3)$ and k_9 (Figure 7.c). During this period maximum water transfer must be made to the future. If the reservoir can be maintained full during this period without the marginal instantaneous benefit of releases being decreased the solution will be optimum. Trajectories Ω_3 and Ω_4^1 have local maximum values occurring respectively at the beginning of the time intervals $k(\Omega_3)$ and k_9 while no minimum value occurs in these trajectories between these time intervals. As $\Omega_3 < \Omega_4^1$, trajectories with marginal values greater than Ω_3 and less than Ω_4^1 have local maximum values that occur between the time intervals $k(\Omega_3)$ and k_9. The sequence of occurrence of these maximum values is a non-decreasing function of the marginal value of their trajectories. Therefore, the referred optimal solution obtained with the reservoir full exists.

Another analogous solution exists for the period defined by the time intervals k_{10} and $k'(\Omega_2)$. The objective of the operation in this period is to transfer maximum water to the past. Therefore if the reservoir can be maintained empty during this period and the marginal instantaneous benefit of releases does not increase the solution is optimal. In this case, trajectories Ω_4^1 and Ω_2 have local minimum values occurring respectively at the beginning of time intervals k_{10} and $k'(\Omega_2)$. No local maximum value occurs in any one of the trajectories during the same period. As $\Omega_4^1 > \Omega_2$, trajectories with marginal values less than Ω_4^1 and greater than Ω_2 have local minimum values that occur between time intervals k_{10} and $k'(\Omega_2)$. The sequence of these minimum values is a non-increasing function of the marginal value of their trajectories. Therefore, the referred optimal solution with the reservoir empty exists.

The definition of such singular solutions restricts the remaining period of operation to that defined by time intervals k_9 to k_{10}. A new trajectory is defined for the operation in this period with violations similar to those of trajectory Ω_4^1. Therefore, it defines an extension of the singular solution to both sides of the period for more time intervals (Figure 7.c). Finally, a trajectory is computed to link the time intervals that define the remaining period of operation without violating the storage constraints (Figure 7.d).

Relative position of local extreme (maximum or minimum) values in trajectories Ω_3 , Ω_4 and Ω_2 defines the sequence of marginal values in Figure 8.d along the entire operation. Since the marginal values of the trajectories increase along the operation when the reservoir is full and decrease when empty, the solution is optimal.

A second example is presented in Figure 9. In this case an initial trajectory Ω_1 has its resultant operation tightened by the maximum storage constraints on two occasions. In other words, the optimal

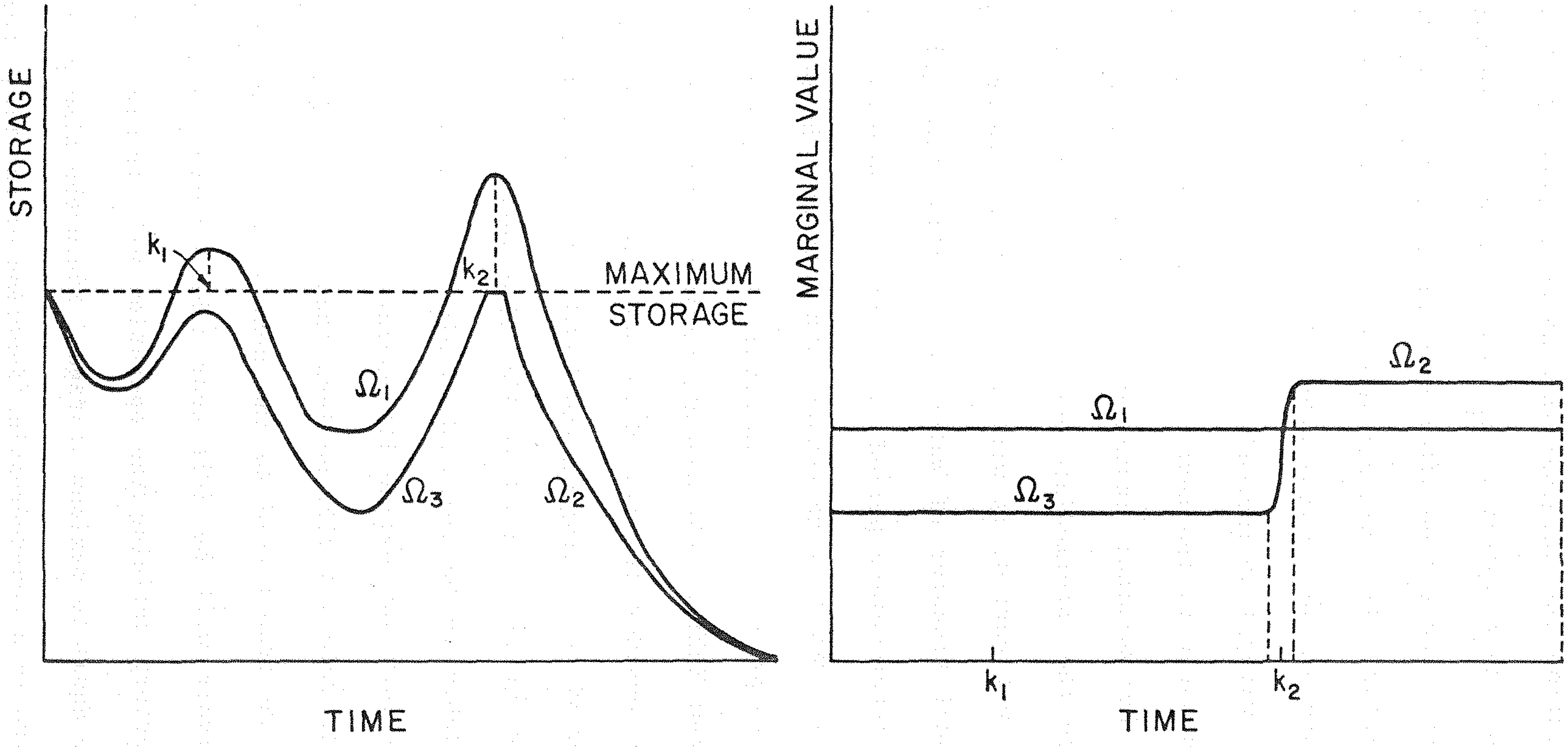


Figure 9. Algorithm Application, Example 2

transfer of water to the future is inhibited twice. In the optimal operation the marginal instantaneous benefit of releases will decrease before time interval k_1 since there is more water available for release. The period after time interval k_2 has less water to release. Therefore, the marginal instantaneous benefit of its releases will increase in the optimal operation. Finally, the intermediate period has less water transferred from the previous period, but it also transfers less water to the next period. Therefore, the effect of the variation of the marginal instantaneous benefit of releases is undetermined. Since the intermediate period has limitations in transferring water to the next period, it will have additional water available. In this case optimal transfer of water from initial time intervals of operation to the intermediate period may eventually be made without being limited by the maximum storage constraints. Hence, the period of operation which occurs prior to time interval k_1 cannot have its operation defined without taking into account operation in the intermediate period.

Consider the period of operation beginning with the initial time interval and ending at the beginning of time interval k_2. Since less water is transferred to the time intervals after k_2, more water is available to be released within this period. Therefore, if a unique trajectory may define its operation the marginal instantaneous benefit of its releases will be less than Ω_1. If a maximum quantity of water must be transferred to the next period in the optimal operation, the reservoir will be full near time interval k_2. In other words, the trajectory defining the operation will terminate with a local maximum value equal to maximum storage. Since the time of occurrence of a local maximum is a non-decreasing function of the trajectories' marginal value and the marginal value of this trajectory is less than Ω_1, time interval k_2 is an upper bound for the time of occurrence of the maximum value of the trajectory. Applying the iterative procedure defined in the previous example, a unique trajectory is eventually obtained for definition of the operation in the period (Figure 9).

The remaining period of operation begins with time interval k_2. Let the optimal trajectory in this period have a marginal value equal to Ω_2. Since the storage is maximum at the beginning of time interval k_2, Ω_2 is greater than Ω_1. Since the time of occurrence of a local maximum is a non-decreasing function of the trajectories' marginal value, time interval k_2 becomes the lower limit for the time of occurrence of the maximum value where trajectory Ω_2 begins. Therefore, the iterative procedure can be applied once more in computation of Ω_2 completing the computation of the operation in the example. Summarizing the operating period can be partitioned in two sub-periods of operation. The optimal operation of each of these sub-periods can be computed independently if a maximum transfer of water from the first to the second sub-period is promoted.

The third example is illustrated by Figure 10. In this case the optimal water transfer is restricted by the minimum storage constraint on two occasions. Therefore, water transfers to the past are restricted twice. Enforcement of the violated constraints results in less water available for the operating period prior to time interval k_1. This also implies that more water is available for the operating period after time interval k_1. Therefore, in the optimal operation the marginal instantaneous benefit of releases before time interval k_1 is greater than Ω_1 and the marginal instantaneous benefit of releases is less

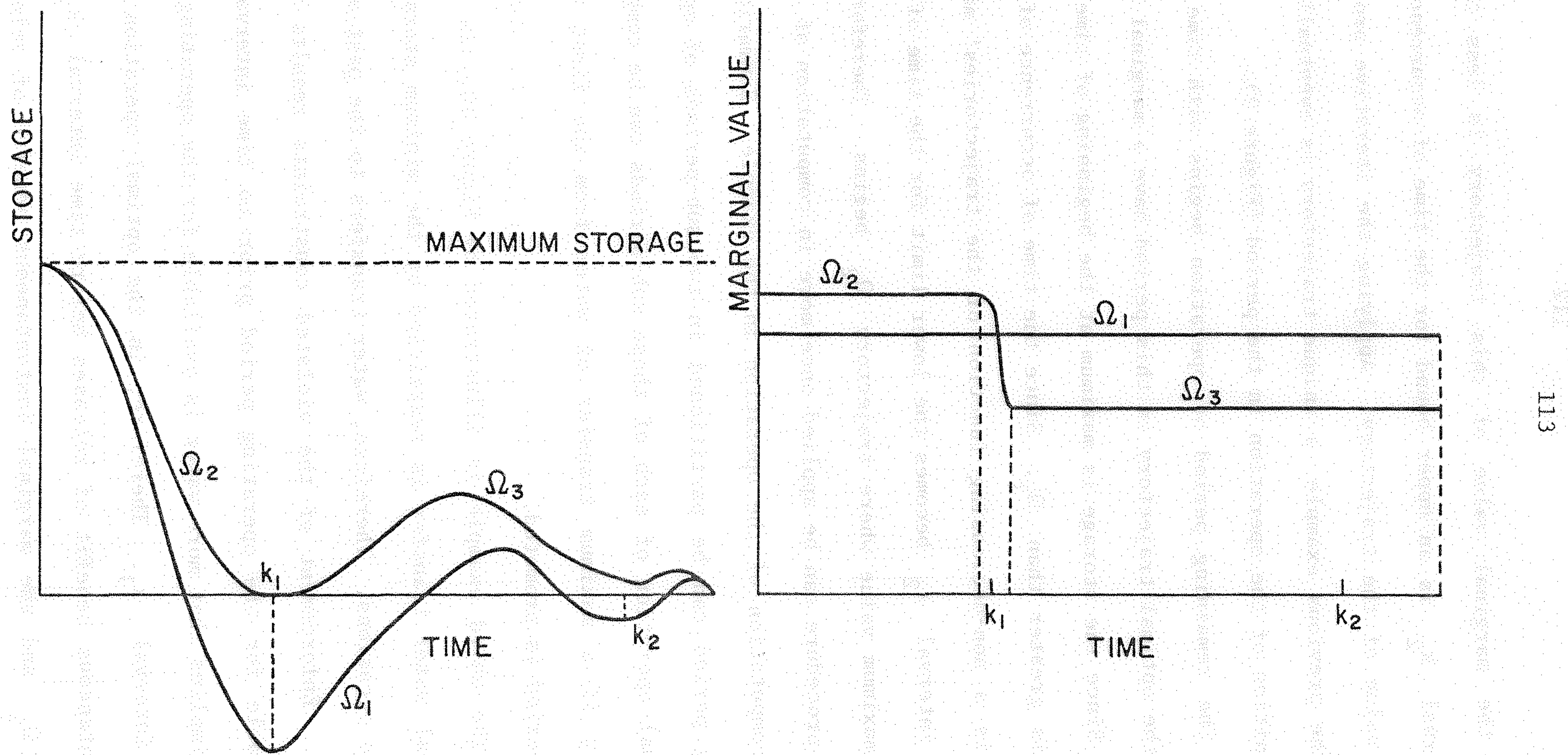


Figure 10. Algorithm Application, Example 3

than Ω_1 after time interval k_1. Operation of the period located after time interval k_2 depends on operation of the previous period. Therefore, the algorithm must be initially applied in the computation of the operation before and after time interval k_1.

The next step in the algorithm is identification of upper or lower limits for the time interval of occurrence of maximum or minimum values in the trajectories that define the operation in both periods. The same analysis applied in the two previous examples indicate that the time interval k_1 defines an upper bound for the trajectory to its left and a lower bound for the trajectory to its right.

Identification of the upper bound for the trajectory to the left of time interval k_1 permits computation of trajectory Ω_2 in Figure 10. The lower bound for the trajectory to the right of time interval k_1 determines trajectory Ω_3. In this case, water transfer in this period is not restricted by minimum storage constraints in proximity to time interval k_2.

The analysis performed utilizing the past three examples allows generalization in the procedure used to define periods where the operation may be independently computed. This generalization is considered in two examples given in Figure 11. In the first case (Figure 11.a), trajectory Ω_1 starts with multiple violations of maximum storage constraints and terminates with multiple violations of the minimum storage constraints. In this problem time intervals k_1 and k_2 define the partition of the entire period of operation in sub-periods that have their operation initially computed. The basis for this definition was stated in previous examples. For definition of k_1 refer to definition of k_2 in the second example (Figure 9) and for definition

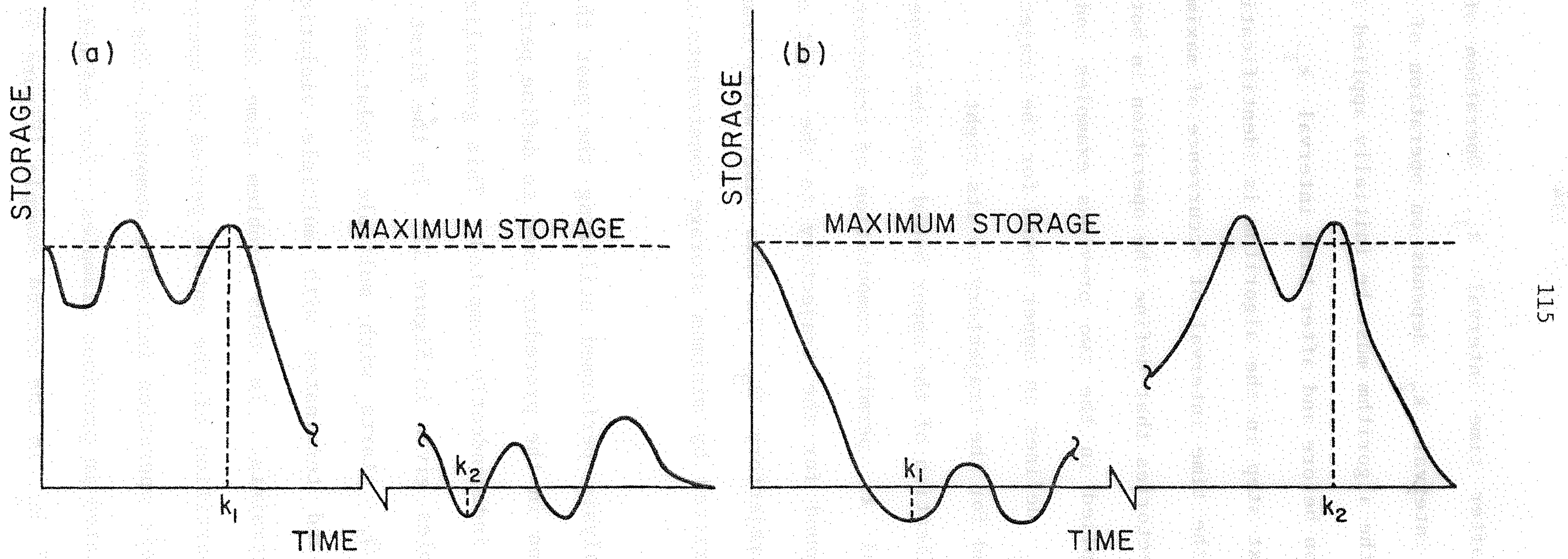


Figure 11. Algorithm Application, Example 4

of k_2 refer to definition of k_1 in the third example (Figure 10). The sequence of computation of optimal operation starts with the period before time interval k_1 and with the period following time interval k_2. The remaining period of operation, the intermediate period, has its operation computed in sequence.

In Figure 11.b trajectory Ω_1 starts with multiple violations of minimum storage constraints and ends with multiple violations of maximum storage constraints. Time intervals of partition are defined in this case by k_1 and k_2. For definition of k_1 refer to definition of time interval k_1 in the third example (Figure 10) and for definition of k_2 refer to definition of k_2 in the second example (Figure 9).

In conclusion, the general rule applied to the partition of periods is that when multiple violations of maximum storage constraints occur, the time interval of occurrence of the maximum value of the last violation defines the partition of the period. Conversely, when multiple violations occur with minimum storage constraints the partition is defined by the time of occurrence of the minimum value of the first violation. In both cases the optimization of water transfers in a constrained optimal operation is assured.

At this point it is interesting to identify similarities and differences between this algorithm and the one proposed by Laufer (1977) and Laufer and Morel-Seytoux (1979). This last algorithm assumes that the marginal instantaneous benefit of releases is given by an inverse exponential law. This assumption permits definition of a trajectory through the solution of an equation presenting marginal value of the trajectory as an explicit function of the summation of inflows and initial and final reservoir storages in a given period of operation.

The partition of the operating period is based in a delineation of each year of operation into drawdown and refill phases. Assuming the reservoir is full at the end of the season, each phase is separately optimized with violated constraints enforced stepwise. Correction of the final storage value is performed assuming in the following season the occurrence of a sequence of inflows equals the average value of recorded weekly inflows. Finally, the transition between drawdown and refill phases is tested. A later date of emptiness of the reservoir is assumed when the marginal instantaneous benefit of releases does not decrease after the minimum storage, as it is required in the optimal operation.

The seasonal characteristics of the hydrologic processes in alpine regions determined that the reservoir is usually full and empty in a very restricted range of pre-defined weeks. It makes the partition proposed by the algorithm viable and efficient in the determination of the optimal operation.

In the case being studied the function of the marginal instantaneous benefit of releases cannot be represented by an inverse exponential law. Hence, the marginal value of a trajectory must be computed by an iterative procedure. However if an inverse exponential law produced a good fitting of the function the same procedure used in the Laufer-Morel-Seytoux algorithm may be applied. Therefore, under these conditions the algorithms are identical although the present situation is more general.

The seasonal characteristic exploited in the Laufer-Morel-Seytoux algorithm cannot be extrapolated elsewhere without previous analysis. In the present study carryover storage is expected to be highly utilized in order to optimize water transfers. The dates of extreme storages are expected to randomly occur. Therefore, the partition of the operating period proposed in the Laufer-Morel-Seytoux algorithm seems to be not applicable to this study. As an alternative, a procedure of partition is proposed based on the analysis of a totally relaxed solution where no storage constraint is considered. This prevents the occurrence of non-optimal sequence of marginal instantaneous benefits of releases in the operation. In other words, the Laufer-Morel-Seytoux algorithm's test for transition between drawdown and refill phases is not required in the proposed algorithm.

A simplified version of the presented algorithm was coded in FORTRAN. In this version the last time interval of partition (time interval k_1, Figure 12) is initially identified. Next, the relaxed operation to the left of this time interval is computed and another partition is defined by the time interval k_2. This procedure is successively applied until a feasible (and optimal) trajectory is computed for the beginning of operation (trajectory Ω_2, Figure 12). The remaining operating period starts at the end of the computed trajectory (time interval k_3, Figure 12) and ends with the final time interval of the entire operating period. Its operation is defined by identical approach. The simplified version does not utilize other computational artifices presented in the discussion of the complete version of the algorithm. Therefore, the simplified version requires less programming effort but will probably require more computation time.

A limited number of tests were performed to evaluate computational efficiency of the algorithm. In Table 7.a each test is presented according to the number of time intervals of operation, the reservoir useful capacity, and the tolerance of convergence of each trajectory to

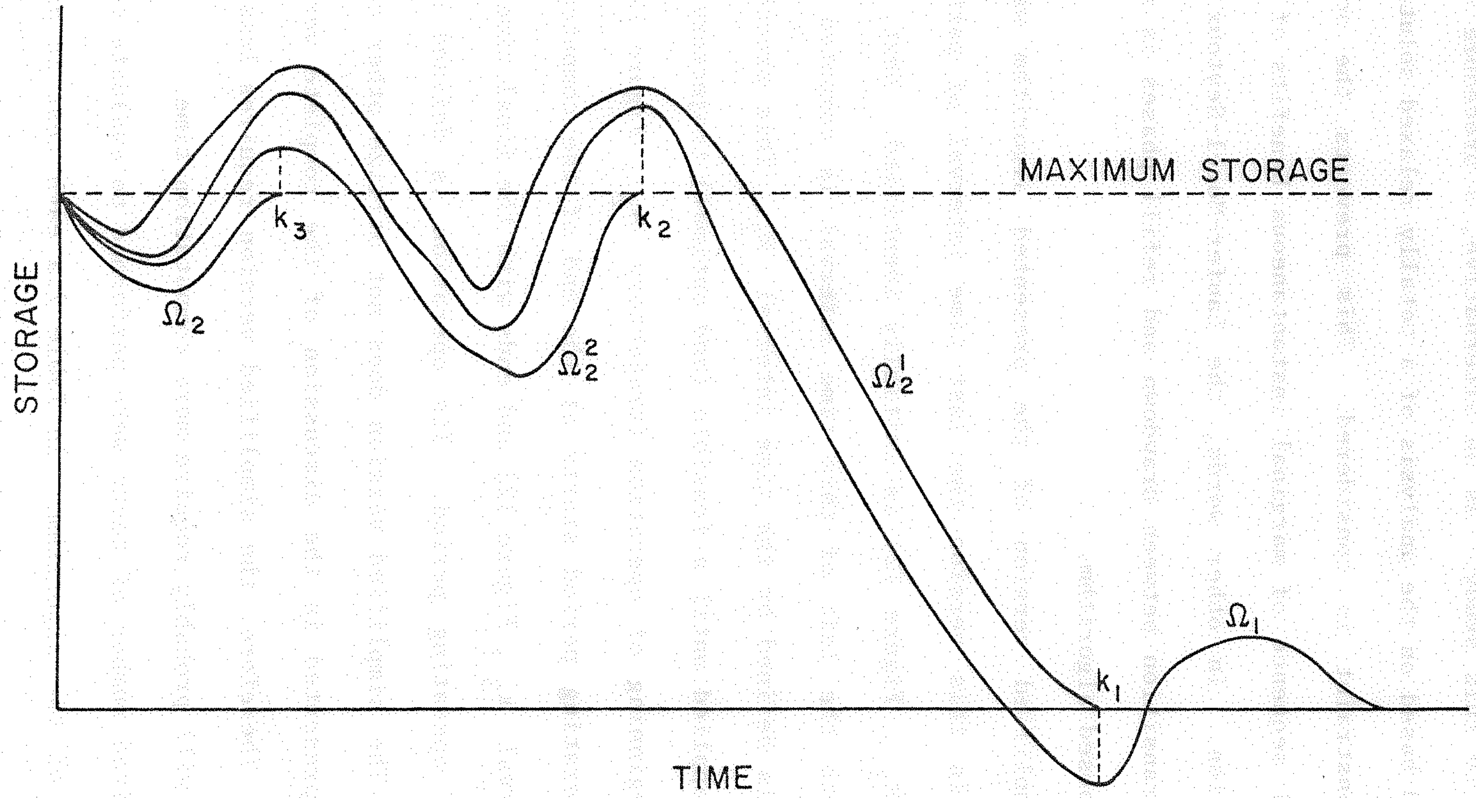


Figure 12. Algorithm's Simplified Version

the final storage. The Central Processing time (CP time) for each test is presented with the number of trajectories that defined the optimal operation. The first two tests used simulated data. The remaining tests are based on applications to the case study. In these tests, the time interval of operation equals one week.

In Table 7.b the same problems of test 1 and 2 were solved by a Discrete Dynamic Programming code (Labadie, 1977). The results verify the computational efficiency of the proposed algorithm. The Cumulative Minimum Surplus Analysis was not used in the tests since it may be applied independently of the algorithm used to define the optimal operation.

The memory requirements of the algorithm are low and independent of the tolerance value used. The vector of the initial trajectory Ω_1 with length equal to the number of time intervals must be stored. The optimal reservoir storage variation may be optionally stored in the same vector. Finally, memory space is required for a vector containing the optimal sequence of marginal values with dimension not greater than the number of time intervals. The remaining memory requirements refer to data. Therefore, they are common to any algorithm. The memory requirements of the algorithm are approximately 1/M of the memory requirements of the Discrete Dynamic Programming. M is the average number of discrete intervals representing the state variable in the Discrete Dynamic Programming.

The algorithm is based on the analysis of the marginal values of alternative decisions under constrained operation. Therefore, it will be referred to as the Constrained Marginal Analysis.

TABLE 7

a. Constrained Marginal Analysis tests.

run	time intervals (weeks)	reservoir capacity (hm^3)	tolerance (hm^3)	number of trajectories	CP time (sec)
1	36	20600	20	3	0.56
2	40	20600	20	14	1.15
3	92	42700	20	2	1.17
4	132	42700	20	4	1.83
5	137	42700	20	1	1.43
6	182	42700	20	10	3.84
7	248	42700	20	3	2.71
8	1377	42700	200	28	53.27
9	1377	42700	20	48	72.11

b. Dynamic Programming tests.

run	time intervals (weeks)	reservoir capacity (hm^3)	interval of discretization (hm^3)	CP time (sec)
1	36	20600	200	97.19
2	40	20600	200	105.99

1) Computer used: Cyber 172 of Colorado State University.

2) The interval of discretization on Table 7.b refers to the discrete representation of the state variable in the Dynamic Programming approach.

5.2 Multiple Reservoir Operation Optimization

In the theoretical analysis of optimal conditions of operation periods of water surplus are identified. In these periods water inflow into the reservoir is sufficient to allow its operation with maximum release. An algorithm was developed to identify such periods in the operation of a single reservoir. This algorithm, referred to as Cumulative Minimum Surplus Analysis, may have its application extended to the case of operation of multiple reservoirs. In this extension some particularities of a system of multiple reservoirs have to be considered.

If for instance the system is composed of only unlinked reservoirs, each reservoir is analyzed individually to identify isolated water surplus periods. When the reservoirs are serially linked the analysis will be applied to the upstream reservoirs first. During identified periods of water surplus it is possible to define the inflow to the immediately downstream reservoir originated from releases from the upstream reservoir. Therefore, the downstream reservoir may be analyzed only during periods of water surplus in the upstream reservoir.

When a reservoir has a period of water surplus the generated energy in this reservoir is first used to supply the demand since it would otherwise be lost by water spills. Consequently the problem is reduced to defining operation of the remaining reservoirs in order to provide the remaining energy demand. The remaining energy demand is given by energy demand less the part of the demand provided by reservoirs in periods of water surplus. Eventually, the remaining energy demand is null. In this case, the operation of the system is trivially defined.

In the operation of multiple reservoirs the marginal instantaneous benefit of the releases in each reservoir depends on the operated releases in all reservoirs of the system. Its mathematical formulation is

$$\frac{\partial}{\partial w(j,t)} \left\{ \sum_{t=1}^{52} F_t[\dot{x}(t)] \right\} = - \text{alpha}(j,t) * f[t, D(t) - \sum_j \text{alpha}(j,t) * w(j,t)] \qquad (5.22)$$

where the terms on both sides of the equation represent the first derivative of the objective function with respect to the release from the reservoir j during the t-th time interval or, as previously defined, the marginal instantaneous benefit of the releases from the reservoir j during the t-th time interval. The term $\dot{x}(t)$ is the total hydroenergy generation during the t-th time interval, which is given by

$$\dot{x}(t) = \sum_j \text{alpha}(j,t) * w(j,t) \qquad (5.23)$$

where w(j,t) is the release from the reservoir j during the t-th time interval and alpha(j,t) is the energy conversion coefficient in the reservoir j during the time interval t. This coefficient is a function of hydraulic head during the t-th time interval, hydraulic losses and efficiency of the operation of the power plant in reservoir j.

The function $f[t, D(t) - \sum_j \text{alpha}(j,t) * w(j,t)]$ is the marginal cost of operation during the t-th time interval. It depends on the thermoenergy generation (given by the expression in brackets or by the difference between the energy demand D(t) and the total hydroenergy generation) during the same time interval.

It may be noted that the marginal instantaneous benefit of releases in each reservoir is proportional to the marginal cost of operation (Equation 5.22). The proportionality factor is given by the energy conversion coefficient alpha(j,t). The hydraulic head variation during a weekly time interval can be considered insignificant in any reservoir of the system. Therefore, given a total hydroenergy generation value in the system, the marginal instantaneous benefit of release in a reservoir is defined independently of the reservoir's release. This characteristic is also derived from a property of operation of interconnected hydropower systems that the hydroenergy provided in a given time interval implies the same benefit independently of which reservoir it is produced from.

At this point of analysis it is helpful to recall the optimality conditions of operation derived in Chapter 4. The optimal variation of marginal instantaneous benefit of releases in each reservoir of a multiple reservoir system is similar to the variation in single reservoir operation during periods with no water surplus. The marginal instantaneous benefit is constant in each reservoir as long as there is no extreme (maximum or minimum) storage simultaneously in all reservoirs. It increases only after all reservoirs simultaneously reach the maximum storage and it decreases only after all reservoirs simultaneously reach the minimum storage. This interrelated variation of the marginal instantaneous benefit of the releases is not surprising, though. Equation 5.22 shows that the variation does not occur under other conditions since it is function of the total hydrogeneration and not of the isolated reservoir energy production.

Variation of the marginal future benefit of storage and of the marginal benefit of water transfer in serially linked reservoirs has characteristics dependent on the referenced reservoir. These marginal values determine the optimal release from a reservoir in situations 1) when the reservoir is operated under a period of surplus of water and 2) when the chain of the extreme storages occurs. In this last situation, the upstream reservoirs have their operation defined to optimize the transfer of water between reservoirs. It occurs during a period prior to simultaneous extreme storage occurrence. Nevertheless, the marginal instantaneous benefit of the releases remains proportional to the marginal operating cost of the system.

Relative freedom in the partition of total hydroenergy generation among the reservoirs of the system suggests a simplified approach for determining optimal operation. It is based on the premise that the optimal hydroenergy generation in the system is not significantly related to each reservoir's storage and inflows but depends mainly on the total value of these figures. However, the total water storage and inflow to the system are poor estimators of the system's actual hydroenergy generation capability. For instance, the same volume of water produces different amounts of energy in different reservoirs due to the distinct hydraulic heads, losses and efficiencies. Moreover, a volume of water stored in the upstream reservoir of a serially-linked reservoir system has more "energetic value" than the same volume stored in the downstream reservoir. Therefore, the total storage and inflows to the system must be expressed by energy equivalent values in order to be good estimators of the system's energy capability.

The result is a formulation of an aggregated representation for the system. In this formulation a unique reservoir with a given energy storage capacity receives energy inflows and promotes energy releases. Computation of numeric values for these variables depends on the hydraulic head variation in each reservoir. As the hydraulic heads are adjusted iteratively the formulation of the aggregated representation must be updated at each iteration.

Optimal operation of the unique, equivalent reservoir is defined by the Cumulative Minimum Surplus Analysis and by the Constrained Marginal Analysis. Results of the operation are presented in terms of the total hydro energy generation in each time interval.

The partition of these total hydro energy generation values among the reservoirs of the system is defined by a disaggregation procedure that is based on a heuristic criterion. This criterion seeks the enforcement of an "optimal state" (Hall, 1971) of water storage distribution among the reservoirs. In this case, optimal state was considered as a situation that minimizes energy lost from the system due to spills.

The aggregation and disaggregation procedures are presented in the following sections. They were obtained experimentally based on tests performed on deterministic operation studies.

5.2.1 The Aggregated Formulation

The equivalent reservoir storage is supplied by an energy inflow. The stored energy may be released up to its maximum value that is defined by the hydraulic head and the system's installed capacity. If the reservoirs have fixed hydraulic head transformation of water to energy is trivial. Under variable hydraulic head the energy produced by a given water volume has to be estimated under the hydraulic head that is expected when the given water volume is released. This introduces some complexities into the transformation between water inflows and resulting energy production.

Assume an unlinked reservoir has storage $s(t)$ at the beginning of time interval t and that the respective hydraulic head is computed by the I-degree polynomial:

$$h[s(t)] = \sum_{i=0}^{I} [A(i) * s(t)^{i}] \tag{5.24}$$

where A(i), i = 0, 1, ..., I are the polynomial's parameters.

The maximum hydraulic head is represented by

$$Hmax = h[Smax]$$

where Smax is the maximum storage in the reservoir.

Assuming an elementary volume of water ds(t) is released through the turbines the resultant energy production de(t) is given by

$$de(t) = coef * ds(t) * h[s(t)] = coef * \sum_{i=0}^{I} [A(i) * s(t)^{i}] * ds(t) \tag{5.25}$$

where coef is a constant coefficient.

If a non-elementary volume of water Δs(t) is utilized the resultant energy production is given by the integral

$$\Delta e(t) = \int_{s(t)-\Delta s(t)}^{s(t)} de(t) = coef * \left\{ \sum_{i=0}^{I} A(i) * (i+1)^{-1} * \{s(t)^{i+1} - [s(t) - \Delta s(t)]^{i+1}\} \right\} \tag{5.26}$$

Therefore, the total energy storage in the reservoir at the beginning of the time interval t is given when Δs(t) = s(t) - Smin, where Smin is the minimum storage

$$e(t) = coef * \sum_{i=0}^{I} \{A(i) * (i+1)^{-1} * [s(t)^{i+1} - Smin^{i+1}]\} = E[s(t)] \tag{5.27}$$

where E[s(t)] is the total energy storage when the water storage in the reservoir is s(t).

If s(t) = Smax, then e(t) = Emax, the reservoir useful energy storage capacity.

Let the equivalent hydraulic head at the beginning of time interval t be defined by

$$\bar{h}(t) = e(t)/\{[s(t) - Smin] * coef\} \tag{5.28}$$

where $\bar{h}(t)$ is the equivalent hydraulic head at the beginning of time interval t. It represents the constant value of hydraulic head which computes the same energy produced under variable hydraulic head when the total storage is released.

The corresponding maximum equivalent hydraulic head is given by

$$\bar{H}max = Emax/\{[Smax - Smin] * coef\} \tag{5.29}$$

Transformation of water to energy inflow is computed by the following procedure. Suppose a volume of water q(t) enters the reservoir during the time interval t. The stored water will increase from the beginning to the end of the time interval t from s(t) to s(t) + q(t). The stored energy will increase from e(t) to e(t) + qe(t) in the same time interval. Therefore, qe(t) is the corresponding energy inflow during the time interval.

If s(t) + q(t) is less than the storage capacity Smax all the water can be stored. The value of qe(t) is given by

$$qe(t) = E[s(t) + q(t)] - E[s(t)] \tag{5.30}$$

If q(t) is small relative to s(t), the equivalent hydraulic head can be considered invariant during the time interval t. Therefore

$$qe(t) \cong coef * [s(t) + q(t)] * \bar{h}(t) - coef * s(t) * \bar{h}(t) \quad \text{or}$$

$$qe(t) \cong coef * q(t) * \bar{h}(t) \tag{5.31}$$

If s(t) + q(t) is greater than the storage capacity Smax, the water inflow can be stored up to Smax. The remaining inflow must be released with the maximum hydraulic head or it will spill. Thus, the total energy inflow to the reservoir is formed by a storable and a non-storable part. The storable inflow is given by

$$qe_s(t) = coef * [Smax - s(t)] * \bar{h}(t) \cong coef * [Smax - s(t)] * \bar{H}max \quad (5.32)$$

The non-storable part is given by

$$qe_n(t) = coef * [s(t) + q(t) - Smax] * Hmax \quad (5.33)$$

If the reservoir has downstream run-of-river power plants Equations 5.27, 5.32, and 5.33 become Equations 5.34, 5.35, and 5.36 respectively.

$$e(t) = coef(J) * \sum_{i=0}^{I} \left\{ A(J,i) * (i+1)^{-1} [s(J,t)^{i+1} - Smin(J)^{i+1}] \right\} + \sum_{j \varepsilon JD} \left\{ coef(j) * s(J,t) * H(j) \right\} = E[s(J,t)] \quad (5.34)$$

(energy storage)

$$qe_s(t) \cong coef * [Smax(J) - s(J,t)] * \bar{h}(J,t) + \sum_{j \varepsilon JD} \left\{ [coef(j) * H(j)] \right\} * [Smax(J) - s(J,t)] \quad (5.35)$$

(storable water inflow)

$$qe_n(j,t) = coef(j) * [s(J,t) + q(J,t) - Smax(J)] * Hmax(J) + \sum_{j \varepsilon JD} \left\{ coef(j) * [s(J,t) + q(J,t) - Smax(J) + q(j,t)] * H(j) \right\} \quad (5.36)$$

(non-storable water inflow)

where JD is the set of the values of j representing the run-of-river power plants downstream of the reservoir-plant J,

- s(J,t) is the water storage in the reservoir-plant J at the beginning of the time interval t
- H(j) is the fixed hydraulic head in the run-of-river power plants,
- q(J,t) is the water inflow to the reservoir-plant J during the time interval t, and
- q(j,t) is the water inflow to the run-of-river plant j during the time interval t.

In the case of serially-linked reservoir with hydroplants no rigorous solution for such transformations is available. The water inflow to an upstream reservoir has the energy value related to the downstream reservoir estimated only upon arrival there. It will be function of the storage that then occurs in the downstream reservoir. However this storage is function of the future releases from the downstream reservoir that are unknown when the water arrives in the upstream reservoir. An approximate transformation is that the water arriving at an upstream reservoir during a time interval is assumed to be operated downstream with the equivalent hydraulic head prevailing in the downstream reservoir at the beginning of the same time interval. In this case the equation for computation of the stored energy $e(t)$ is identical to Equation 5.34 except for the term $H(j)$. This term represents now the equivalent hydraulic head in the downstream reservoir given by Equation 5.28.

It was also assumed that in the case of water inflow exceeding storage space in the upstream reservoir it will be turbinated downstream according to the following procedure

1. the storable water inflow will be turbinated downstream with the maximum equivalent hydraulic head (Equation 5.32).

2. The non-storable water inflow will be turbinated downstream with the maximum hydraulic head.

The remaining energy demand to be provided by the aggregated system is determined by subtracting the non-storable energy inflow from the energy demand on the system. The equivalent reservoir energy storage capacity is calculated by summation of the individual energy storage capacities in each reservoir. The maximum energy release capacity is computed as the total energy which will be produced if the maximum water releases (section 3.2.2) were operated in each reservoir. The minimum release commitment is not considered in the aggregated formulation. During the disaggregation procedure however, it will be enforced always when possible, according to the actual meaning of such a commitment.

5.2.2 A Criterion of Disaggregation

The purpose of the disaggregation procedure is to distribute the total energy production for each time interval among each reservoir. This energy production is computed in the aggregated system's operation optimization. Some heuristic rules based on the enforcement of an ideal distribution of space available in each reservoir were tested before the criterion of disaggregation was selected. Being heuristic, the rules cannot provide the optimal disaggregation. However it is shown experimentally that the chosen rule gives results very near the true optimal disaggregation for the case study. Additionally, the rule is computationally tractable and allows consideration of some requirements of operation not viable for presentation in a standard Mathematical Programming formulation. Finally it can be employed in operation with deterministic or uncertain future inflows.

The criterion of disaggregation is enforcement, when possible, of the following relation for each time interval in any reservoir:

$$\frac{Smax(j) - s(j,t)}{E[q(j,t+1)] + qu(j,t+1)} * \frac{Smax(j) - s(j,t+1)}{Smax(j) - Smin(j)} = \text{Constant} \qquad (5.37)$$

where $Smax(j) - s(j,t)$ is the difference between the maximum and the actual water content in reservoir j at the beginning of time interval t or the space available in reservoir j at the beginning of time interval t; $E[q(j,t+1)]$ is the expected value of the water inflow to reservoir j during time interval $t+1$; $qu(j,t+1)$ is the water inflow to reservoir j during time interval $t+1$ caused by previous release in the upstream reservoir; $Smin(j)$ is the minimum storage in reservoir j.

The criterion promotes an increase in the available storage space in a reservoir at the beginning of the next time interval which is proportional to the expected total inflow to the reservoir during this next time interval. Its basic objective is to avoid spills.

The water balance equation states

$$s(j,t+1) = s(j,t) + q(j,t) + qu(j,t) - w(j,t) - spill(j,t) \quad (5.38)$$

where $s(j,t+1)$ is the water storage in reservoir j at the beginning of time interval $t+1$;

$s(j,t)$ is the water storage in reservoir j at the beginning of time interval t;

$q(j,t)$ is the water inflow to reservoir j during time interval t;

$qu(j,t)$ is the water inflow to reservoir j during time interval t caused by releases from a reservoir upstream of reservoir j;

$w(j,t)$ is the water release from reservoir j during time interval t;

and $spill(j,t)$ is the water spill in reservoir j during time interval t.

The reservoir storage constraints are

$$Smin(j) \leq s(j,t+1) \leq Smax(j) \quad (5.39)$$

where $Smin(j)$ is the minimum storage in reservoir j

and $Smax(j)$ is the maximum storage in reservoir j.

The water release constraints state

$$w(j,t) \leq Wmax(j,t) \quad (5.40)$$

where $Wmax(j,t)$ is the maximum release from reservoir j during the time interval t, which is function of the hydraulic head in the interval.

$$w(j,t) \geq Wmin(j) \quad \text{if and only if}$$

$$s(j,t) + q(j,t) + qu(j,t) \geq Wmin(j) + Smin(j) \qquad (5.41)$$

where Wmin(j) is the minimum release commitment in reservoir j. This conditional imposition states that the minimum release will be enforced if and only if enough water is available in the reservoir.

Finally, total energy production is computed by

$$\sum_j coef(j) * w(j,t) * h(j,t) + xloss(t) - xdef(t) = x(t) \qquad (5.42)$$

whre coef(j) represents a constant coefficient

xloss(t) is the excess of energy produced during time interval t

xdef(t) is the deficit in energy production during time interval t.

The Equation 5.42 states that total energy production must equal the energy production established by the optimization of the aggregated system's operation, unless the water balance equation implies an excess or deficit of energy production. Energy deficit or loss are to be avoided whenever possible.

The chain of extreme storages must also be considered in the disaggregation in order to optimize temporal and spatial water transfers. The enforcement of this chain is complicated in a strategic operation since an overall extreme storage in the system has just a chance of occurring. Consequently, a reservoir with downstream linkages will have only a probability of reaching the corresponding extreme storage before the occurrence of overall extreme storage in the optimal operation. In the case being studied this fact is not very significant since the travel times of the water between reservoirs are small, not exceeding one time interval (one week). Hence, the non-enforcement of the chain of extreme storages in serially-linked reservoirs will not significantly affect the results.

The disaggregation procedure starts by computing the maximum release capacity in each reservoir as defined in the section 3.2.2. It is assumed the maximum release computed at the beginning of the time interval will persist through the interval. This assumption is acceptable when hydraulic head variations are not significant in each time interval, what may eventually occur if the time interval is small. The total water inflow to each reservoir is computed as the summation of the inflow q(j,t) and the inflow caused by upstream releases, qu(j,t).

The computation of qu(j,t) involves two cases. When the travel time of the water is equal or greater than one time interval qu(j,t) is known at the beginning of time t. When travel time is less than one time interval, qu(j,t) is not known since it depends on the release being computed for the upstream reservoir. An iterative adjustment of qu(j,t) is performed in this case. Initially it is set equal to the release in the upstream reservoir during the previous time interval. Then the disaggregation is performed and the estimated qu(j,t) is compared with the computed one. A new iteration is done if the difference between estimated and computed values is greater than a tolerance value. The estimated value of qu(j,t) in the new iteration is the computed value of qu(j,t) in the previous iteration.

The expected total water inflow in the following time interval is given by the summation of the expected inflow E[q(j,t+1)] and the inflow caused by upstream releases qu(j,t+1). The expected inflow is defined by a forecasting model based on statistical analysis.

At this point relation 5.37 may be defined for each reservoir forming a system of N - 1 linear equations where the releases in each one of the N reservoirs are the unknown variables. The Equation 5.42 gives the last linear equation required to define the values of the releases. Some constraints are present, however. Minimum releases in each reservoir are defined as the maximum between the minimum release commitment and the minimum release in each reservoir to avoid spill. If these minimum values are able to fulfill the total required energy production the disaggregation is therefore defined and there is no need to solve the linear system of equations. In the same way, the maximum releases in each reservoir are computed as the minimum between the maximum release capacity and the total water availability in each reservoir. This last term is given by the summation of initial water content and the total inflow during a given time interval. In serially-linked reservoirs the release is also limited in order to avoid spills in the downstream reservoir. If the maximum energy production corresponding to the maximum releases is less or equal to the required energy production the disaggregation is defined and there is no need to solve the linear equation system. These two previous extreme cases correspond to an eventual occurrence of spills in the first case and deficits in the second case. In order to avoid or decrease the deficits in the energy production, the reservoirs with downstream power plants are allowed to spill water in some plants as long as the other plants are still producing energy below their maximum capacities. In the joint operation of the run-of-river plants with upstream reservoir plant (section 3.2.3) when the reservoir plant spills water, the spill will be used to produce energy downstream, if possible.

For intermediate cases the determination of releases from each reservoir is accomplished by solving the linear equation system. The computed releases which fall outside the range formed by the minimum and maximum releases are equalled to the nearest limit value. The resultant energy production is computed. If the energy production is greater than the requirement the releases above their minimum limit are decreased. This decrease is defined in the inverse proportion of the expected water to enter the reservoir in the next interval and in the direct proportion of the space available in the reservoir. Otherwise, if the energy production is less than the requirement, the releases below their maximum limit are increased. This increase is made in the direct proportion of the expected water inflows in the next interval and in the inverse proportion of the space available in the correspondent reservoir. This second order adjustment is repeated iteratively until the releases are feasible and the produced energy equals the requirement for that time interval.

5.2.3 Comments

The aggregation-optimization-disaggregation procedure for the optimization of multiple reservoir systems allows the reduction of the dimensionality of the optimization problem. It also provides a way to consider the optimal operation requirements and criteria which can not be considered in a standard Mathematical Programming formulation. To obtain these computational and practical advantages the procedure has to pay the price of derivation of relaxed solutions. A relaxed solution is caused by the aggregated representation of the reservoirs' constraints in the aggregated formulation. It may result in violations of the maximum release constraints and of the minimum and maximum reservoir storage constraints in the operation of the real system. As a consequence the computed optimal energy production level and the final equivalent reservoir content obtained in the optimization of the aggregated system may not be achieved in the real operation. Hence, during low storage periods the actual energy production may be below the required level. During high storage periods, energy surplus and consequent spills may occur. Since these spills are not considered in the aggregated formulation, it causes an overestimate of the water actually stored in the system.

A characteristic of relaxed solutions is that the computed minimum cost of operation represents a lower bound for the real theoretical minimum cost. It provides an economic criterion which can be used to verify the efficiency of the aggregation-optimization-disaggregation procedure in the derivation of near-optimal solutions.

5.3 Application

The aggregation-optimization-disaggregation procedure was applied to the computation of the optimal deterministic operation of the studied multiple reservoir system. Observed series of inflows into the reservoirs for the period from 1931 to 1966 were used in this application. The optimal operation was computed as if the characteristics of the system in 1983 were invariable during a 36 year period. The inflows during this period were identical to the ones observed from 1931 to 1966. These assumptions obviated the necessity of establishing the initial and final reservoir storage for each annual inflow series. It permitted study of the characteristics of the optimal operation during several probable series of inflows and storages in the reservoirs.

Initial and final storage values were assumed for the beginning of the 1931 inflow series and for the end of the 1966 inflow series. Several initial and final storage values were tested. The application of the Cumulative Minimum Surplus Analysis to each test verified that 1931 and 1966 were years of water surplus in the system. This is a valid conclusion regardless of the initial and final storages in the reservoirs. Consequently, the optimal operation during these years will be defined by the maximum release tactics. And the initial and final reservoir's storage have little significance in the study. Therefore, the initial storages were assumed with the reservoirs being 73 percent full. The final storage in each reservoir were maximum.

The initial estimate of the hydraulic head variation in each reservoir was given by the rated heads in each reservoir. The reservoir's energy storage capacities are independent of the assumed hydraulic heads. Their values are presented in Table 8.

The aggregation-optimization-disaggregation procedure was applied using the following steps

STEP 1. Given: estimated hydraulic heads

Compute: storable energy inflow
non-storable energy inflow
maximum energy release capacity for the aggregated formulation

STEP 2. Given: the aggregated formulation of step 1

Compute: the periods of water surplus by the Cumulative Minimum Surplus Analysis

STEP 3. Given: The water inflows in each reservoir
the maximum water release in each reservoir

Table 8. Energy storage capacity in the system.

Reservoirs	Energy storage capacity related to each power plant (Eq. MW)									
	PR	JC	IT	FA	SS	SO	PF	CC	Total	%
PR	1354	3099	2954	--	--	--	--	--	7407	16.5
FA	--	--	--	9546	6912	4645	--	--	21103	47.
SS	--	--	--	--	5640	3790	--	--	9430	21.
SO	--	--	--	--	--	408	--	--	408	.9
PF	--	--	--	--	--	--	4955	--	4955	11.
CC	--	--	--	--	--	--	--	1614	1614	3.6
Total	1354	3099	2954	9546	12552	8843	4955	1614	44917	100.
%	3.	6.9	6.6	21.3	27.9	19.7	11.	3.6	100	

Abbreviations used for power plants: Passo Real (PR); Jacuí (JC); Itaúba (IT); Foz do Areia (FA); Salto Santiago (SS); Salte Osório (SO); Passo Fundo (PF); Capivari-Cachoeira (CC).

the period where water surplus does not occur (from step 2)

Compute: the periods of isolated water surplus in each reservoir by the Cumulative Minimum Surplus Analysis

STEP 4. Given: the periods where water surplus does not occur (from step 2)

the reservoirs where the maximum release tactics are not optimal (from step 3)

Update: the aggregated formulation if needed

STEP 5. Given: the aggregated formulation in each period where water surplus does not occur (from step 4)

Compute: the optimal aggregated operation by the Constrained Marginal Analysis

STEP 6. Given: the optimal operation of the aggregated system (from steps 2 to 5)

Compute: the actual operation of the system by the disaggregation procedure

STEP 7. Given: the actual operation of the system (from step 6)

Compare: the computed and assumed hydraulic heads

If: convergence does not occur, make estimated hydraulic heads equal to the computed ones and return to step 1

If: convergence occurs, STOP.

The tolerance assumed for the convergence of the final storage in each trajectory in the Constrained Marginal Analysis was 20 Eq. MW, about 0.04 percent of the total energy storage capacity. After 5 iterations the computed hydraulic heads in each reservoir did not differ from the assumed ones by more than 2 percent. Table 9 presents the average computation time in each step of the aggregation-optimization-disaggregation procedure.

At the final iteration the breakdown between water surplus periods and period with no water surplus is given in Table 10. This breakdown proved to be almost insensitive to hydraulic head variations. A good estimate of it was available after the first iteration. No variation occurred after the second iteration. Isolated periods of water surplus did not occur in any reservoir. The cause may be attributed to the homogeneity of the hydrology in the region. Hence, an optimization problem with 6 dimensions and 1872 weekly time intervals (or stages) was divided into 5 one-dimensional problems with 92 to 248 weekly time intervals.

Table 10 presents the optimum tactic of operation of the system. The optimum tactic is defined through marginal values in a very concise and accurate manner. This characteristic can be considered another advantage of the Constrained Marginal Analysis in the deterministic operation of reservoirs.

The disaggregation procedure was applied at each iteration in order to define each reservoir's operation and the resultant hydraulic head variation. The convergence between the estimated and computed hydraulic heads occurred faster in the periods of surplus of water. Two reasons existed for it; the optimal operation is defined by the maximum release tactics during such periods and their locations are not very sensitive to the actual values of the hydraulic heads. It caused the convergence of the hydraulic heads to occur during the third iteration in periods of surplus of water. Therefore, after this iteration, only the periods

Table 9. Average computation time for the aggregation-optimization-disaggregation procedure.

Phase	Average central processing time
1 Aggregation procedure	8 seconds
2 Cumulative minimum surplus analysis . . .	0.3 seconds
3 Optimization procedure	12 seconds
4 Disaggregation procedure	
- for the entire period	60 seconds
- for non-surplus periods	45 seconds
Total for each iteration:	65 to 81 seconds

Note: a) computer used: Cyber 172
b) period of operation: 1872 time intervals (weeks)
c) the Cumulative Minimum Surplus Analysis is applied to the aggregated formulation of the problem only

Table 10. Optimal tactic of operation for the aggregated system. Storage capacity: 44,917 Eq. MW.

Interval (week)	Initial reservoir content Eq. MW	Marginal value $/MWh	Interval (week)	Initial reservoir content Eq. MW	Marginal value $/MWh
1-104	32789	0.0	781	empty	39.8
105	full	30.1	782	empty	8.4
106-234	full	50.5	783-931	empty	0.0
235	empty	39.5	932	full	19.2
236	empty	15.7	933-1127	full	37.9
237-600	empty	0.0	1128-1178	empty	37.7
601	full	6.8	1179	empty	17.3
602	full	16.1	1180-1401	empty	0.0
603	full	27.2	1402-1538	full	7.9
604-614	full	29.1	1539-1610	empty	0.0
615	full	41.2	1611-1625	full	7.2
616-690	full	42.4	1626-1702	full	12.6
691-780	full	99.5	1703-1872	empty	0.0
			1873	full	

Note: the periods of water surplus are the ones with zero marginal values.

Total variable cost of operation including deficits: U.S. $1305.8 * 10^6

Average annual variable cost : U.S. $ 36.3 * 10^6

with no water surpluses needed to have their hydraulic heads corrected after each iteration. It represented a substantial computational saving.

The production in actual operation of the total hydroenergy generation computed by the aggregated formulation presented some expected discrepancies. During periods of low storage the actual hydroenergy generation could not eventually attain the required level or hydroenergy deficits occurred. It resulted in an increase of the thermoenergy generation which totaled less than 1 percent of the total energy inflow during periods with no water surplus. This percentage is insignificant during periods of water surplus. During high storage periods the observed spills in the real operation were somewhat above the levels computed by the aggregated formulation or hydroenergy losses occurred. They achieved the order of 4 percent of the total hydroenergy inflow during periods with no water surplus. During periods of surplus of water the spills were about 11 percent of the total energy inflow. It resulted that the energy storage given by the aggregated formulation was consistently above the actual energy storage obtained with application of the disaggregation procedure. This situation was not surprising, though. When the aggregated formulation was presented it was understood that this kind of formulation would overestimate the system's actual energy storage capacity. It happens because spills occur in the aggregated formulation only when all reservoirs are simultaneously full. This formulation does not account for spills which may occur in an isolated reservoir independently of the other reservoirs' storage.

In order to consider this deficiency of the aggregated formulation an artifice was used. The total energy storage capacity of the system was reduced to 95 percent of its computed value. The optimal operation under this new formulation was computed. The resultant optimal tactic is presented in Table 11.

The periods of water surplus had their location unchanged with the decrease of the total storage. The tactic itself has very few modifications. The marginal values eventually increased after the reservoir has filled and decreased after the reservoir has emptied. This behavior can be explained by the concept of water transfer in reservoir operation. When the storage capacity decreases relatively less water can be transferred to the future. This implies that less water will be used after a maximum storage and consequently the marginal values increase. The excedent water has to be used before the time when the reservoir becomes full; this decreases the respective marginal values. Tables 10 and 11 show such an occurrence.

Some discrepancies are still noticed between the operation computed by the aggregated formulation and the actual operation of the system. The percent values of the hydroenergy deficits decreases during periods of low storage when compared with the figures obtained in the previous results. During periods of high storage the hydroenergy losses occur with roughly the same percentage level for both results. The variable cost of operation increases about 6 percent with a decrease on the aggregated energy storage capacity of the system.

The trajectories of the variation of the energy storage each time interval are presented in Figures 13.a to 13.f. The curves represent the results obtained with the aggregated formulation with decreased

Table 11. Optimal tactic of operation for the aggregated system. Storage capacity: 42,700 Eq. MW.

Interval (week)	Initial reservoir content Eq. MW	Marginal value $/MWh	Interval (week)	Initial reservoir content Eq. MW	Marginal value $/MWh
1-104	32789	0.0	616-690	full	42.4
105	full	30.1	691-780	full	101.0
106-234	full	66.6	781	empty	39.4
235	empty	39.5	782	empty	6.8
236	empty	15.7	783-931	empty	0.0
237-600	empty	1.0	932-1127	full	38.2
601	full	6.8	1128-1178	empty	37.3
602-603	full	16.1	1179	empty	16.8
604	full	27.2	1180-1401	empty	0.0
605-614	full	29.1	1402-1538	full	8.8
615	full	41.2	1539-1610	empty	0.0
			1611-1702	full	14.4
			1703-1872	empty	0.0
			1873	full	

Note: the periods of water surplus are the ones with zero marginal values.

Total variable cost of operation including deficits: U.S. $1393.1 * 10^6

Average annual variable cost : U.S. $ 38.7 * 10^6

Figure 13.a. Energy Storage Trajectories, Aggregated and Actual Formulations, 1931 to 1936

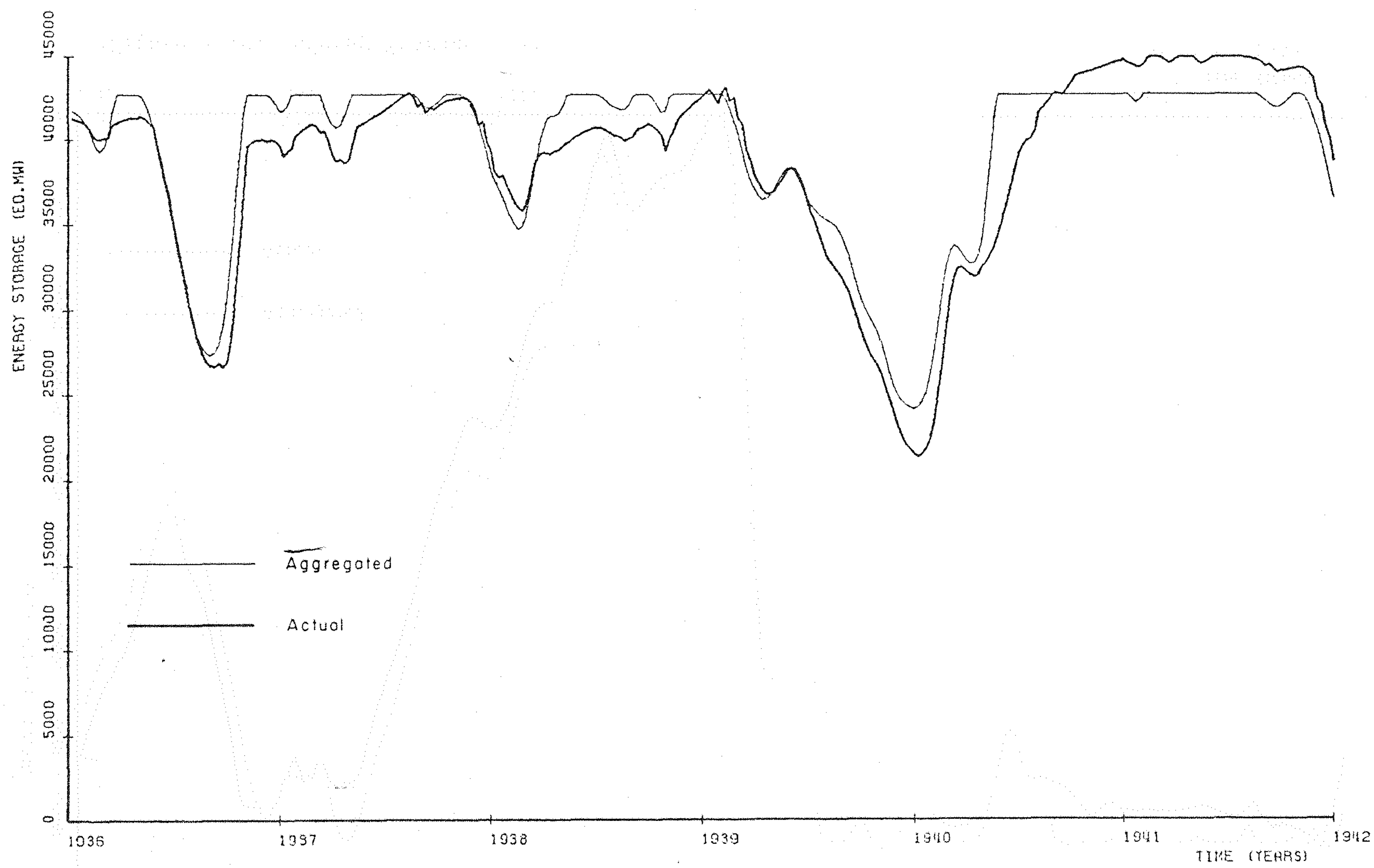


Figure 13.b. Energy Storage Trajectories, Aggregated and Actual Formulations, 1937 to 1942

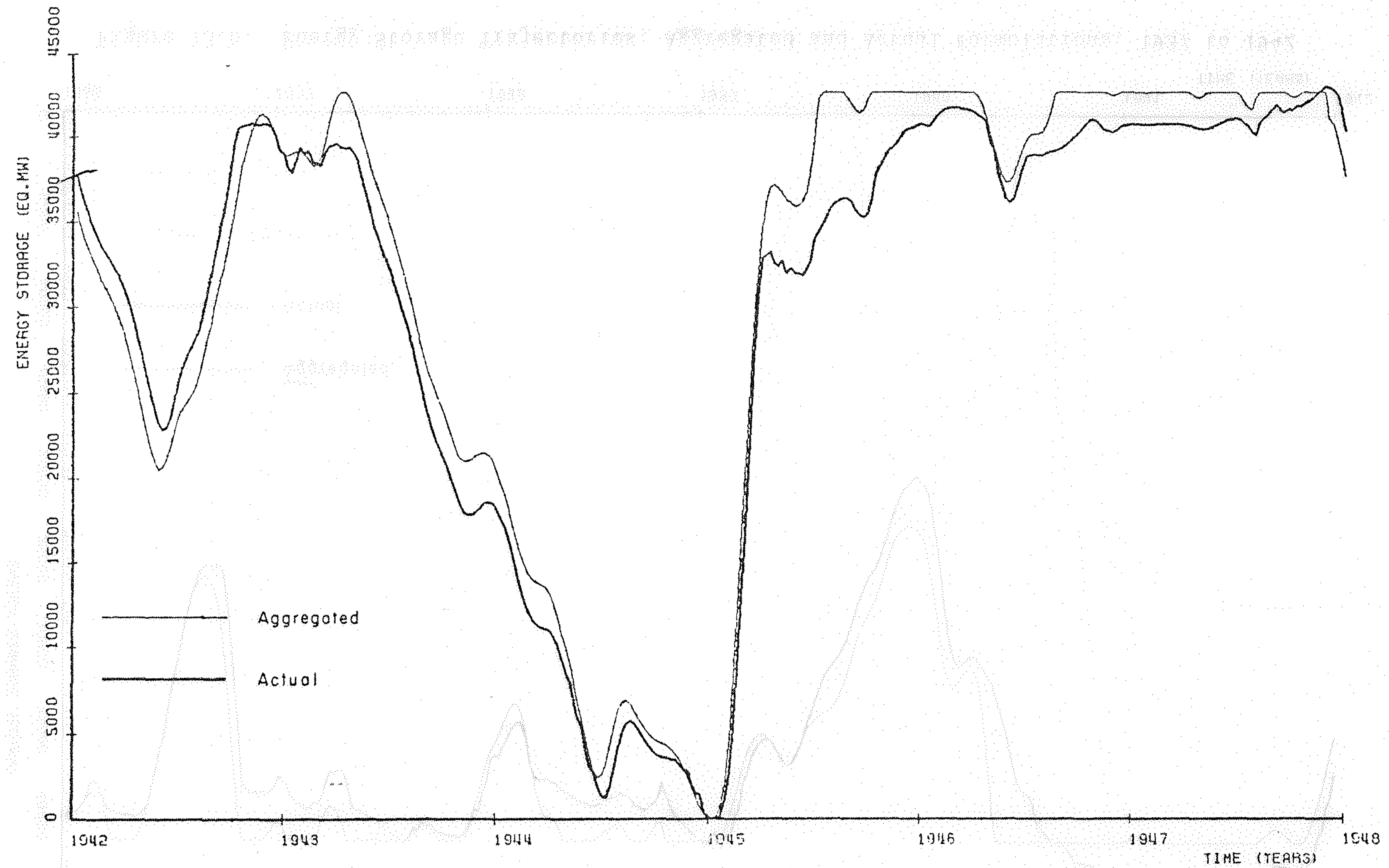


Figure 13.c. Energy Storage Trajectories, Aggregated and Actual Formulations, 1943 to 1948

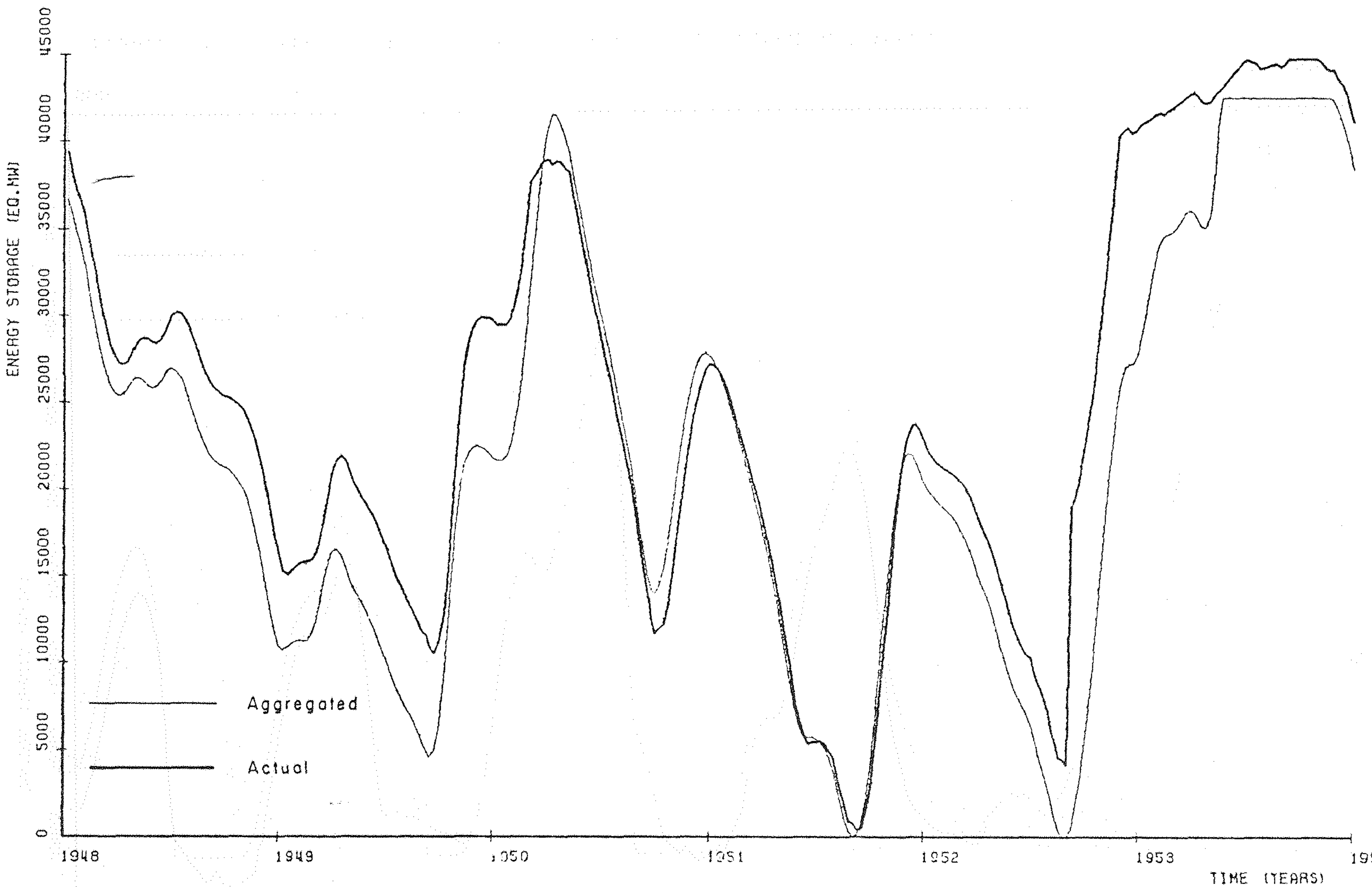


Figure 13.d. Energy Storage Trajectories, Aggregated and Actual Formulations, 1949 to 1954

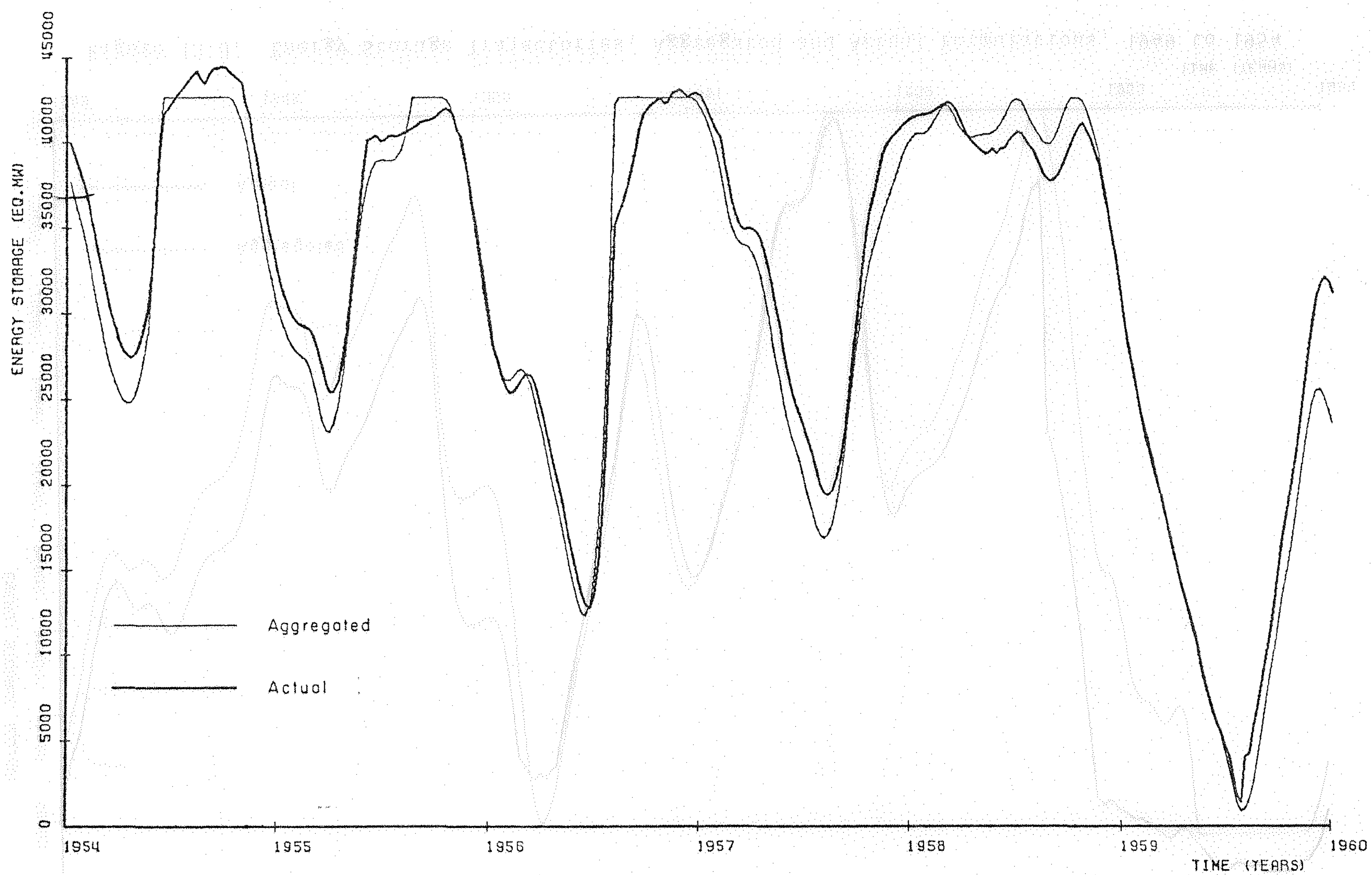


Figure 13.e. Energy Storage Trajectories, Aggregated and Actual Formulations, 1955 to 1960

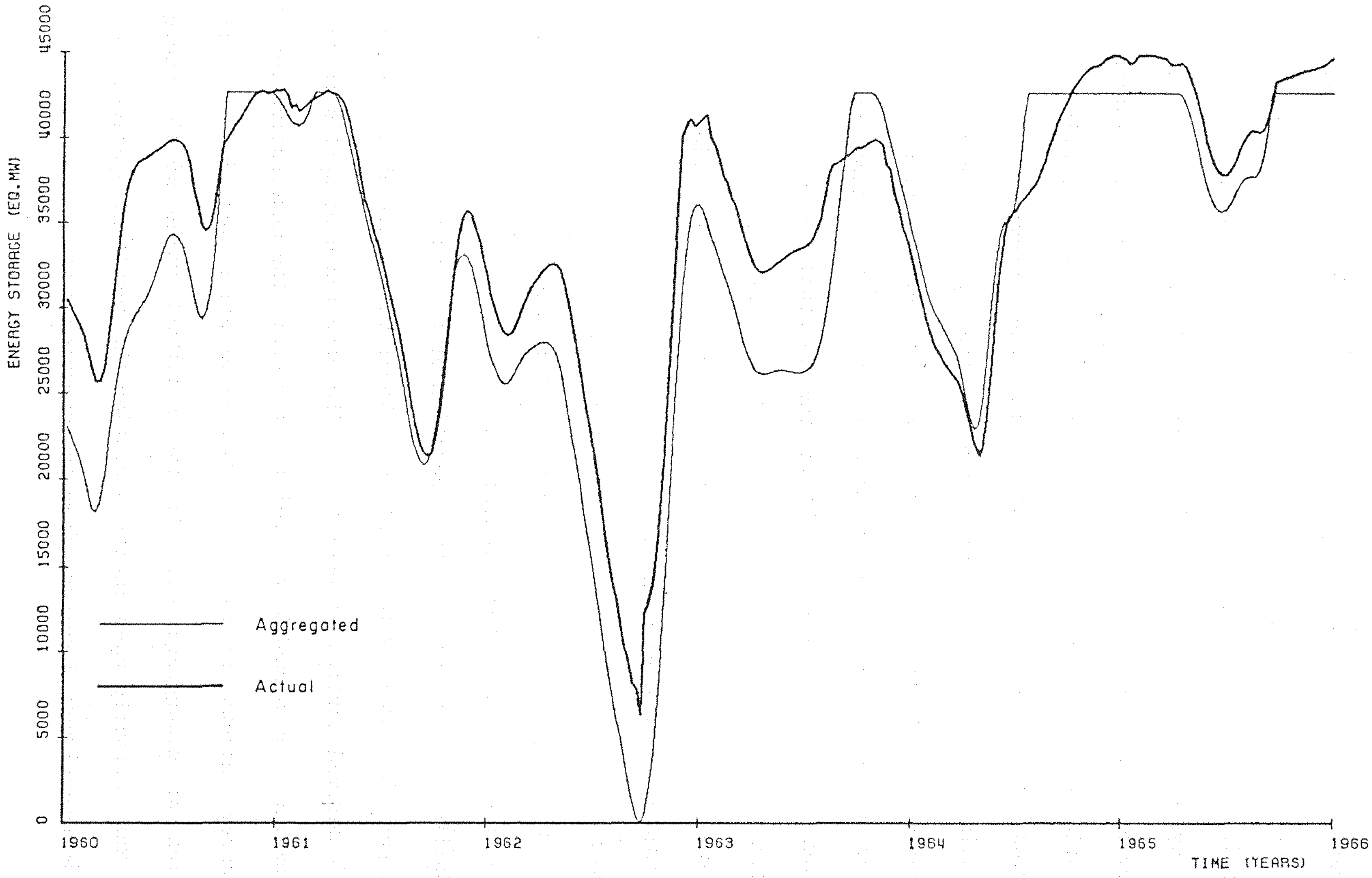


Figure 13.f. Energy Storage Trajectories, Aggregated and Actual Formulations, 1961 to 1966

energy storage capacity and the actual values of total energy storage in the real system's operation. The greatest deviations between the trajectories occur during the periods when the storages are being refilled. It is apparently caused by underestimation of energy inflows to the system in the aggregated formulation. During the presentation of the aggregation procedure it was observed that no rigorous solution for computation of energy inflows to serially-linked reservoirs exists. The criterion then adopted resulted in the previously referred underestimation. In the computation of the system's operation the underestimation of energy inflows must be of concern during periods with no water surplus. During periods of water surplus the optimal operation is given by maximum release tactic; the eventual correction of the procedure of energy inflow estimate will not cause any change in the operation. Figures 13.a to 13.f show that the deviation between trajectories is more pronounced during refilling periods which occurred in times of water surplus. This minimizes the significance of the errors on the optimal operation caused by the underestimation of energy inflows into the aggregated formulation.

Spills were observed in the operation of the individual reservoirs during periods with no water surpluses. Their values are about the same in both derived tactics of operation. The occurrence of spills in such a circumstance is caused by two related factors: the underestimation of the energy inflows in the aggregated formulation and the heuristic criterion of disaggregation. If the resultant value of the energy losses was optimally used in the operation the resultant operating cost would decrease about 5 percent from the computed cost.

A property of the aggregated formulation mentioned previously is the derivation of a lower bound for the minimum value of the objective function. It is caused by the representation of storage constraints in each reservoir by a lumped equivalent storage constraint. Therefore, the cost of operation presented in Table 10 may be considered such a lower bound for the actual minimum cost of operation. A more stringent value for this lower bound may be obtained if the 5 percent observed cost decrease is taken to account for the optimal usage of spills occurring in periods with no water surplus.

The reduction of the aggregated storage value in the second application (Table 11) increased the cost of operation by 6 percent. It also significantly improved the adherence between the trajectories of overall energy storage obtained in the aggregated formulation and in the actual operation (Figures 13.a to 13.f). The resultant cost of operation is probably higher than the actual minimum cost. Notice that no allowance is made in this case for the optimal usage of spills occurring in periods of no water surplus. Therefore, the actual minimum cost of the system's operation is in the range of $1240.5 million to $1393.1 million, or a $34.5 million to $38.7 million per year average. If the deterministic operation had a purpose other than its use as a screening study, more rigorous solution may need to be devised. An improvement of the estimates of energy inflows to the aggregated system would be required. For the purpose of screening, however, the approximation of solutions is considered satisfactory.

The tactics of operation presented in Tables 10 and 11 were analyzed in terms of the utilization of thermal units. The results are shown in Table 12. In both cases the thermal units are working at a

Table 12. Use of thermo units in the optimal deterministic operation.

Unit	Fuel used	Number of idle intervals tactic 1	tactic 2	Percentage tactics 1	tactics 2
	-	0	0	0	0
# 1	coal	1081	1081	58	58
# 2	coal	1316	1316	70	70
# 3	coal	1317	1317	70	70
# 4	coal	1329	1327	71	71
# 5	coal	1330	1328	71	71
# 6	transfer 1	1330	1329	71	71
# 7	oil	1654	1653	88	88
# 8	oil	1782	1653	95	88
# 9	oil	1782	1782	95	95
# 10	transfer 2	1782	1782	95	95
	shortage	1872	1872	100	100

Note: tactic 1--aggregated formulation with full storage
tactic 2--aggregated formulation with 95 percent storage

minimum load 58 percent of the time. The less efficient oil-fueled units are idle 88 percent of the time. The second kind of energy transfer is not used 95 percent of the time and is never used at full load. No shortage occurs.

The values of energy storage and inflow to each reservoir in the system were roughly identical in both cases of operation. Some regularity was observed in the partition of these variables among reservoirs of the system. Figures 14 through 25 show the pattern obtained with the second tactic of operation. In Figures 14 to 19 energy storage in each reservoir is plotted with the total energy stored in the system in some weeks. A regular pattern is observed in the dominant reservoirs of the system, Passo Real, Foz do Areia and Salto Santiago. Their total storages count for 85 percent of the total energy storage of the system.

Figures 20 to 25 represent the relationship between the energy inflows into each reservoir and the total energy inflow into the system for some weeks. A regular pattern is shown for the reservoirs on the dominant river of the system, the Iguaçú River. The energy inflows to these reservoirs (Foz do Areia, Salto Santiago and Salto Osório) represent about 62 percent of the total energy inflows to the system on the average.

Tables 13 and 14 present some statistical parameters of the distribution of energy inflows and storages in each reservoir. The non-storable energy inflow to the run-of-river power plants is also considered in Table 13 (site RR). This resulted in an insignificant part of the energy demand, less than 2 percent, on the average. Weekly energy inflows showed a strong variance around the mean. The correlation coefficient was also high between adjacent weeks. These

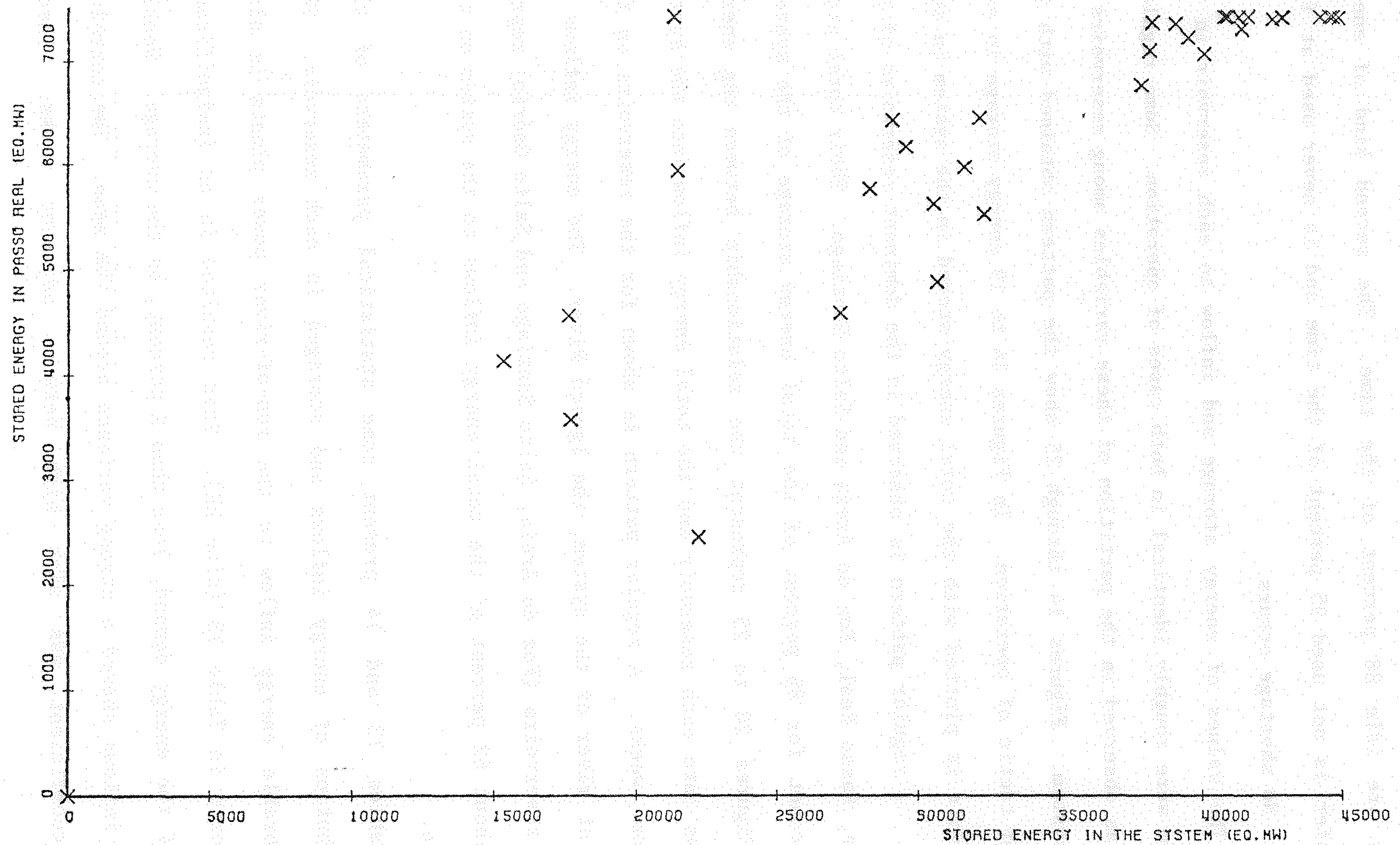


Figure 14.a. Energy Storage Distribution in Passo Real, 1st week

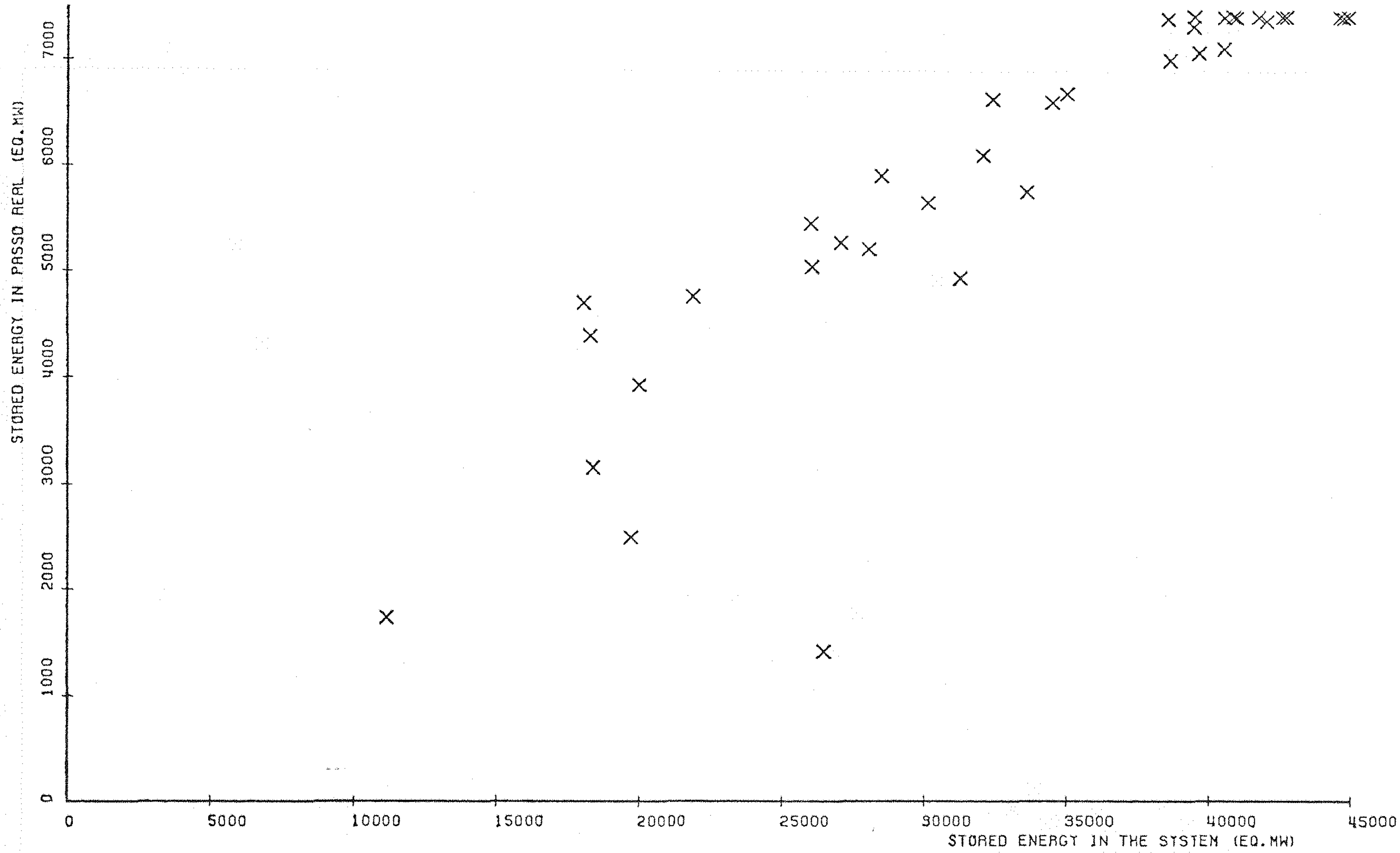


Figure 14.b. Energy Storage Distribution in Passo Real, 11th week

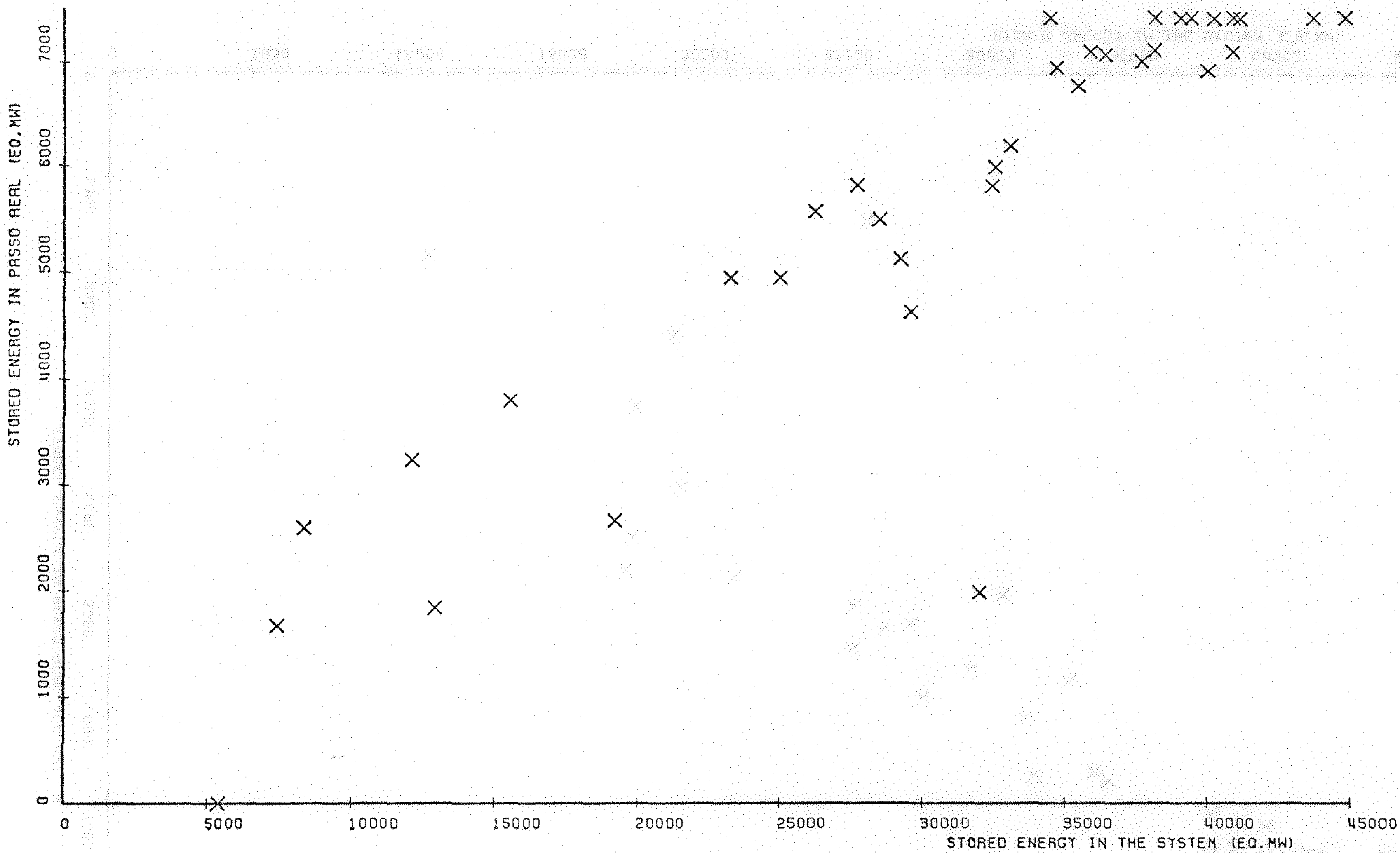


Figure 14.c. Energy Storage Distribution in Passo Real, 21st week

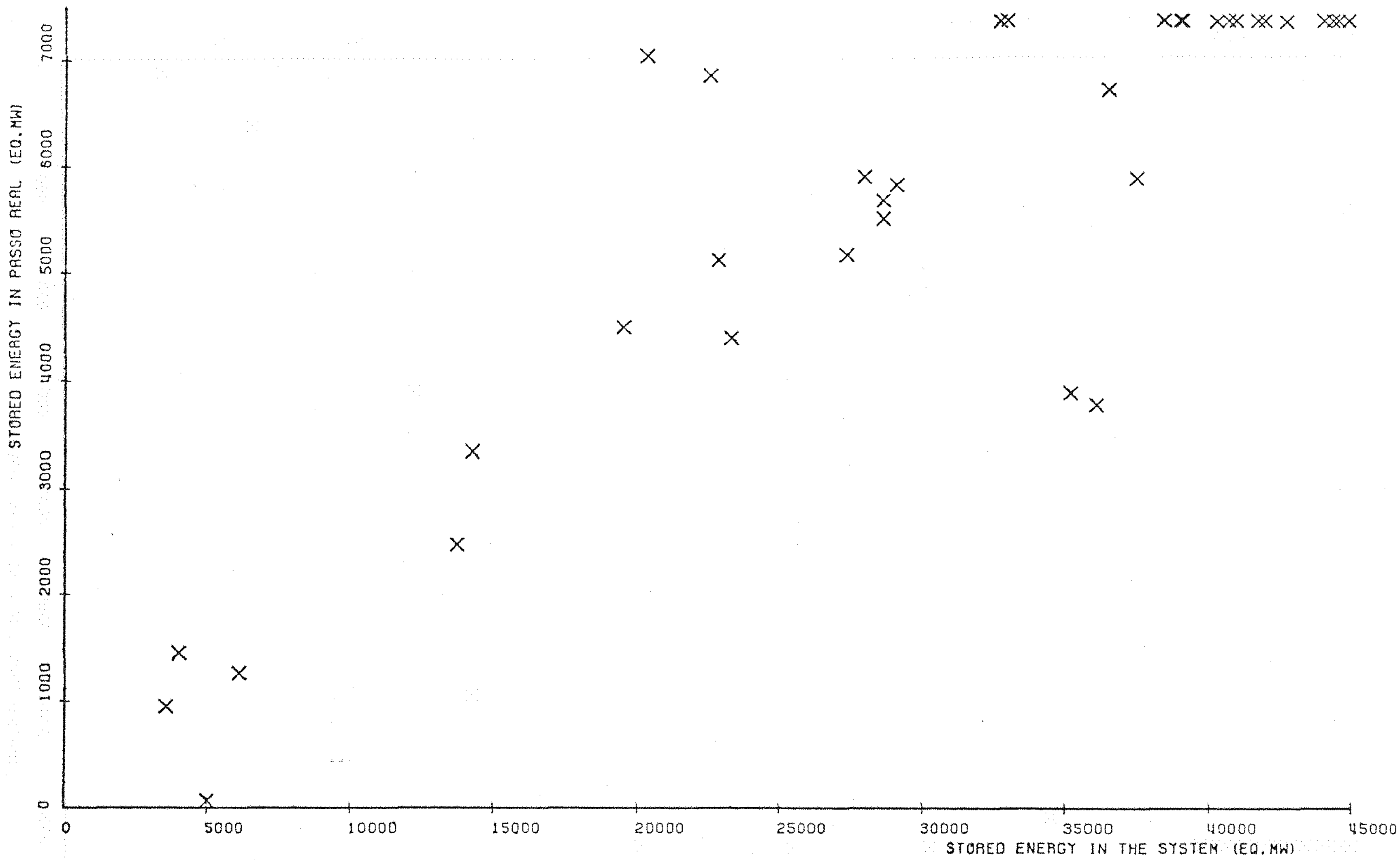


Figure 14.d. Energy Storage Distribution in Passo Real, 31st week

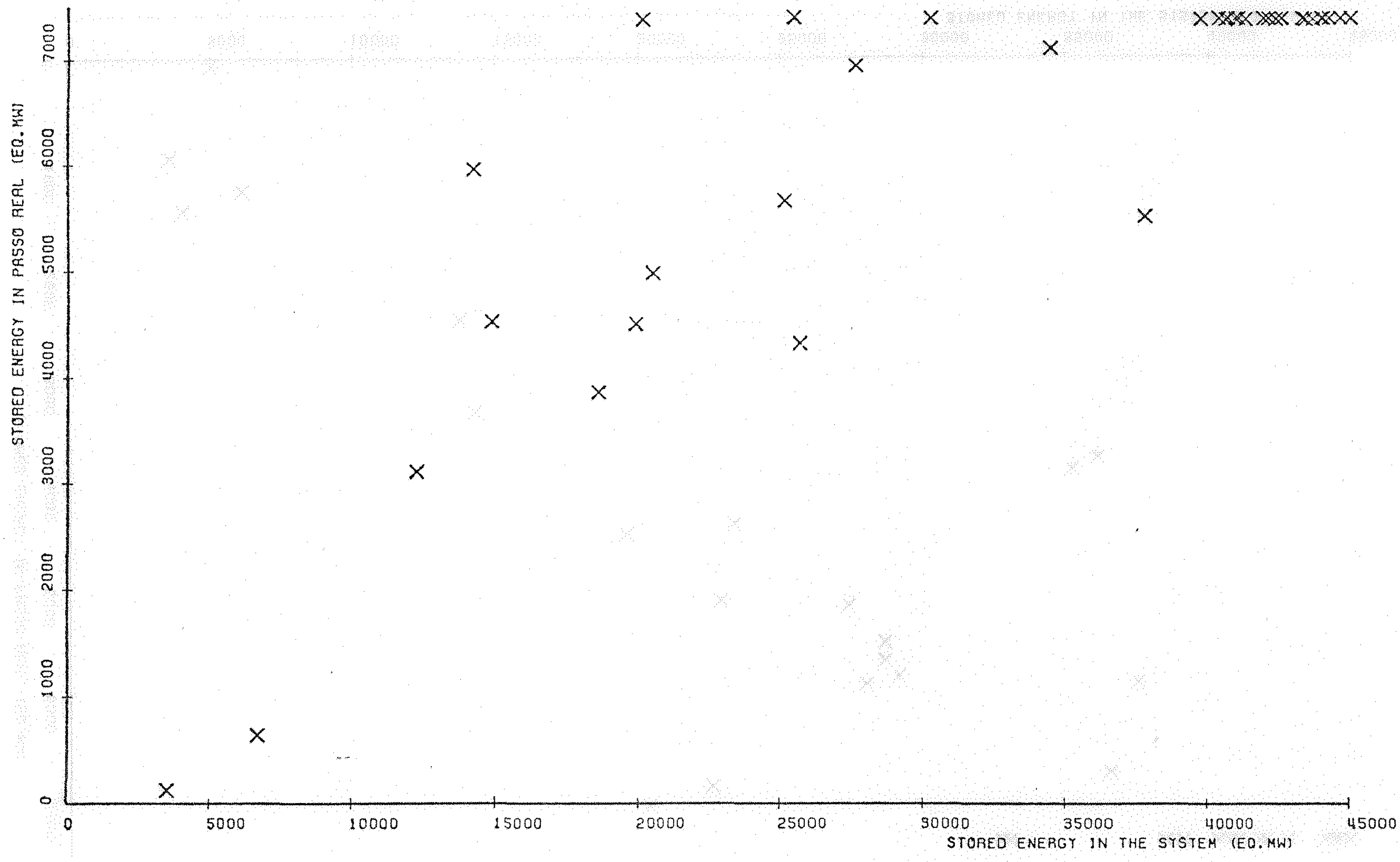


Figure 14.e. Energy Storage Distribution in Passo Real, 41st week

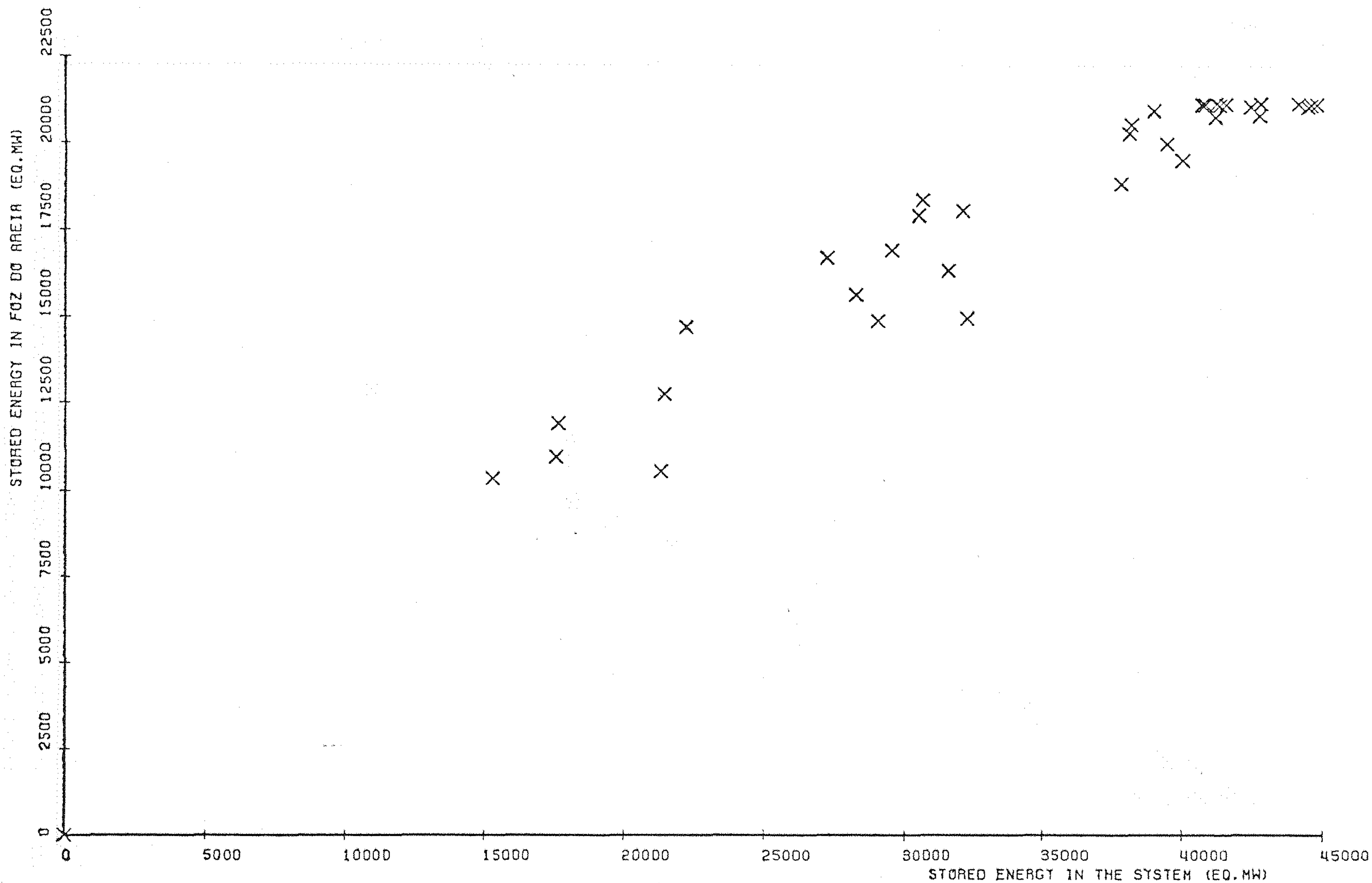


Figure 15.a. Energy Storage Distribution in Foz do Areia, 1st week

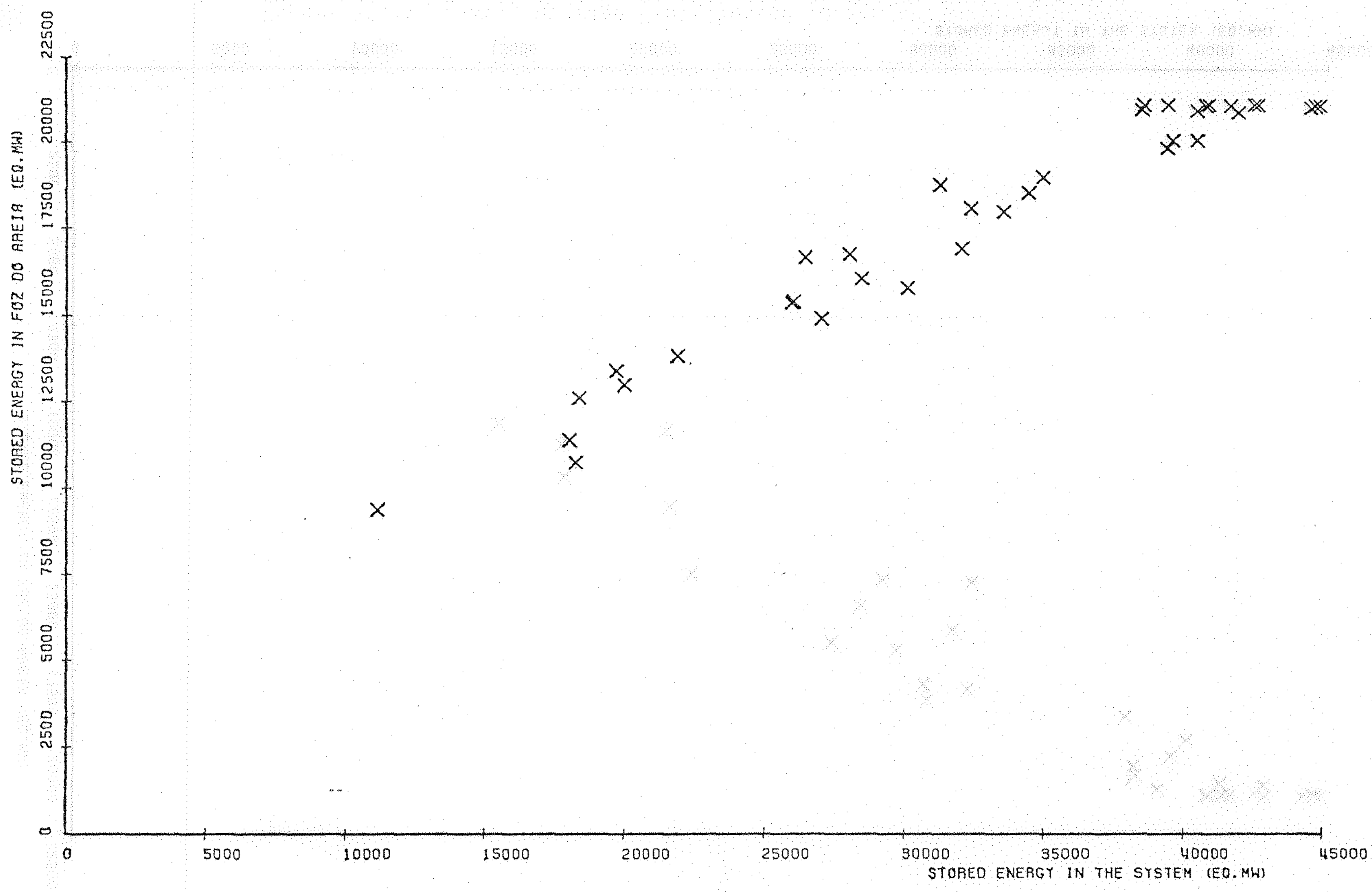


Figure 15.b. Energy Storage Distribution in Foz do Areia, 11th week

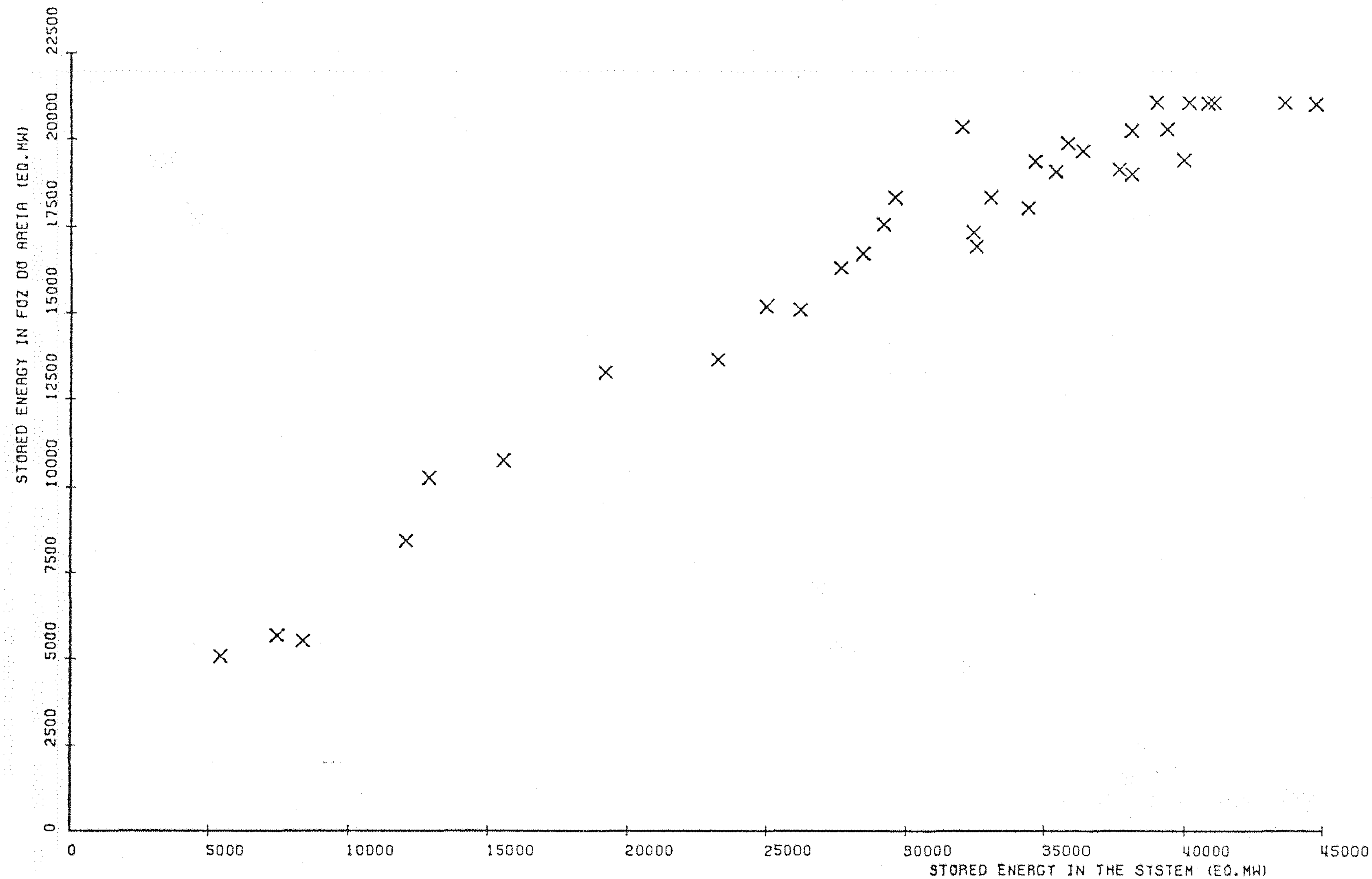


Figure 15.c. Energy Storage Distribution in Foz do Areia, 21st week

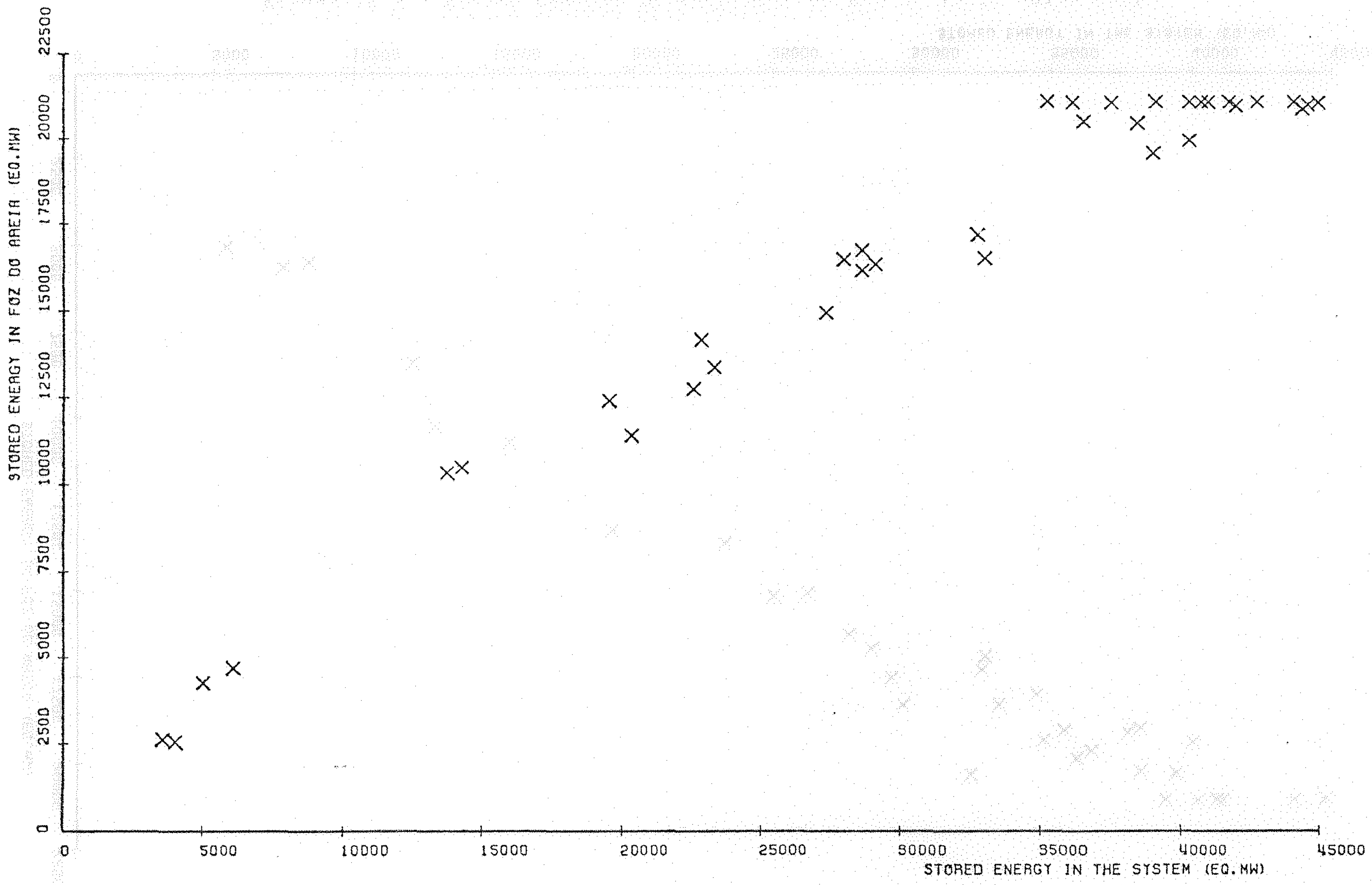


Figure 15.d. Energy Storage Distribution in Foz do Areia, 31st week

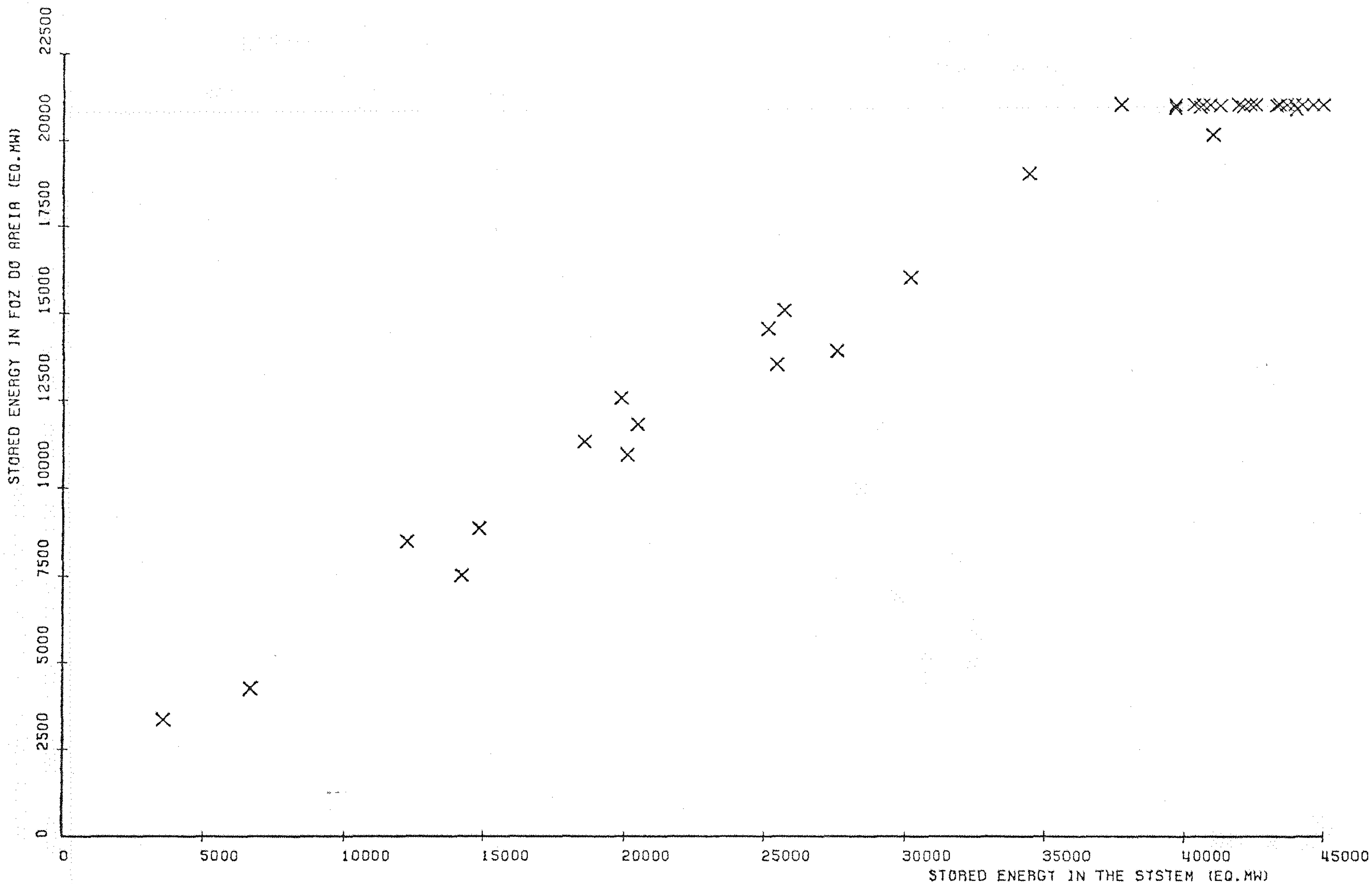


Figure 15.e. Energy Storage Distribution in Foz do Areia, 41st week

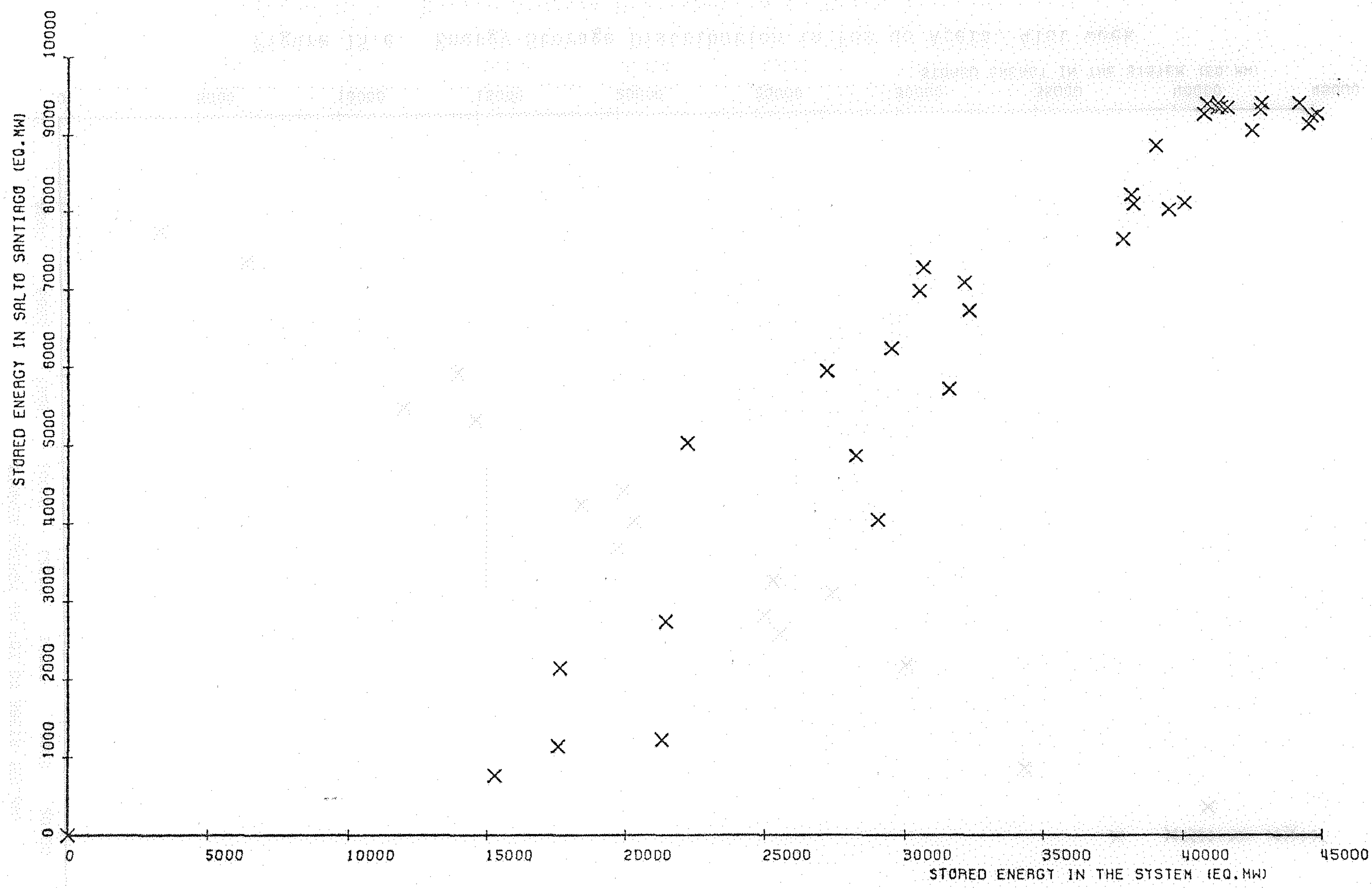


Figure 16.a. Energy Storage Distribution in Salto Santiago, 1st week

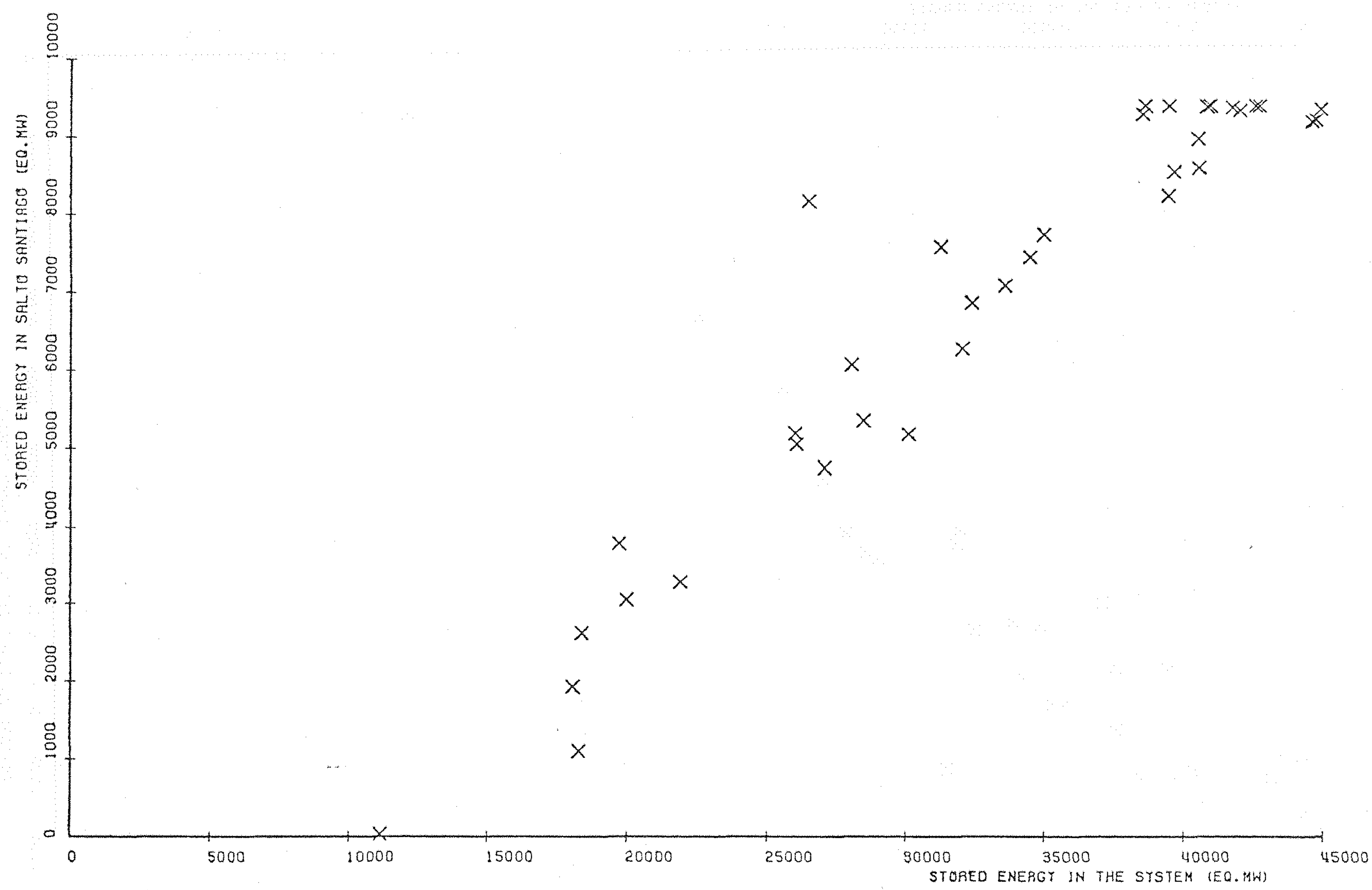


Figure 16.b. Energy Storage Distribution in Salto Santiago, 11th week

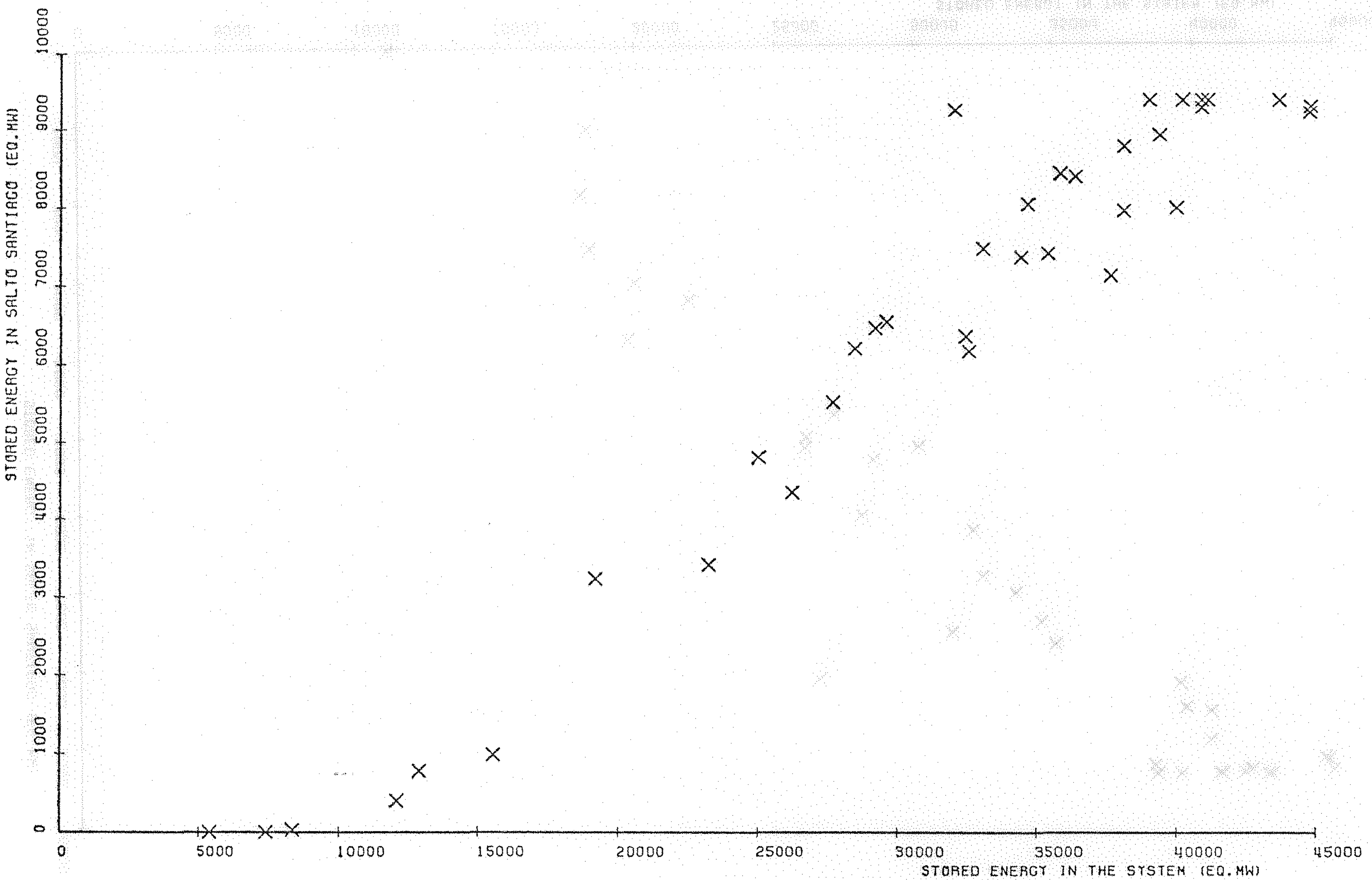


Figure 16.c. Energy Storage Distribution in Salto Santiago, 21st week

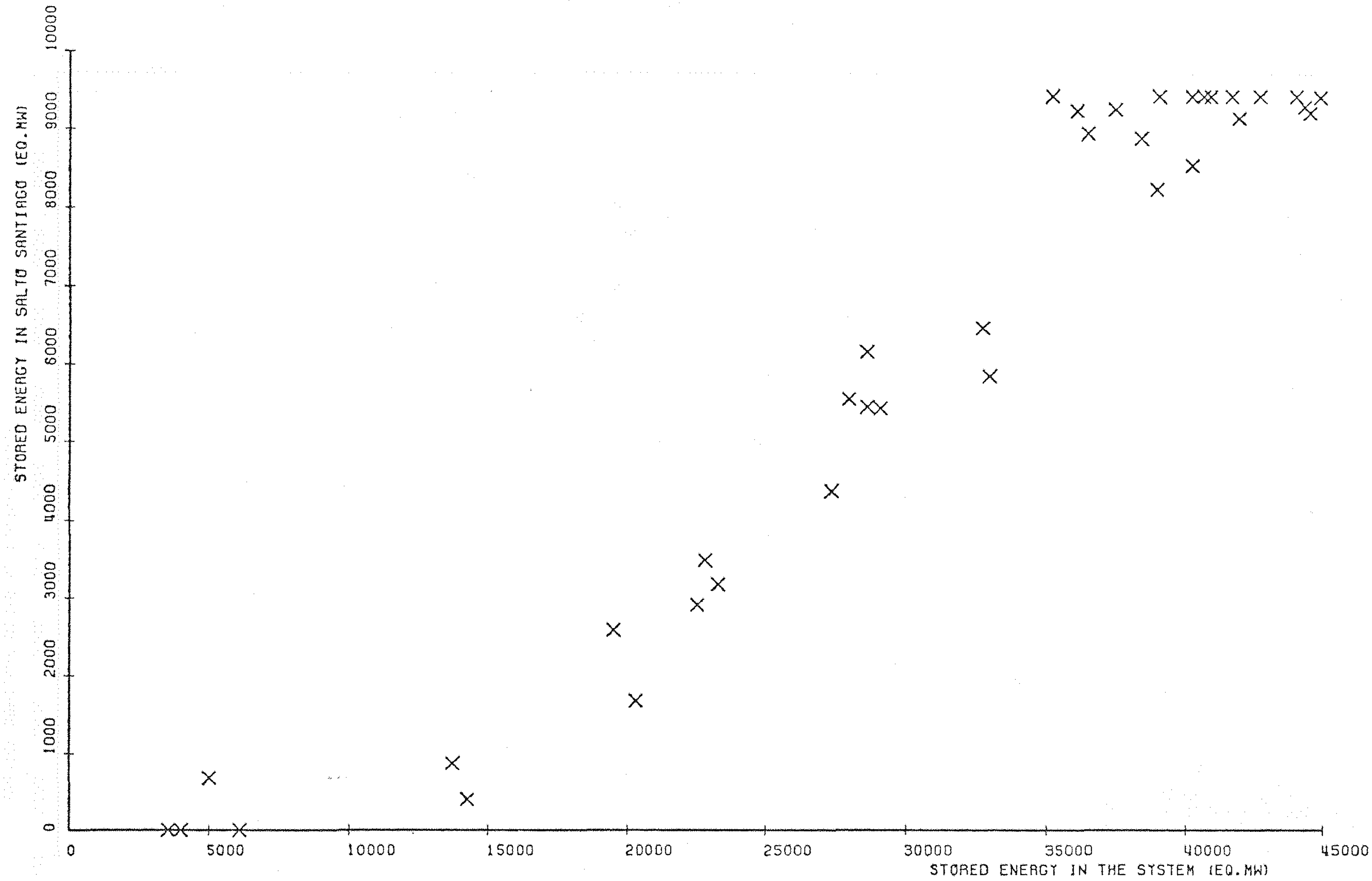


Figure 16.d. Energy Storage Distribution in Salto Santiago, 31st week

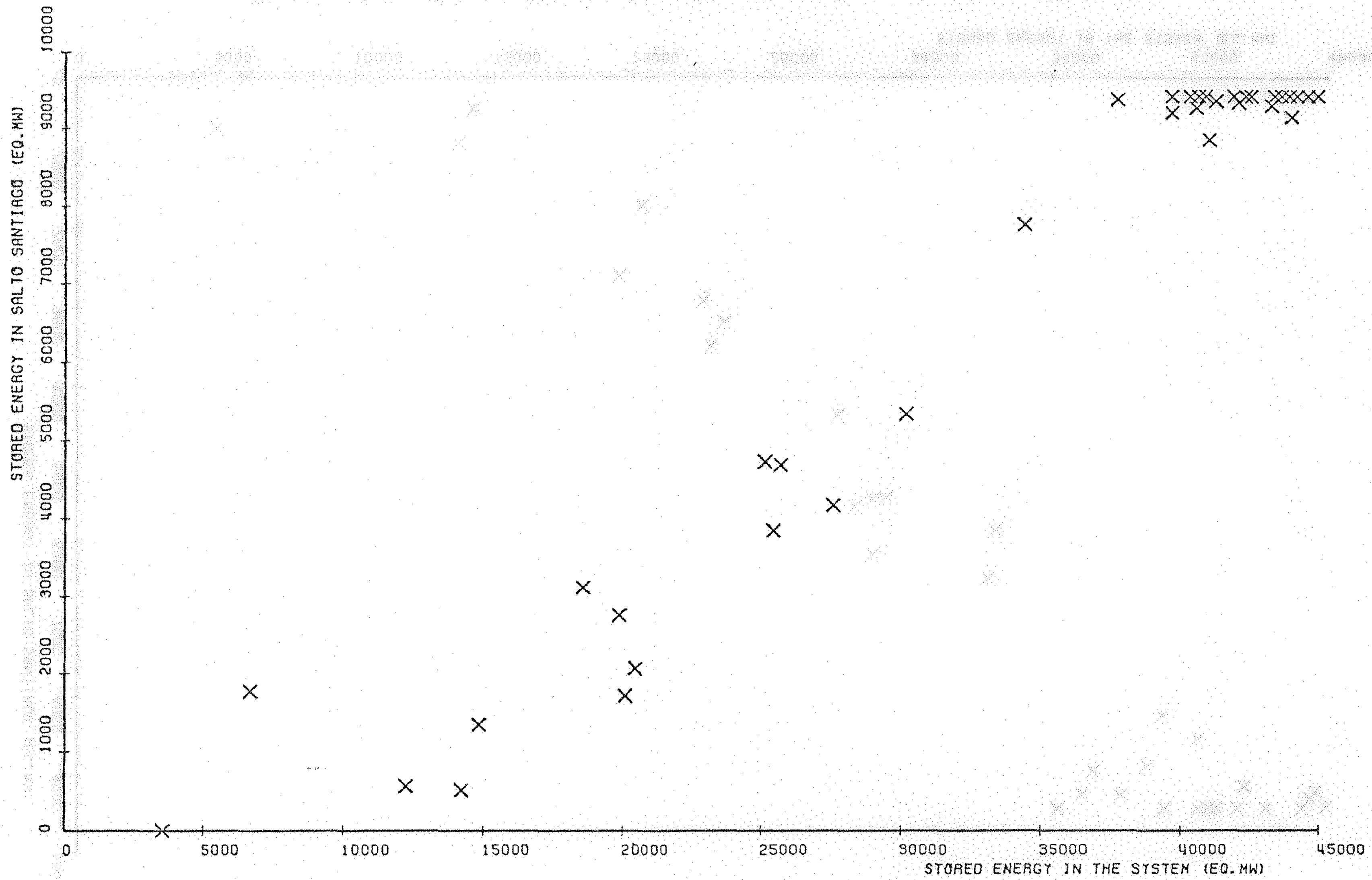


Figure 16.e. Energy Storage Distribution in Salto Santiago, 41st week

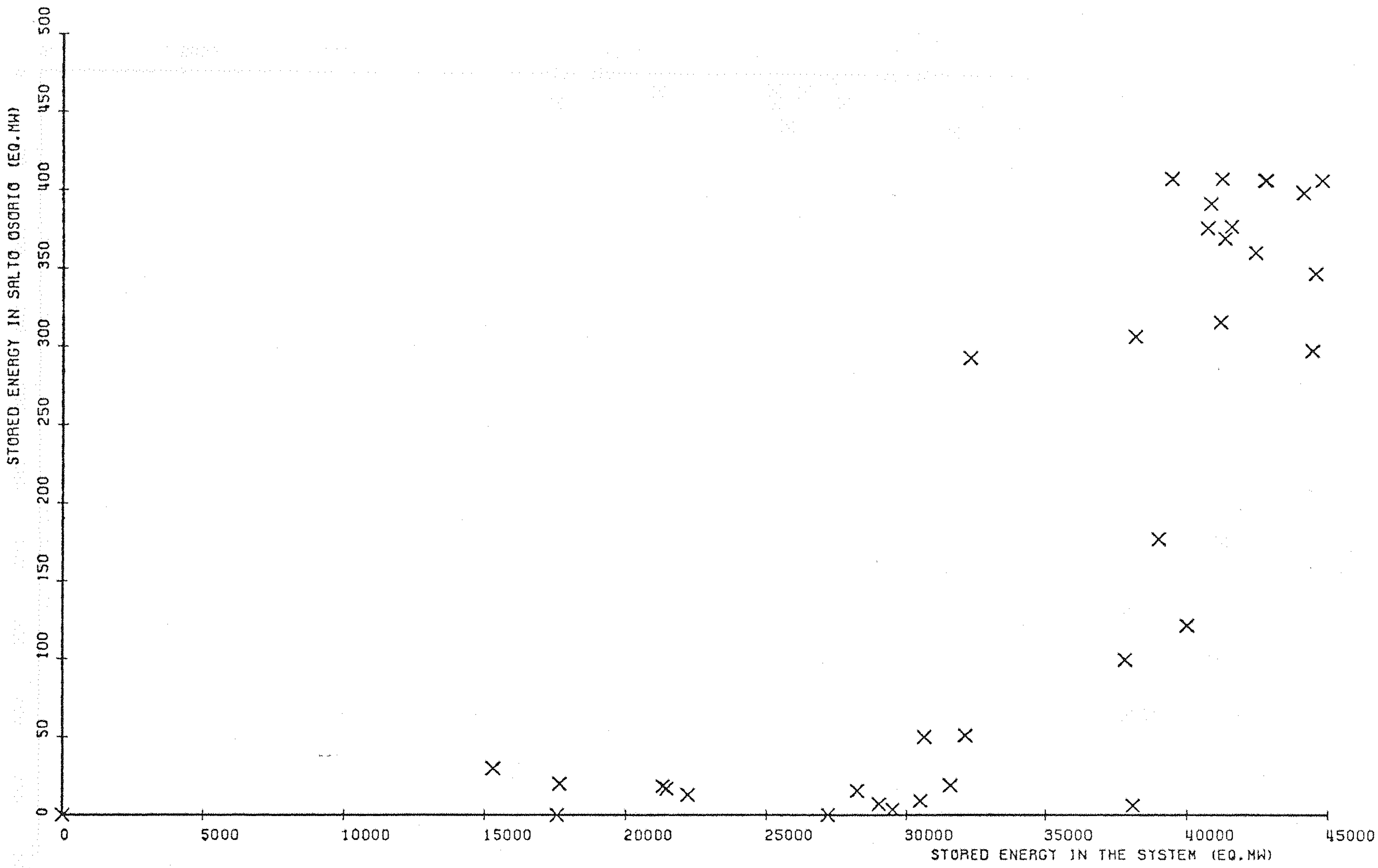


Figure 17.a. Energy Storage Distribution in Salto Osorio, 1st week

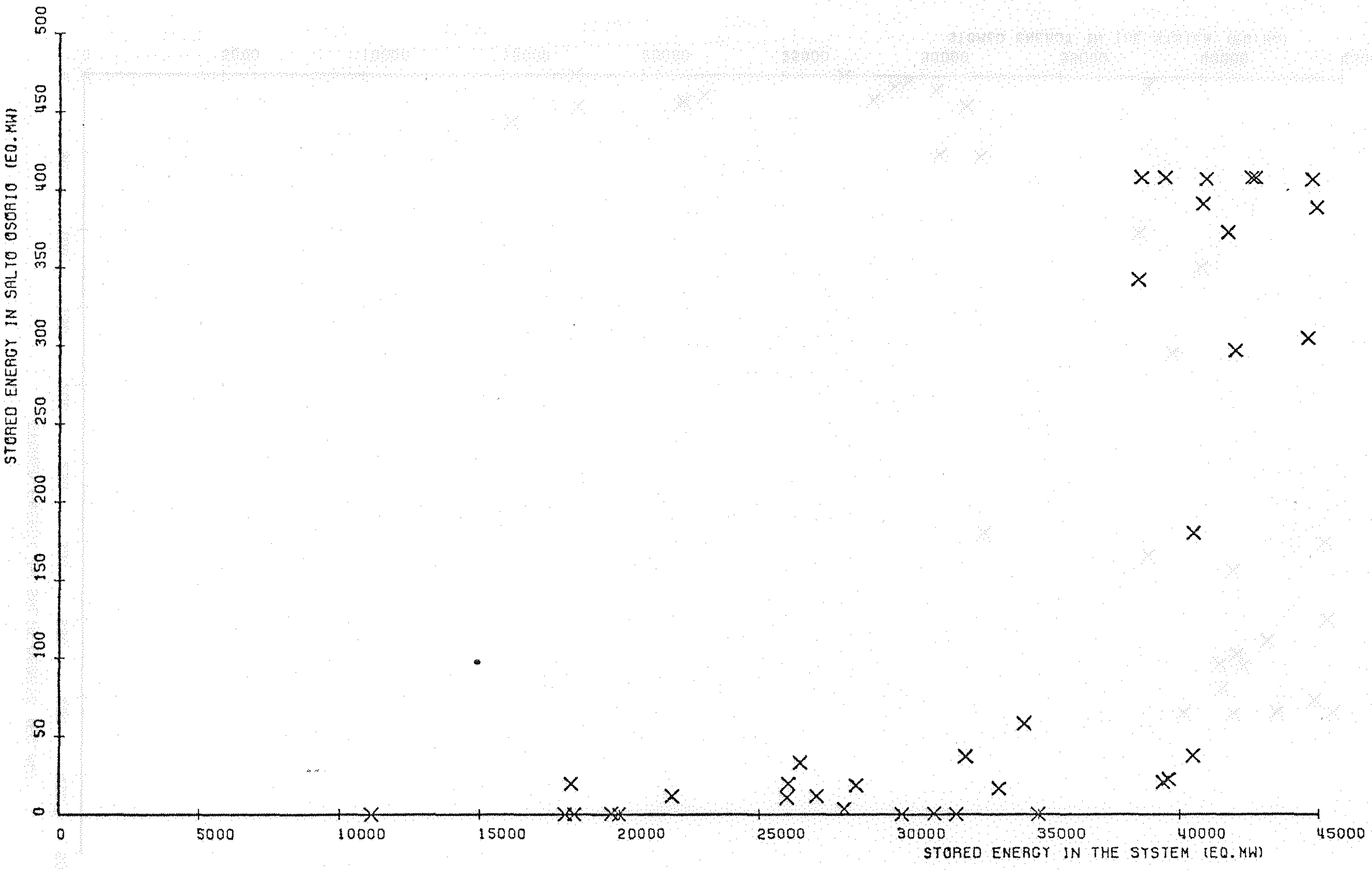


Figure 17.b. Energy Storage Distribution in Salto Osorio, 11th week

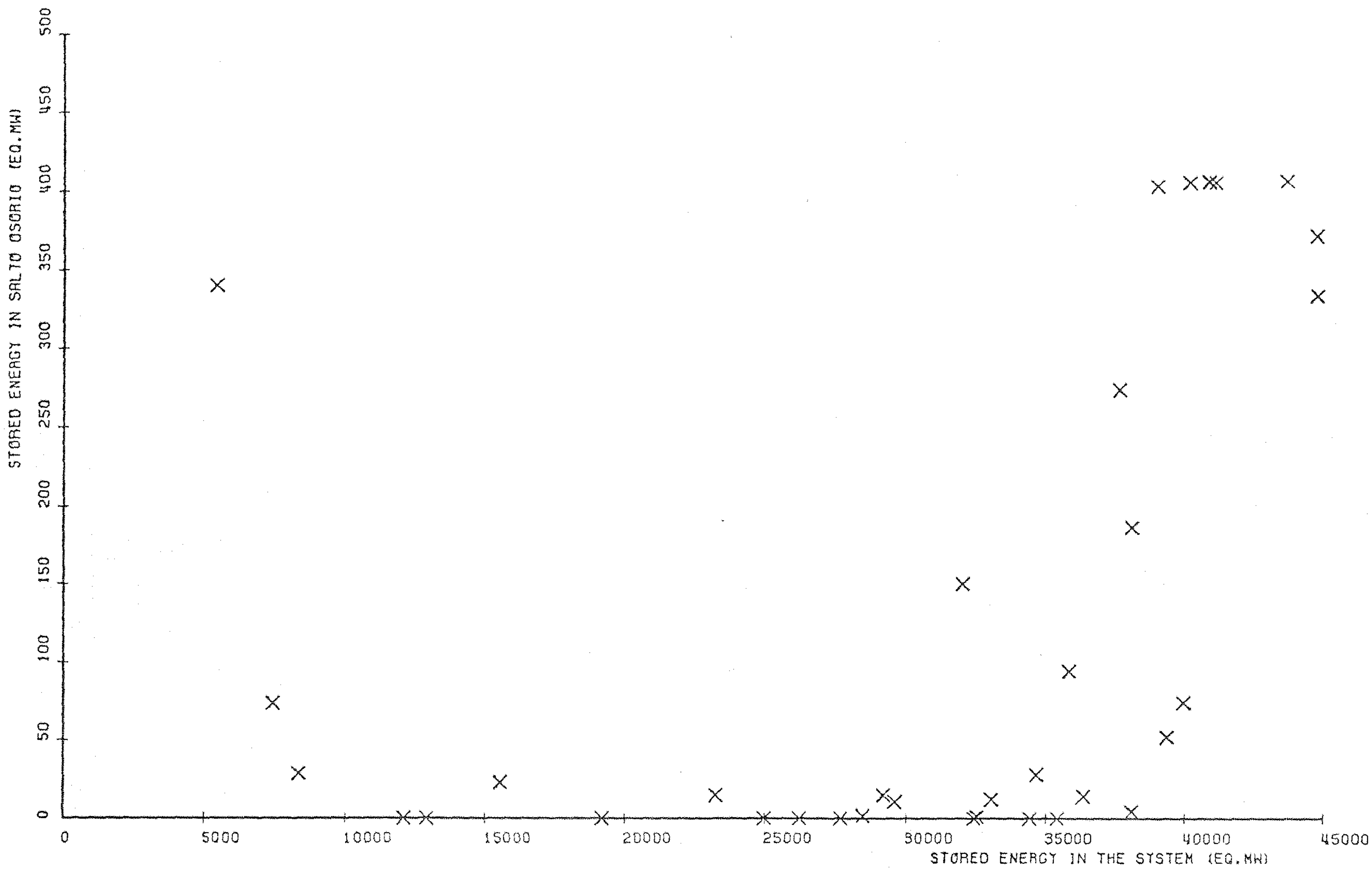
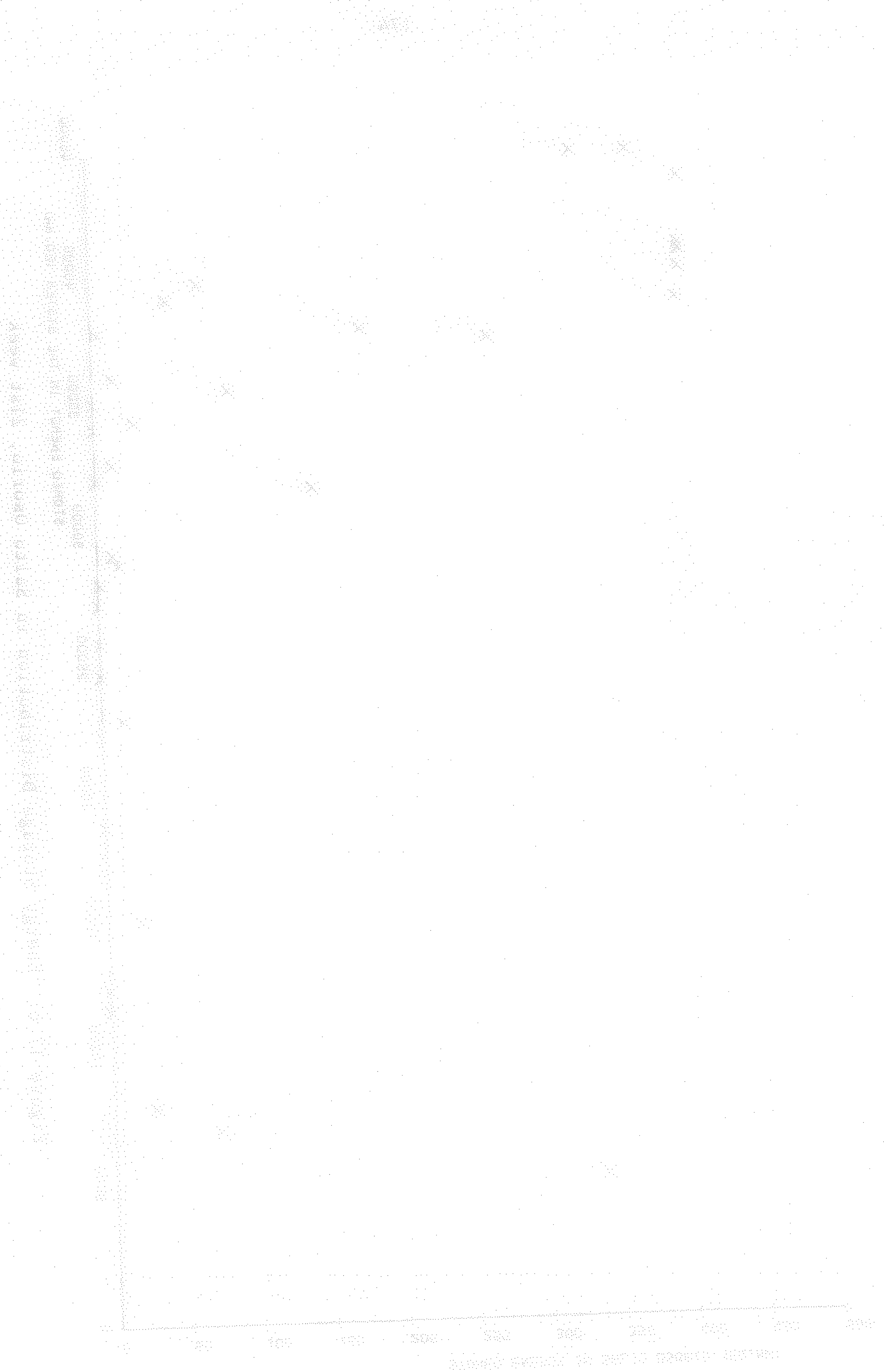


Figure 17.c. Energy Storage Distribution in Salto Osorio, 21st week



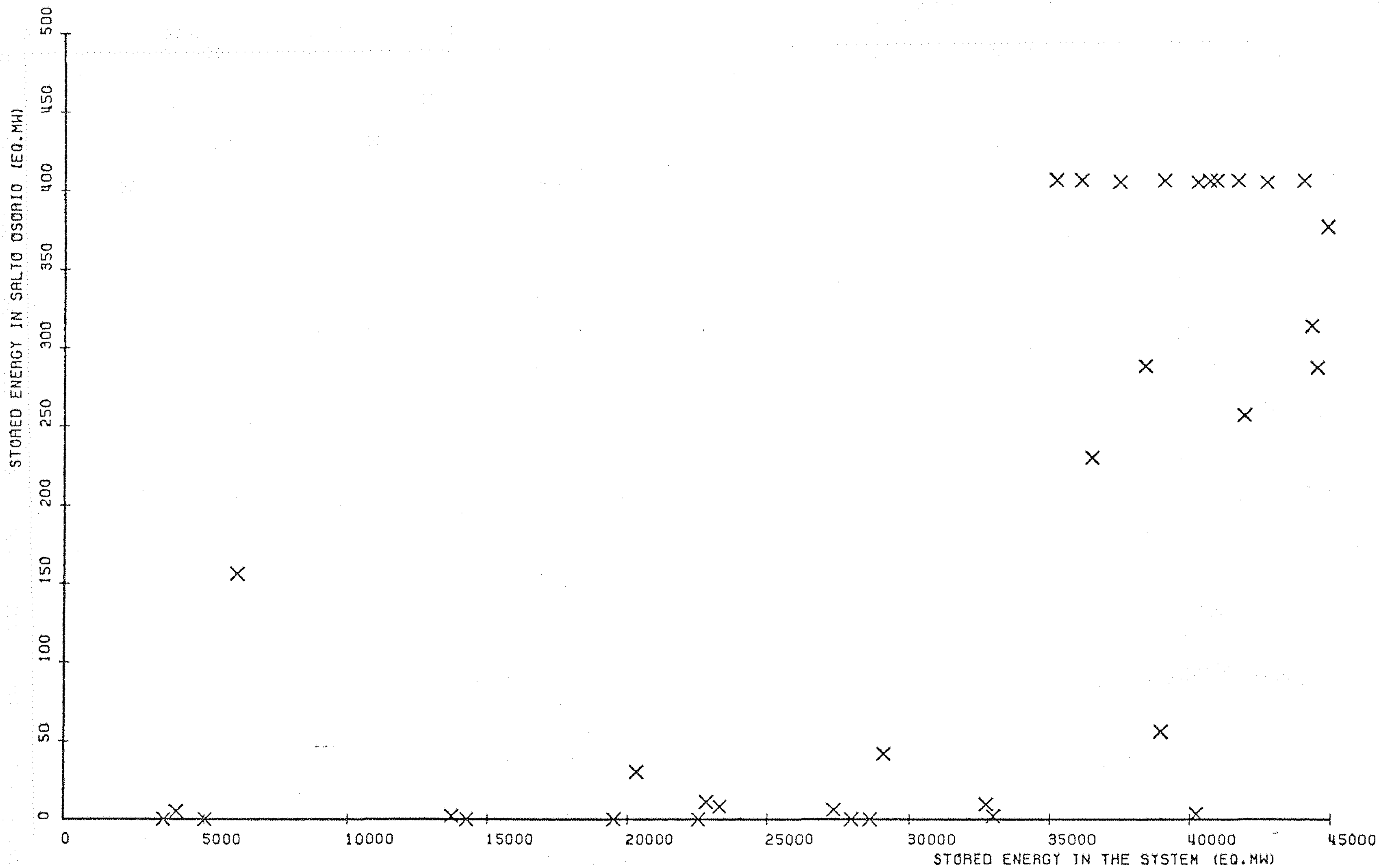


Figure 17.d. Energy Storage Distribution in Salto Osorio, 31st week

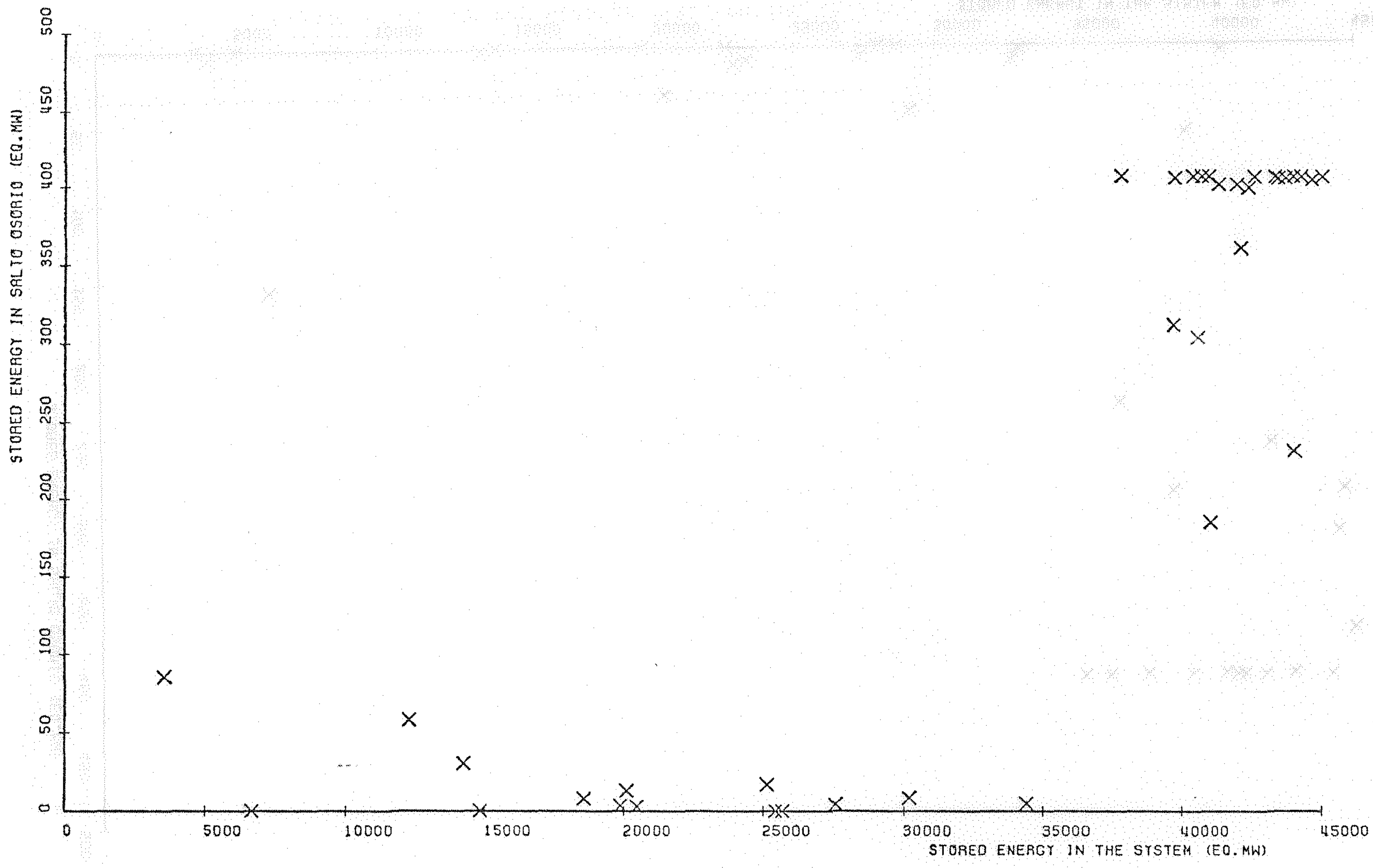


Figure 17.e. Energy Storage Distribution in Salto Osorio, 41st week

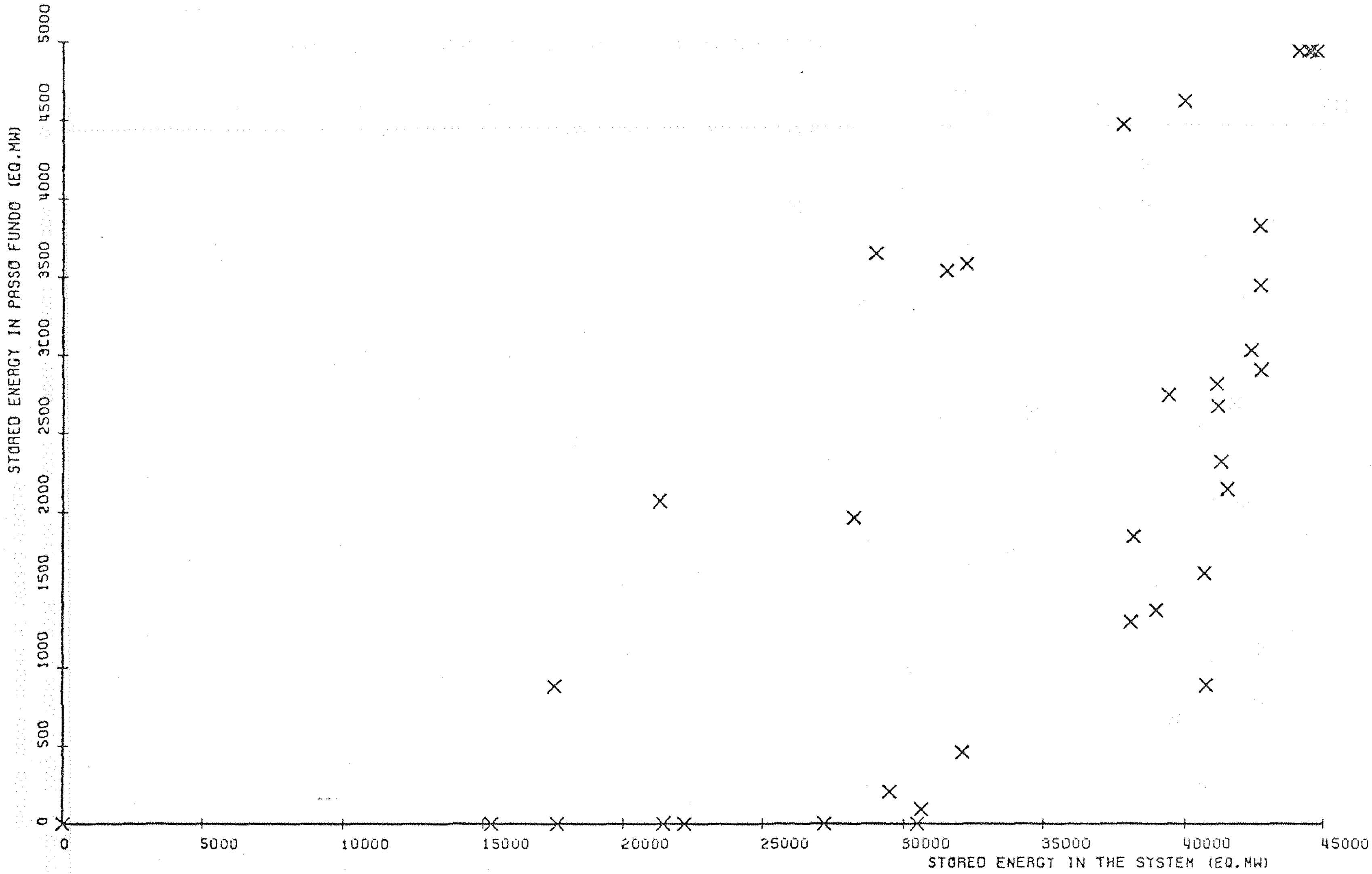


Figure 18.a. Energy Storage Distribution in Passo Fundo, 1st week

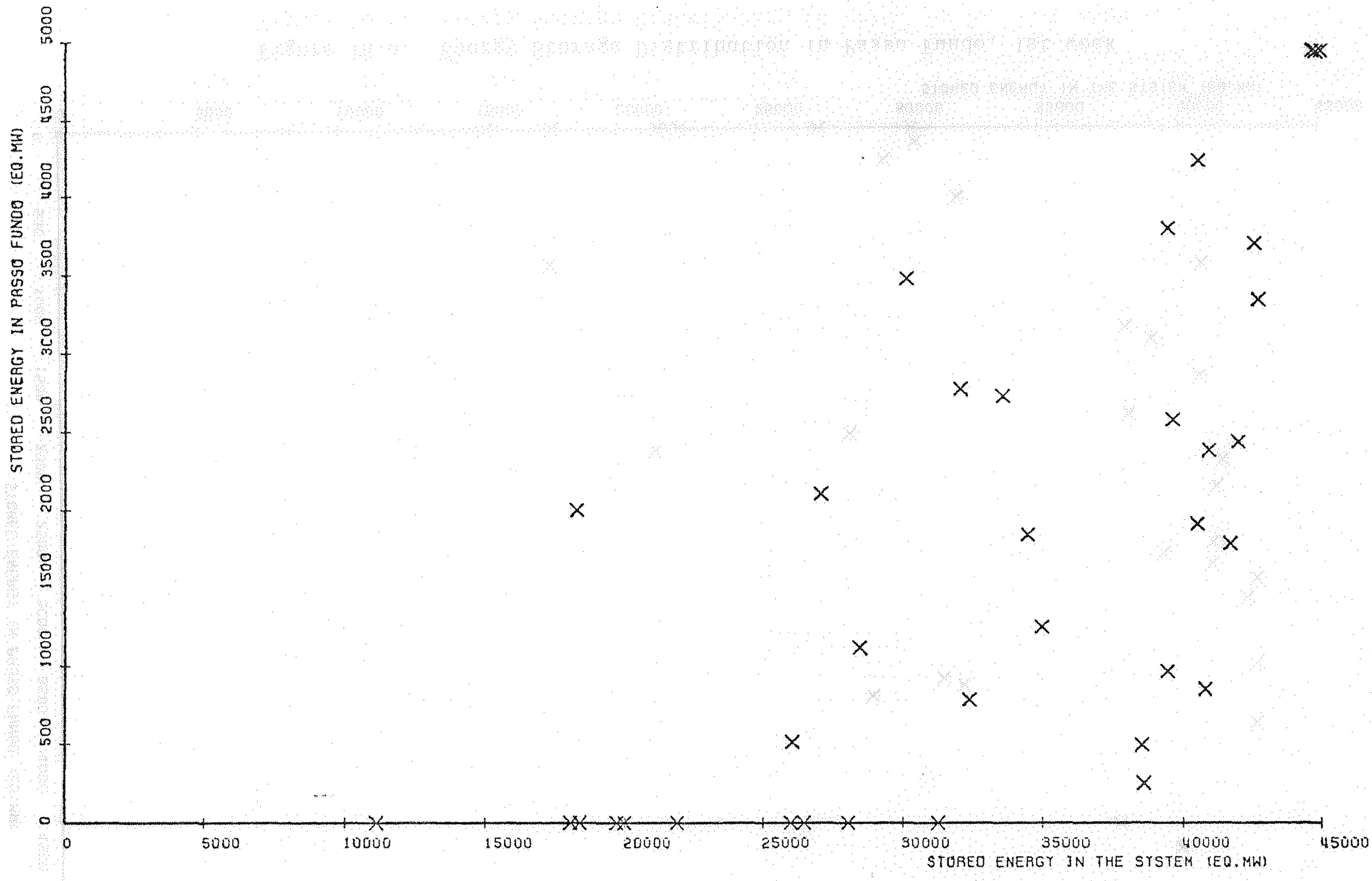


Figure 18.b. Energy Storage Distribution in Passo Fundo, 11th week

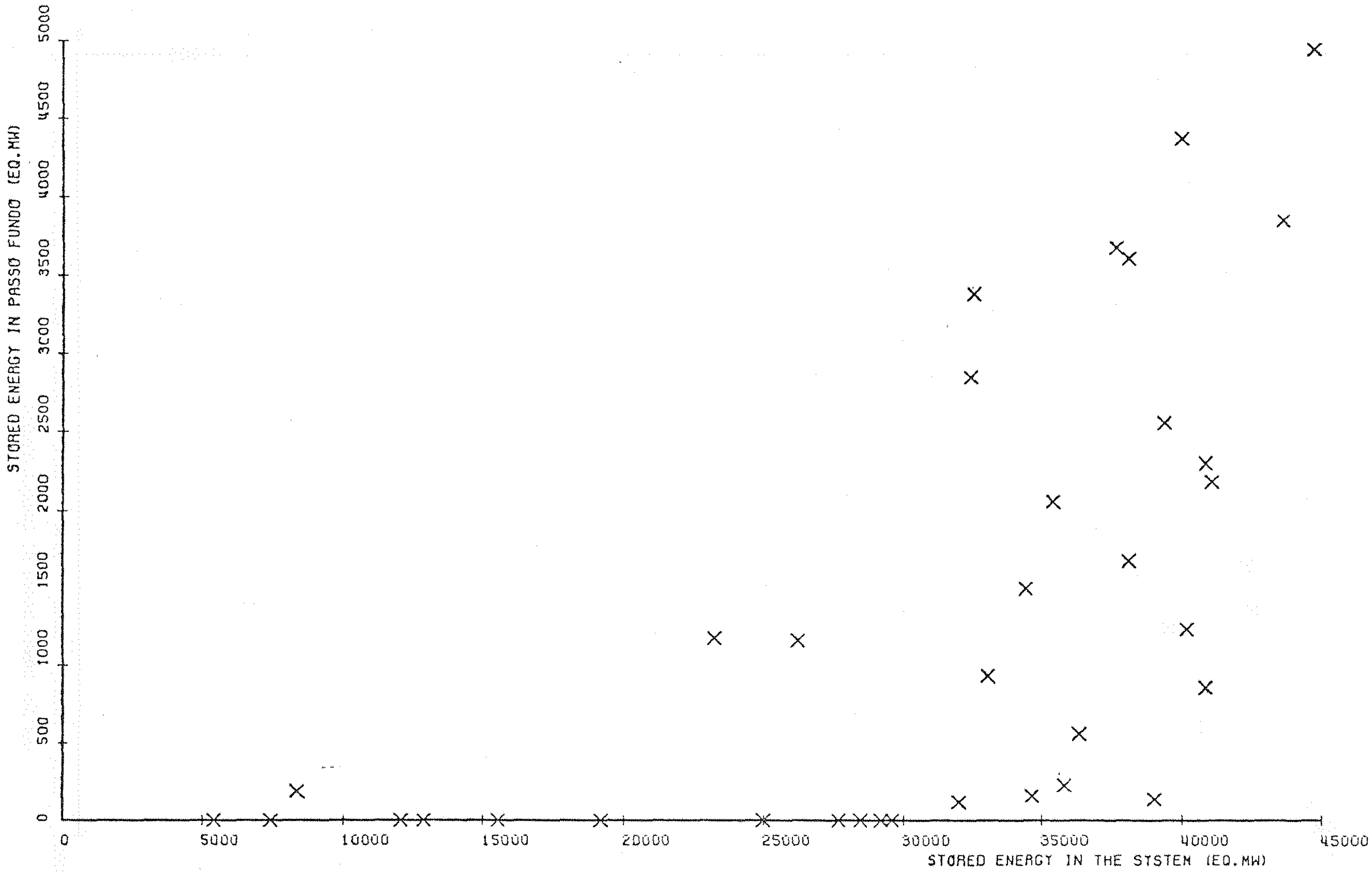


Figure 18.c. Energy Storage Distribution in Passo Fundo, 21st week

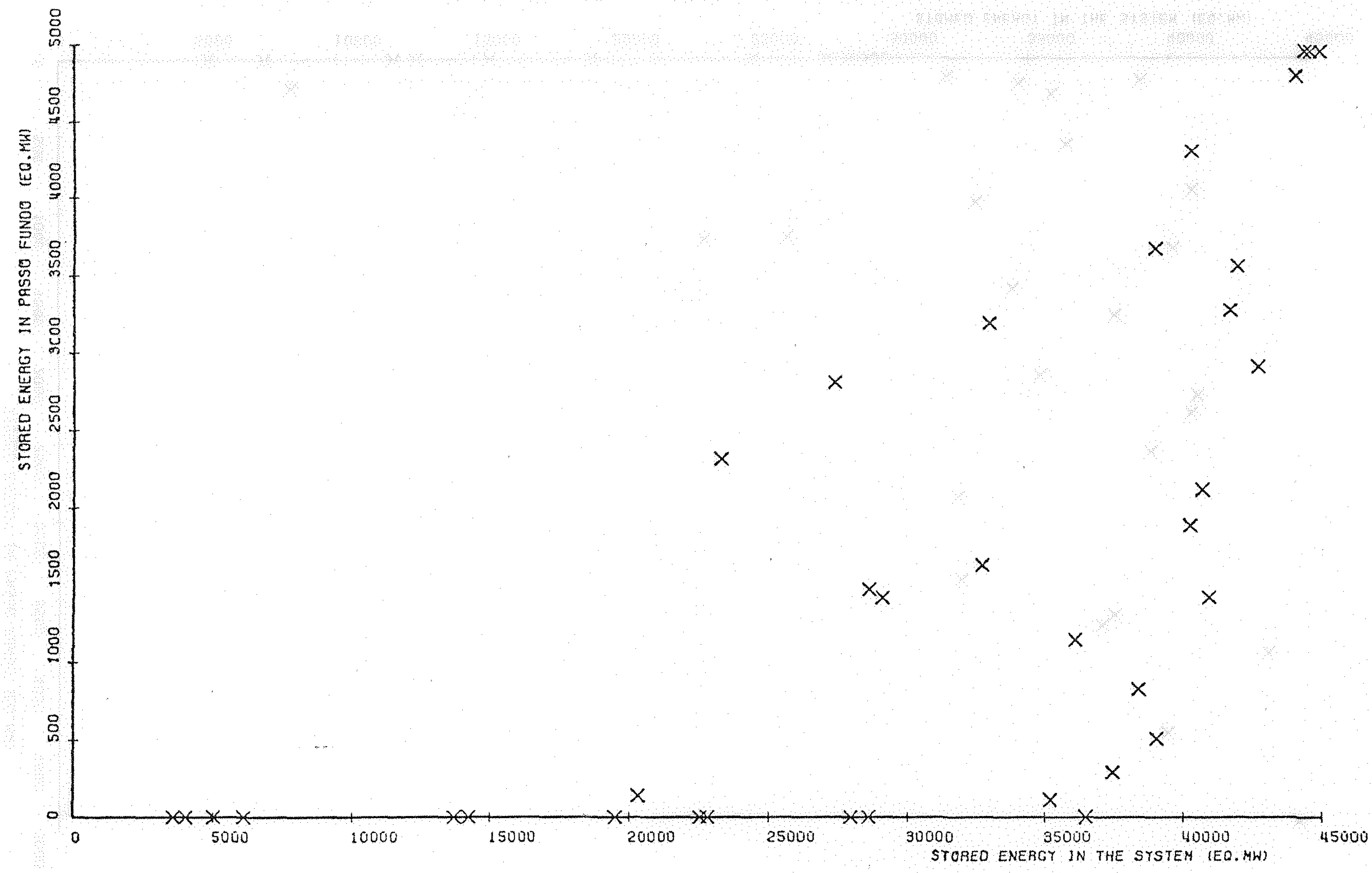


Figure 18.d. Energy Storage Distribution in Passo Fundo, 31st week

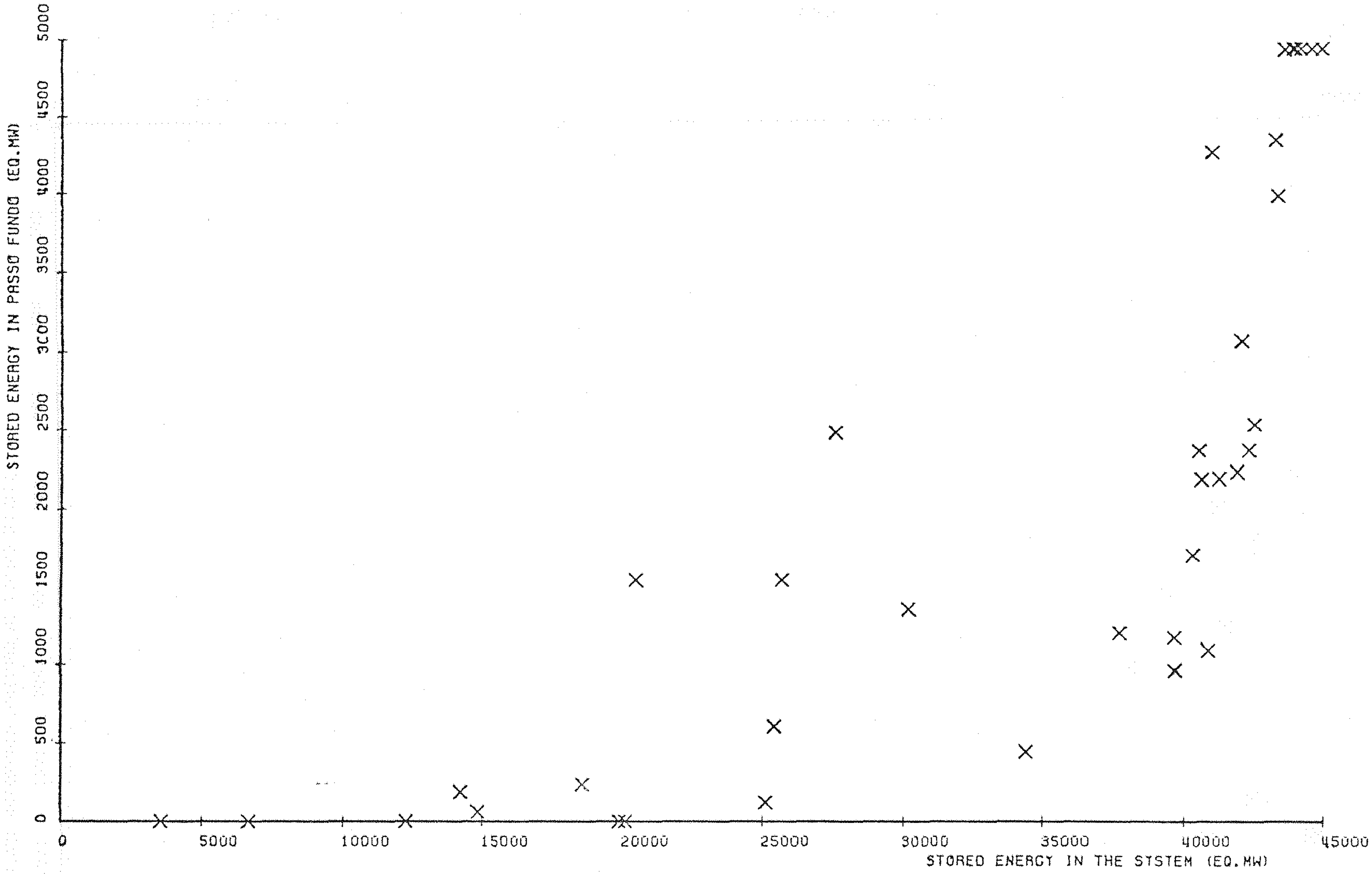


Figure 18.e. Energy Storage Distribution in Passo Fundo, 41st week

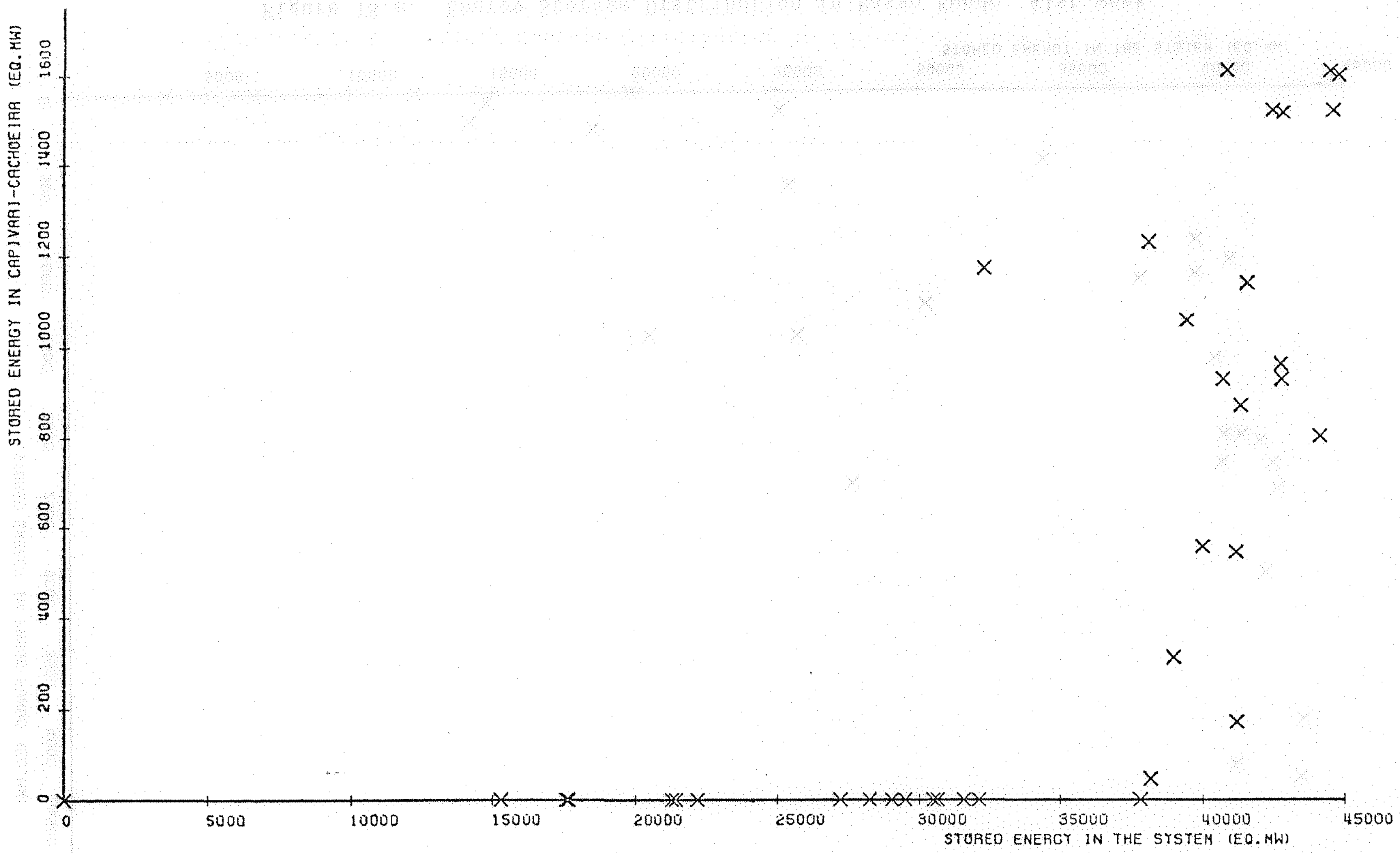


Figure 19.a. Energy Storage Distribution in Capivari-Cachoeira, 1st week

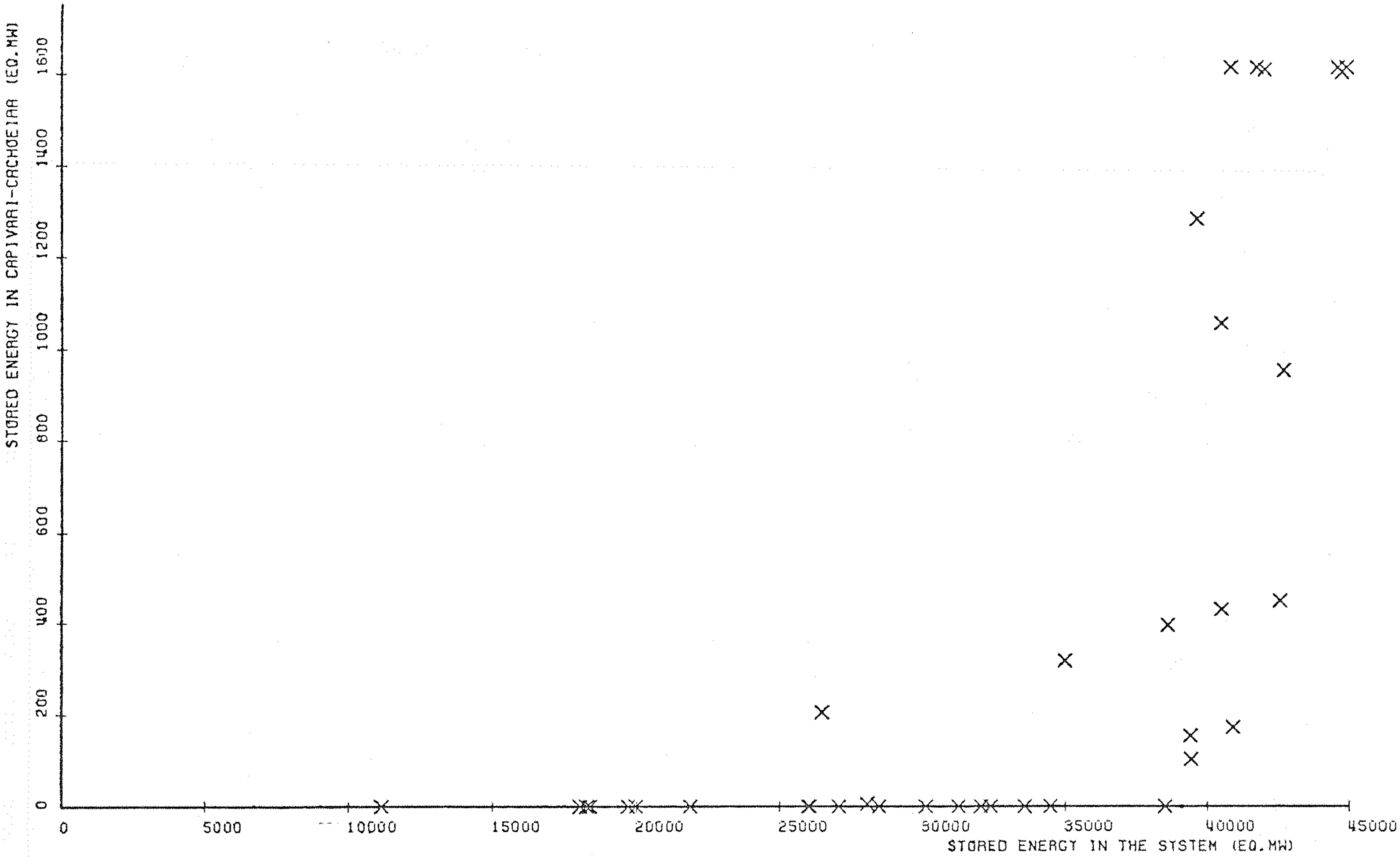


Figure 19.b. Energy Storage Distribution in Capivari-Cachoeira, 11th week

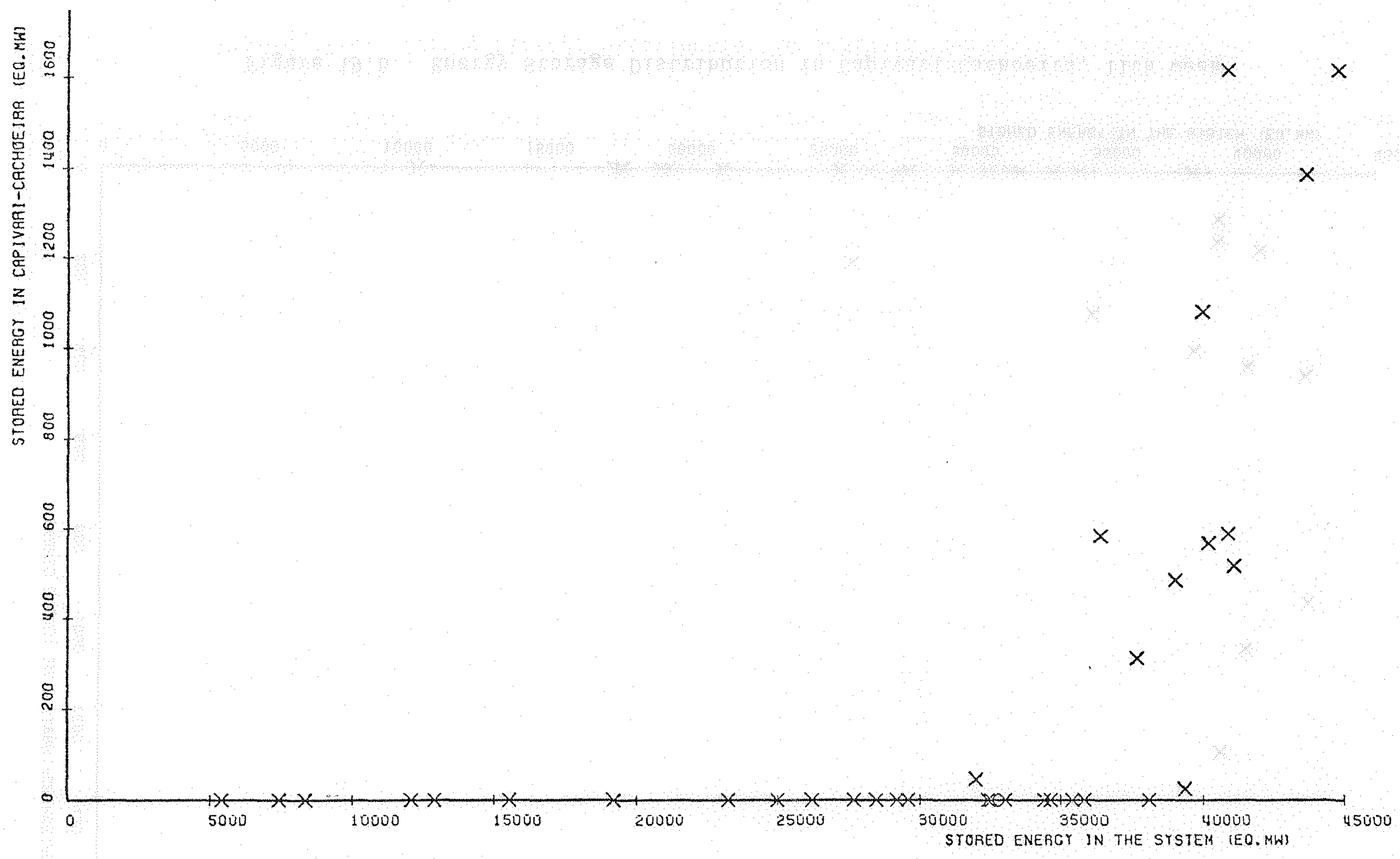


Figure 19.c. Energy Storage Distribution in Capivari-Cachoeira, 21st week

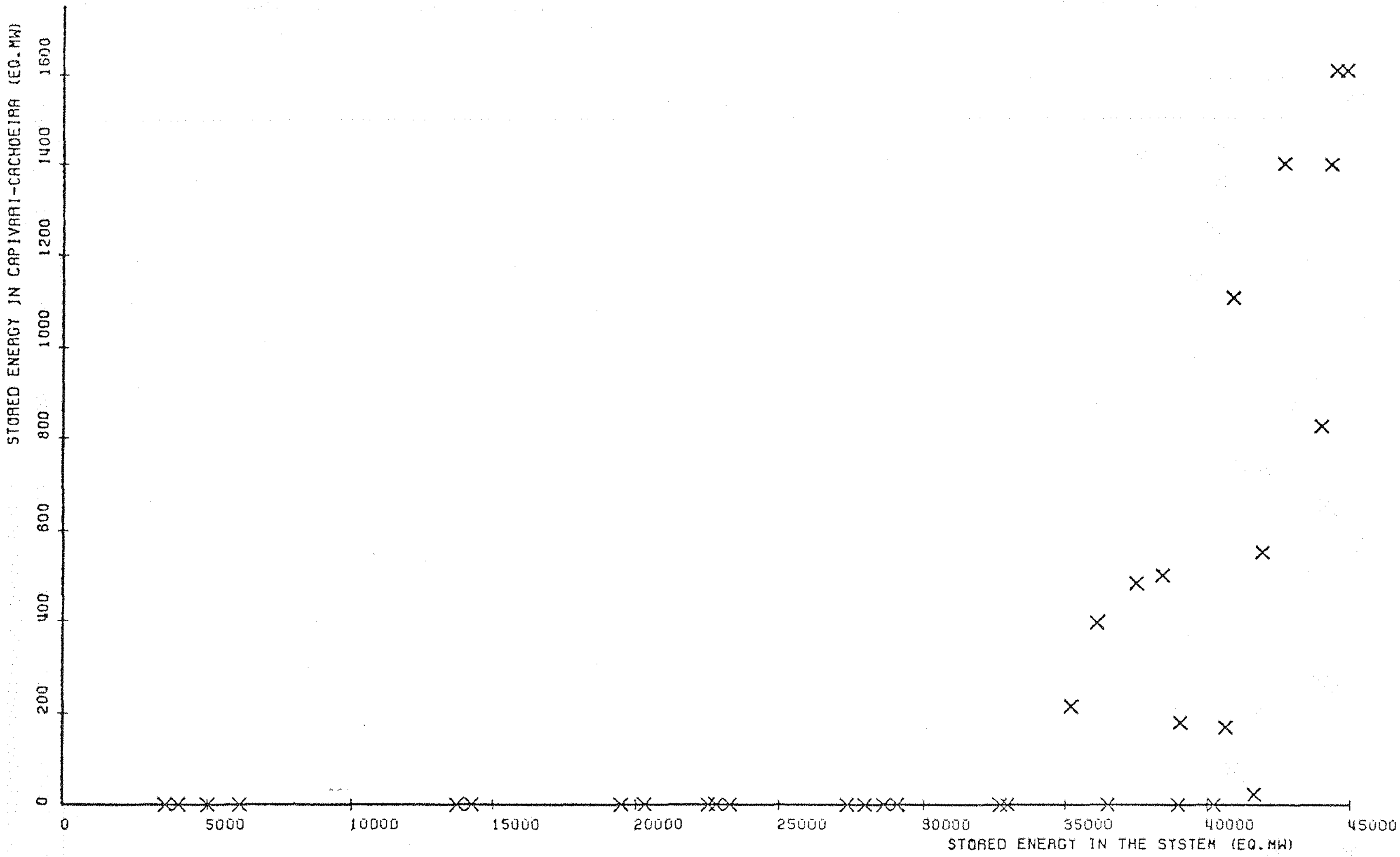


Figure 19.d. Energy Storage Distribution in Capivari-Cachoeira, 31st week

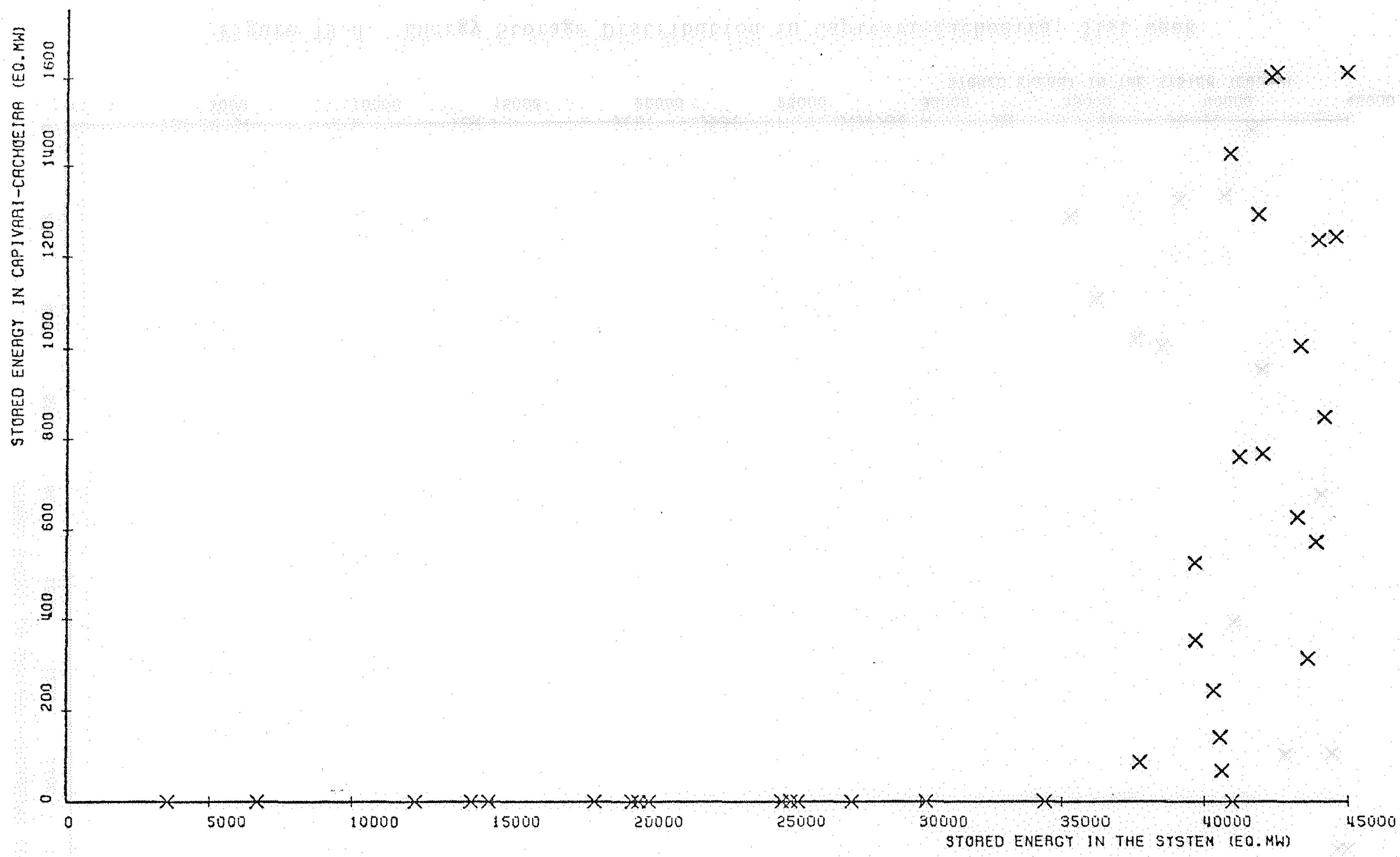


Figure 19.e. Energy Storage Distribution in Capivari-Cachoeira, 41st week

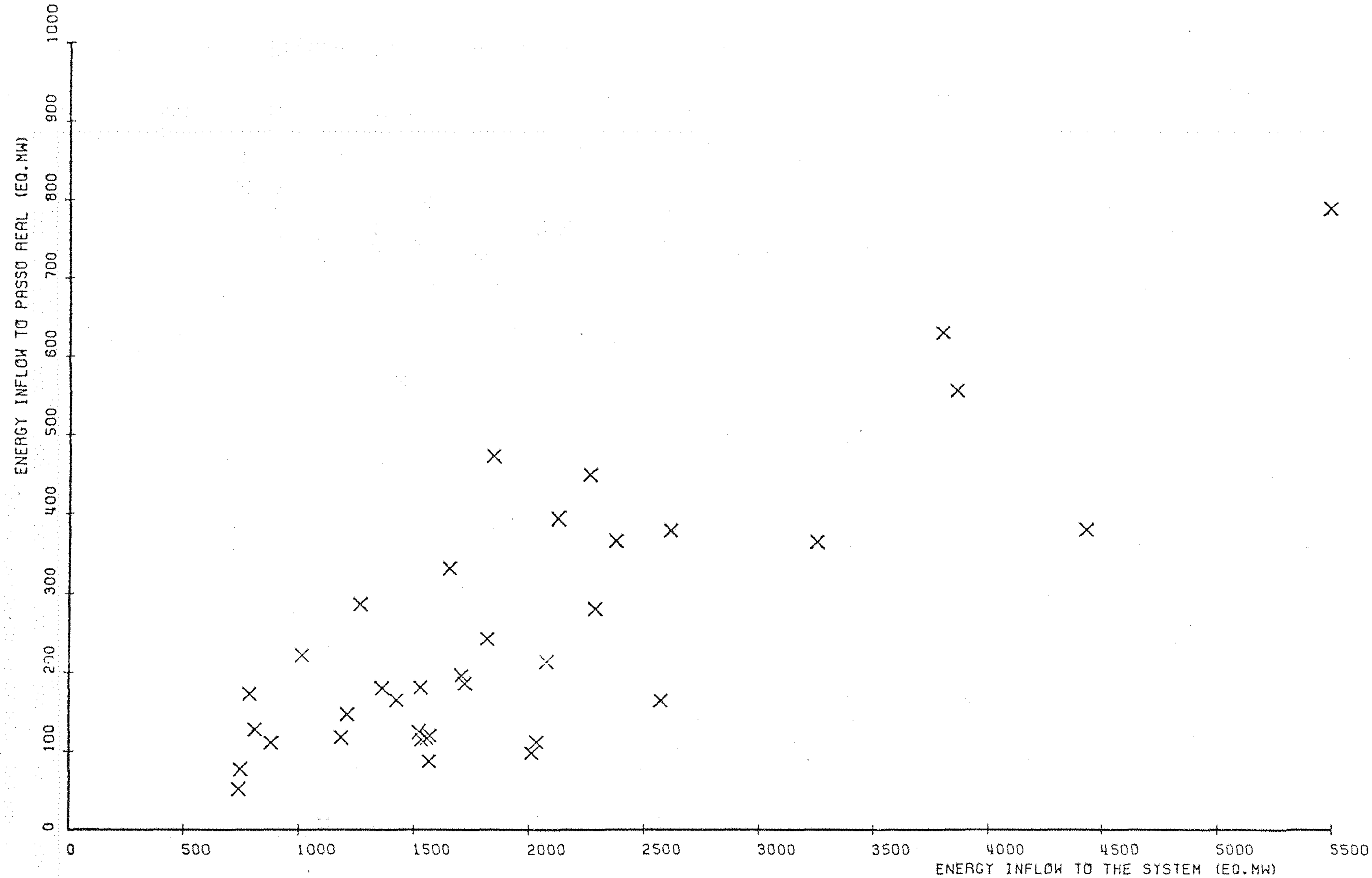


Figure 20.a. Energy Inflow Distribution in Passo Real, 1st week

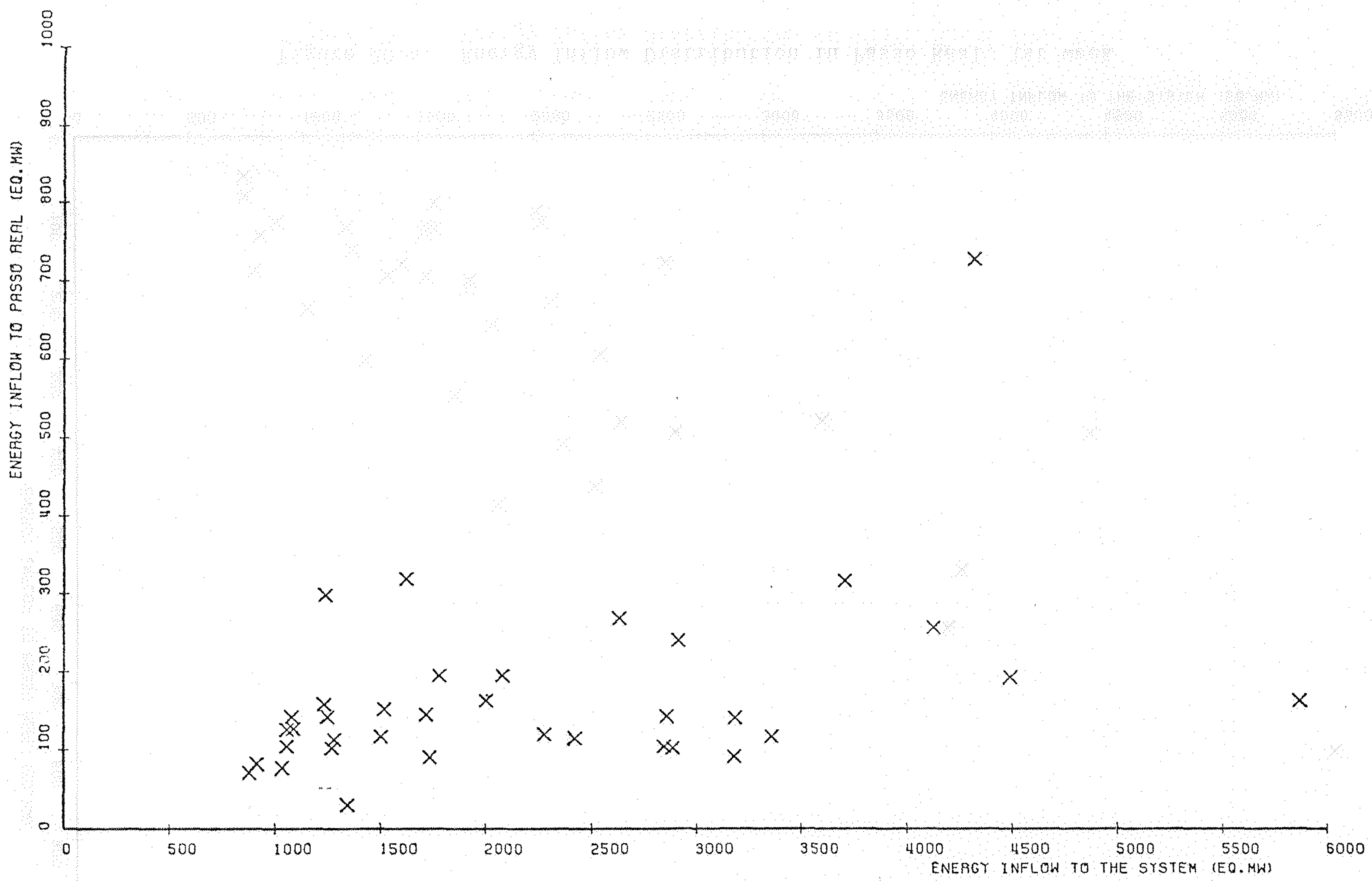


Figure 20.b. Energy Inflow Distribution in Passo Real, 11th week

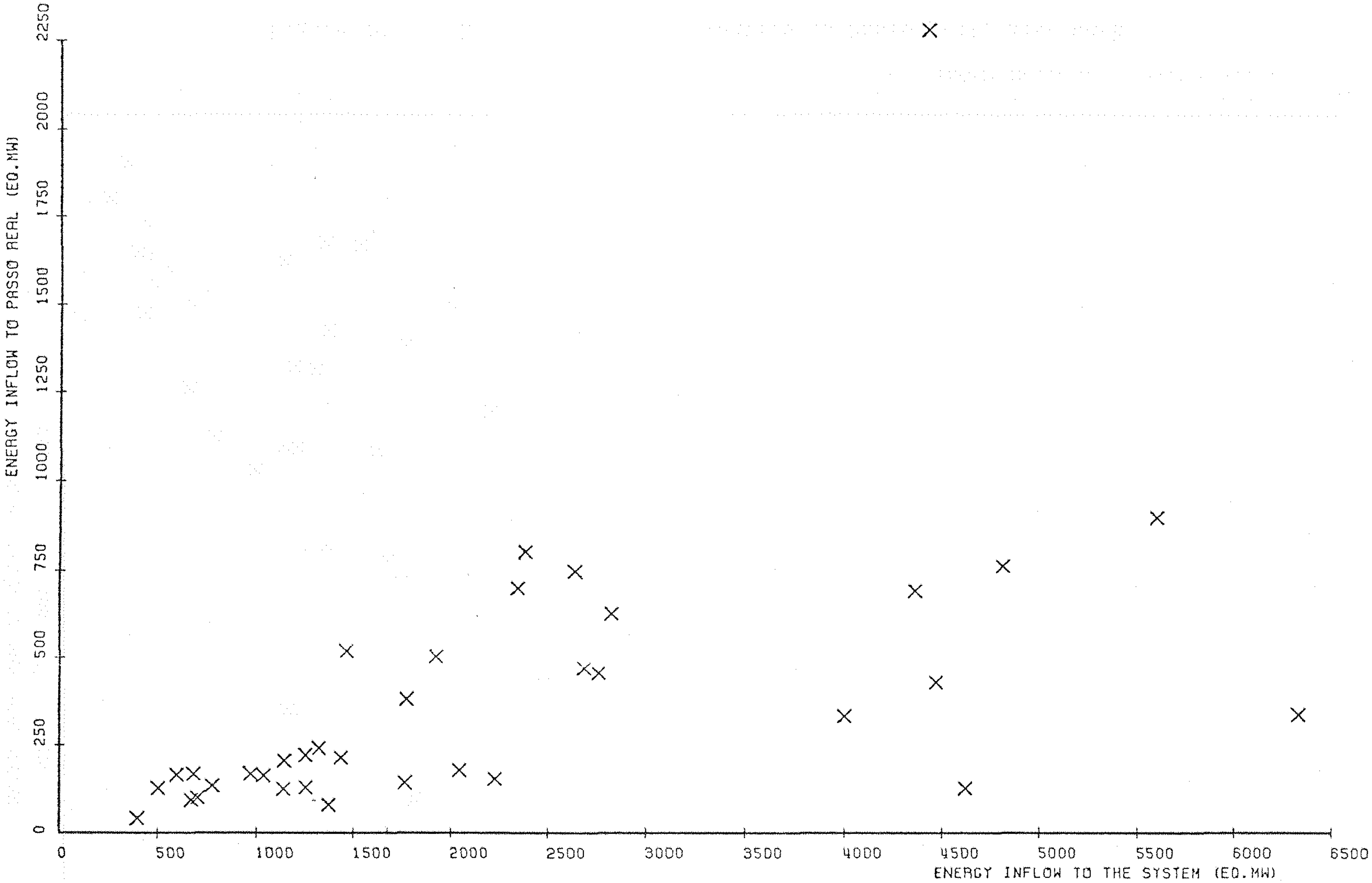


Figure 20.c. Energy Inflow Distribution in Passo Real, 21st week

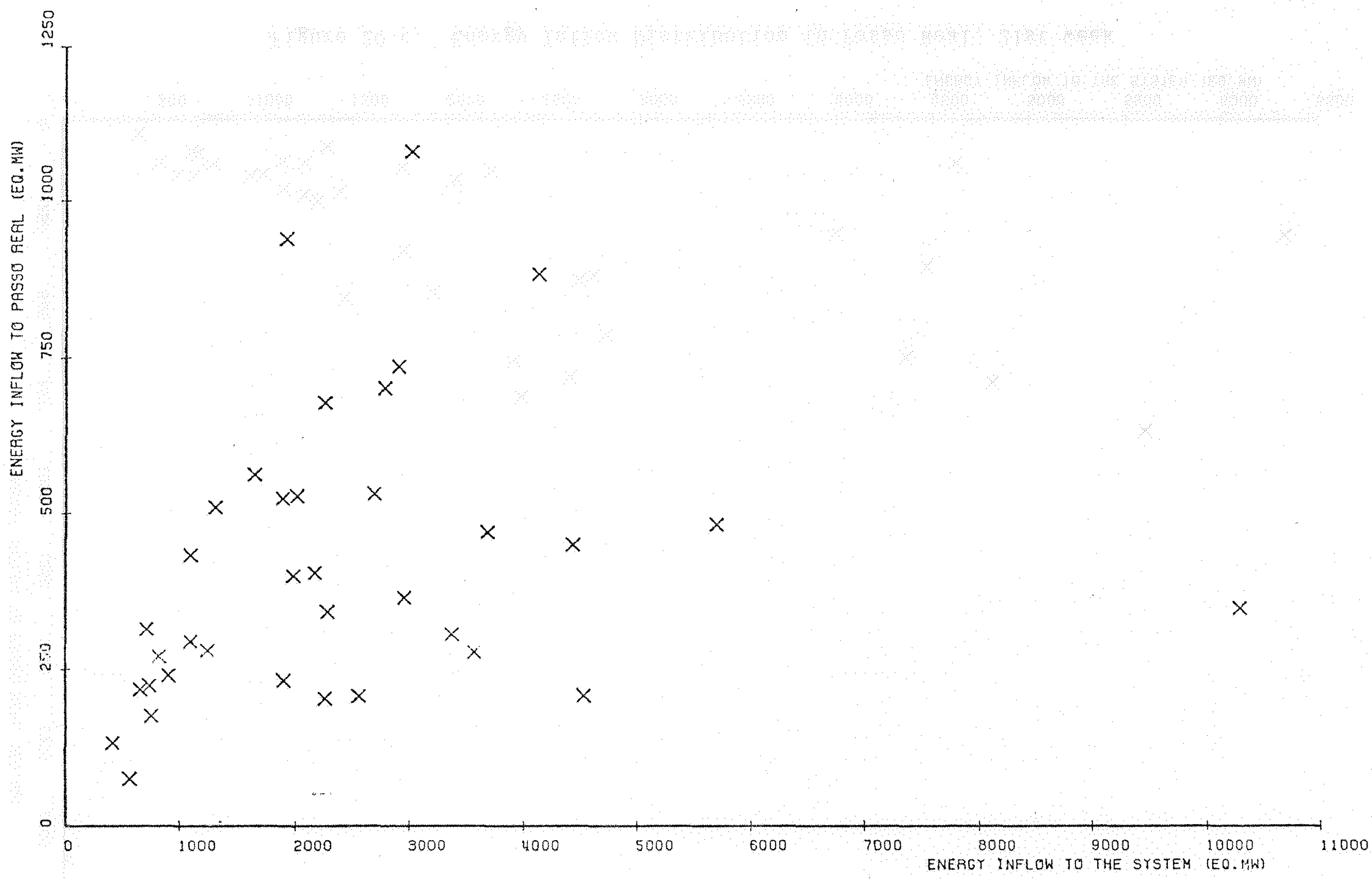


Figure 20.d. Energy Inflow Distribution in Passo Real, 31st week

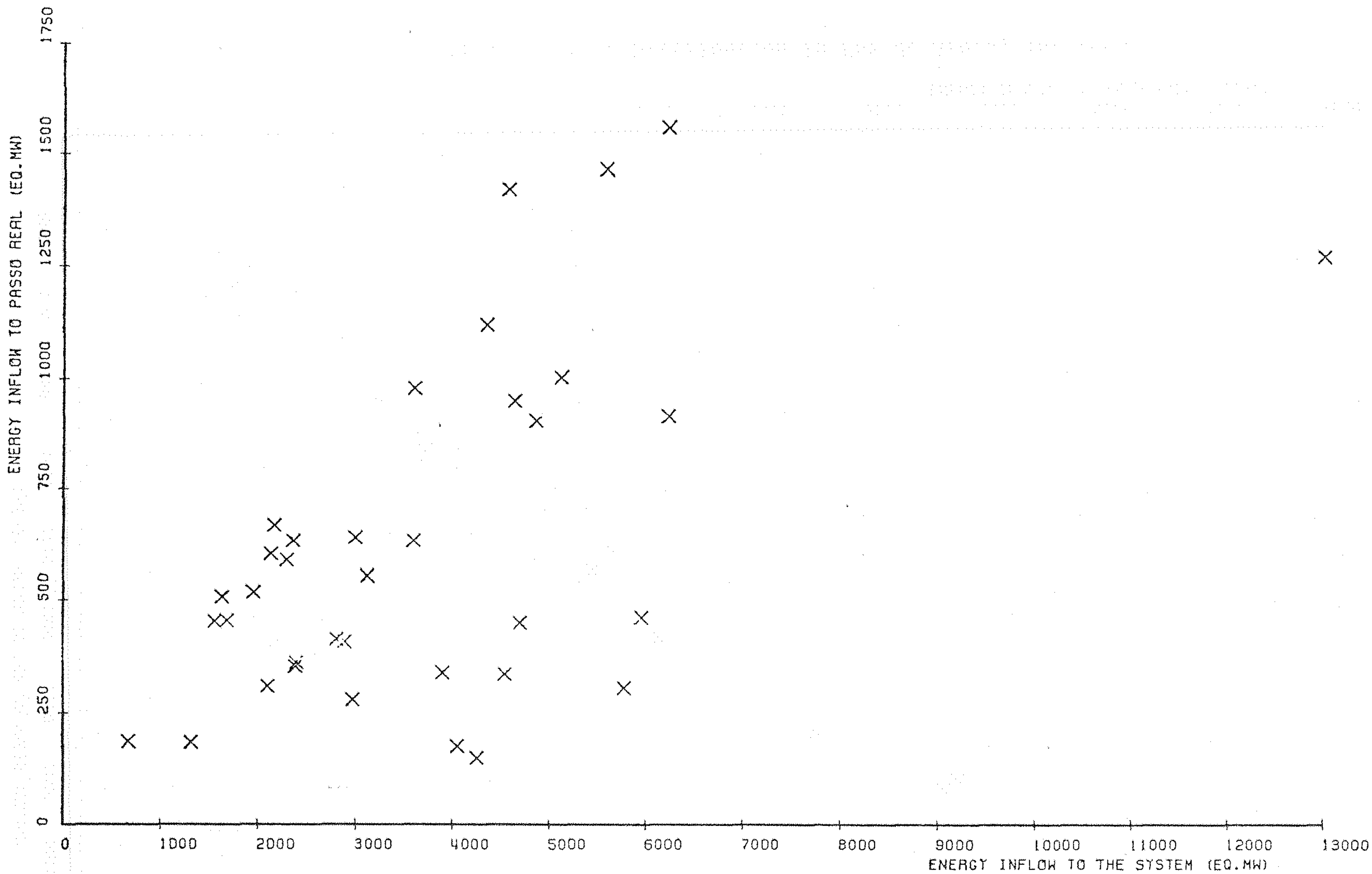


Figure 20.e. Energy Inflow Distribution in Passo Real, 41st week

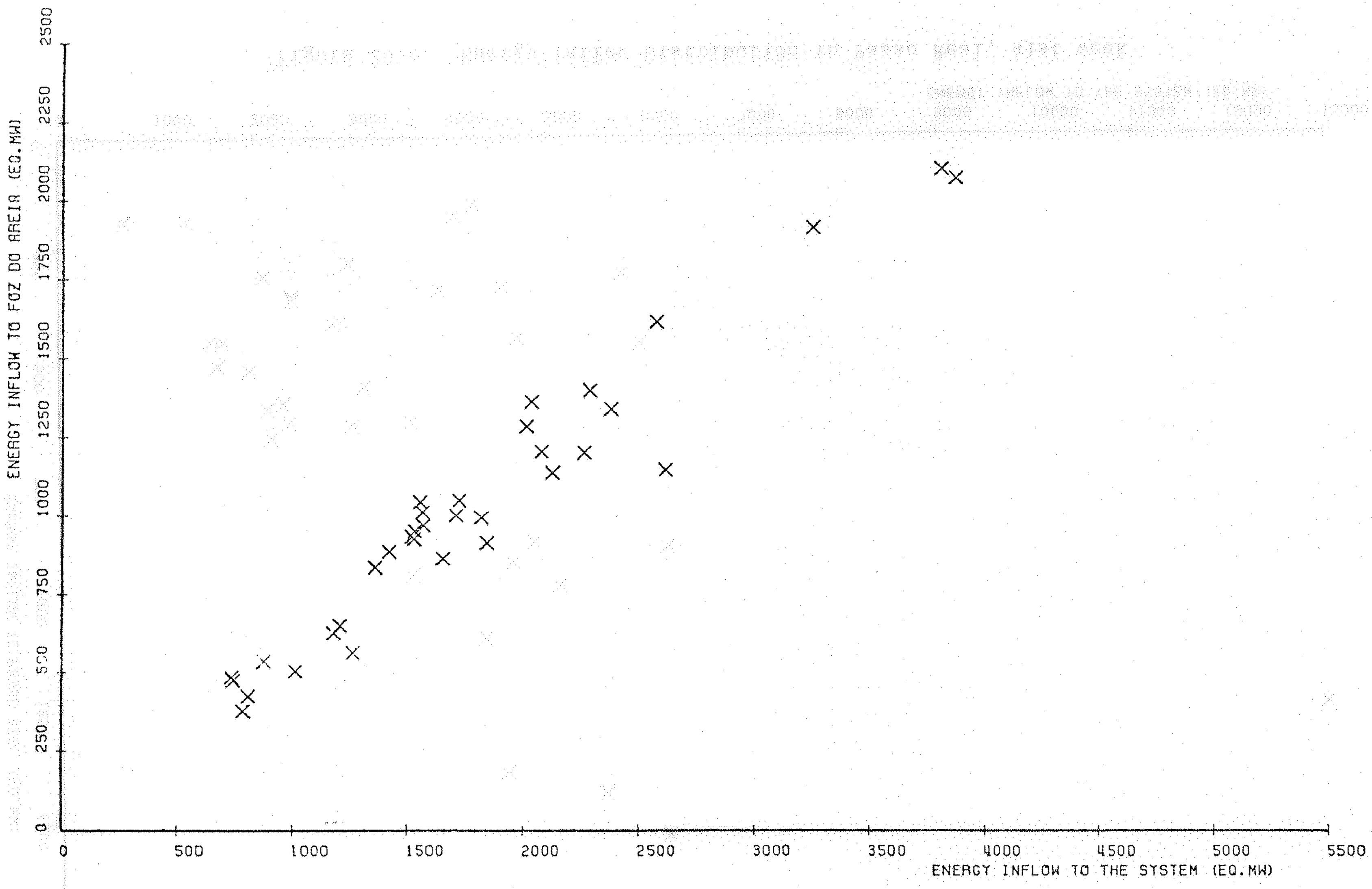


Figure 21.a. Energy Inflow Distribution in Foz do Areia, 1st week

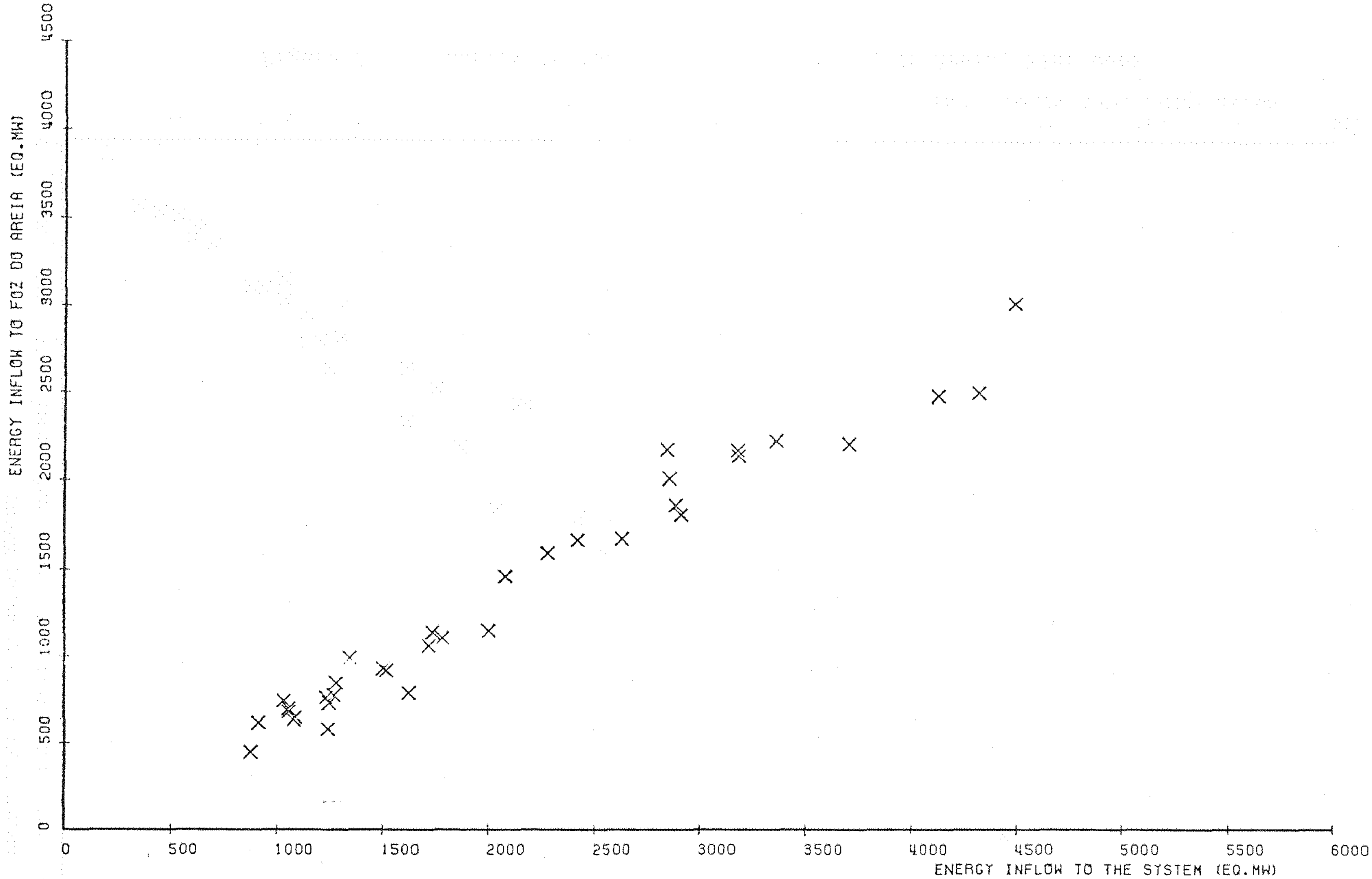


Figure 21.b. Energy Inflow Distribution in Foz do Areia, 11th week

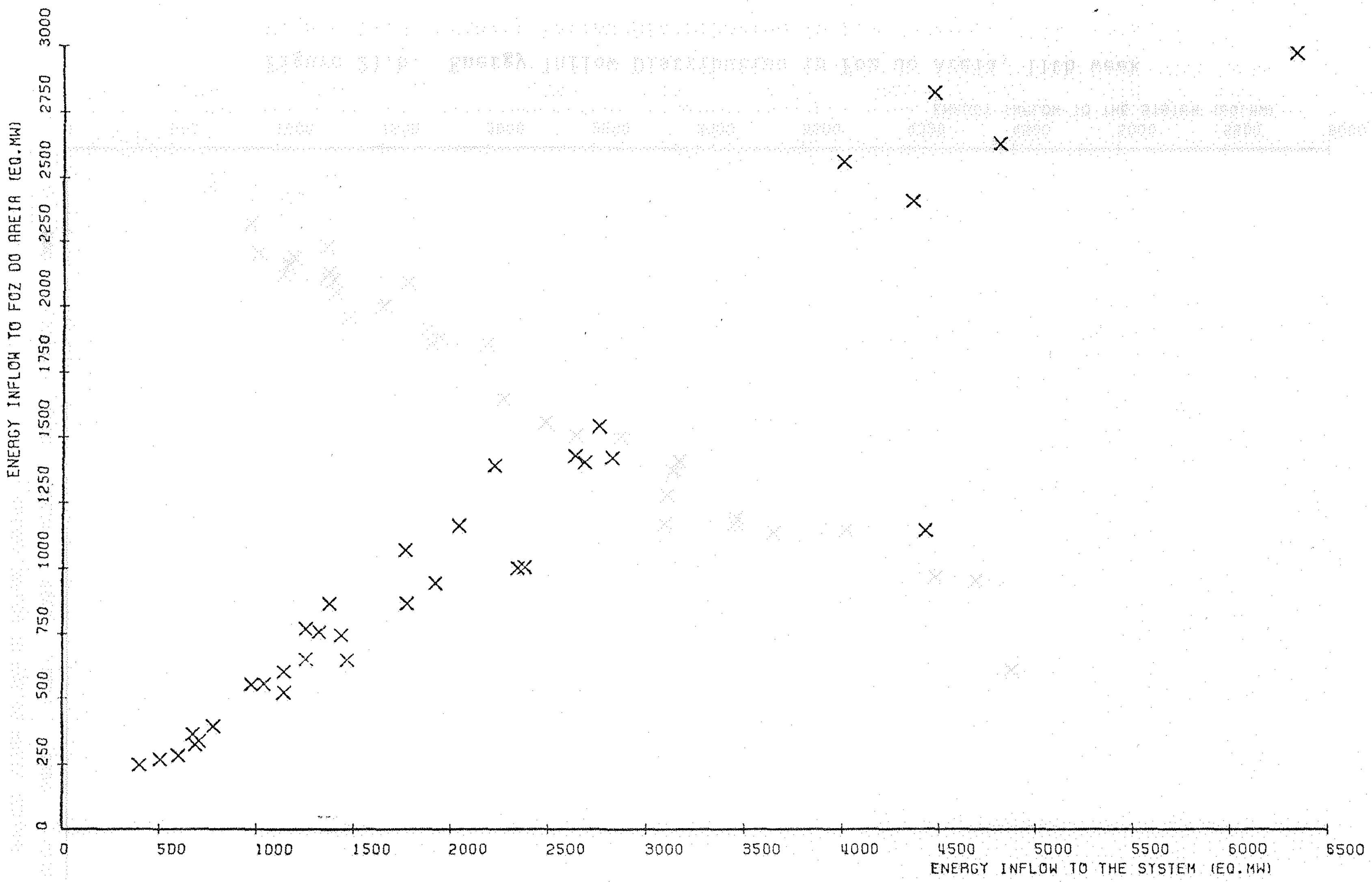


Figure 21.c. Energy Inflow Distribution in Foz do Areia, 21st week

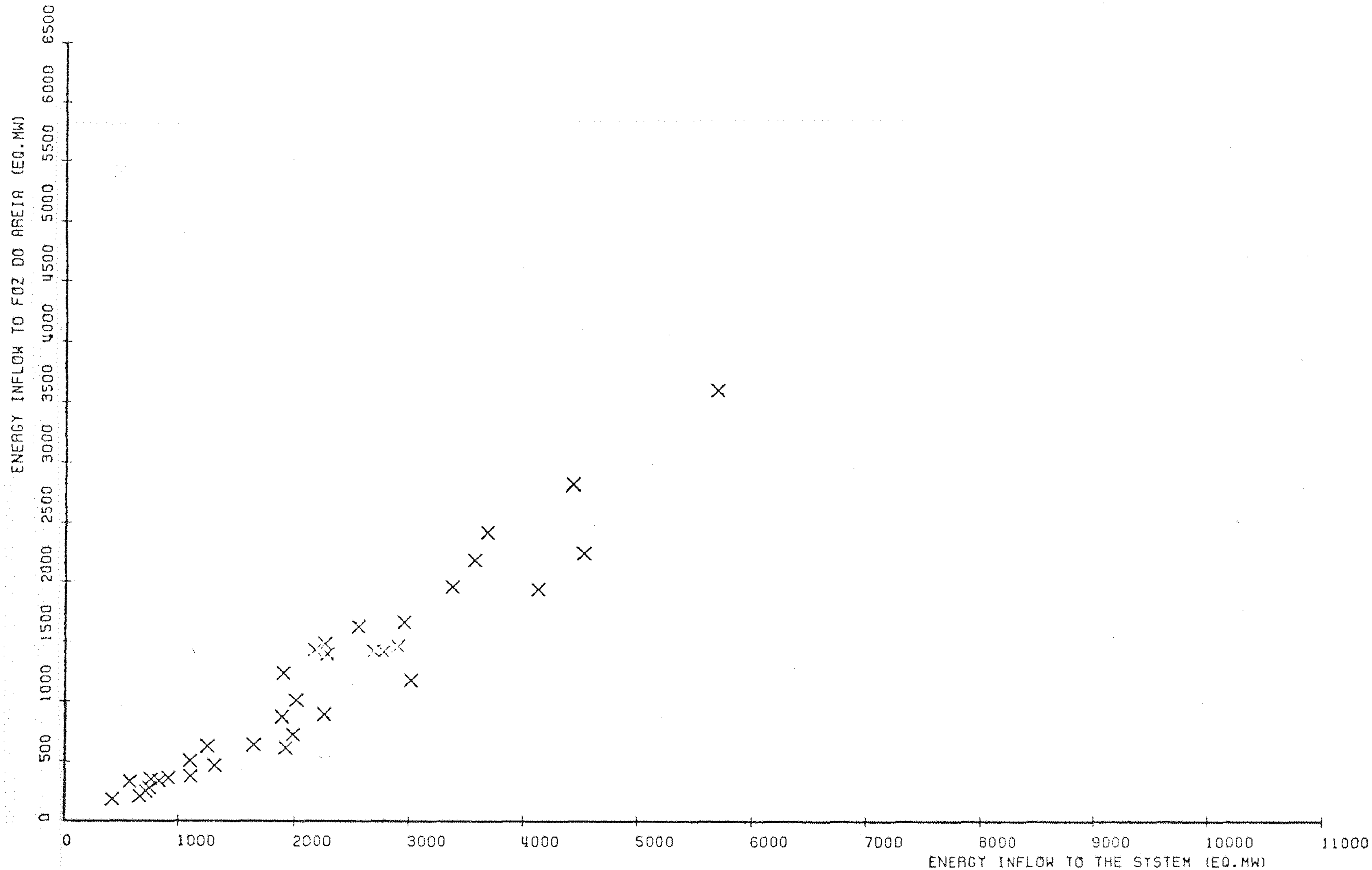


Figure 21.d. Energy Inflow Distribution in Foz do Areia, 31st week

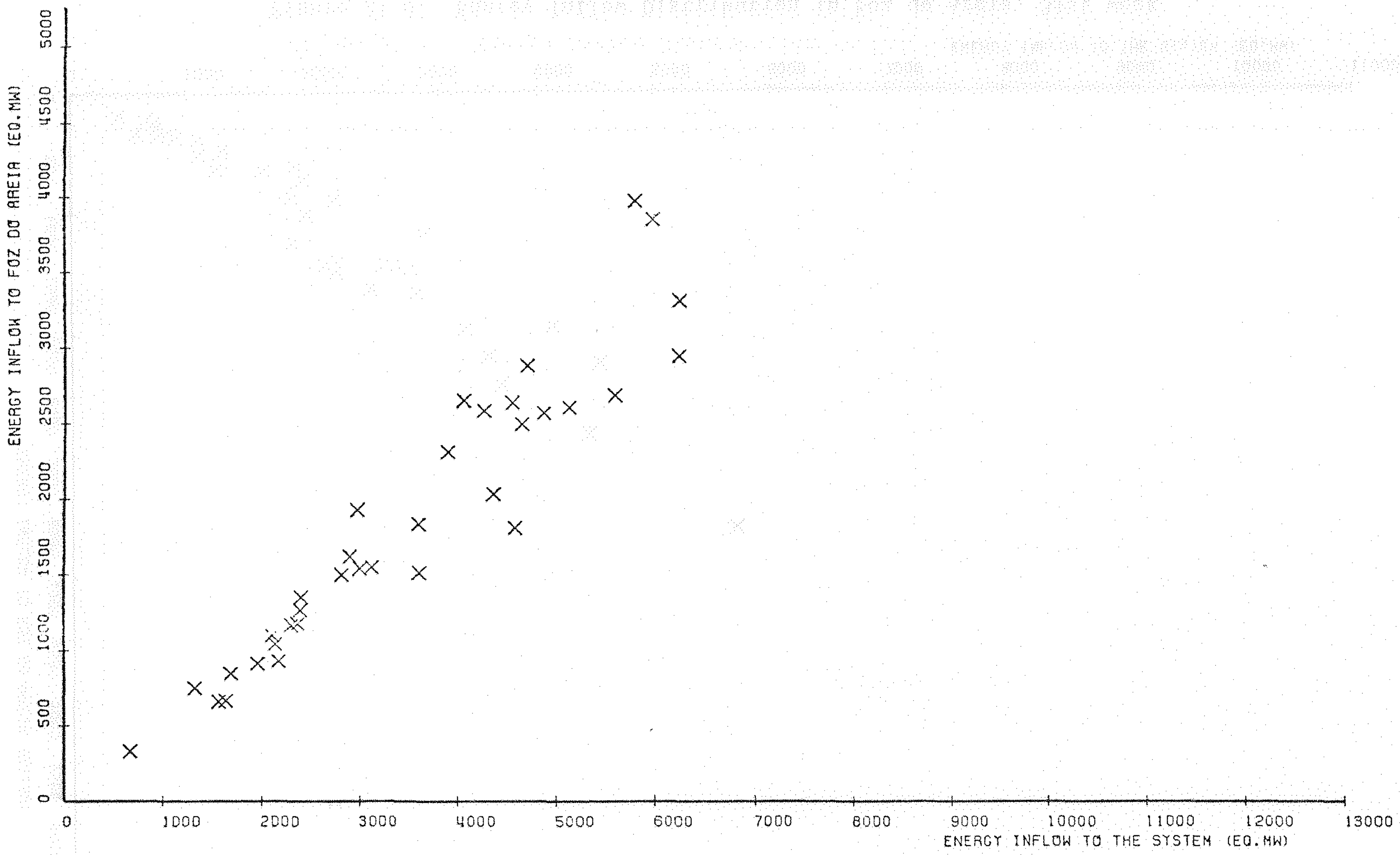


Figure 21.e. Energy Inflow Distribution in Foz do Areia, 41st week

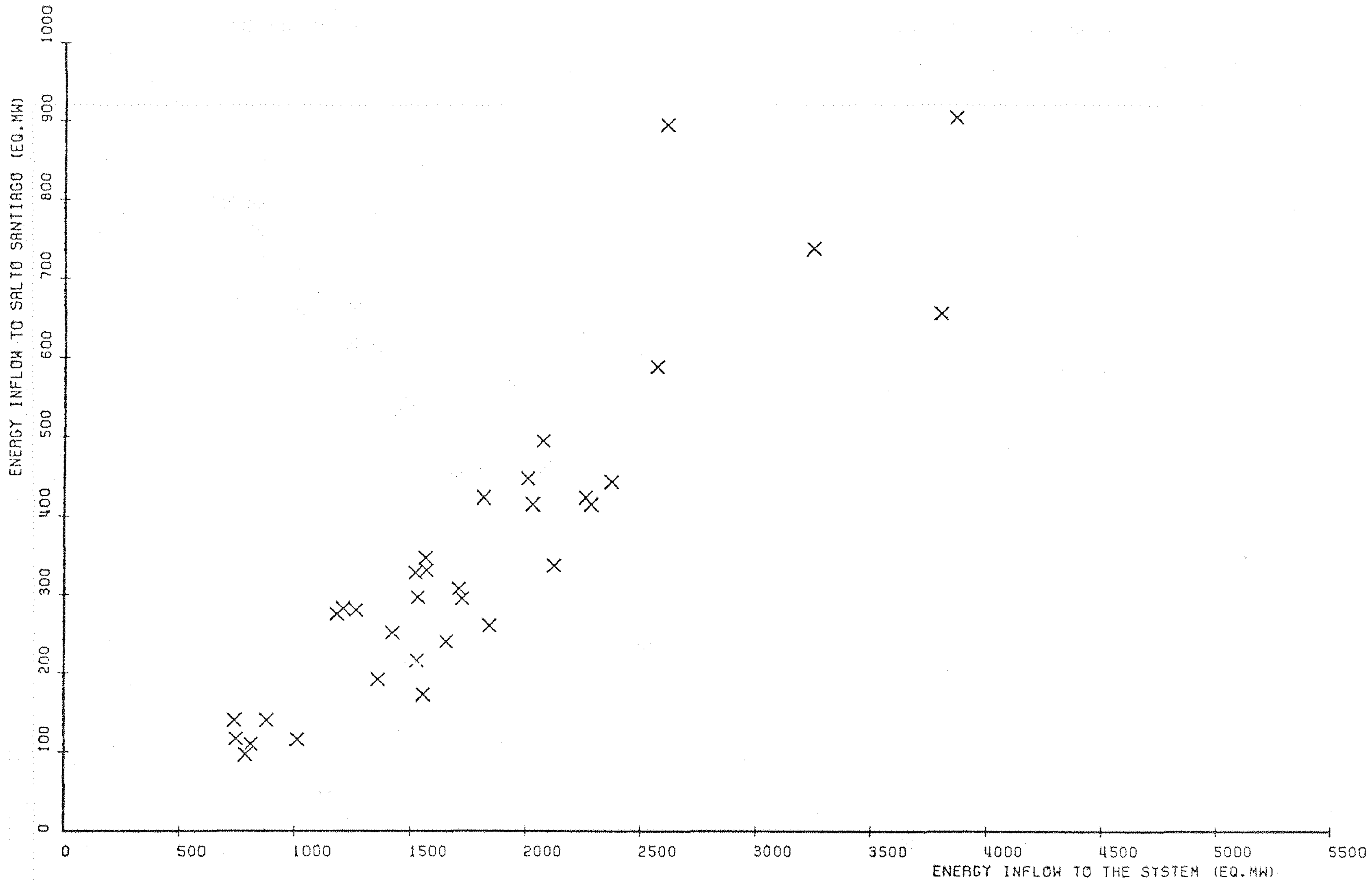


Figure 22.a. Energy Inflow Distribution in Salto Santiago, 1st week

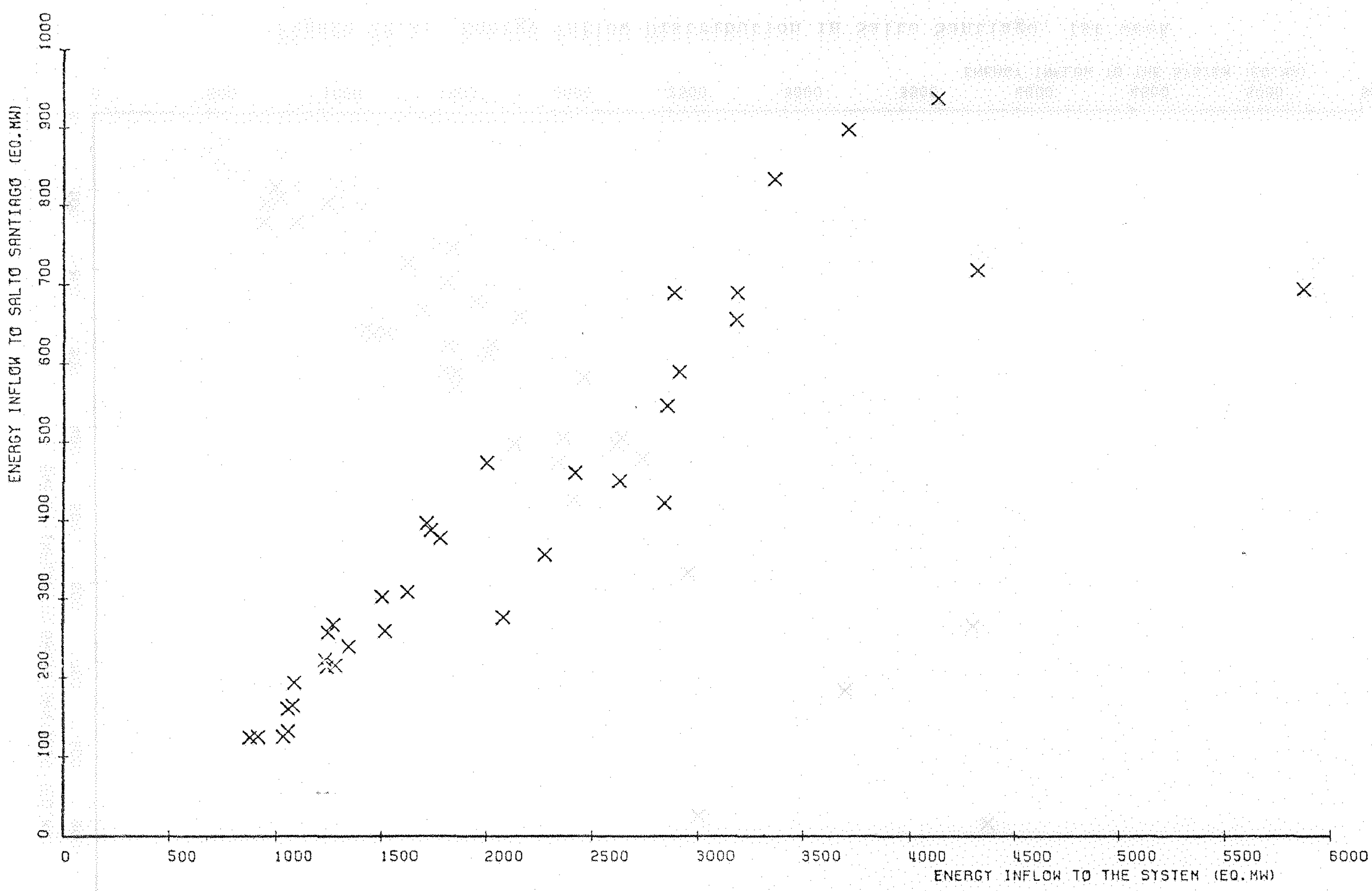


Figure 22.b. Energy Inflow Distribution in Salto Santiago, 11th week

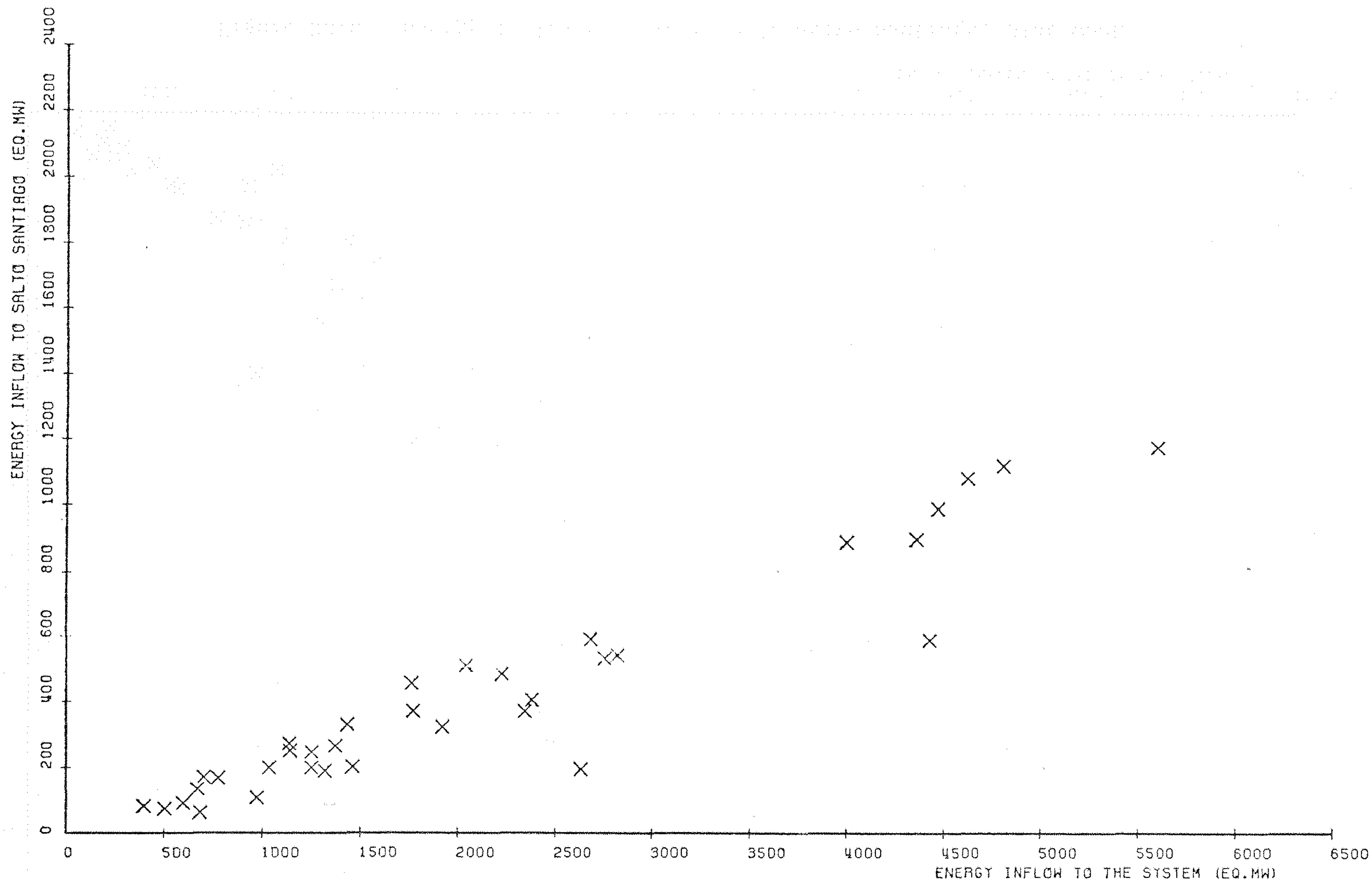


Figure 22.c. Energy Inflow Distribution in Salto Santiago, 21st week

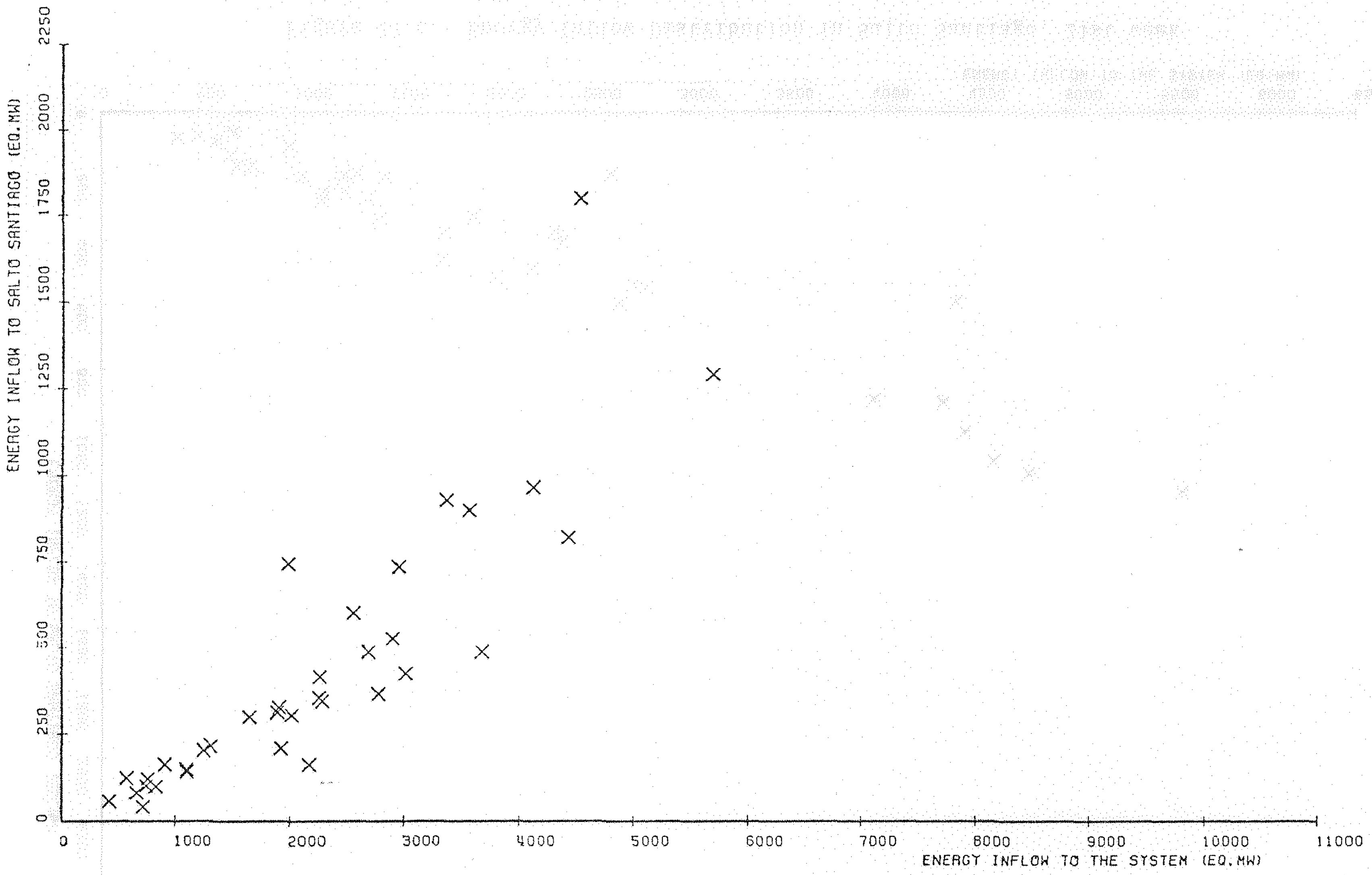


Figure 22.d. Energy Inflow Distribution in Salto Santiago, 31st week

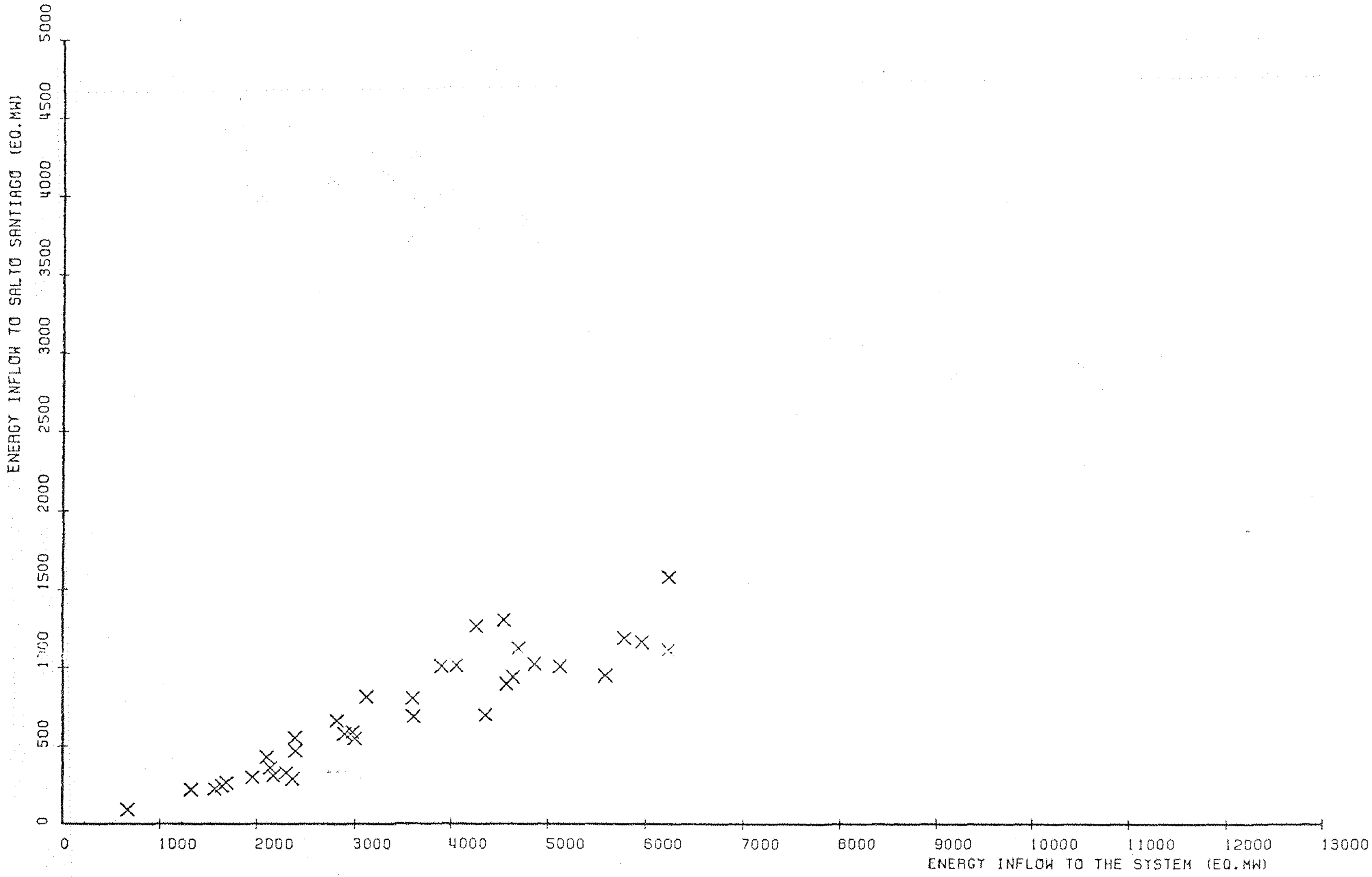


Figure 22.e. Energy Inflow Distribution in Salto Santiago, 41st week

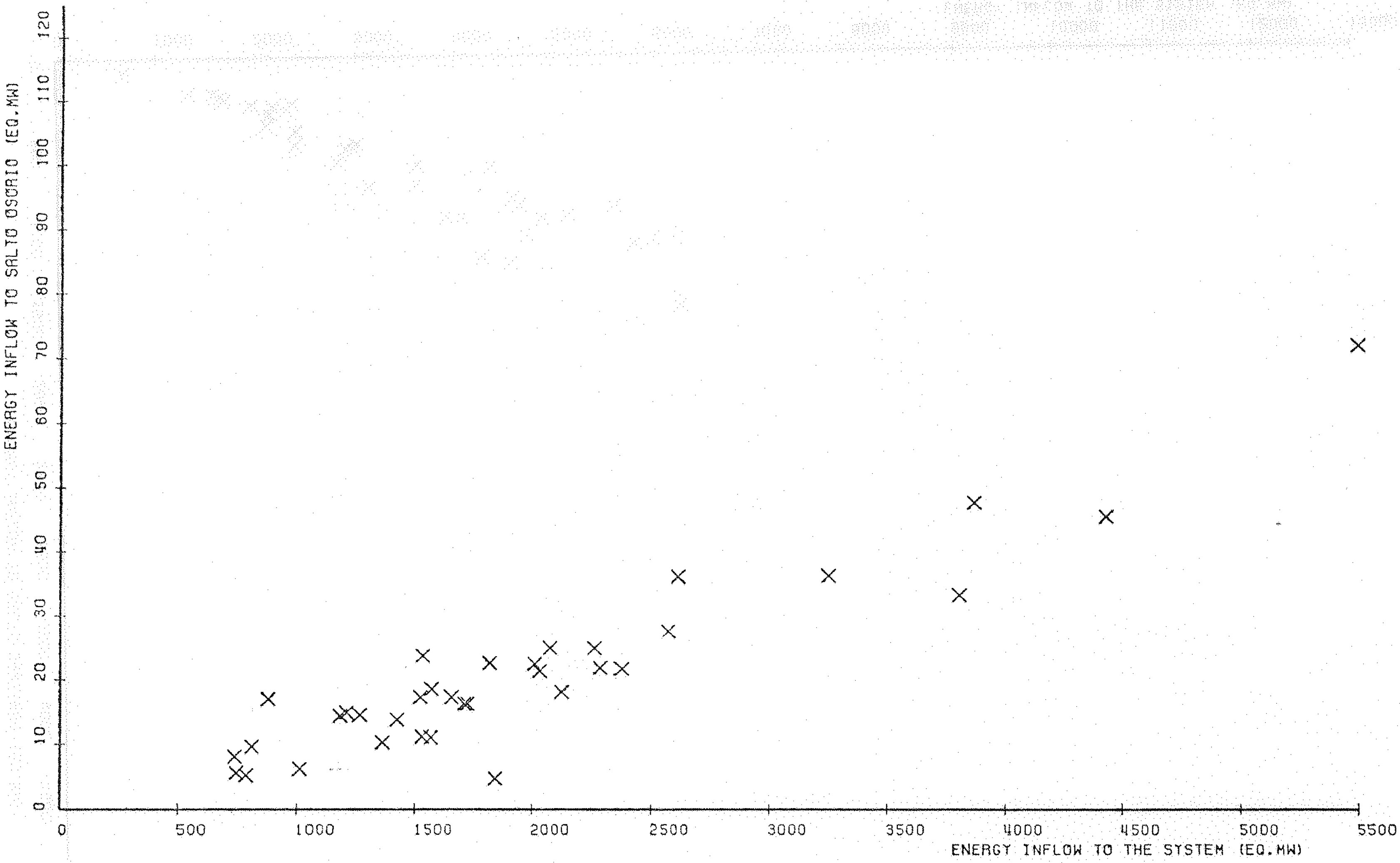


Figure 23.a. Energy Inflow Distribution in Salto Osorio, 1st week

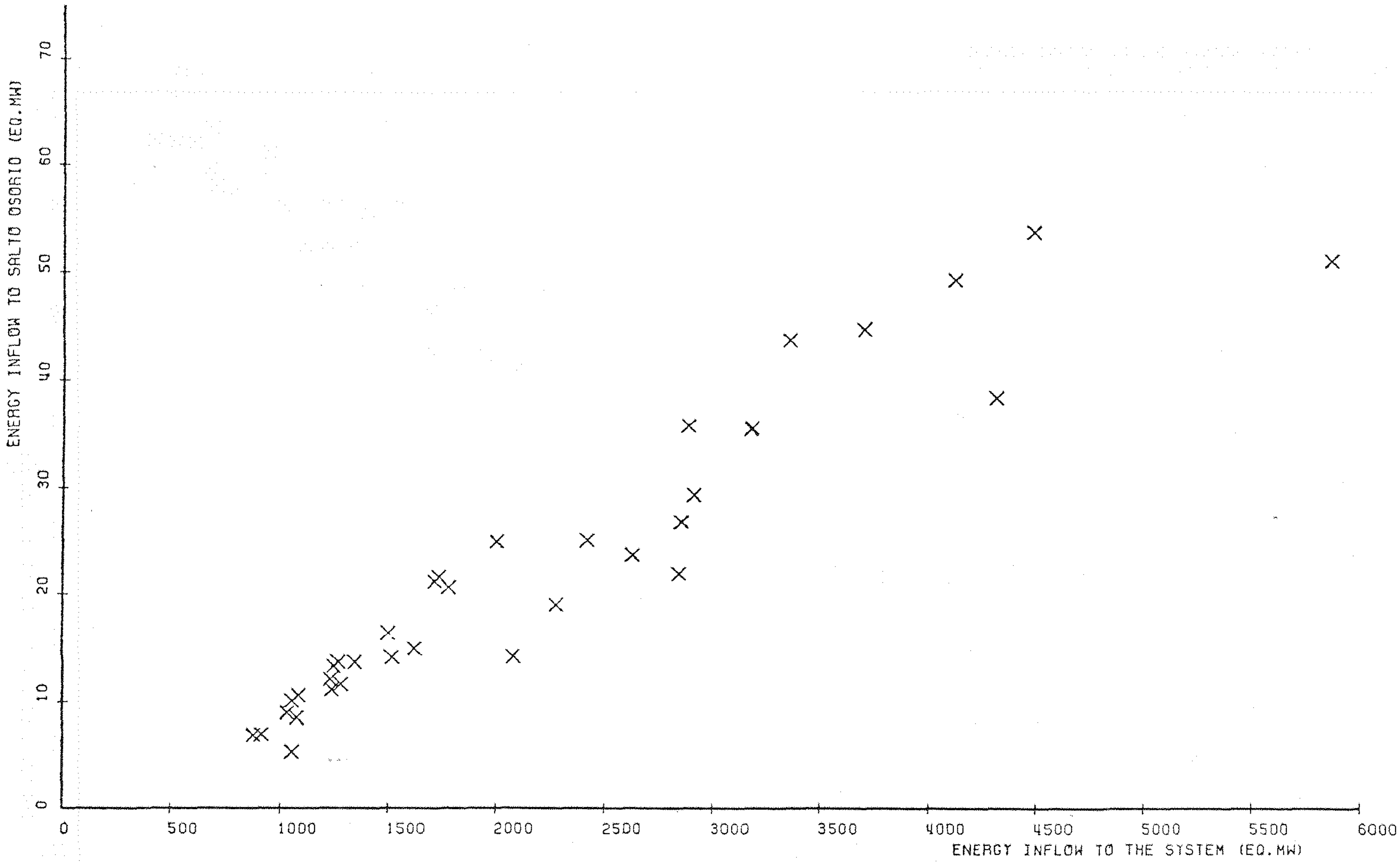


Figure 23.b. Energy Inflow Distribution in Salto Osorio, 11th week

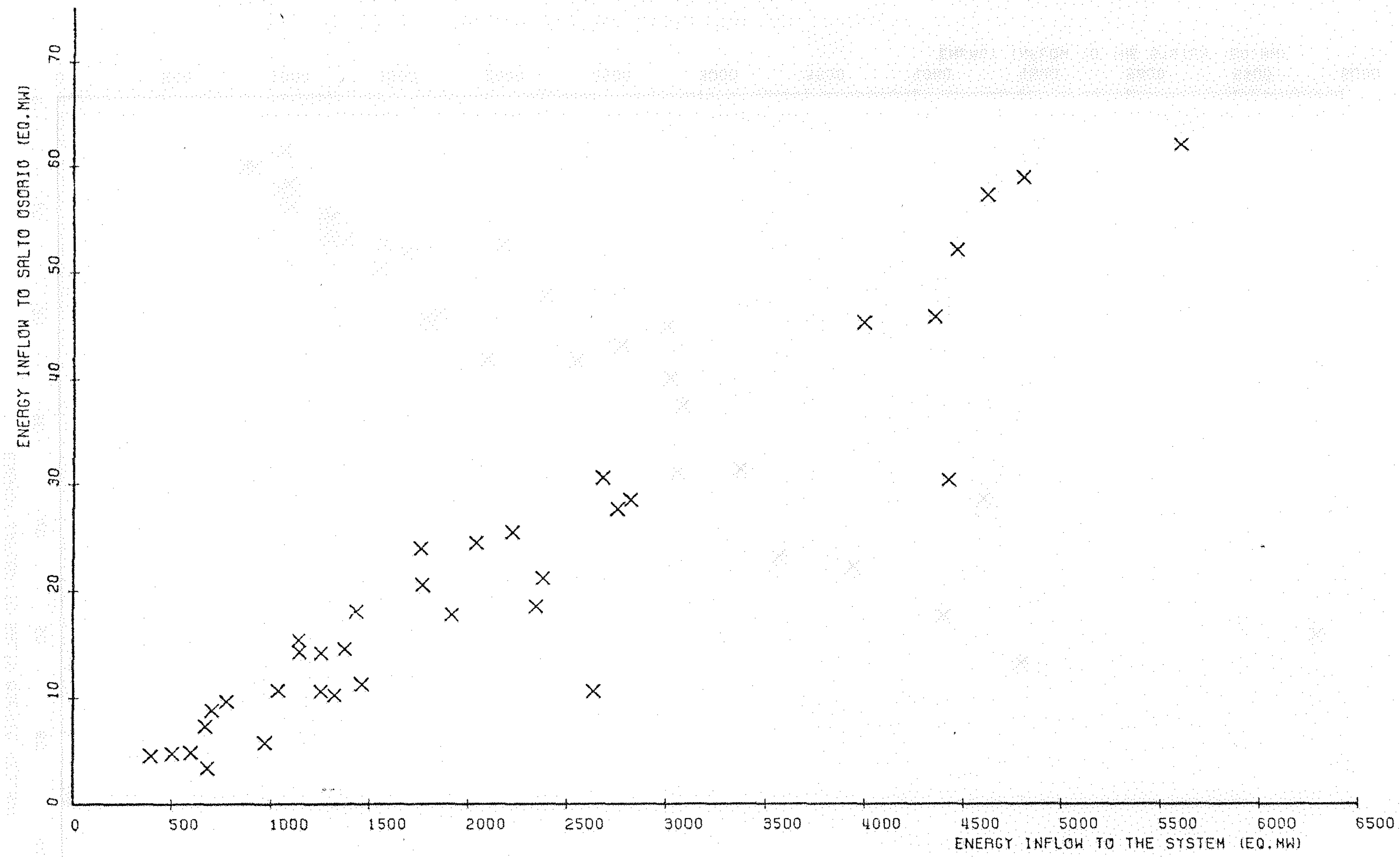


Figure 23.c. Energy Inflow Distribution in Salto Osorio, 21st week

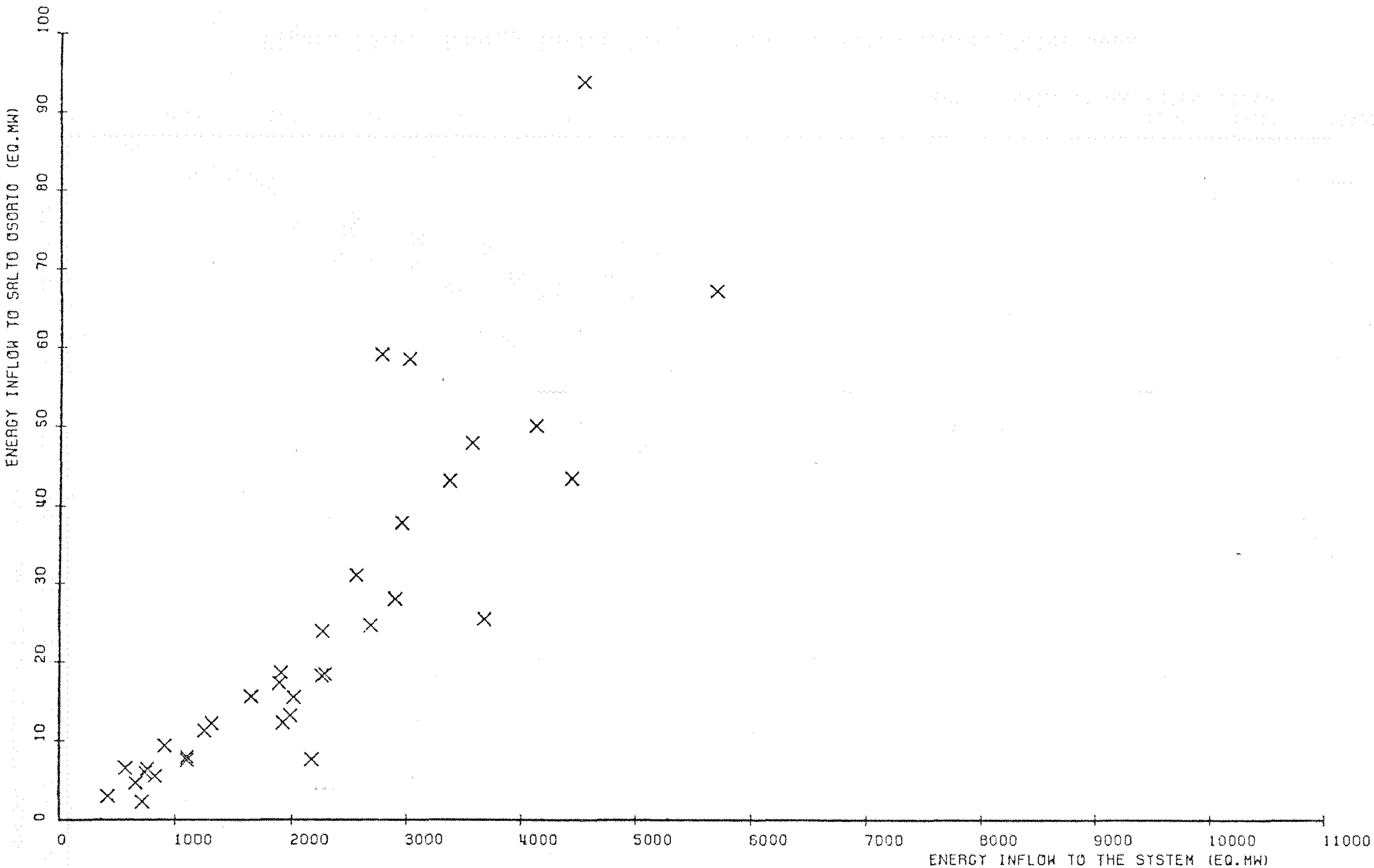


Figure 23.d. Energy Inflow Distribution in Salto Osorio, 31st week

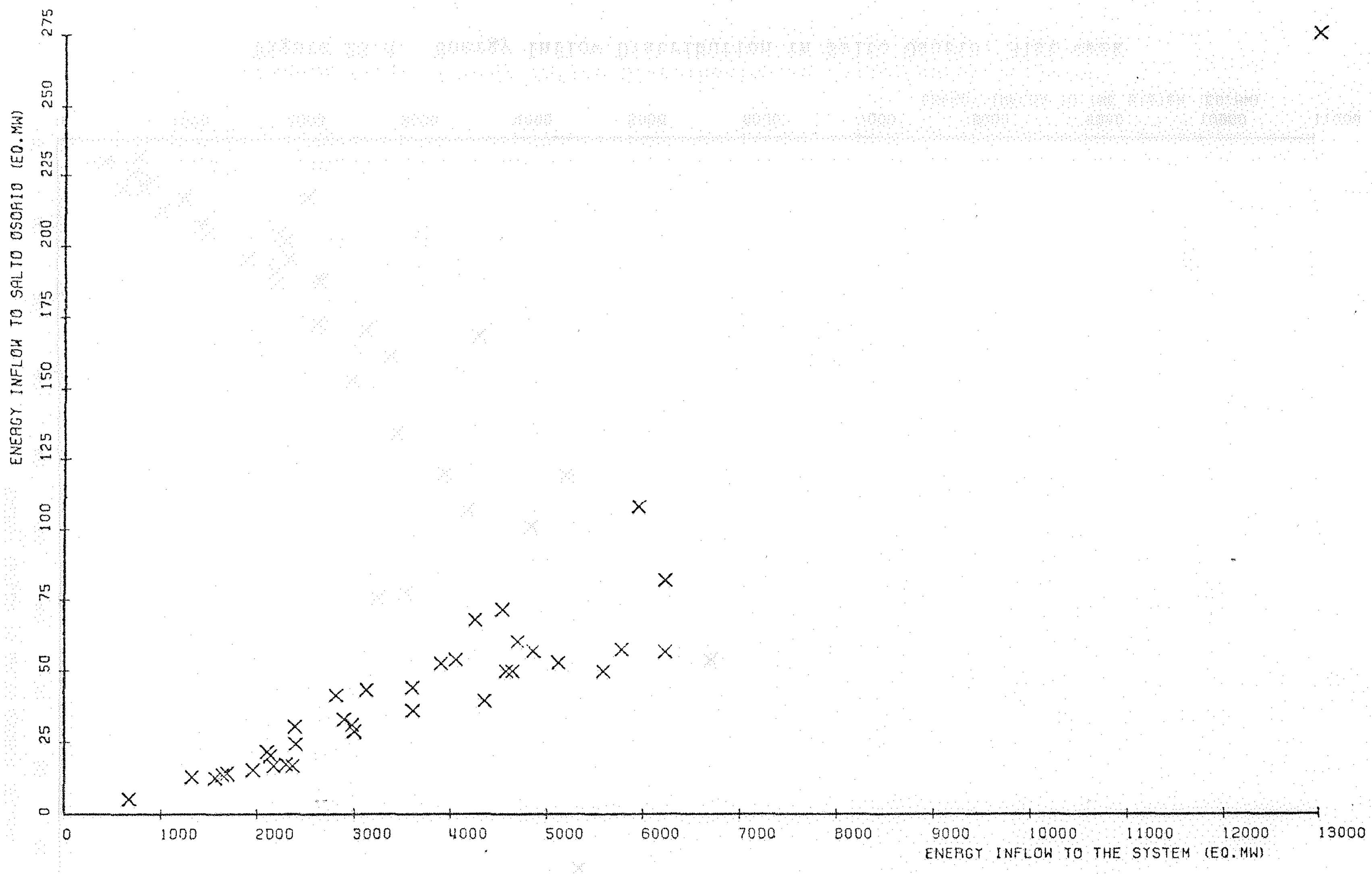


Figure 23.e. Energy Inflow Distribution in Salto Osorio, 41st week

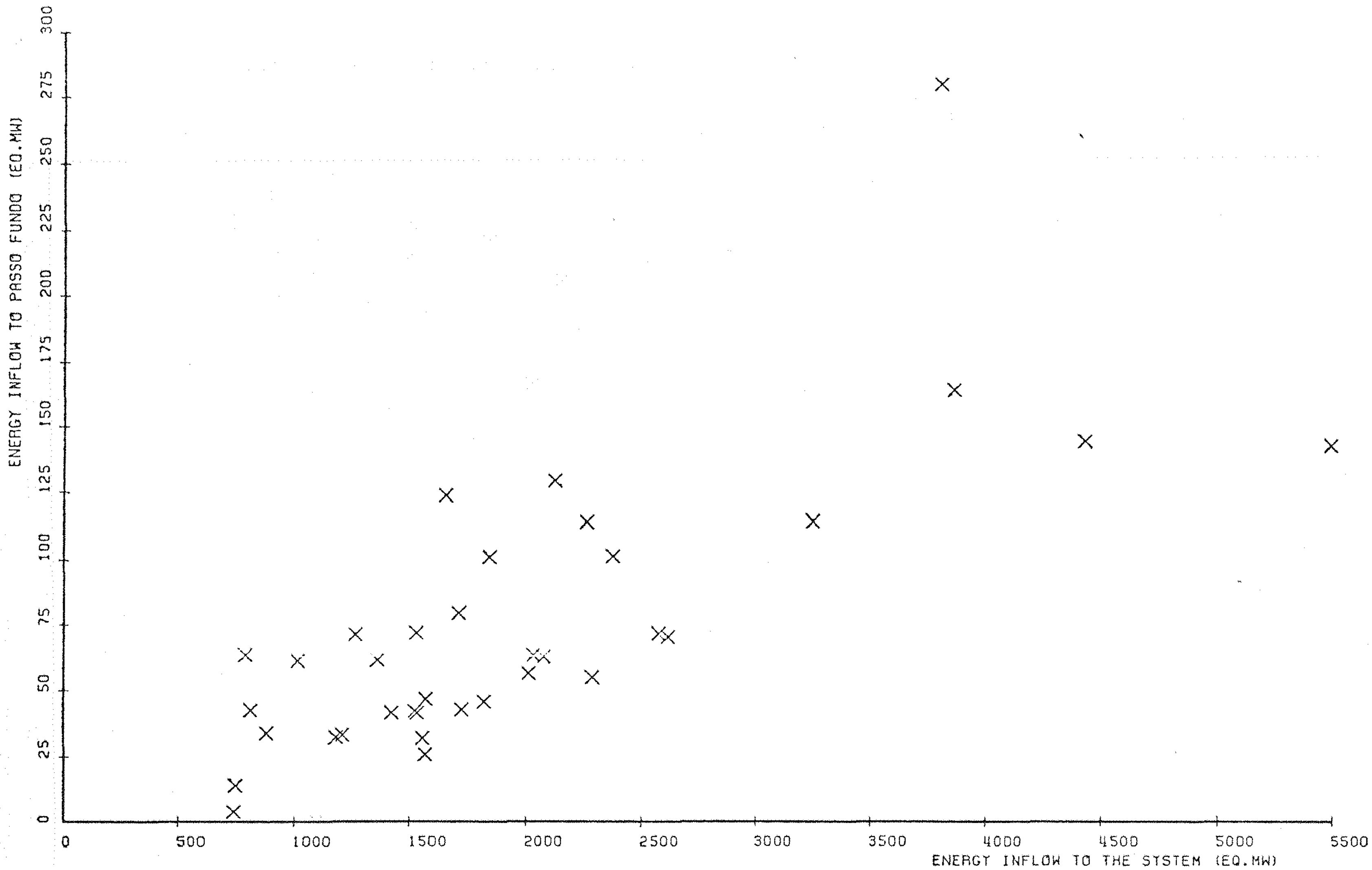


Figure 24.a. Energy Inflow Distribution in Passo Fundo, 1st week

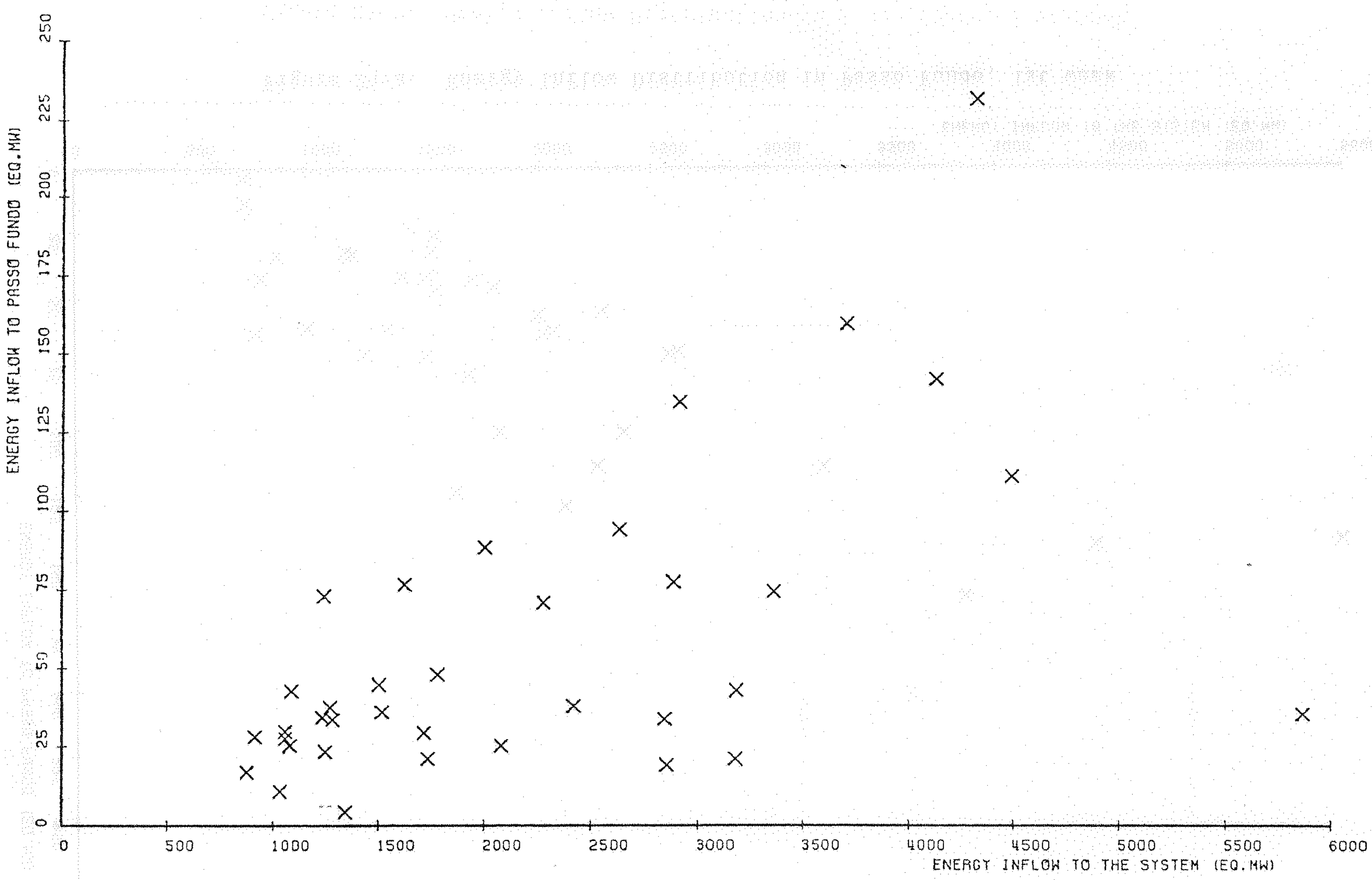


Figure 24.b. Energy Inflow Distribution in Passo Fundo, 11th week

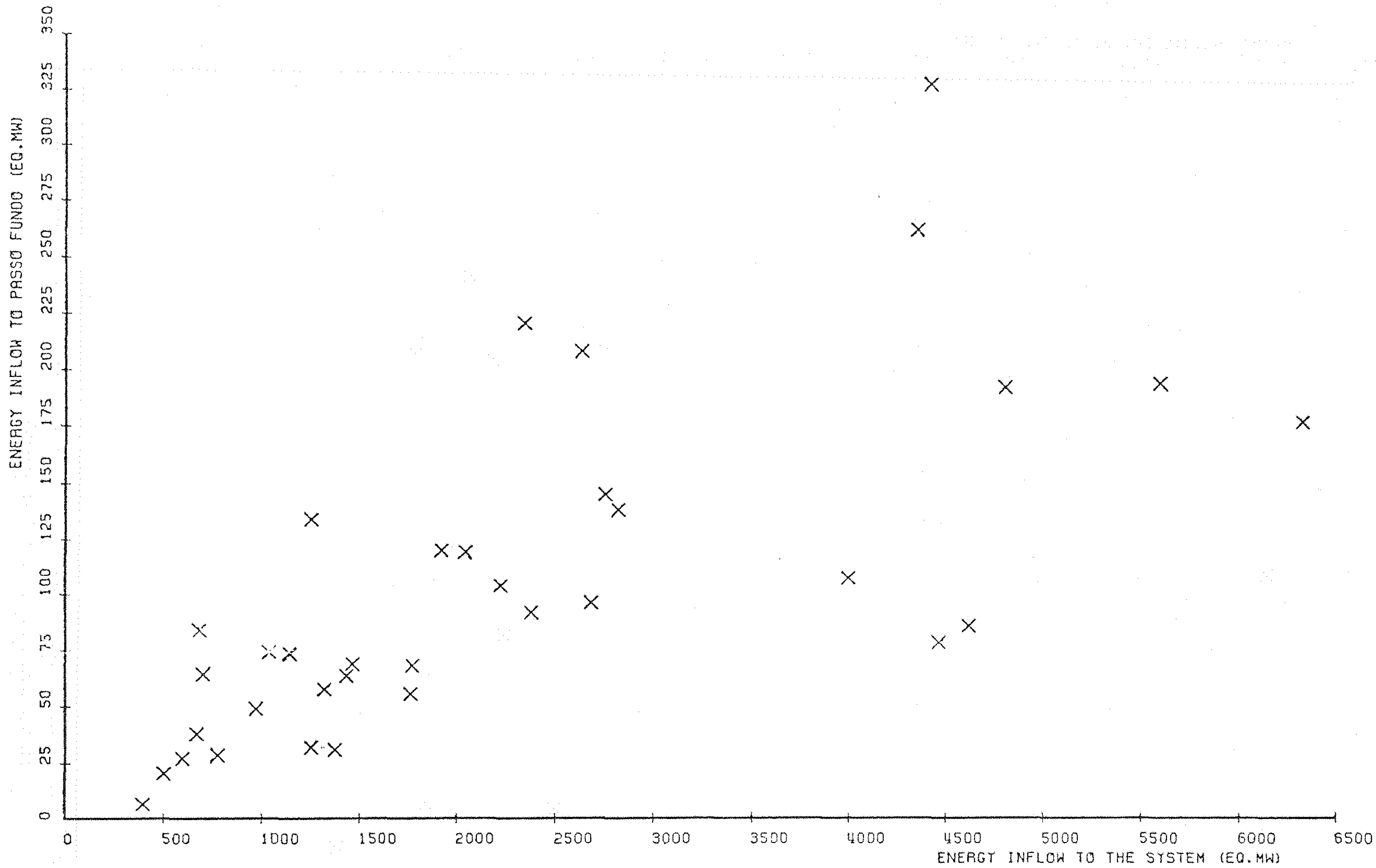


Figure 24.c. Energy Inflow Distribution in Passo Fundo, 21st week

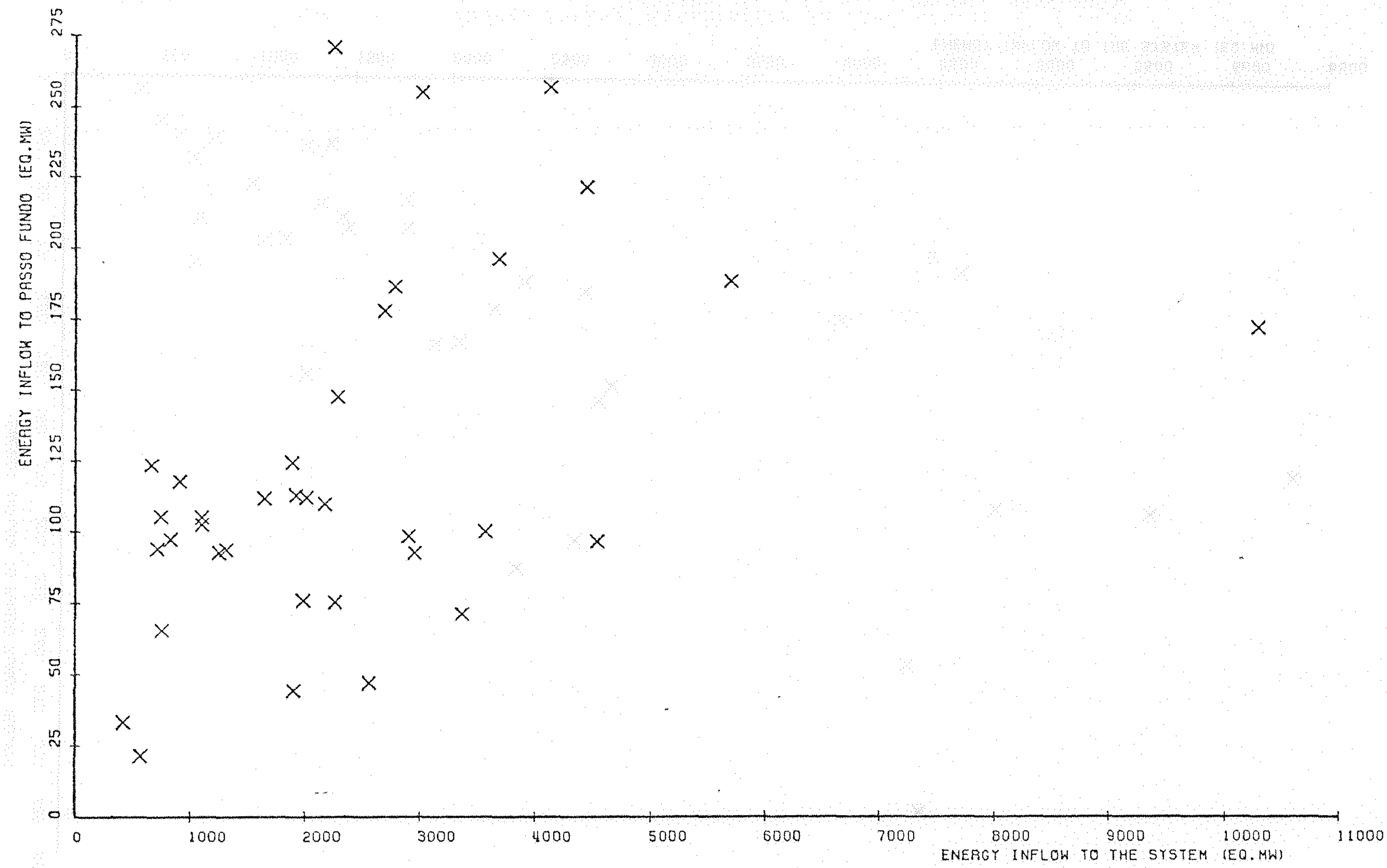


Figure 24.d. Energy Inflow Distribution in Passo Fundo, 31st week

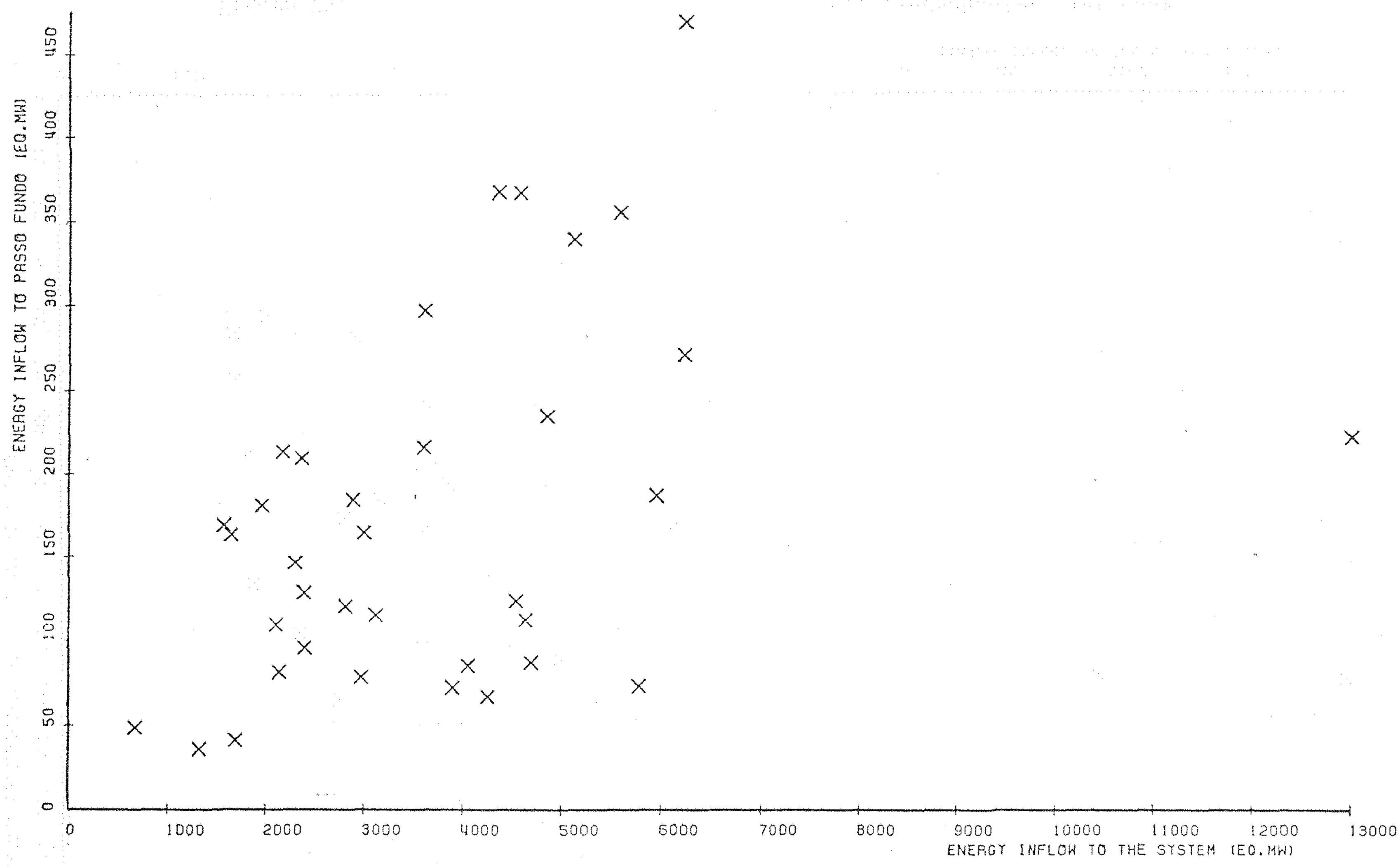


Figure 24.e. Energy Inflow Distribution in Passo Fundo, 41st week

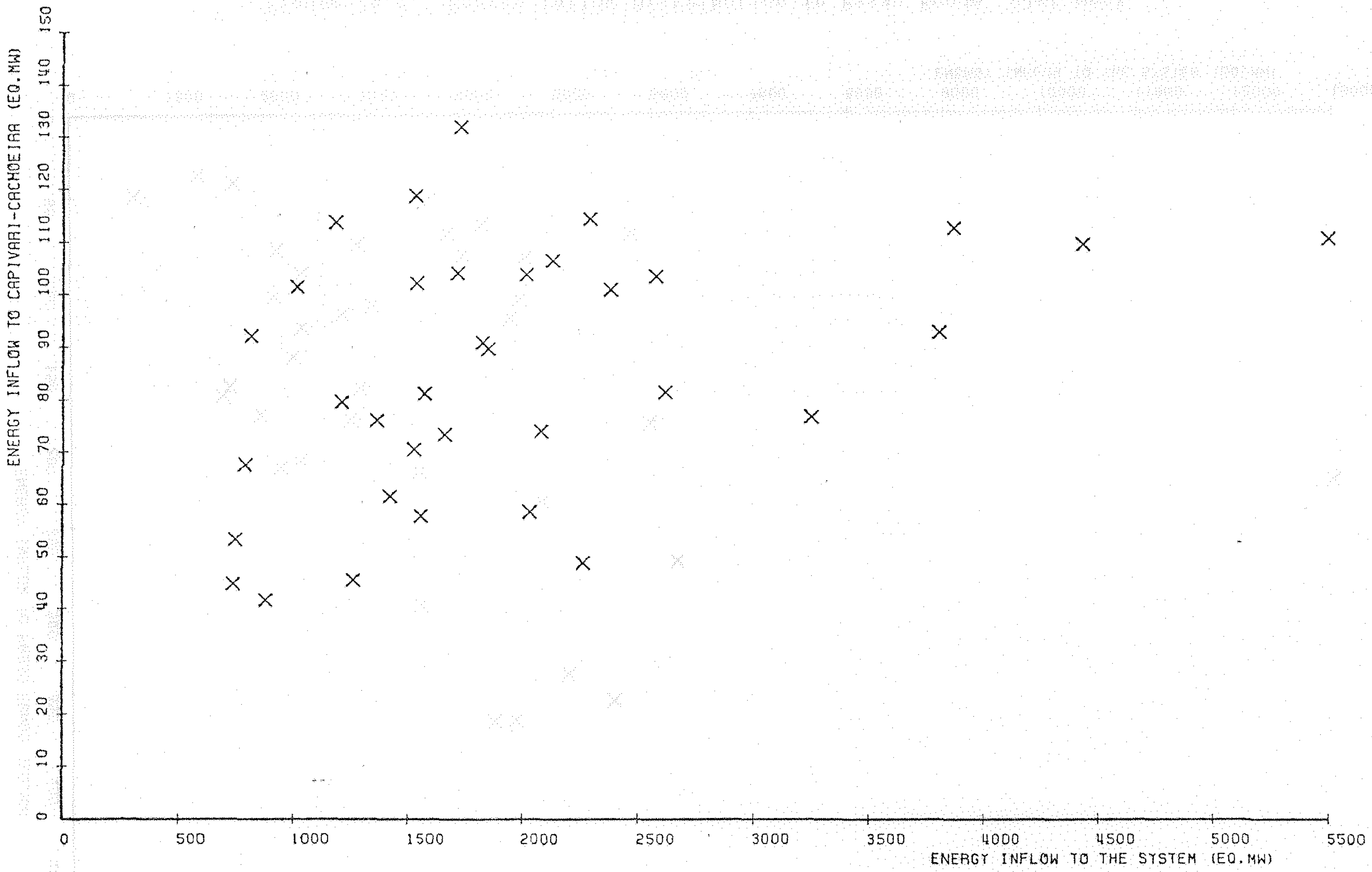


Figure 25.a. Energy Inflow Distribution in Capivari-Cachoeira, 1st week

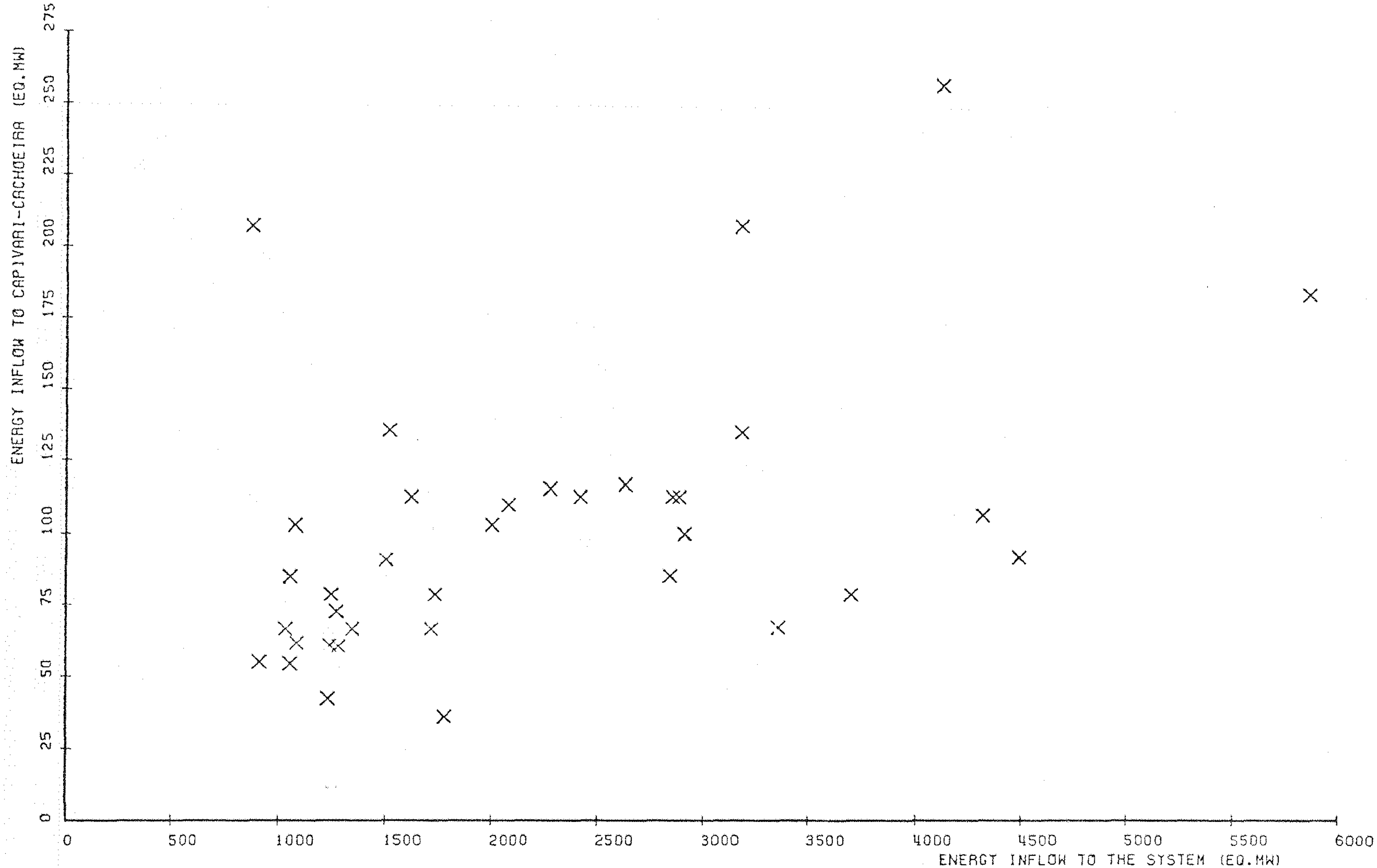


Figure 25.b. Energy Inflow Distribution in Capivari-Cachoeira, 11th week

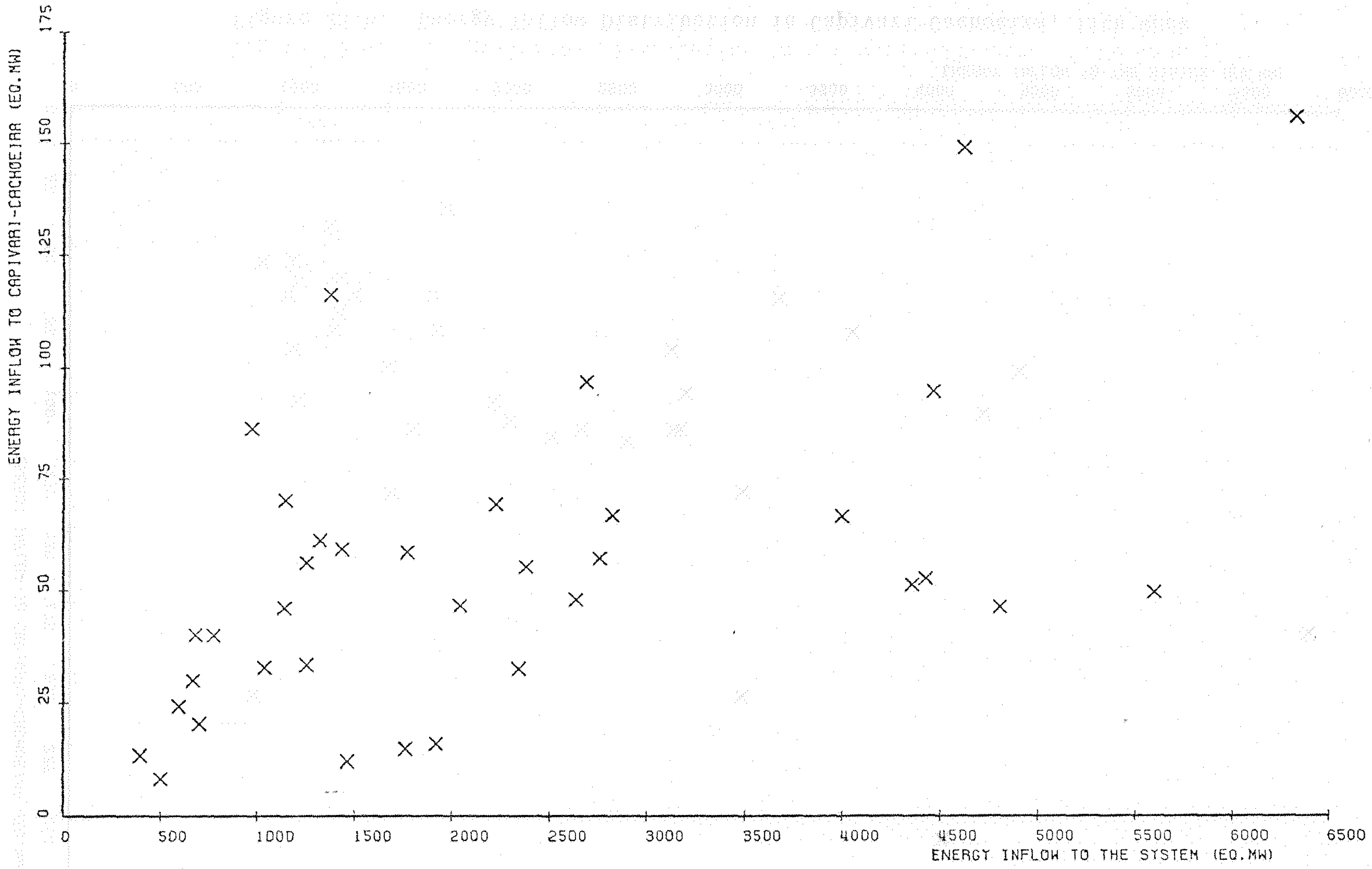


Figure 25.c. Energy Inflow Distribution in Capivari-Cachoeira, 21st week

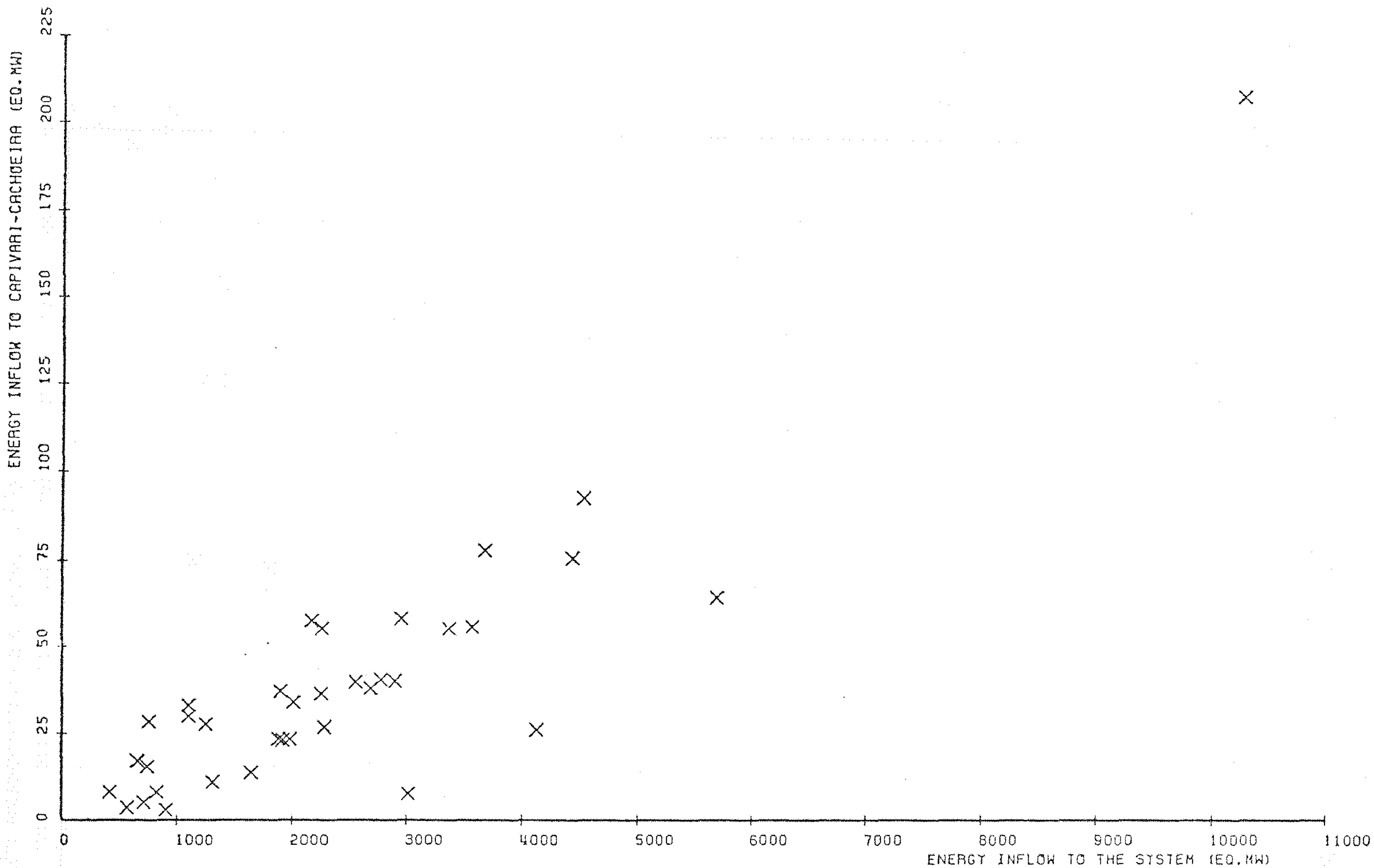


Figure 25.d. Energy Inflow Distribution in Capivari-Cachoeira, 31st week

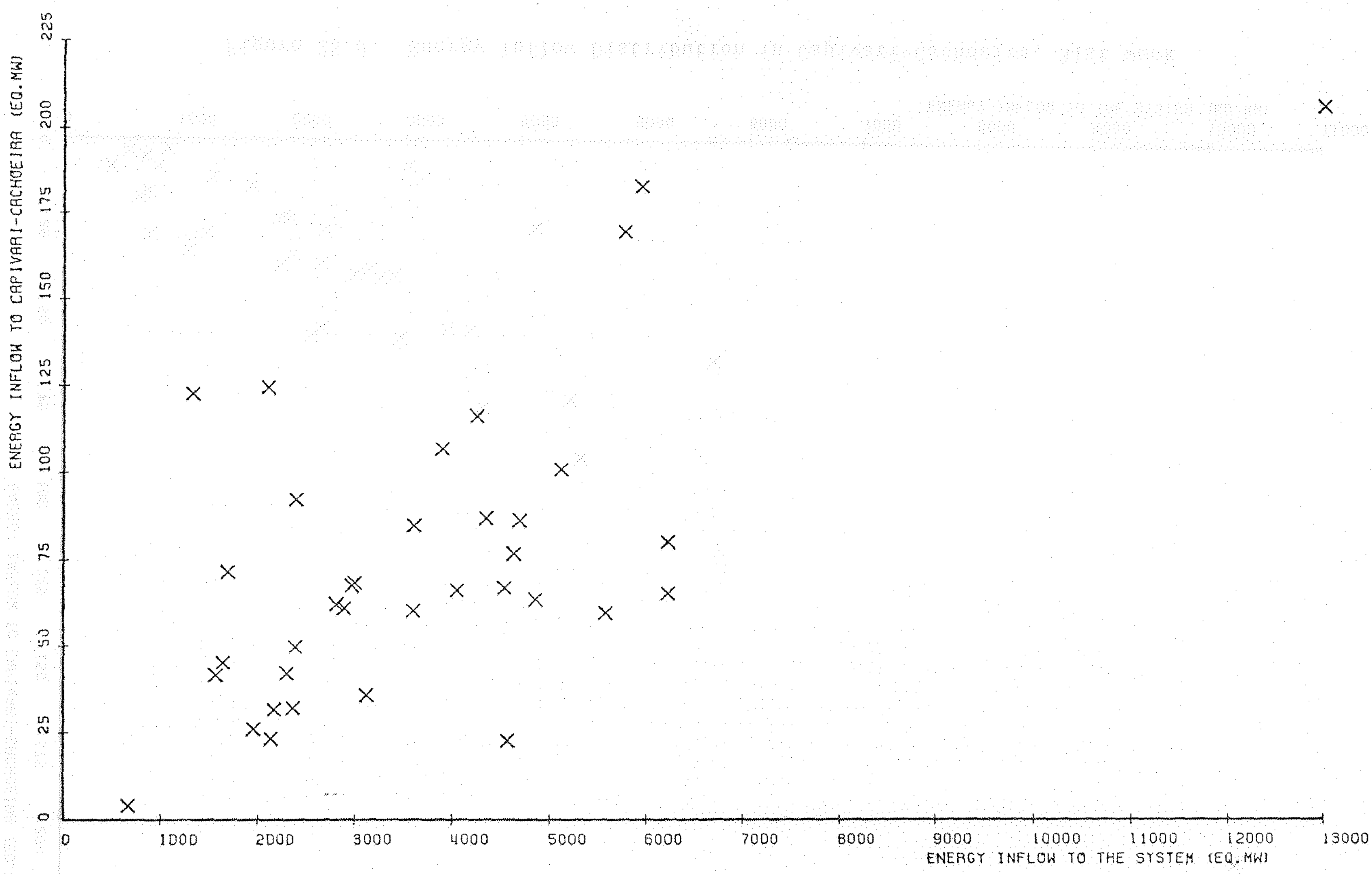


Figure 25.e. Energy Inflow Distribution in Capivari-Cachoeira, 41st week

Table 13. Statistical parameters of the energy inflows to the reservoirs (Eq. MW).

Site		Weeks 1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
PR	mean	250.6	196.6	168.0	261.2	385.9	426.1	417.7	564.6	627.1	329.6	290.0
	st dv	170.6	131.6	118.6	266.6	406.8	263.9	232.9	331.5	381.9	187.9	220.2
	skew	1.38	1.66	3.22	1.83	3.11	0.65	1.12	0.45	1.01	0.87	1.28
	corr	0.93	0.98	0.92	0.95	0.98	0.98	0.97	0.99	0.99	0.97	0.99
FA	mean	1122	1335	1442	1021	1193	1360	1324	1538	1962	1606	1315
	st dv	593.9	715.1	890.2	624.5	892.6	1121	1286	1418	1120	948.1	905.2
	skew	1.38	0.98	1.62	2.11	1.12	1.86	2.82	3.32	1.20	0.64	2.15
	corr	0.93	0.98	0.99	0.95	0.97	0.98	0.98	0.99	0.98	0.97	0.99
SS	mean	403.5	490.4	421.6	377.8	476.7	532.0	489.3	541.4	844.5	635.0	484.9
	st dv	297.7	420.5	254.6	270.2	481.6	537.7	497.1	479.2	853.1	429.1	408.8
	skew	1.92	2.48	0.81	2.02	2.72	2.39	2.17	2.23	4.25	1.00	2.63
	corr	0.95	0.99	0.96	0.90	0.98	0.98	0.95	0.98	0.99	0.96	0.99
SO	mean	23.9	26.5	22.6	19.7	23.8	28.0	26.5	37.9	46.1	32.1	25.2
	st dv	22.5	18.3	13.8	13.9	19.7	28.1	24.8	65.2	45.4	20.2	21.2
	skew	3.27	1.17	0.85	2.13	1.50	2.38	1.54	5.05	3.89	0.74	2.71
	corr	0.97	0.93	0.97	0.94	0.98	0.99	0.94	0.99	0.96	0.95	0.96
PF	mean	74.6	61.1	56.8	70.8	103.2	113.5	122.0	157.6	173.3	111.0	83.3
	st dv	52.5	36.7	48.5	56.0	72.7	72.6	62.0	106.4	107.4	80.1	64.8
	skew	1.95	1.41	1.89	1.21	1.25	1.92	0.89	1.42	0.99	1.41	1.52
	corr	0.93	0.97	0.97	0.98	0.96	0.98	0.95	0.99	0.98	0.98	0.99
CC	mean	85.3	114.4	100.8	64.9	54.9	46.7	38.9	52.0	74.9	70.9	78.0
	st dv	23.9	51.6	48.1	33.4	34.6	31.3	36.5	45.4	44.6	39.3	37.6
	skew	- 0.19	1.44	1.55	0.93	1.29	1.14	2.98	2.44	1.22	1.16	0.65
	corr	0.90	0.99	0.98	0.95	0.95	0.96	0.98	0.99	0.98	0.97	0.99
RR	mean	39.7	30.4	29.7	40.0	55.7	60.3	69.5	87.8	89.4	50.0	44.0
	st dv	26.2	19.4	22.7	37.9	53.8	39.4	39.3	49.1	47.7	26.2	31.4
	skew	1.34	1.40	2.46	1.64	2.54	0.80	0.88	0.35	0.98	0.80	0.96
	corr	0.93	0.98	0.94	0.96	0.98	0.98	0.96	0.98	0.99	0.97	0.99

Table 14. Statistical parameters of the energy storage in the reservoirs (Eq. MW).

Site		Weeks 1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
PR	mean	6273	6081	5920	5765	5620	5607	5685	5942	6324	6564	6472
	st dv	1680	1664	1716	1908	2082	2224	2202	2052	1917	1735	1665
	skew	-2.03	-1.53	-1.17	-1.22	-1.11	-1.10	-1.19	-1.45	-1.99	-2.46	-2.38
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
FA	mean	17640	17373	17716	17499	16901	16509	16358	16185	17042	18361	18296
	st dv	4611	4301	3515	3833	4801	5787	5787	5934	5533	4293	4392
	skew	-1.93	-1.38	-0.76	-0.82	-1.35	-1.39	-1.12	-1.09	-1.08	-1.86	-2.15
	corr	0.99	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
SS	mean	6877	6697	6827	6503	6333	6275	6132	6061	6689	7445	7355
	st dv	2941	2892	2765	2977	3197	3451	3537	3624	3477	2795	2863
	skew	-1.08	-0.80	-0.88	-0.82	-0.95	-0.79	-0.59	-0.48	-0.73	-1.23	-1.33
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
SO	mean	193	144	140	139	126	167	171	170	226	214	212
	st dv	175	170	175	164	164	179	182	187	187	173	163
	skew	0.01	0.70	0.71	0.53	0.92	0.36	0.32	0.37	-0.22	-0.09	-0.03
	corr	0.85	0.85	0.97	0.84	0.69	0.88	0.91	0.85	0.99	0.85	0.73
PF	mean	2179	1975	1730	1488	1409	1511	1629	1808	2058	2296	2312
	st dv	1729	1707	1634	1600	1605	1684	1754	1780	1784	1748	1761
	skew	0.17	0.28	0.59	0.96	0.91	0.77	0.71	0.66	0.51	0.31	0.20
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
CC	mean	560	461	422	396	290	292	292	338	454	532	595
	st dv	624	609	623	570	521	541	510	536	570	610	643
	skew	0.53	0.94	1.20	1.25	1.77	1.80	1.73	1.41	0.94	0.74	0.49
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99

statistical characteristics of energy inflows agree with the statistical characteristics of water inflows to the system presented in Table 6.

The analysis of the trajectories of the reservoir storages strengthens the initial supposition about the importance of the inter-seasonal carryover storage in the operation of the system. No regular pattern for reservoir trajectories was defined. In other words, high and low levels of storage were computed for any season. The occurrence of maximum and minimum storage was randomly distributed along the weeks. Despite that, one remarkable peculiarity was identified. This was the pronounced regularity of the partition of energy storage and inflows among some of the most important reservoirs in the system. These characteristics will be explored in the strategic studies of the operation under uncertain future inflows.

Chapter 6

AN ALGORITHMIC APPROACH TO THE OPERATION UNDER UNCERTAINTY OF THE FUTURE INFLOWS

In this chapter the optimality conditions for operation of the studied multireservoir system are derived in the case of uncertain future inflows. The strategic objective is the minimization of the expected operating cost of the system. This situation was analyzed by Massé (1946b) in his Theory of the Marginal Expectations in the problem of operation of a single reservoir. Most of the material in the first part of this chapter is taken from this report.

An algorithm was developed for a single reservoir operation optimization based on the Theory of the Marginal Expectations. This is an improved version of the Explicit Stochastic Approach of the Dynamic Programming to convex problems.

In the second part of the chapter an extension of the Theory of the Marginal Expectations to the operation of a system of multiple reservoirs is investigated. In order to restrict the dimension of the problem an aggregated formulation is proposed. The problem of a relaxed solution with its consequent losses of water is analyzed and a procedure to cope with its effects is proposed using some characteristics of the deterministic operation.

The application of the method to the derivation of the optimal strategy of operation is performed and its results are discussed at the end of the chapter.

6.1 Single Reservoir Operation Optimization

The first optimality condition for the operation of a single reservoir stated that in the optimal operation the marginal instantaneous benefit of releases is equal to the marginal future benefit of remaining storage. In decisions under complete certainty of the future the marginal future benefit of storage is known since present and future decisions can and must be defined at the beginning of the operation. In decisions under uncertainty of the future, the decisions are made for the next time interval of operation only, reflecting the strategical need to wait until additional information about the future is collected. Therefore, the marginal future benefit of storage is uncertain when the decision is made.

Suppose, however, that the residual benefit of the remaining storage is known at the end of the operating period. This is given by the function $R[n+1, s(n+1)]$, where $s(n+1)$ is the storage at the end of the last interval of operation n. The transition between storage at the beginning of the interval and storage at the end is defined by the hydrologic balance equation

$$s(n+1) = s(n) + q(n) - w(n) - spill(n) \tag{6.1}$$

where $s(n)$ is the storage at the beginning of time interval n

$q(n)$ is the inflow during time interval n

$w(n)$ is the operated release during time interval n

$spill(n)$ is the eventual spill during time interval n.

The restrictions on the operation are given by

$$Smin \leq s(n+1) \leq Smax \tag{6.2}$$

$$Wmin \leq w(n) \leq Wmax(n) \tag{6.3}$$

where $Smin$ and $Smax$ are the minimum and maximum storage in the reservoir, respectively, and $Wmin$ and $Wmax(n)$ are the minimum and maximum release during the time interval n , respectively. $Wmax(n)$ is a function of the reservoir's hydraulic head during the time interval n.

The value of $q(n)$ is associated with a given conditional probability function

$$P[n, q|z(n)] = P[q(n) = q|Z(n) = z(n)] \quad (6.4)$$

where $Z(n)$ is a set of relevant informations to be used in the forecasting of $q(n)$

$z(n)$ is one of the possible realizations of $Z(n)$

and q is a realization of $q(n)$ defined as a finite interval of inflow values.

For each possible realization of $q(n)$ the definition of the optimal decision will follow optimality condition 1 of the deterministic operation. The marginal instantaneous benefit of the release must be equal to the marginal future benefit of the remaining (or residual) storage after the release. This decision is optimal if the assumed realization of $q(n)$ actually occurs. Therefore, if such a realization is associated with a probability level, the defined decision will be optimal with this same probability level. If expected values may be used the following equation computes the expected benefit of storage at the beginning of time interval n

$$R[n,s(n)|z(n)] = \int_q P[n,q|z(n)] * \{F[n,w(n)] + R[n+1,s(n+1)]\} \, dq \quad (6.5)$$

where $R[n,s(n)|z(n)]$ is the expected future benefit of storage $s(n)$ at the beginning of interval n, such that the set of information to forecast $q(n)$ is $z(n)$

$F[n,w(n)]$ is the instantaneous benefit of release $w(n)$.

The equation states that the expected future benefit of storage at the beginning of interval n is given by the expected value of the sum of the instantaneous benefit of releases and the benefit of the remaining storage after the release.

It is assumed that the length of the time interval of operation is small enough to allow a nearly accurate forecast of inflow $q(n)$ when a decision about release $w(n)$ is made. In this case the following procedure is used in the computation of the optimal release during time interval n.

A. Define a release $w(n)$ such that $Wmin \leq w(n) \leq Wmax(n)$.

B. Compute its marginal instantaneous benefit given by the first derivative of function $F[n,w(n)]$ with respect to $w(n)$.

C. Compute the remaining storage at the end of time interval n by hydrologic balance equation (6.1).

D. Compute the marginal benefit of the remaining (or residual) storage by the first derivative of function $R[n+1,s(n+1)]$ with respect to $s(n+1)$.

E. Release $w(n)$ is optimum if the marginal instantaneous benefit of the release is equal to the marginal benefit of the remaining (or residual) storage $s(n+1)$, or

$$\frac{\partial F[n,w(n)]}{\partial w(n)} = \frac{\partial R[n+1,\ s(n+1)]}{\partial s(n+1)} \tag{6.6}$$

When one or more constraints are tightening the optimal operation, equality 6.6 may eventually not be attained. To analyze such a case it is convenient to express Equation 6.5 in terms of its derivatives with respect to $s(n)$

$$\frac{\partial R[n,s(n)|z(n)]}{\partial s(n)} = \int_q P[n,q|z(n)] \cdot T \cdot dq$$

where $$T = \frac{\partial F[n,w(n)]}{\partial w(n)} \cdot \frac{\partial w(n)}{\partial s(n)} + \frac{\partial R[n+1,s(n+1)]}{\partial s(n+1)} \cdot \frac{\partial s(n+1)}{\partial s(n)} \tag{6.7}$$

The derivative with respect to $s(n)$ from the hydrologic balance equation is

$$\frac{\partial s(n+1)}{\partial s(n)} = \frac{\partial s(n)}{\partial s(n)} + \frac{\partial q(n)}{\partial s(n)} - \frac{\partial w(n)}{\partial s(n)} - \frac{\partial spill(n)}{\partial s(n+1)} \cdot \frac{\partial s(n+1)}{\partial s(n)} \quad \text{or}$$

$$\frac{\partial s(n+1)}{\partial s(n)} = 1 - \frac{\partial w(n)}{\partial s(n)} - I \cdot \frac{\partial s(n+1)}{\partial s(n)} \tag{6.8}$$

If spill occurs only when the reservoir is full

$$I = \frac{\partial spill(n)}{\partial s(n+1)} = \begin{cases} 0 \ , \ \text{if spill} = 0 \\ 1 \ , \ \text{if spill} > 0 \end{cases} \tag{6.9}$$

Collecting identical terms

$$(1+I) \cdot \frac{\partial s(n+1)}{\partial s(n)} + \frac{\partial w(n)}{\partial s(n)} = 1 \tag{6.10}$$

The derivatives of the restrictions may be defined when they are tightened

$$\frac{\partial s(n+1)^{+}}{\partial s(n)} = 0 \quad \text{if} \quad s(n+1) = Smax \tag{6.11}$$

$$\frac{\partial s(n+1)^{-}}{\partial s(n)} = 0 \quad \text{if} \quad s(n+1) = Smin \tag{6.12}$$

where the plus and minus signs mean the derivatives being evaluated respectively for increasing or decreasing values of $s(n+1)$ or to the "right" or "left" of $s(n+1)$.

$$\frac{\partial w(n)^{+}}{\partial s(n)} = 0 \quad \text{if} \quad w(n) = Wmax(n) \tag{6.13}$$

$$\frac{\partial w(n)^{-}}{\partial s(n)} = 0 \quad \text{if} \quad w(n) = Wmin \tag{6.14}$$

Suppose the optimality condition given by Equation 6.6 can be attained without violating any constraints. If no spill occurs the value of the variable I is zero (relations 6.9). The equality given by the optimality condition (Equation 6.6) and the equality given by Equation 6.10 imply for expression T given by Equation 6.7

$$T = \frac{\partial F[n,w(n)]}{\partial w(n)} = \frac{\partial R[n+1,\ s(n+1)]}{\partial s(n+1)} \tag{6.15}$$

Suppose the optimality condition given by Equation 6.6 cannot be enforced due to violations of the restrictions. The following cases may occur

A. The release is tightened by the maximum release constraint and the marginal instantaneous benefit of release is still greater than the marginal future benefit of the remaining storage. In this case improvements in the results of the operation might be obtained if the maximum release constraint did not tighten the operation. Substituting the value of derivatives given by Equations 6.13 and 6.10 into Equation 6.7, the value of T becomes

$$T = \frac{\partial R[n+1,\ s(n+1)]}{\partial s(n+1)} \tag{6.16}$$

B. The release is tightened by the minimum release constraint and the marginal instantaneous benefit of release is still less than the marginal future benefit of the remaining storage. In this case improvements in the results of the operation might be obtained if the minimum release constraint did not tighten the operation. Substituting the value of the derivatives given by Equations 6.14 and 6.10 into Equation 6.7, the value of T is still given by Equation 6.16.

C. The storage is tightened by the minimum storage constraint and the marginal instantaneous benefit of release is still greater than the marginal future benefit of the remaining storage. Improvements might be obtained in the results of the operation if the maximum storage constraint did not tighten the operation. The value of T in Equation 6.7 is defined in this case by substituting in 6.7 the value of the derivatives in Equations 6.12 and 6.10

$$T = \frac{\partial F[n,w(n)]}{\partial w(n)} \tag{6.17}$$

D. The storage is tightened by the minimum storage constraint and the marginal instantaneous benefit of release is still less than the marginal future benefit of the remaining storage. Improvement of the solution in this case would be obtained if the minimum storage constraint did not tighten the operation. The value of T in Equation 6.7 is defined by substituting the value of the derivatives given in Equations 6.12 and 6.10 into Equation 6.7. It will be given by an expression identical to 6.17.

The computation can be extended to other time intervals. The expected future benefits of storage at the beginning of time interval n-1 will be given by an equation similar to 6.5. It will be a function of the expected future benefits of storage at the beginning of the time interval n and the instantaneous benefit of release during the time interval n-1. Generalizing, the expected future marginal benefits of storage at the beginning of time interval t is given by

$$\frac{\partial R[t,s(t)\ z(t)]}{\partial s(t)} = \int_q P[t,q|z(t)] \cdot T \cdot dq \tag{6.18}$$

The value of the variable T will be given by equations similar to those previously defined, or

Case 1. None of the restrictions are tightening the optimal operation

$$T = \frac{\partial F[t,w(t)]}{\partial w(t)} = \frac{\partial R[t+1,s(t+1)|\,z(t+1)]}{\partial s(t+1)} \tag{6.19}$$

Case 2. If the marginal instantaneous benefit of release is greater (less) than the marginal expected future benefit of the remaining storage at the end of interval t and the release is maximum (minimum)

$$T = \frac{\partial R[t+1, s(t+1)\ z(t+1)]}{\partial s(t+1)} \tag{6.20}$$

Case 3. If the marginal instantaneous benefit of release is greater (less) than the marginal expected future benefit of the remaining storage at the end of interval t and the remaining storage is minimum (maximum)

$$T = \frac{\partial F[t, w(t)]}{\partial w(t)} \tag{6.21}$$

Equations 6.19 to 6.21 have a very concrete meaning. When the future is uncertain one is willing to postpone, in some circumstances, an instantaneous and certain operating benefit in order to obtain a greater but uncertain future benefit. The expected value of future benefits is in the case an acceptable utility function for the future satisfaction derived from the use of the reserves in the optimal operation. Then, in the optimal operation, the marginal instantaneous benefit of releases is equal to the marginal expected future benefits (or marginal expectation for short) of the remaining storage. Equation 6.19 states that when this equality may be enforced the marginal expectation of storage at the beginning of the time interval is equal to the marginal expectation of storage at the end of the same interval. It occurs because an infinitesimal value of the initial storage is worth the same whether released in the interval or in the future, when the equality occurs.

When the marginal instantaneous benefit of release is greater than the marginal expectation of the remaining storage, the instantaneous release may be optimally increased. Otherwise, the remaining storage may be optimally increased. Suppose the release reaches the maximum and marginal instantaneous benefit of the release is still greater than the marginal expectation of the remaining storage. Then the marginal expectation of the storage at the beginning of the interval will equal its value at the end of the interval (Equation 6.20). It means that in the optimal operation an infinitesimal volume of the initial storage will be evaluated where it can be feasibly used, that is, after the interval. Suppose, the release is minimum and the marginal instantaneous benefit of the release is less than the marginal expectation of the remaining storage. Then, an infinitesimal volume of the initial storage must be evaluated in the optimal operation where it will be worth more. The marginal expectation of the storage at the beginning of the interval will then be equal to its value at the end of the interval (Equation 6.20).

Suppose the reservoir is full at the end of the interval and the marginal instantaneous benefit of the release is less than the marginal expectation of the remaining storage. In this case Equation 6.21 defines the marginal expectation of storage at the beginning of the interval as equal to the marginal instantaneous benefit of the release. Or, since no additional water may be stored without being spilled, an infinitesimal volume of the initial storage must be evaluated for its value during the interval. Otherwise, suppose the reservoir is empty at the end of the interval and the marginal instantaneous benefit of the release is greater than the marginal expectation of the null remaining storage. In this case there is no gain in increasing the remaining storage, therefore, the infinitesimal volume of the initial storage is evaluated by its value during the interval (Equation 6.21).

This recursive computational procedure was first proposed by Massé with the name of Chain of Marginal Expectations (1946b). It assumes that in the optimal operation whatever the initial state the decisions made up to instant t and the state achieved by the system, the decisions from t to the end of the operating period must be optimal for this remaining period of operation. This was the first explicit recognition of Bellman's Principle of Optimality.

The development is still incomplete, however, since it assumes prior knowledge of the marginal benefit function of residual storage at the end of the operating period. Suppose this function is not known and an arbitrary choice is made. Let the maximum absolute error of estimate be given by $u(n+1)$. Then

$$u(n+1) = \max_{s(n+1)} \{u[n+1,s(n+1)]\} = \max_{s(n+1)} \left\| \frac{\partial R[n+1,s(n+1)]}{\partial s(n+1)} - \frac{\partial R'[n+1,s(n+1)]}{\partial s(n+1)} \right\| \quad (6.22)$$

where $\frac{\partial R'[n+1,\ s(n+1)]}{\partial s(n+1)}$ is the correct marginal benefit function of the residual storage.

Let $z(n)$ be a given realization of $Z(n)$, the set of information used to forecast the inflow during the n-th time interval. The basic axiom of the probabilities states

$$\int_q P[n,q|z(n)] \cdot dq = 1 \quad (6.23)$$

The optimal transition between the functions $\frac{\partial R[n+1,s(n+1)]}{\partial s(n+1)}$ and $\frac{\partial R[n,s(n)|z(n)]}{\partial s(n)}$ was defined by Equation 6.7 and Equation 6.15 to 6.17. Suppose the computation of $\frac{\partial R[n,s(n)|z(n)]}{\partial s(n)}$ for given values of $s(n)$ and $z(n)$ is accomplished without the storage constraint ever being tightened or T is given either by Equation 6.15 or 6.16. If $u[n,\ s(n),\ z(n)]$ is the absolute error transmitted to $\frac{\partial R[n,s(n)|z(n)]}{\partial s(n)}$ in the computations the axiom of the probabilities given in 6.23 implies

$$u[n,\ s(n),\ z(n)] \le u(n+1)$$

and

$$u(n) = \max_{s(n),z(n)} \{u[(n),\ s(n),\ z(n)]\} \le u(n+1) \quad (6.24)$$

Suppose, however, the storage constraints at the beginning of time interval n+1 tighten the operation for some values of inflow q. In this situation the transition between the known function $\frac{\partial R[n+1,s(n+1)]}{\partial s(n+1)}$ to function $\frac{\partial R[n,s(n)|z(n)]}{\partial s(n)}$ will be given by the Equation 6.17. In these cases no error will be transmitted in the transition. Let p[n, s(n), z(n)] be the probability of occurrence of such cases in the optimal operation for given values of s(n) and z(n). Or, p[(n), s(n), z(n)] is the probability of occurrence of values of inflows which will cause the operation be tightened by the storage constraints at the beginning of time interval n+1 when the storage at the beginning of time interval n is s(n) and Z(n) = z(n). The axiom of probabilities given in Equation 6.24 implies

$$u[(n),\ s(n),\ z(n)] \leq \{1 - p[n,\ s(n),\ z(n)]\} \cdot u(n+1)$$

and $$u(n) = \max_{s(n),z(n)} \{u[n,\ s(n),\ z(n)]\} \leq u(n+1) \qquad (6.25)$$

Applying the reasoning recursively the maximum absolute error in the computation of the marginal expected future benefit of the storage will present the following relation

$$u(1) \leq u(2) \leq \ldots \leq u(t) \leq \ldots \leq u(n) \leq u(n+1) \qquad (6.26)$$

In other words, error in the arbitrary marginal benefit of the storage at the end of the operating period is not amplified along the iterations. The sufficient condition for its attenuation is that p[k, s(k), z(k)] is non-zero for any s(k) and z(k) or the probability of the operation be tightened by the storage constraints is non-null in any state of the operation. This situation is the rule instead of being the exception in the operation of reservoirs. Therefore, if the computations start enough in advance the computed marginal expected future benefit (or the marginal expectation) of the storage will converge to its correct value. In stationary (or non-evolutive) systems a cyclic iterative procedure may be applied. The computed marginal expectation of the storage at the beginning of the first time interval of operation updates the assumed marginal expectation of the storage at the end of the last interval of operation. For instance, in the case of weekly time intervals, the computed marginal expectation of the storage at the beginning of the first week updates the assumed marginal expectation of the storage at the end of the 52nd week. The iterations are repeated until no signficant difference occurs between the marginal expectations of the storage in two successive iterations.

The analytical solution of the Chain of Marginal Expectations may be accomplished only in very special cases. Generally the marginal instantaneous benefit of releases cannot be formulated by an analytical function and the conditional probability density function has a complex mathematical structure. Under the scope of Dynamic Programming this difficulty suggested the consideration of a discrete approach. In this approach the mentioned functions are represented by a discrete form. The optimal value of the decision in each time interval (or stage) is sought by enumeration of its feasible discrete values. The principle of optimality is applied recursively until all stages are analyzed. Artifices were devised to define the optimal decision in each stage by search techniques, implying what may be called by a systematic enumeration (Erickson et al., 1969). The Theory of the Marginal Expectations shows that the optimal solution may be enforced according to an optimality condition. A systematic search for the optimal decision cannot be avoided. But the optimality conditions given by Equations 6.19 to 6.21 establish a criterion and a stopping-rule for searching for the optimal decision. The enumeration of decision is consequently reduced to a minimum.

An algorithm was defined to implement the computations of the Chain of Marginal Expectations in a digital computer. An auto-regressive model of forecasting the inflows was tested with the algorithm. Therefore, the set of information which forecasts the inflow during a time interval is formed by the previous interval's inflow.

The Chain of Marginal Expectations is given by

$$\frac{\partial R[t,s(t)|q(t-1)]}{\partial s(t)} = \int_q P[t,q\ q(t-1)] \cdot T \cdot dq \qquad (6.27)$$

The values of T are computed as

Case 1: if $\frac{\partial R[t+1,\ s(t+1)\ q(t)]}{\partial s(t+1)} = \frac{\partial F[t,\ w(t)]}{\partial w(t)}$ and $Smin \leq s(t+1) \leq Smax$

and $Wmin \leq w(t) \leq Wmax(t)$

then: $$T = \frac{\partial R[t+1,\ s(t+1)|q(t)]}{\partial s(t+1)} = \frac{\partial F[t,\ w(t)]}{\partial w(t)} \qquad (6.28)$$

Case 2: if $\frac{\partial F[t,\ w(t)]}{\partial w(t)} < \frac{\partial R[t+1,\ s(t+1)|q(t)]}{\partial s(t+1)}$ and $w(t) = Wmin$

or $\frac{\partial F[t,\ w(t)]}{\partial w(t)} > \frac{\partial R[t+1,\ s(t+1)|q(t)]}{\partial s(t+1)}$ and $w(t) = Wmax(t)$

then: $$T = \frac{\partial R[t+1,\ s(t+1)|q(t)]}{\partial s(t+1)} \qquad (6.29)$$

Case 3: if $\frac{\partial F[t,\ w(t)]}{\partial w(t)} < \frac{\partial R[t+1,\ s(t+1)|q(t)]}{\partial s(t+1)}$ and $s(t+1) = Smax$

or $\frac{\partial F[t,\ w(t)]}{\partial w(t)} > \frac{\partial R[t+1,\ s(t+1)|q(t)]}{\partial s(t+1)}$ and $s(t+1) = Smin$

then: $$T = \frac{\partial F[t,\ w(t)]}{\partial w(t)} \qquad (6.30)$$

The algorithm presents a structure similar to the Explicit Stochastic Approach to Dynamic Programming. The computations in each stage however are performed with marginal values.

Given a value of previous inflow $q(t-1)$ the range where the inflow in the next time interval $q(t)$ may occur is divided into discrete intervals of inflows q. The conditional probability function $P[q\ q(t-1)]$ has its value defined for each value of q. Variable q represents the discrete intervals of occurrence of the inflow $q(t)$ during the t-th time interval.

For each pair of values for $q(t)$ and $s(t)$ the maximum and minimum release strategies are tested for optimality. This verifies the occurrence of the Case 2 of optimal operation. It is done by

A. Maximum Release Strategy

$$\begin{aligned} w'(t) &= \min\{Wmax(t),\ s(t) - Smin + q(t)\} \\ s'(t+1) &= \min\{\max[Smin,\ s(t) + q(t) - w'(t)],\ Smax\} \end{aligned} \quad \ldots (6.31)$$

The optimality condition in this case is

$$FT1 = \frac{\partial F[t,\ w'(t)]}{\partial w(t)} \geq \frac{\partial R[t+1,\ s'(t+1)|q(t)]}{\partial s(t+1)} = RT1 \qquad (6.32)$$

B. Minimum Release Strategy

$$\begin{aligned} w''(t) &= \max\{Wmin,\ s(t) + q(t) - Smax\} \\ s''(t+1) &= \min\{\max[Smin,\ s(t) + q(t) - w''(t)],\ Smax\} \end{aligned} \qquad (6.33)$$

The optimality condition in this case is

$$FT2 = \frac{\partial F[t,\ w''(t)]}{\partial w(t)} \leq \frac{\partial R[t+1,\ s''(t+1)|q(t)]}{\partial s(t+1)} = RT2 \qquad (6.34)$$

In both strategies the optimality may be obtained under a given tolerance TOL if

$$\begin{aligned} & FT1 - RT1 \leq TOL \\ \text{or}\quad & FT2 - RT2 \leq TOL \end{aligned} \qquad (6.35)$$

The values of the marginal expectations of the remaining storage $s'(t+1)$ or $s''(t+1)$ are computed by interpolation.

If an extreme strategy is not optimal a cycle of iterations begins with the new release being given by linear interpolation

$$w'''(t) = w''(t) + (FT2 - RT2) \cdot [w'(t) - w''(t)]/[RT1 - RT2 - FT1 + FT2] \quad \ldots \quad (6.36)$$

and $s'''(t+1) = \min\{\max[Smin,\ s(t) + q(t) - w(t)],\ Smax\}$

$$\left.\begin{aligned} &\text{Defining} \qquad FT3 = \frac{\partial F[t,\ w'''(t)]}{\partial w(t)} \\ &\text{and} \qquad RT3 = \frac{\partial R[t+1,\ s'''(t+1)\ q(t)]}{\partial s(t+1)} \end{aligned}\right\} \quad (6.37)$$

the optimality condition is given by the approximation

$$FT3 - RT3 \leq TOL \quad (6.38)$$

If the optimality condition is not enforced the following iteration will be

A. If $FT3 > RT3$ it implies that the release must increase. Its upper bound is given by $w'(t)$. Therefore

$$w''(t) = w'''(t), \quad FT2 = FT3, \quad RT2 = RT3 \quad (6.39)$$

A new $w'''(t)$ is defined and tested by 6.36, 6.37 and 6.38.

B. If $FT3 < RT3$ it implies that the release must decrease. Its lower bound is given by $w''(t)$. Therefore

$$w'(t) = w'''(t), \quad FT1 = FT3, \quad RT1 = RT3 \quad (6.40)$$

A new $w'''(t)$ is defined and tested by 6.36, 6.37 and 6.38.

When the optimal solution is computed for all q the marginal expectations of storage at the beginning of the time interval t is given by

$$\frac{\partial R[t,\ s(t)|q(t-1)]}{\partial s(t)} = \sum_q P[q\ q(t-1)] \cdot T \quad (6.41)$$

where $T = FT1$ when the maximum release strategy is optimal

$T = RT2$ when the minimum release strategy is optimal

$$T = \frac{FT3 + RT3}{2} \quad \text{otherwise.}$$

A limited number of tests were made with the algorithm. Table 15 presents their results. Tests 1 and 2 refer to a situation where the marginal instantaneous benefit of release function is constant along the time intervals. In tests 3 and 4 the marginal instantaneous benefit of release function varies through the year. A marginal expectation of the remaining storage function was assumed for the last interval of operation. The algorithm was applied recursively. The computed marginal expectation of storages at the beginning of the first interval of operation updated the assumption for the last interval. After 4 to 5 iterations the computed marginal expectations converged to their previously assumed values.

6.2 Multiple Reservoir Operation Optimization

In Dynamic Programming the number of variables used to define the state of the problem (or the number of state variables) is referred to as the dimension of the problem. It is observed that when the dimension of the problem increases linearly, the computation time increases exponentially. The Chain of Marginal Expectations is a more elaborate version of the Explicit Stochastic Approach of Dynamic Programming. Nevertheless the problem of dimensionality will limit its extension to problems with large dimensions. This limitation caused the extension of the Chain of Marginal Expectations to the operation of a multireservoir system to be accomplished through an aggregated formulation.

The expense of having this computational advantage will be the derivation of relaxed solutions. In other words, the spilling occurrences and deficits in the actual operation of the system will be above

Table 15. Chain of marginal expectations.
Computational tests

Test	NS	NQP	NQ	Range U.S. $/MWh	Tolerance U.S. $/MWh	Average CP time per stage (sec)
1	3	3	10	100	3	.10
2	5	5	10	100	3	.27
3	15	8	11	700	10	1.6
4	14	8	11	700	5	2.3

NS : number of discrete values of s(t)
NPQ : number of discrete values of q(t+1)
NQ : number of discrete values of q(t)
Range : range of the values of the marginal instantaneous benefit of releases
Tolerance: tolerance for enforcement of the optimality condition
CP time : central processing time--computer Cyber 172

the levels defined by the aggregated formulation. Another problem is the need of the aggregated system formulation to be expressed in terms of energy variables. The problem is related to the identification of the probability distribution of energy inflows to the aggregated system. Energy inflow depend on the existent hydraulic head in each reservoir. But, in the operation under uncertainty of the future (strategical operation), the hydraulic heads are not known previously.

At this point, however, the optimal deterministic operation of the system is known for a given sample of inflows. It allows an estimate of the conditional probability distribution of energy inflows into the system to be derived from deterministic operation. The use of such a probability distribution in the strategic operation must be the subject of further analysis.

A basic difference exists between tactical and strategic decisions: no allowance for risk is included in a tactical decision since the future is known when decision is made. For instance, in the deterministic operation, no energy shortage occurred. Hence, no matter what the cost of the shortage was the optimal operation would be the same. In the strategic operation, however, the decision must consider all possible future occurrences. Therefore, an increase in the cost of shortages would probably imply an increase in the storage in the system. In marginal economic terms, if the cost of shortage increases it will cause an increase in the marginal expectation of the storage. The optimal releases must then be made with higher marginal instantaneous benefits and the system's energy storage will increase.

These different approaches in the consideration of risk may drive the storage in the strategic operation to be consistently above the levels obtained in the deterministic operation. The cause is related to the sudden increase of the marginal cost of operation of the system when energy shortage occurs (Figure 2). This implies the derivation of a strategy of operation with a pronounced aversion to the risks of shortages with the consequent higher levels of storage. Therefore, the hydraulic heads observed in the deterministic operation are probably below the values which will occur in the operation under uncertainty of the future inflows. But the variation of hydraulic head in the reservoirs of the system is not too pronounced. It is on the order of 15 percent and, never exceeds 30 percent of the gross head of the reservoir. Consequently, the probability distribution of the energy inflows to the system, inferred from the results of the deterministic operation, apply to the strategic operation. It will probably give an underestimate of the real values of energy inflow in the strategic operation, but it should not be too far from the real figures.

A more important deficiency of the aggregated formulation is related to the derivation of relaxed solutions or a relaxed strategy. In the deterministic operation a corrective measure was used to cope with this problem. The total aggregated storage capacity was decreased in order to account for isolated spills from the reservoirs of the system. This correction method cannot be applied in the strategic operation though. The decrease in the aggregated storage capacity will mean that the strategy is undefined for values of aggregated storage close to the maximum value.

An alternative approach which avoids this deficiency is proposed. It is based on estimates of the partition of the storage and inflows to each reservoir in the system, given the state of the aggregated formulation. The resulting spills from each reservoir can be then computed and used to correct the marginal expectations of the aggregated storage.

The partition estimates are based on the results provided by the deterministic operation. It was observed that a substantial part of the energy storage and inflow to each reservoir may be deterministically explained as a function of their aggregated values. It suggests defining statistical models to explain the partition of the energy in the system. The parameters of the models can be inferred based on the results of the deterministic operation.

This extrapolation of results from a situation where the future is certain to another where it is uncertain needs further comment. The partition of the aggregated storage among the system's reservoirs in the deterministic operation is defined by the disaggregation procedure. This disaggregation procedure will be used for the same purpose in the strategic operation. Therefore, the partition of the aggregated storage will be based on the same information in both cases. Thus, the results are expected to be identical, statistically speaking.

The partition of the energy inflows among the reservoirs is more dependent on the temporal and spatial correlation of the water inflows than on the hydraulic heads. This occurs because the hydraulic head variation in each reservoir is not pronounced in the operation of the system. Then partition of the energy inflows is more dependent on natural causes than on the operation of the system. Consequently, the partition of the energy inflows in the deterministic and strategic operation must be identical, statistically speaking.

A final consideration refers to the viability of expressing the marginal expectations of the storage in terms of its aggregated values. The energy storage will have a different marginal expectation according to its partition among the system's reservoirs. The marginal expectations can be expressed as a function of aggregated values only if they are not significantly sensitive to variations of storage and inflow partitions around their deterministic trend. This aspect of the aggregated formulation will be analyzed in more detail during the application to the case study.

6.2.1 Statistical Model of Partition

This model was first proposed by Valencia and Schaake (1973) under the scope of synthetization of a hydrologic series. A generalization was presented by Mejia and Rousselle (1976). This application seems to be the first extension to a multi-reservoir operation. Its mathematical presentation is based on the second reference.

The study of the deterministic operation of the system has defined sample values of the reservoir's energy storage and inflows. Suppose Y is a vector with the computed value of one of the variables in a given time interval. Let X be the aggregated representation of Y. Then

$$X = \sum_{j} y(j) \qquad (6.42)$$

where $y(j)$ are the elements of the vector Y, $j = 1, NY$; and NY is the number of reservoirs. Let Z be a vector of independent variables with dimension NZ. The relationship between Y, X and Z is given by the linear model

$$Y = A.X + D.Z + U \qquad (6.43)$$

where A is a NY dimension vector of parameters

D is a NY * NZ matrix of parameters

U is a NY vector of independent normal residuals of the estimation of Y by the model.

The objective of the analysis is to define a method of computation of A, D and U such that the first and second moments of the joint distribution of Y, X and Z are maintained for each time interval.

Suppose the samples of values of Y, X and Z have been transformed in order to have their means equal to zero. Then

$$X = C^t.Y \qquad (6.44)$$

where C^t is the transpose of a NY dimension vector in which the j-th element is given by

$$c(j) = \bar{y}(j)/\bar{X} \qquad (6.45)$$

where $\bar{y}(j)$ and $\bar{X}$ are the mean of $y(j)$ and X before the transformation, respectively.

If the first moments are to be maintained

$$E[Y] = A.E[X] + D.E[Z] + E[U] \qquad (6.46)$$

where E[.] is the expectation operator.

The expected values of Y, X and Z after transformation are zero; therefore

$$E[U] = 0 \qquad (6.47)$$

If the second moments are going to be maintained the following equalities occur

$$S(YX^t) = A.S(XX^t) + D.S(ZX^t) + S(UX^t) \qquad (6.48)$$

$$S(YZ^t) = A.S(XZ^t) + D.S(ZZ^t) + S(UZ^t) \qquad (6.49)$$

$$S(YY^t) = A.S(XY^t) + D.S(ZY^t) + S(UY^t) \qquad (6.50)$$

where $S(YX^t)$ is the covariance between Y and X, for instance.

As U is a vector of independent residuals

$$S(UX^t) = S(UZ^t) = 0 \tag{6.51}$$

Equation 6.44 yields

$$S(YX^t) = S(YY^t).C \tag{6.52}$$

$$S(XY^t) = C^t.S(YY^t) \tag{6.53}$$

and $$S(XX^t) = C^t.S(YY^t).C \tag{6.54}$$

Substituting relations 6.52 and 6.54 into 6.48

$$S(YY^t).C = A.C^t.S(YY^t).C + D.S.(ZX^t) \tag{6.55}$$

Equations 6.49 and 6.55 may be solved in terms of A and D. After a lengthy and tedious manipulation it yields

$$A = S(YY^t).C - S(YZ^t).S(ZZ^t)^{-1}.S(ZX^t) *$$

$$C^t.S(YY^t).C - S(XZ^t).S(ZZ^t)^{-1}.S(ZX^t)^{-1} \tag{6.56}$$

$$D = [S(YZ^t) - A.S(XZ^t)].S(ZZ^t)^{-1} \tag{6.57}$$

At this point the deterministic part of the model has been solved. The random part is a function of the independent normal residual U. The following equality occurs

$S(YU^t) = A.S(XU^t) + D.S(ZU^t) + S(UU^t) = S(UU^t)$ since U is independent of X and Z. Because $S(UU^t)$ is a symmetrical matrix

$$S(YU^t) = S(UU^t) = S(UY^t) \tag{6.58}$$

Substituting 6.58 into 6.50

$$S(YY^t) = A.S(XY^t) + D.S(ZY^t) + S(UU^t) \tag{6.59}$$

Suppose $U = B.V$ where V is a vector of independent random normal variables with a zero mean and unit variance, and B is a $NY*NY$ matrix of coefficients. Then

$$S(UU^t) = B.S(VV^t).B^t = B.B^t \tag{6.60}$$

and
$$S(YY^t) = A.S(XY^t) + D.S(ZY^t) + B.B^t \tag{6.61}$$

Using equality 6.54

$$B.B^t = S(YY^t) - A.C^t.S(YY^t) - D.S(ZY^t) \tag{6.62}$$

and finally

$$B.B^t = (I - A.C^t).S(YY^t) - D.S(ZY) \tag{6.63}$$

where I is the identity matrix.

The computation of B may be performed using eigenvalues and eigenvectors. Let E be the matrix of the eigenvalues of $B.B^t$ and F a vector with the associated eigenvectors. It can be shown that

$$B = F * (E)^{\frac{1}{2}} \tag{6.64}$$

Therefore, a necessary condition for the existence of a real matrix B is that its eigenvalues are non-negatives. This implies that matrix $B.B^t$ must be positive semidefinite. Valencia and Schaake (1973) showed that estimate of $B.B^t$, based in records of equal length of Y, X and Z, produces a positive semidefinite matrix. Therefore, since this is always the condition in the case study, matrix B will be a real matrix. This completes the derivation of the partition model.

6.2.2 Corrective Procedure for the Relaxed Strategy

The algorithm developed to compute the Chain of Marginal Expectations was adapted to include the partition models. The correction of relaxed strategy is made through the following steps:

Step 1. Preliminaries: Given the aggregated values of the energy storage and inflow into the system, their partition among the reservoirs of the system is computed. The partition models are used in this step. A random number generation routine must be introduced in the algorithm to define vector U of normal distribution residuals (Equation 6.43). The energy storages and inflows are transformed in each reservoir to water storage and inflows. The resultant hydraulic head and maximum release capacity in each reservoir is computed.

Step 2. Maximum and minimum releases definition: In serially-linked reservoirs the maximum releases are corrected to avoid spills in the downstream reservoirs. The minimum release in each reservoir is defined in order to enforce the minimum release commitment. If the inflow to a reservoir cannot be totally stored, the value of the excedent water is a lower limit to the value of the minimum release from the reservoir.

Step 3. Spill computation: If the computed minimum release is greater than the maximum release, the excess of water is spilled. In serially-linked reservoirs this spilled water is stored downstream if possible. When run-of-river plants are located downstream the spilled water is used there to produce energy whenever possible. The part of the spill which cannot be used to produce energy forms the energy losses.

Step 4. Maximum and minimum energy generation: The maximum energy generation during a time interval is given by the maximum release in each reservoir transformed into energy. The minimum energy generation is computed by the summation of the following components

A. energy generated by the minimum release in each reservoir (step 2)

B. energy produced by the upstream reservoir spills in the downstream run-of-river hydroplants (step 3).

Step 5. Net energy inflow into the system: The net energy inflow into the system is given by the total energy inflow less the part of this inflow which is spilled (step 3). It defines the energy inflow which can actually be used in the operation.

The steps 4 and 5 correct values of variables to be used in the Chain of Marginal Expectations. The computation is made as explained in section 6.1. The Central Processing time for each stage's computation increases substantially with the modifications. For tests 3 and 4 (Table 15) it becomes 7 and 10 seconds, respectively on the average.

6.3 Application

The error in estimating marginal expectations of the storage at the end of the operation is attenuated along the computations of the Chain of Marginal Expectations. The convergence of the marginal expectation function to its actual value can be accomplished through two alternative approaches. In the first approach computations must start several time intervals after the operating period. This allows the error of estimate to be attenuated before computations reach the operating period. The second approach can be applied to stationary (or non-evolutive) systems. These systems have their characteristics invariant in time or usually being periodically repeated each year. The characteristics of a hydro-thermal electric generating system refer to configuration, units, operating cost, energy demand, etc.... In non-evolutive systems the marginal expectation function is invariant or varies periodically. For instance, the marginal expectation of storages at the beginning of the first week and at the end of the 52-th week are equal if the period of variation equals one year. In this case the convergence of the marginal expectation function is accomplished through a cyclic iterative approach. Each iteration updates the marginal expectation function computed in the previous iteration until convergence is obtained.

The studied system is a non-stationary or evolutive system. Energy demand increases about 12 percent from the beginning to the end of the studied year (Table 1). This increase is followed by a similar increase of installed thermal unit capacity (Table 4). If such a trend is maintained over a period of years, no drastic variation in the marginal expectation function in the same weeks of two consecutive years is expected. Therefore, the equality of these marginal expectation functions is a simplifying assumption that may be accepted eventually.

The other alternative is to extend the analysis to the future. The work involved in preparation of data and computations increases proportionally to the number of years of study. Additional work required may be justified in a real word application. For research purposes it demands more time and computational resources without significant gain. Therefore, it is assumed in this study that the marginal expectations of the storages varies within an annual period but the values of the marginal expectations of storages in a given week is the same any year of operation.

6.3.1 The Aggregated Chain of Marginal Expectations

The high value of the autocorrelation coefficient of the energy inflow series (Table 16) implied the use of an autoregressive model to forecast future inflows. The conditional probability distribution

Table 16. Statistical parameters of the aggregated energy inflow (Eq. MW).

1. Computed Series

parameters	Weeks 1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
average	1960	2224	2212	1816	2237	2507	2419	2891	3728	2785	2276
st deviation	1066	1238	1214	1040	1586	1826	1858	2122	2190	1566	1522
skewness	1.59	1.24	1.12	1.61	1.00	1.91	2.34	2.80	2.19	0.60	2.01
correlation	0.95	0.99	0.98	0.95	0.98	0.98	0.97	0.99	0.99	0.97	0.99

2. Logarithm of the computed series

parameters	Weeks 1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
average	3.24	3.29	3.29	3.19	3.24	3.31	3.28	3.36	3.51	3.37	3.27
st deviation	2.15	2.35	2.26	2.48	3.20	2.83	3.13	3.20	2.42	2.67	2.72
skewness	0.25	0.00	0.29	-0.23	-0.08	0.16	-0.12	-0.85	-0.30	-0.22	-0.02
correlation	0.95	0.98	0.97	0.96	0.98	0.99	0.98	0.99	0.99	0.98	0.99

$P[q|q(t-1)]$ was accepted to be a log-normal distribution. The forecasting model is

$$lq(t) = \bar{lq}(t) + \frac{lsd(t)}{lsd(t-1)} \cdot [lq(t-1) - \bar{lq}(t-1)] + lsd(t) \cdot \sqrt{1 - lqr(t)^2} \cdot tn \qquad (6.65)$$

where $lq(t)$ is the logarithm of the forecasted inflow during the time interval t,

$lq(t-1)$ is the logarithm of the previous time interval's inflow,

$\bar{lq}(.)$, $lsd(.)$ are the mean and the standard deviation of the logarithms of the inflows in the indexed time interval,

$lqr(t)$ is the correlation coefficient between the logarithm of the inflows during the time interval t and $t-1$, and

tn is a standard normal deviation.

The discrete representation of the conditional probability distribution $P[q|q(t-1)]$ is obtained each week of operation by the following procedure. NPQ intervals for the random normal deviation tn are defined in a range given by values $-tne$ and tne. The probability of values of tn occurring outside that range must be practically null. After the intervals of tn have been defined, the corresponding intervals of energy inflow during the t-th week are calculated using Equation 6.65 for each value of previous energy inflow $q(t-1)$. Average value of energy inflow in each interval defines the values q in the conditional probability distribution $P[q|q(t-1)]$. The probability of occurrence of inflow in the interval q during time interval t is given by the probability associated to the occurrence of tn in the respective interval.

In the application, 11 values of q were considered in the discretization of the conditional probability distribution. Standard normal deviations of tn ranged from -3.5 to 3.5. The set of intervals of tn and the associated conditional probabilities are given in Table 17a.

The discrete values of the aggregated energy storage and previous inflows were defined to allow good representation of the marginal expectation function. The need for smaller intervals between discrete values of total storage at extreme (high or low) storages was noted in experimental computations. Variation in the marginal expectations was observed to be more regular when previous inflow value varies. As a result 14 discrete values for the storage variation and 8 values for the previous inflows were defined to represent the marginal expectation function. Discrete values of energy storage are presented in Table 17b. Discrete values of previous energy inflows were assumed according to the observed inflow variation in each time interval. The adopted discretization of storage and previous inflows represented (or mapped) the marginal expectation function by 112 points each week.

The considered energy demand was the value of the total energy demand less minimum thermal production, less firm energy production of small hydropower developments and less the mean value of non-storable run-of-river energy inflow. Values of the non-storable energy inflows are so small compared with the total energy inflow (Table 13) that consideration through mean values is assumed acceptable.

6.3.2 The Relaxed Strategy of Operation

An arbitrary function for marginal expectation of storages at the end of the 52nd weekly interval was assumed. Marginal expectations of storages at the previous weeks were recursively computed. After 5 cycles of iteration the convergence of the marginal expectations was

Table 17a. Discretization for the function $P[q|q(t-1)]$.

Interval for tn	$P[q \mid q(t-])]$ %	Interval for tn	$P[q \mid q(t-1)]$ %
-3.50 to -2.86	0.19	0.32 to 0.95	20.53
-2.86 to -2.22	1.09	0.95 to 1.59	11.41
-2.22 to -1.59	4.29	1.59 to 2.22	4.29
-1.59 to -0.95	11.41	2.22 to 2.86	1.09
-0.95 to -0.32	20.53	2.86 to 3.50	0.19
-0.32 to 0.32	24.98		

Table 17b. Discretization of the equivalent reservoir storage (Eq. MW).

0	25000
2000	30000
4000	35000
6000	39000
10000	41000
15000	43000
20000	44920

obtained with a tolerance of U.S. $5. per MWh. Table 18 presents the results for several weeks.

6.3.3 The Models of Statistical Partition

In section 6.2.1 a general model of partition that has the ability to maintain the variance-covariance matrix of considered variables was presented. Selection of these variables and the consequent definition of partition models for the aggregated energy inflow and storage are made in this section. In each case selection of the dependent variables is made considering the covariance (or cross correlations) that are more important to maintain in the partition.

For partition of the total energy inflow the following model was chosen

$$[q'(1,t),\ldots,q'(j,t),\ldots]^t = A.q'(t) + D.[s'(1,t),\ldots,s'(j,t),\ldots]^t \tag{6.66}$$

where $q'(j,t)$ is the transformed value of the energy inflow to the reservoir j during the t-th week;

$q'(t)$ is the transformed value of the total energy inflow to the system during the t-th week;

$s'(j,t)$ is the transformed energy storage in the reservoir j at the beginning of the t-th week;

$[.]^t$ stands for the transpose of the vector $[.]$.

The transformations are

$$q'(j,t) = q(j,t)/\bar{q}(j,t) - 1 \tag{6.67}$$

$$s'(j,t) = s(j,t)/\bar{s}(j,t) - 1 \tag{6.68}$$

$$q'(t) = q(t)/\bar{q}(t) - 1 \tag{6.69}$$

where $\bar{q}(j,t)$ and $\bar{q}(t)$ are the mean of the energy inflows to each reservoir and the mean of the total energy inflows to the system during the t-th week, respectively; and $s(j,t)$ is the mean of the storages at the beginning of t-th week.

Table 18a. Marginal expected future benefit of the remaining storage with relaxed formulation.
(U.S. $/MWh)

WEEK NO. 1

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW) 650.	1000.	1500.	2250.	3000.	3500.	4000.	4500.
0	568.3	513.7	371.9	234.5	151.7	111.0	77.8	49.7
2000	526.9	450.2	337.7	222.0	144.9	106.1	73.9	47.1
4000	503.3	423.1	321.0	212.7	138.6	101.3	70.3	44.6
6000	485.5	405.0	308.2	204.3	132.7	96.5	66.8	42.1
10000	457.8	377.5	286.0	188.4	121.1	87.3	59.6	36.9
15000	426.1	348.1	261.4	169.8	107.1	75.8	50.8	30.6
20000	393.8	319.8	237.8	151.5	93.1	64.5	42.1	24.5
25000	362.2	292.4	215.0	133.2	78.9	53.0	33.1	17.8
30000	332.6	266.4	192.8	114.8	64.4	41.1	24.1	12.5
35000	304.3	241.1	170.1	94.7	48.0	27.8	14.8	6.0
39000	282.5	220.5	149.0	74.2	31.1	15.0	5.5	1.8
41000	271.7	209.2	135.2	60.1	20.6	8.3	2.1	0.5
43000	260.8	196.7	119.0	43.7	10.4	2.3	0.5	0.1
44920	250.2	185.3	104.5	18.3	0.5	0.0	0.0	0.0

Table 18b. Marginal expected future benefit of the remaining storage with relaxed formulation.
(U.S. $/MWh)

WEEK NO. 11

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	700.	1000.	1750.	2400.	3050.	3750.	4500.	5000.
0	578.7	531.3	366.3	248.0	165.6	106.0	63.8	42.1
2000	546.9	489.2	334.2	229.2	155.2	100.1	60.3	39.7
4000	524.4	462.6	312.6	216.1	147.3	95.0	57.0	37.4
6000	504.9	440.7	297.0	206.0	140.3	90.4	54.0	35.3
10000	474.1	407.6	272.8	188.4	127.6	81.5	47.9	31.2
15000	440.2	374.6	247.4	169.0	112.9	71.1	41.0	26.2
20000	405.1	343.0	223.6	150.7	99.2	61.1	34.1	21.3
25000	370.9	312.7	201.3	133.3	86.0	51.3	27.4	16.5
30000	339.6	284.9	180.3	116.6	73.0	41.5	20.3	11.3
35000	311.0	259.2	159.7	99.7	59.5	31.2	13.6	7.0
39000	289.2	239.0	142.4	84.9	47.1	20.9	6.0	2.1
41000	278.5	228.8	132.3	74.8	37.5	12.9	2.3	0.6
43000	267.9	218.3	119.4	59.5	22.9	5.1	0.5	0.1
44920	257.6	208.4	104.2	26.2	0.8	0.0	0.0	0.0

Table 18c. Marginal expected future benefit of the remaining storage with relaxed formulation.
(U.S. $/MWh)

WEEK NO. 21

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	400.	1000.	2000.	3000.	4500.	5500.	6500.	7000.
0	608.0	518.7	251.5	140.6	60.1	31.0	14.1	8.6
2000	565.3	435.7	233.5	133.0	56.3	28.4	12.4	7.4
4000	539.9	400.5	220.4	126.0	52.7	25.9	10.2	5.4
6000	518.0	374.5	209.5	119.6	49.2	23.6	9.2	4.8
10000	482.1	340.5	190.8	107.3	42.2	19.1	6.8	3.5
15000	441.5	308.3	169.7	92.8	34.0	13.5	3.7	1.5
20000	402.8	279.8	149.7	79.3	26.6	8.6	1.6	0.5
25000	368.7	253.4	130.7	66.4	19.8	4.9	0.6	0.1
30000	339.6	228.6	112.5	53.6	13.2	3.0	0.1	0.1
35000	311.1	204.5	94.1	40.3	7.2	0.5	0.0	0.0
39000	289.5	185.5	77.8	27.5	2.0	0.1	0.0	0.0
41000	278.9	175.4	67.0	18.3	0.5	0.0	0.0	0.0
43000	267.9	163.8	51.9	6.6	0.0	0.0	0.0	0.0
44920	257.0	150.5	24.2	0.0	0.0	0.0	0.0	0.0

Table 18d. Marginal expected future benefit of the remaining storage with relaxed formulation.
(U.S. $/MWh)

WEEK NO. 31

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	500.	1000.	2000.	3000.	4000.	5000.	6000.	7000.
0	595.3	525.1	267.6	153.6	91.3	51.7	26.6	11.4
2000	534.9	421.9	242.3	144.9	85.6	47.7	23.9	9.9
4000	496.0	381.7	229.8	137.3	80.1	43.8	21.4	8.7
6000	469.9	363.7	220.0	130.0	74.7	39.8	18.4	7.5
10000	437.4	338.5	201.2	115.6	64.3	32.8	14.4	4.6
15000	401.4	308.6	178.0	98.0	51.9	24.4	9.0	2.0
20000	368.4	280.5	155.0	81.1	40.1	16.7	4.8	0.6
25000	339.4	253.3	132.4	64.8	29.2	9.8	1.8	0.4
30000	311.2	226.8	110.2	49.2	19.3	4.7	0.5	0.0
35000	283.8	200.1	88.4	34.3	10.6	1.5	0.1	0.0
39000	261.2	177.7	70.8	23.2	4.4	0.4	0.0	0.0
41000	249.4	165.2	60.9	16.5	1.6	0.1	0.0	0.0
43000	237.0	150.7	48.5	7.1	0.2	0.0	0.0	0.0
44920	224.4	138.5	26.4	0.3	0.0	0.0	0.0	0.0

Table 18e. Marginal expected future benefit of the remaining storage with relaxed formulation.
(U.S. $/MWh)

WEEK NO. 41

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	1000.	2000.	3200.	4400.	5600.	6800.	8000.	9000.
0	568.0	422.3	285.5	195.1	129.0	80.4	45.6	24.5
2000	537.7	400.2	273.5	186.8	122.4	75.3	41.9	21.9
4000	515.0	382.4	262.9	178.8	116.0	70.3	38.2	19.4
6000	494.1	367.6	253.0	170.9	109.6	65.2	34.2	17.1
10000	461.4	342.2	233.9	155.2	96.6	54.7	26.7	11.6
15000	432.8	313.8	211.1	135.4	79.9	41.5	17.4	7.0
20000	392.2	287.4	188.3	115.0	62.9	28.2	9.0	2.0
25000	361.8	261.5	164.3	93.5	45.8	16.7	3.7	0.5
30000	334.6	235.6	137.8	70.3	28.3	6.1	0.6	0.0
35000	306.5	208.0	106.6	43.4	10.7	0.7	0.0	0.0
39000	285.5	182.1	73.2	16.3	0.6	0.0	0.0	0.0
41000	274.8	164.1	49.8	5.8	0.1	0.0	0.0	0.0
43000	264.2	139.0	24.3	0.4	0.0	0.0	0.0	0.0
44920	254.5	104.3	0.0	0.0	0.0	0.0	0.0	0.0

The purpose of the model is to maintain correlation between energy inflows and energy storage in each reservoir. This correlation has a physical meaning derived from the interrelation among energy inflow, hydraulic head and energy storage in each reservoir. The cross correlation among the energy inflows to each reservoir is also maintained in the partition.

A model of partition was defined for each week. The statistical analysis of the results is presented for several weeks in Table 19.

The partition of the energy storage was accomplished with the following model

$$[s'(1,t),\ldots,s(j,t),\ldots]^t = A.s'(t) + D.q'(t-1) \qquad (6.71)$$

where $s'(t)$ is the transformed value of the total energy stored at the beginning of the t-th week and the other variables were defined previously.

The transformation is now

$$s'(j,t) = [s(j,t) - \bar{s}(j,t)]/C(j) \qquad (6.71)$$

$$s'(t) = [s(t) - \bar{s}(t)]/C \qquad (6.72)$$

where $\bar{s}(t)$ is the mean value of the total storage at the beginning of the t-th week, and C and $C(j)$ are respectively the aggregated and the individual reservoir capacities.

The transformation used for the previous inflows is given in Equation 6.69.

Introduction of the previous aggregated energy inflow in the model considers the response of the reservoir's storage partition to regional extreme inflow occurrences.

Table 19. Statistical analysis: Model of partition of energy inflows.

		SITE					
WEEK		PR	FA	SS	SO	PF	CC
5	average inflow (Eq. MW)	202	1252	461	26	62	109
	explained variance (%)	50.5	96.1	93.3	57.1	65.1	59.0
	st dev of residuals	.531	.109	.216	.512	.391	.274
10	average inflow (Eq. MW)	168	1464	447	23	57	107
	explained variance (%)	43.3	98.2	93.7	89.8	62.8	46.8
	st dev of residuals	.594	.085	.173	.218	.557	.359
15	average inflow (Eq. MW)	236	1074	374	20	66	70
	explained variance (%)	28.2	91.5	90.7	91.6	52.8	29.6
	st dev of residuals	.910	.192	.237	.218	.575	.440
20	average inflow (Eq. MW)	365	1136	459	23	100	57
	explained variance (%)	39.1	88.0	75.6	91.3	55.1	36.3
	st dev of residuals	1.09	.293	.610	.286	.590	.629
25	average inflow (Eq. MW)	430	1356	527	28	112	48
	explained variance (%)	50.7	97.9	95.1	95.0	56.0	41.3
	st dev of residuals	.496	.131	.228	.227	.480	.543
30	average inflow (Eq. MW)	413	1333	499	27	120	40
	explained variance (%)	45.5	96.8	83.3	82.1	41.8	86.0
	st dev of residuals	.480	.188	.489	.481	.443	.391
35	average inflow (Eq. MW)	524	1461	511	34	148	47
	explained variance (%)	45.3	97.6	96.1	92.3	41.7	83.4
	st dev of residuals	.443	.169	.208	.472	.545	.396
40	average inflow (Eq. MW)	639	1906	790	46	175	71
	explained variance (%)	49.8	92.5	87.2	82.3	39.8	65.6
	st dev of residuals	.471	.177	.360	.473	.520	.415
45	average inflow (Eq. MW)	392	1714	700	35	125	72
	explained variance (%)	79.2	97.1	89.3	88.1	69.2	39.3
	st dev of residuals	.277	.107	.253	.266	.421	.436
50	average inflow (Eq. MW)	290	1315	485	25	83	78
	explained variance (%)	58.0	97.8	94.5	94.7	36.5	28.9
	st dev of residuals	.570	.113	.217	.216	.692	.454
52	average inflow (Eq. MW)	264	1176	435	24	74	82
	explained variance (%)	70.2	97.8	95.1	68.3	55.6	34.3
	st dev of residuals	.437	.099	.202	.517	.483	.305

The relations between energy stored in each reservoir and total energy stored in the system (Figures 13 to 15) showed a light curvature in the dominant reservoirs of the system. This curvature was more pronounced at the extremities. This is attributed to the influence of the maximum and minimum storage and release capacities in the partition of the energy in the system. Models I and II consider this curvature. Model I was defined for total storage values from 37,500 Eq. MW to 44,920 Eq. MW and Model II for values of total storage from 15,000 Eq. MW to 37,500 Eq. MW. The lower levels of storage were not considered in the partition because the probability of spills there is practically null.

When the equivalent reservoir storage reaches the region of Model I the probability of spills is high. Factors that determine the distribution of the water in the system are mainly related to maximal storage and release capacity in each reservoir. It refers to the situation in the disaggregation procedure (section 5.2.2) where the decision defined by the system of linear equations cannot be enforced. Therefore in this region, one partition model was defined for the entire year.

The decision defined by the system of linear equations is expected to be enforced a substantial part of the time when the aggregated storage is in the range defined by the model II. Consequently the partition of the storage in the system is expected to be driven mainly by the forecasted inflows during the next time interval. This implies the definition of a model of partition for each season of the year. The seasons of the year were identified by the following procedure. First, a partition model was defined for each week of the year. Then the weeks where the parameters of the model were statistically identical defined a season. They are

Season I : 48-th to the 7-th weeks

Season II : 8-th to the 29-th weeks

Season III : 30-th to the 38-th weeks

Season IV : 39-th to the 47-th weeks

Table 20 presents a statistical analysis of the results.

The models were tested using as samples of the aggregated energy inflow and storage variables the series obtained in the deterministic operation. Two tests were made. The first test considered only the deterministic component of the partition models and the second test considered the deterministic and random components.

Negative inflows and storages, and storages greater than the reservoirs' capacities may be obtained in the partitions. This occurred more often when the random component of the partition models was included. In these cases, a correction was applied distributing the deficit or excess of energy among the remaining reservoirs proportionally to their storage capacities. As violations above 5 percent of the respective variable's mean are rare, such a correction did not significantly alter the results.

The series of energy inflows and storages in each reservoir of the system have some of their statistical parameters presented in Tables 21 to 24. Comparison between these parameters and parameters obtained in the deterministic operation (Tables 13 and 14) give interesting results. The mean values are statistically identical in every case. The deterministic partition defined standard deviations smaller than the observed ones. When the random component of the partition model is introduced the standard deviations of the series are statistically identical to the observed ones. The most interesting result was related to the

Table 20. Statistical analysis: Model of partition of energy storage.

a. Region I: 37,500 Eq. MW to 44,920 Eq. MW

	RESERVOIRS					
	PR	FA	SS	SO	PF	CC
average storage (Eq. MW)	7340	20778	9127	306	2251	661
explained variance (%)	17.0	33.4	34.4	25.5	18.4	27.0
st dev of residuals	.024	.021	.038	.286	.210	.294

b. Region II: 15,000 Eq. MW to 37,500 Eq. MW

	PR	FA	SS	SO	PF	CC
Season I						
average storage (Eq. MW)	5452	15314	4930	28	1187	55
explained variance (%)	32.4	87.6	85.7	12.4	30.4	20.1
st dev of residuals	.143	.048	.094	.114	.246	.110
Season II						
average storage (Eq. MW)	5284	16345	5677	36	934	32
explained variance (%)	48.4	89.2	89.3	16.7	22.2	8.6
st dev of residuals	.140	.042	.075	.200	.210	.070
Season III						
average storage (Eq. MW)	5757	15533	4930	48	947	42
explained variance (%)	10.7	93.2	94.1	48.4	18.3	49.0
st dev of residuals	.151	.038	.063	.194	.183	.061
Season IV						
average storage (Eq. MW)	5714	14152	4224	16	777	0
explained variance (%)	21.4	91.1	81.4	19.0	24.8	—
st dev of residuals	.188	.042	.096	.096	.150	—

Table 21. Deterministic partition: Statistical parameters of the energy inflow (Eq. MW).

Site		Weeks 1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
PR	aver	253.0	200.1	164.2	266.7	384.7	421.8	417.0	538.7	596.0	331.1	291.7
	st dv	136.0	89.8	87.3	128.1	192.6	140.0	108.4	156.4	217.2	163.4	162.0
	skew	1.41	1.11	0.65	0.83	0.54	0.99	0.18	1.24	1.73	0.65	1.52
	corr	0.94	0.96	0.87	0.90	0.93	0.96	0.95	0.91	0.97	0.96	0.97
FA	aver	1120	1330	1437	1022	1195	1365	1336	1572	2006	1627	1314
	st dv	591	698	813	599	870	1097	1260	1361	1068	910	898
	skew	1.58	1.08	1.21	1.56	0.97	1.87	2.40	2.88	2.11	0.48	2.03
	corr	0.95	0.98	0.98	0.94	0.98	0.98	0.97	0.99	0.99	0.97	0.99
SS	aver	402.8	492.5	432.8	386.0	480.7	529.8	476.8	540.1	836.5	617.5	482.9
	st dv	289.0	413.0	248.1	261.5	445.6	513.4	438.1	459.1	803.2	408.6	396.4
	skew	1.61	2.62	0.97	1.53	1.07	1.96	2.29	2.88	2.33	0.77	2.09
	corr	0.96	0.99	0.94	0.94	0.97	0.98	0.96	0.98	0.99	0.97	0.99
SO	aver	23.6	27.1	22.9	20.0	23.8	27.9	27.5	43.4	46.7	31.7	25.1
	st dv	10.5	15.3	13.5	13.5	19.4	26.8	23.1	57.9	42.6	19.6	20.7
	skew	0.80	1.37	1.12	1.59	1.01	1.97	2.12	3.45	2.31	0.70	2.13
	corr	0.66	0.98	0.97	0.93	0.97	0.98	0.97	0.99	0.99	0.97	0.99
PF	aver	75.7	59.7	58.7	70.8	102.4	115.3	122.5	155.7	173.5	108.0	84.5
	st dv	41.9	30.1	40.6	39.6	49.7	49.2	35.4	61.1	52.9	68.7	32.6
	skew	1.40	0.97	0.69	1.88	1.02	1.80	1.67	1.51	1.03	0.69	1.78
	corr	0.94	0.97	0.97	0.93	0.94	0.98	0.94	0.98	0.95	0.96	0.98
CC	aver	84.9	115.5	98.7	64.4	55.1	47.8	40.4	52.7	77.4	69.7	77.9
	st dv	9.96	39.6	30.4	16.6	22.0	23.2	34.0	39.9	29.6	21.0	17.2
	skew	0.49	2.17	1.43	0.10	0.63	1.81	2.01	2.61	1.96	0.48	1.98
	corr	0.87	0.99	0.97	0.90	0.92	0.97	0.97	0.98	0.98	0.94	0.93

Table 22. Random partition: Statistical parameters of the energy inflow (Eq. MW).

Site		Weeks										
		1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
PR	aver	253.2	213.1	174.2	257.5	498.9	377.4	450.7	585.1	617.4	320.2	298.1
	st dv	162.1	151.3	102.4	244.0	286.5	296.6	246.9	323.3	351.3	174.6	234.7
	skew	0.39	1.18	0.50	0.66	-0.23	0.53	0.44	0.50	0.69	0.32	0.96
	corr	0.51	0.41	0.02	0.27	0.26	0.58	0.24	0.20	0.31	0.70	0.68
FA	aver	1132.	1342.	1412.	1050.	1141.	1444.	1308.	1523.	2018.	1679.	1327.
	st dv	620.0	705.9	803.5	660.9	940.7	1046.	1283.	1364.	1110.	998.2	872.3
	skew	0.41	0.88	1.03	1.39	0.85	1.76	2.13	2.79	1.79	0.58	1.90
	corr	0.88	0.95	0.91	0.85	0.92	0.94	0.95	0.94	0.91	0.92	0.98
SS	aver	391.6	477.2	445.0	401.6	461.3	528.7	494.5	529.9	814.6	594.9	488.3
	st dv	299.3	417.3	282.6	278.4	482.3	509.8	443.9	471.4	836.3	403.9	386.5
	skew	1.38	2.16	0.71	1.39	0.87	2.03	2.11	2.77	2.45	0.37	2.12
	corr	0.86	0.90	0.86	0.77	0.83	0.92	0.86	0.93	0.89	0.88	0.92
SO	aver	25.4	28.9	23.7	20.7	22.5	27.6	29.4	43.0	45.2	30.5	25.1
	st dv	22.2	18.5	14.8	14.6	20.6	26.5	21.8	59.5	43.8	20.5	20.2
	skew	0.34	0.39	0.89	1.44	1.01	2.06	2.57	2.89	2.11	0.61	2.09
	corr	0.79	0.75	0.89	0.80	0.91	0.92	0.81	0.87	0.88	0.92	0.90
PF	aver	77.6	59.6	67.2	69.3	112.2	103.4	132.2	177.9	175.9	103.0	83.2
	st dv	52.6	33.6	45.9	54.5	66.3	68.3	65.7	96.1	103.4	73.6	65.6
	skew	0.69	0.57	0.33	0.89	0.15	0.60	0.47	0.61	0.68	0.38	0.80
	corr	0.36	0.44	0.46	0.55	0.64	0.63	0.29	0.26	0.22	0.64	0.55
CC	aver	89.9	113.8	98.9	59.4	54.5	53.4	42.4	52.6	76.2	69.6	75.8
	st dv	18.7	51.8	48.0	30.5	35.5	28.4	35.3	37.1	34.6	33.2	31.1
	skew	0.14	0.14	1.25	-0.09	0.49	1.34	1.47	2.21	0.44	0.46	0.25
	corr	0.17	0.57	0.50	0.15	0.36	0.52	0.89	0.76	0.35	0.20	0.27

Table 23. Deterministic partition: Statistical parameters of the energy storage (Eq. MW)

Site		Weeks										
		1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
PR	aver	6301	6132	5963	5882	5638	5596	5727	5888	6192	6660	6485
	st dv	1137	1495	1518	1505	1855	2052	2065	1986	1852	1367	1352
	skew	-2.18	-1.54	-0.66	-0.72	-1.03	-1.12	-1.57	-1.58	-1.74	-3.16	-1.88
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
FA	aver	17599	17341	17749	17350	16852	16410	16281	16187	16989	18179	18144
	st dv	4453	4294	3494	3824	4671	5787	5679	5803	5342	4268	4390
	skew	-2.07	-1.42	-0.94	-1.00	-1.33	-1.43	-1.21	-1.14	-1.18	-1.87	-2.38
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
SS	aver	6836	6664	6811	6459	6309	6296	6044	5984	6632	7200	7280
	st dv	2810	2868	2644	2945	3209	3310	3459	3572	3262	2745	2714
	skew	-1.05	-0.79	-0.81	-0.88	-0.99	-0.85	-0.64	-0.55	-0.74	-1.24	-1.28
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
SO	aver	187	161	160	135	137	152	165	187	220	234	217
	st dv	162	157	150	140	132	152	150	166	178	165	169
	skew	0.15	0.46	0.41	0.80	0.84	0.66	0.46	0.19	-0.21	-0.36	-0.19
	corr	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
PF	aver	2260	1984	1686	1633	1480	1559	1663	1792	2131	2383	2455
	st dv	1559	1415	1368	1330	1306	1331	1452	1641	1774	1677	1512
	skew	0.62	0.66	0.97	1.19	1.44	1.20	1.12	1.00	0.57	0.56	0.37
	corr	0.99	0.97	0.97	0.99	0.99	0.97	0.98	0.99	0.99	0.99	0.97
	aver	550	465	398	334	280	350	396	477	627	659	667
	st dv	617	547	520	503	470	509	545	599	653	653	602
	skew	0.82	1.04	1.23	1.61	2.07	1.49	1.33	1.03	0.46	0.46	0.41
	corr	0.99	0.98	0.98	0.99	0.97	0.99	0.99	0.99	0.99	0.98	0.98

Table 24. Random partition: Statistical parameters of the energy storage (Eq. MW)

Site		Weeks 1st	6th	11th	16th	21st	26th	31st	36th	41st	46th	50th
PR	aver	6328	6085	6002	5664	5616	5296	5869	5803	6141	6743	6353
	st dv	1703	1592	1578	1648	1821	2240	2185	2217	2015	1473	1629
	skew	-1.99	-1.20	-0.81	-0.68	-0.77	-0.90	-1.60	-1.39	-1.70	-3.15	-1.72
	corr	0.86	0.87	0.78	0.73	0.83	0.88	0.93	0.92	0.77	0.90	0.76
FA	aver	17714	17433	17668	17489	16785	16588	16137	16189	17043	18191	18114
	st dv	4497	4205	3670	3851	5033	5733	5568	5757	5296	4161	4322
	skew	-2.12	-1.61	-0.92	-0.91	-1.38	-1.45	-1.19	-1-09	-1.19	-1.75	-2.47
	corr	0.98	0.96	0.93	0.97	0.98	0.98	0.99	0.99	0.99	0.98	0.96
SS	aver	6928	6802	6707	6534	6331	6534	5933	5939	6643	7209	7244
	st dv	2808	2683	2790	3066	3236	3283	3342	3604	3356	2879	2626
	skew	-1.11	-0.81	-0.81	-0.93	-0.92	-0.87	-0.57	-0.52	-0.85	-1.20	-1.36
	corr	0.97	0.94	0.94	0.97	0.97	0.97	0.98	0.99	0.98	0.98	0.93
SO	aver	1975	1833	1596	1448	1647	1687	1541	2072	2263	2408	2164
	st dv	171	179	149	131	134	154	168	174	185	164	168
	skew	0.14	0.34	0.67	0.64	0.63	0.56	0.62	0.01	-0.19	-0.25	-0.07
	corr	0.89	0.90	0.72	0.79	0.68	0.72	0.79	0.92	0.89	0.79	0.86
PF	aver	2707	1968	1884	1760	1691	1515	1870	1934	2209	2498	2835
	st dv	1778	1633	1537	1671	1492	1478	1604	1648	1798	1745	1742
	skew	0.51	0.48	0.60	0.65	0.72	0.95	0.66	0.75	0.44	0.27	-0.39
	corr	0.87	0.68	0.61	0.73	0.69	0.80	0.74	0.81	0.89	0.83	0.72
CC	aver	610	517	429	345	270	373	421	536	618	647	610
	st dv	630	598	584	522	482	544	586	638	688	664	621
	skew	0.67	0.99	1.18	1.60	2.17	1.57	1.25	0.79	0.47	0.55	0.71
	corr	0.86	0.79	0.88	0.82	0.84	0.90	0.93	0.86	0.94	0.86	0.83

autocorrelation coefficient of the computed series. These coefficients were maintained when only the deterministic component is used. When the random component is introduced the autocorrelation coefficients of the series in the Foz do Areia and Salto Santiago reservoirs are still near the actual figures. This is remarkable since the partition models cannot assure that the autocorrelation coefficients of the variables will be maintained. The results are attributed to the high autocorrelation observed between values of aggregated inflows and storages. This autocorrelation was partially transmitted to the inflows and storages in each reservoir through the partition model.

6.3.4 Correction of the Relaxed Strategy

Correction of the relaxed strategy was performed in two steps. Initially, only the deterministic component of the partition model was used. Later the complete model of partition was considered. The initial function of marginal expectations was provided by the relaxed strategy (section 6.3.2). After the relaxed strategy was corrected with the deterministic partition the correction with the random partition was applied.

Observations made on the deterministic operation indicated that the probability of spillings is practically null when the overall energy content in the system is below 30,000 Eq. MW. Therefore, the partition models were applied only for values of overall energy content greater than this level. This resulted in substantial computation economy.

Table 25 presents the obtained strategy with deterministic partition correction. The convergence with U.S. $5.00 tolerance was obtained after two cycles of computations. The marginal expectations increased in periods of imminent shortage and decreased in periods of

Table 25a. Marginal expected future benefit of the remaining storage with deterministic partition.
(U.S. $/MWh)

WEEK NO. 1

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	650.	1000.	1500.	2250.	3000.	3500.	4000.	4500.
0	568.3	514.1	376.7	235.3	145.8	101.0	66.9	38.8
2000	548.2	465.9	348.0	223.6	139.0	97.2	62.6	36.5
4000	531.8	442.5	332.4	214.1	132.2	91.3	59.3	33.9
6000	518.1	426.2	319.5	204.9	125.4	85.9	55.0	30.8
10000	492.2	399.9	297.3	187.1	112.3	75.3	46.9	24.9
15000	459.4	369.9	271.5	166.3	96.7	62.0	37.1	19.2
20000	423.1	338.7	244.4	145.2	80.0	48.8	28.1	14.2
25000	388.0	307.3	217.4	122.8	62.8	36.1	19.6	9.7
30000	353.1	276.6	190.1	99.8	44.9	23.7	11.1	4.7
35000	320.2	245.5	162.4	75.7	27.2	11.6	4.3	1.4
39000	295.0	221.7	137.7	53.1	13.6	3.0	0.6	0.1
41000	283.2	209.6	122.7	44.5	8.9	1.2	0.2	0.0
43000	269.5	193.9	105.1	31.4	5.4	0.8	0.1	0.0
44920	255.0	168.3	72.9	10.7	0.2	0.0	0.0	0.0

Table 25b. Marginal expected future benefit of the remaining storage with deterministic partition.
(U.S. $/MWh)

WEEK NUMBER 11

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	700.	1000.	1750.	2400.	3050.	3750.	4500.	5000.
0	581.7	533.1	370.1	246.8	159.3	97.7	53.1	32.7
2000	567.5	506.6	340.7	229.0	149.9	90.9	49.1	30.2
4000	549.6	482.2	320.7	216.8	141.6	85.0	45.7	28.0
6000	534.2	462.0	305.9	206.7	134.2	79.7	42.7	25.5
10000	508.8	432.5	282.5	188.0	121.6	71.1	36.8	21.8
15000	474.7	399.5	255.3	166.9	104.8	60.3	30.2	17.5
20000	435.7	364.7	228.5	145.7	88.8	48.3	23.4	12.4
25000	397.1	330.3	202.7	126.5	74.3	37.7	16.7	8.9
30000	361.7	298.3	178.1	106.4	60.1	28.2	10.5	4.8
35000	328.9	268.3	149.5	87.3	43.7	17.5	5.0	1.8
39000	303.9	246.0	135.9	55.9	27.9	7.9	0.3	0.1
41000	291.2	234.7	126.0	51.3	22.0	3.7	0.0	0.1
43000	278.9	222.1	109.5	46.7	16.2	1.9	0.0	0.0
44920	266.2	204.0	70.0	12.8	0.4	0.0	0.0	0.0

Table 25c. Marginal expected future benefit of the remaining storage with deterministic partition.
(U.S. $/MWh)

WEEK NO. 21

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	400.	1000.	2000.	3000.	4500.	5500.	6500.	7000.
0	608.0	519.0	251.6	134.3	47.4	20.2	7.5	3.9
2000	582.2	447.2	234.7	125.9	44.5	17.9	4.9	3.4
4000	564.7	415.1	221.6	118.0	40.3	15.5	4.5	1.9
6000	548.6	391.6	210.4	112.6	36.8	14.1	4.1	1.7
10000	518.9	358.3	190.8	100.2	30.5	11.1	3.3	1.4
15000	476.0	324.8	167.4	83.3	22.7	7.1	1.4	0.4
20000	433.4	292.4	144.5	66.9	15.9	4.0	0.5	0.1
25000	396.3	261.6	122.1	52.0	11.2	1.6	0.1	0.0
30000	361.1	234.2	103.2	39.3	5.2	0.5	0.0	0.0
35000	328.8	207.9	84.6	26.9	3.3	0.3	0.0	0.0
39000	303.2	183.6	65.4	15.7	0.0	0.0	0.0	0.0
41000	290.4	173.3	55.0	7.1	0.0	0.0	0.0	0.0
43000	277.5	160.7	41.3	4.7	0.0	0.0	0.0	0.0
44920	264.4	139.2	12.8	0.0	0.0	0.0	0.0	0.0

Table 25d. Marginal expected future benefit of the remaining storage with deterministic partition.
(U.S. $/MWh)

WEEK NO. 31

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW)							
	500.	1000.	2000.	3000.	4000.	5000.	6000.	7000.
0	595.3	525.5	269.8	148.0	80.4	40.4	16.7	7.0
2000	551.6	436.3	246.9	138.4	74.9	35.3	14.9	4.8
4000	523.1	401.7	234.3	130.8	69.2	31.8	13.3	4.3
6000	503.6	385.9	223.1	121.9	62.4	28.9	11.9	3.8
10000	471.7	359.5	201.4	104.9	51.3	22.2	8.0	1.8
15000	432.9	325.5	174.9	87.2	39.3	14.8	4.2	0.6
20000	395.3	292.3	148.8	68.4	27.6	8.9	1.7	0.4
25000	361.4	261.1	123.8	51.4	18.9	4.6	0.6	0.0
30000	328.3	230.6	100.6	35.9	11.8	3.2	0.4	0.0
35000	295.9	198.7	74.6	22.6	4.3	0.5	0.0	0.0
39000	268.0	169.3	53.4	12.1	0.5	0.0	0.0	0.0
41000	254.8	158.5	49.9	6.7	0.4	0.0	0.0	0.0
43000	240.8	146.6	40.0	5.2	0.1	0.0	0.0	0.0
44920	225.3	121.2	19.7	0.2	0.0	0.0	0.0	0.0

Table 25e. Marginal expected future benefit of the remaining storage with determinstic partition.
(U.S. $/MWh)

WEEK NO. 41

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW) 1000.	2000.	32000.	4400.	5600.	6800.	8000.	9000.
0	579.2	438.7	293.6	194.6	120.5	67.1	31.9	15.0
2000	565.8	419.3	281.5	185.1	113.3	62.3	28.3	11.9
4000	547.0	402.1	270.4	175.6	105.6	56.3	24.6	10.4
6000	529.2	387.6	261.0	167.3	99.2	50.1	21.6	9.0
10000	496.9	361.6	239.3	149.5	84.3	40.1	15.4	5.0
15000	455.6	330.9	213.6	127.6	67.2	27.8	8.6	1.9
20000	421.2	301.0	187.1	103.4	48.1	16.4	3.9	0.5
25000	388.2	271.4	158.2	78.9	31.2	8.5	0.7	0.1
30000	356.2	239.9	122.0	48.7	13.8	1.7	0.1	0.0
35000	323.5	205.6	79.6	18.7	1.6	0.0	0.0	0.0
39000	298.8	169.4	53.1	0.8	0.0	0.0	0.0	0.0
41000	285.9	149.1	24.3	0.0	0.0	0.0	0.0	0.0
43000	272.9	110.5	17.6	0.0	0.0	0.0	0.0	0.0
44920	261.5	41.4	0.0	0.0	0.0	0.0	0.0	0.0

imminent spillage. Nevertheless, the resultant corrected strategy did not differ drastically from the relaxed strategy.

Table 26 presents the resultant strategy after the random partition correction. This was determined after 2 cycles of computation using the same sequence of random normal deviations to define the random component of the partition models in each cycle. Practically no difference from the strategy obtained with the deterministic partition correction was verified. Other tests were applied to verify the contributions of the random component of the partition in the strategy. One test was performed with different sequences of random deviations per cycle. No sensible modification of the strategy was observed. In another test the initial marginal expectations of the storages at the end of the 52-nd week were given by the relaxed strategy. The random partition correction was applied with the same series of random normal deviations per cycle. After two iterations it was observed that the marginal expectations converged to the previously corrected values. Those tests allow the conclusion that the random component of the partition models does not introduce any new information into the strategy. In other words, the strategy is insensitive to the variations of the reservoirs' storages and inflows around their deterministic trend. This implies that aggregated variables provide good representation of the state of the system. Consequently the strategy of operation can be defined in terms of aggregated values of energy inflows and storages.

Summarizing, the resultant corrected strategy explicitly considers the informations about the aggregated reservoir storage and inflow and the autocorrelation on the aggregated inflows. It also accounts implicitly for the cross correlation among reservoir's energy inflow and

Table 26a. Marginal expected future benefit of the remaining storage with random partition.
(U.S. $/MWh)

WEEK NO. 1

Reservoir storage (Eq. MW)	Previous Inflows (Ew. MW)							
	650.	1000.	1500.	2250.	3000.	3500.	4000.	4500.
0	568.3	514.1	377.1	236.2	147.1	101.9	67.9	40.4
2000	548.2	466.6	348.6	225.0	140.4	97.9	63.9	37.3
4000	531.8	442.6	332.7	214.9	133.5	93.3	60.2	34.8
6000	518.0	426.4	320.4	206.3	126.7	87.1	56.9	32.3
10000	492.2	400.9	298.4	189.4	115.0	78.3	48.9	27.9
15000	459.2	370.6	272.9	169.2	99.6	65.5	40.5	21.6
20000	423.3	340.1	246.1	148.6	84.4	54.1	31.3	16.4
25000	388.3	308.3	220.0	127.4	67.5	41.0	22.6	11.4
30000	354.6	279.0	194.9	106.2	50.7	28.2	14.6	5.7
35000	321.9	249.2	166.9	80.5	32.3	15.2	7.3	1.9
39000	296.4	223.6	140.0	57.2	19.1	8.5	2.2	0.6
41000	283.4	209.4	123.1	46.1	9.9	1.4	0.1	0.0
43000	269.9	193.3	103.9	30.6	5.3	0.8	0.0	0.0
44920	253.8	166.1	69.0	9.8	0.2	0.0	0.0	0.0

Table 26b. Marginal expected future benefit of the remaining storage with random partition.
(U.S. $/MWh)

WEEK NO. 11

Reservoir storage (Eq. MW)	Previous Inflows (Ew. MW) 700.	1000.	1750.	2400.	3050.	3750.	4500.	5000.
0	581.7	533.1	370.5	247.6	161.1	98.5	54.0	33.4
2000	567.4	506.4	341.1	230.0	150.7	91.8	49.7	30.9
4000	549.6	482.3	320.8	218.2	143.4	86.3	47.0	28.8
6000	534.3	462.0	306.2	207.0	135.3	82.5	43.7	26.8
10000	508.8	432.7	283.1	190.2	123.1	73.4	37.7	22.4
15000	474.2	399.5	256.7	169.1	106.7	61.4	30.8	18.3
20000	436.4	365.7	231.0	148.6	91.6	49.6	24.3	13.2
25000	397.4	331.5	204.8	129.0	77.9	40.7	18.5	10.0
30000	362.6	300.3	181.4	111.2	62.6	31.4	13.5	7.3
35000	329.1	269.8	157.5	91.9	48.6	21.4	8.2	4.0
39000	302.7	246.2	137.9	73.3	33.9	8.8	1.0	0.0
41000	291.4	234.8	125.9	63.4	18.1	0.7	0.0	0.0
43000	279.4	222.0	107.5	45.8	15.8	0.0	0.0	0.0
44920	265.2	201.3	60.5	12.0	0.4	0.0	0.0	0.0

Table 26c. Marginal expected future benefit of the remaining storage with random partition.
(U.S. $/MWh)

WEEK NO 21

Reservoir storage (Eq. MW)	Previous Inflows (Ew. MW) 400.	1000.	2000.	3000.	4500.	5500.	6500.	7000.
0	608.0	519.0	252.7	134.8	49.0	21.2	7.9	4.1
2000	582.2	446.9	236.0	127.1	46.0	19.1	7.0	3.6
4000	564.7	414.9	222.8	119.5	41.5	16.5	4.8	2.0
6000	548.6	392.0	211.8	113.4	38.7	14.9	4.3	1.8
10000	518.6	358.4	192.1	101.4	32.1	11.8	3.4	1.4
15000	475.6	325.1	169.3	85.0	24.3	7.7	1.5	0.5
20000	433.3	293.5	147.3	70.0	18.6	4.4	0.5	0.1
25000	396.2	263.3	126.4	56.6	13.1	3.3	0.4	0.1
30000	361.1	235.4	105.4	42.7	7.8	1.3	0.1	0.0
35000	328.8	207.8	86.4	29.9	4.0	0.2	0.0	0.0
39000	302.6	185.1	65.9	21.4	0.0	0.0	0.0	0.0
41000	289.7	173.0	54.2	11.4	0.0	0.0	0.0	0.0
43000	277.1	159.9	40.7	4.5	0.0	0.0	0.0	0.0
44920	264.3	135.2	15.9	0.0	0.0	0.0	0.0	0.0

Table 26d. Marginal expected future benefit of the remaining storage with random partition.
(U.S. $/MWh)

WEEK NO. 31

Reservoir storage (Eq. MW)	Previous Inflows (Eq. MW) 500.	1000.	2000.	3000.	4000.	5000.	6000.	7000.
0	595.3	525.5	270.0	148.9	81.4	40.4	17.2	7.1
2000	551.6	436.2	247.2	139.9	75.7	35.7	15.2	4.9
4000	523.2	401.4	234.6	131.4	68.3	32.3	13.5	4.3
6000	503.5	385.2	224.0	122.8	63.5	29.4	12.1	3.9
10000	471.1	359.0	202.2	107.3	52.9	22.7	8.1	1.8
15000	432.8	325.5	176.7	87.0	39.5	15.3	4.3	0.6
20000	394.9	292.5	150.1	70.0	29.0	10.9	3.2	0.4
25000	361.3	262.4	125.5	53.9	20.2	6.6	0.6	0.1
30000	328.6	230.9	103.4	40.1	12.7	3.3	0.4	0.0
35000	296.4	199.8	78.8	24.7	6.3	0.5	0.0	0.0
39000	268.8	173.2	57.8	16.8	0.9	0.0	0.0	0.0
41000	255.0	161.0	49.2	7.7	0.0	0.0	0.0	0.0
43000	240.5	144.7	38.4	5.0	0.0	0.0	0.0	0.0
44920	223.4	118.2	18.7	0.2	0.0	0.0	0.0	0.0

Table 26e. Marginal expected future benefit of the remaining storage with random partition.
(U.S. $/MWh)

WEEK NO. 41

Reservoir storage (Eq. MW)	Previous Inflows (Ew. MW) 1000.	2000.	3200.	4400.	5600.	6800.	8000.	9000.
0	579.2	438.7	293.5	194.4	120.5	67.4	32.4	15.2
2000	565.8	419.3	281.4	185.2	113.7	62.1	28.8	12.0
4000	547.0	402.2	270.8	176.2	106.1	56.1	24.9	10.4
6000	529.2	387.6	260.1	167.4	99.1	50.0	21.7	9.0
10000	496.7	361.6	239.3	149.1	83.7	39.5	15.1	5.0
15000	455.6	331.0	213.3	127.1	65.9	27.2	8.6	1.9
20000	421.2	301.1	187.0	103.1	47.8	16.3	3.9	0.5
25000	388.4	271.2	158.2	78.7	30.1	8.1	0.6	0.1
30000	355.3	240.7	124.6	49.6	13.8	1.7	0.1	0.0
35000	323.4	205.4	83.1	19.3	1.6	0.0	0.0	0.0
39000	297.3	170.3	49.0	0.1	0.0	0.0	0.0	0.0
41000	285.2	144.2	29.2	0.0	0.0	0.0	0.0	0.0
43000	272.4	103.3	15.5	0.0	0.0	0.0	0.0	0.0
44920	263.4	37.3	0.0	0.0	0.0	0.0	0.0	0.0

storage. Finally, a substantial part of the autocorrelation on the reservoirs' energy inflows and storage is indirectly considered. All this information could be explicitly included in the formulation only through a disaggregated multidimensional formulation implying severe computational limitations, however. Therefore, the aggregated formulation considers most important information in the decision process in the derivation of the strategy without having cumbersome computational requirements of the alternative multidimensional formulation. This makes the proposed method attractive from a theoretical and computational points of view.

6.4 Strategy in Transient Periods

Transient periods of operation are periods during which the partition of energy inflow and storage among the reservoirs cannot be explained by the respective partition model. It may be caused by transient effects of failure of equipment, unexpected variations of the energy demand, and rare hydrologic events, among others.

During the transient periods the derived strategy of operation cannot be used since the partition models do not apply. However, tactical and strategic approaches that have been developed are still able to define transient strategies for such periods. The derivation of transient strategies relies on the definition of the optimal operation for several generated series of events likely to occur in the transient period. As the extension of the transient period is unknown it has to be previously assumed. A first guess may be considered as the number of time intervals of the transient variation of the exogenous variable (demand, inflow, equipment outage, etc...). The transient period will be defined if at the end of this guessed period the endogenous variables (reservoir contents) are spatially distributed according to the partition model.

Operation of the aggregated system is defined by the Constrained Marginal Analysis for each generated series. A final storage is initially assumed. The marginal expectation of this storage is known from the strategic studies. If the marginal instantaneous benefit of release in the last time interval of the transient period is different from the marginal expectations of the final storage, the assumed final storage must be corrected. The correction may be easily defined through analysis of the marginal values. If the marginal instantaneous benefit of release at the end of the period is greater than the marginal expectation of final storage more energy must be produced in the transient period. Therefore, the final storage is decreased. Otherwise, when the marginal instantaneous benefit of release at the end of the period is less than the marginal expectation of final storage the final storage has to be increased. When the equality of the marginal values is obtained the optimal operation in the transient period is defined for the considered series of events. The disaggregation procedure is then applied and the resultant partition of the reservoir energy contents at the end of the transient period is verified. If it belongs in the statistical sense to the partition model adopted in the definition of the strategy the assumed length of the transient period was well defined. Otherwise, it will be increased and the procedure repeated.

A reasonable number of generated series of transient events is to be used in the definition of alternative tactics of operation. The resultant tactics define a sample of marginal expectations of storages for each state of the system in the transient period. The marginal expectation function may then be estimated for each possible state of the system in the period.

6.5 The Cost of the Uncertainty

The difference between expected minimum cost of the operation under uncertainty and minimum cost of operation under certainty of future inflows defines what is referred to as the cost of uncertainty. Although the knowledge of its value has little practical worth it is computed to allow better insight into the characteristics of strategic operation.

To avoid unnecessary programming and computation a simple application of the strategy of operation was made. The aggregated formulation of the system was used. Values of the overall energy inflows were those values obtained in the deterministic operation. This allowed a comparison of the results of the deterministic and the strategic operations without too much computational effort.

The same value of the initial storage in the deterministic operation was assumed. The strategy of operation was applied to the 36 years of observed inflows. As in the deterministic operation the initial storage each year was assumed equal to the computed final storage at the end of the previous year.

The aggregated storage trajectories for the strategic and deterministic operations are presented in Figure 26. As expected, the strategic operation defines higher storages than the tactical operation. This reflects the shortage aversion of the strategic operation.

Shortages occurred in about 37 percent of the weeks of operation. The value of the energy shortage was however always less than 50 Eq. MW, or less than 2 percent of the average energy demand. It can not be

Figure 26.a. Energy Storage Trajectories, Deterministic and Strategic Operations, 1931 to 1936

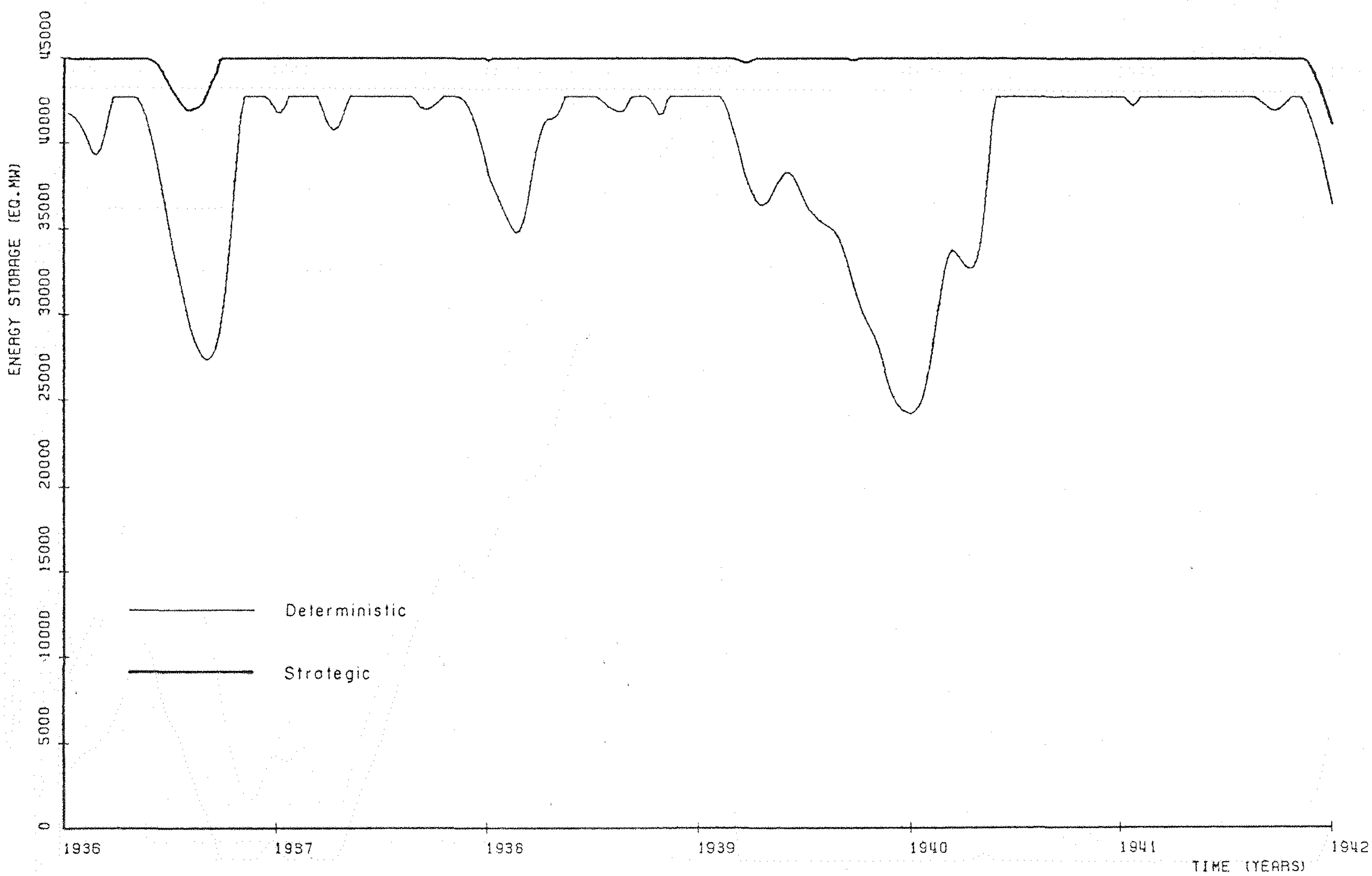


Figure 26.b. Energy Storage Trajectories, Deterministic and Strategic Operations, 1937 to 1942

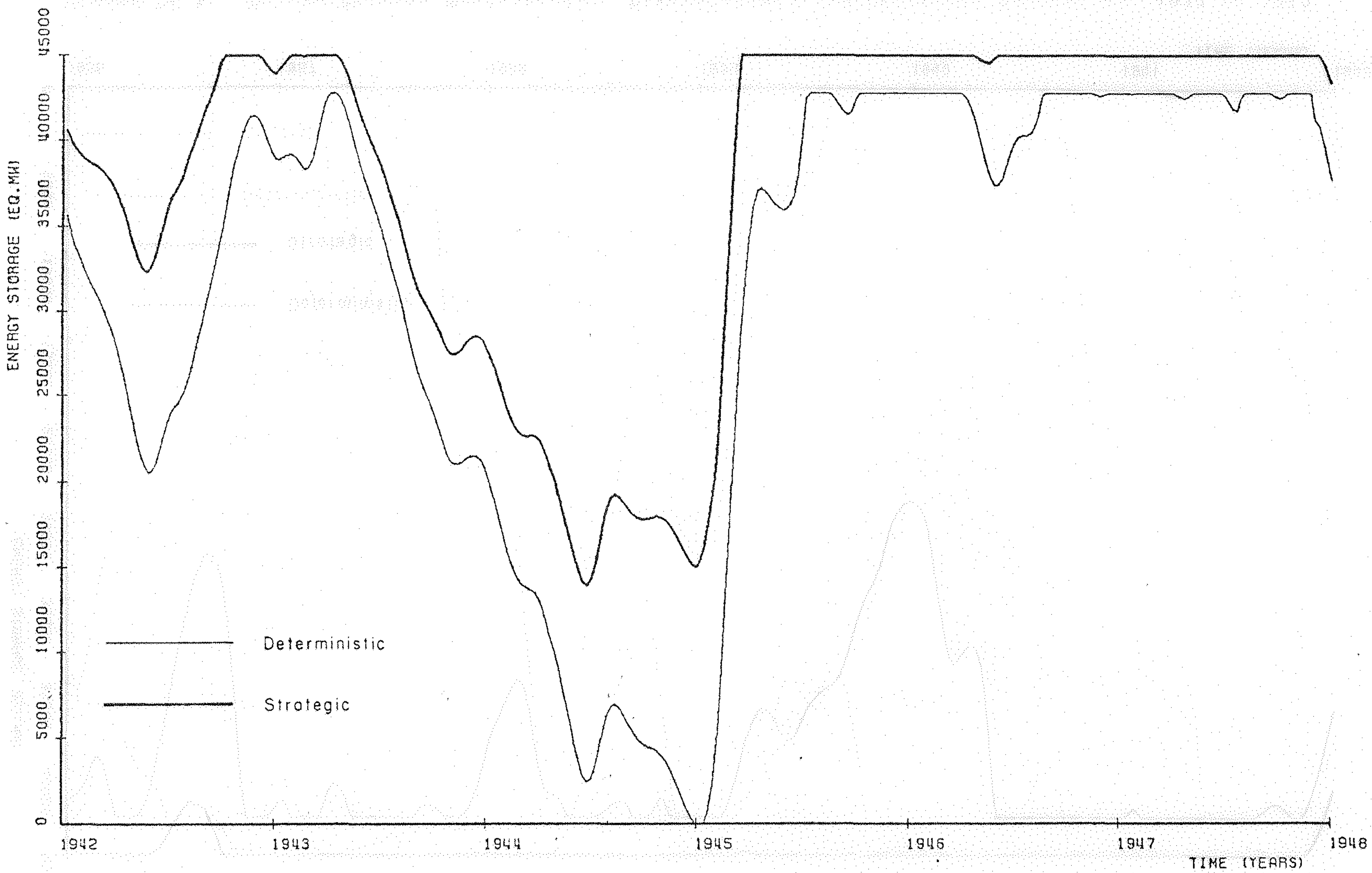


Figure 26c. Energy Storage Trajectories, Deterministic and Strategic Operations, 1943 to 1948

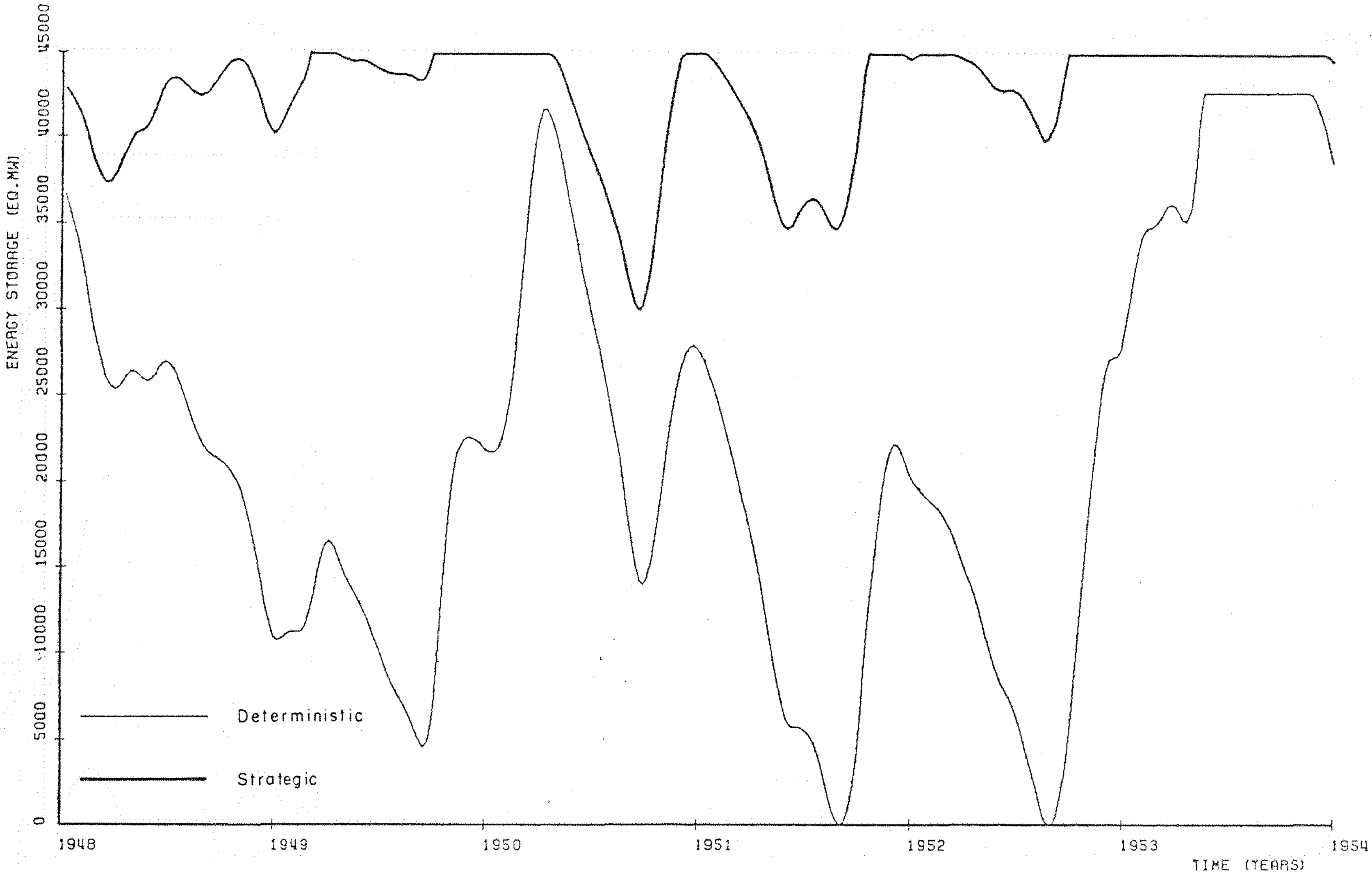


Figure 26.d. Energy Storage Trajectories, Deterministic and Strategic Operations, 1949 to 1954

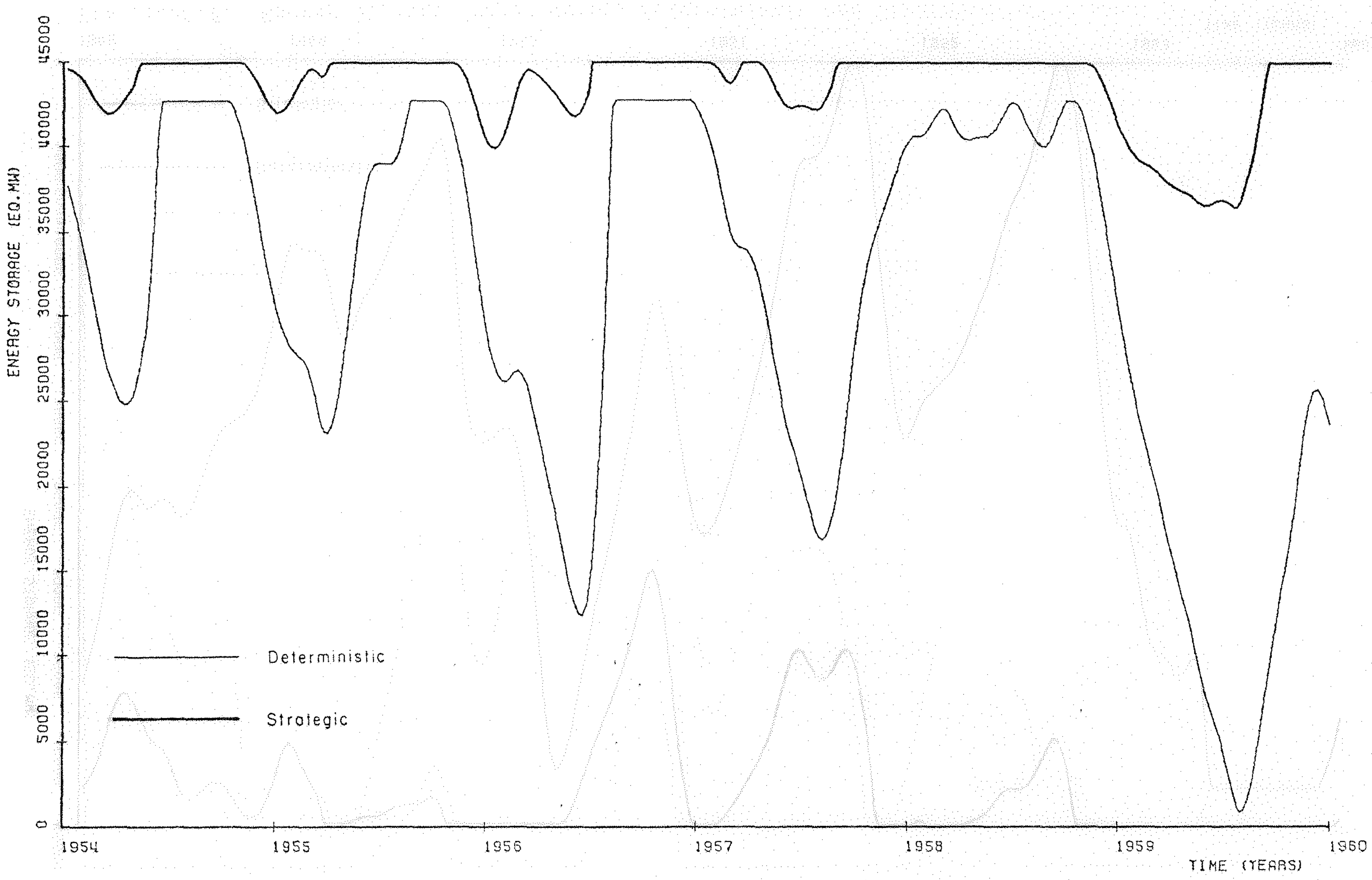


Figure 26.e. Energy Storage Trajectories, Deterministic and Strategic Operations, 1955 to 1960

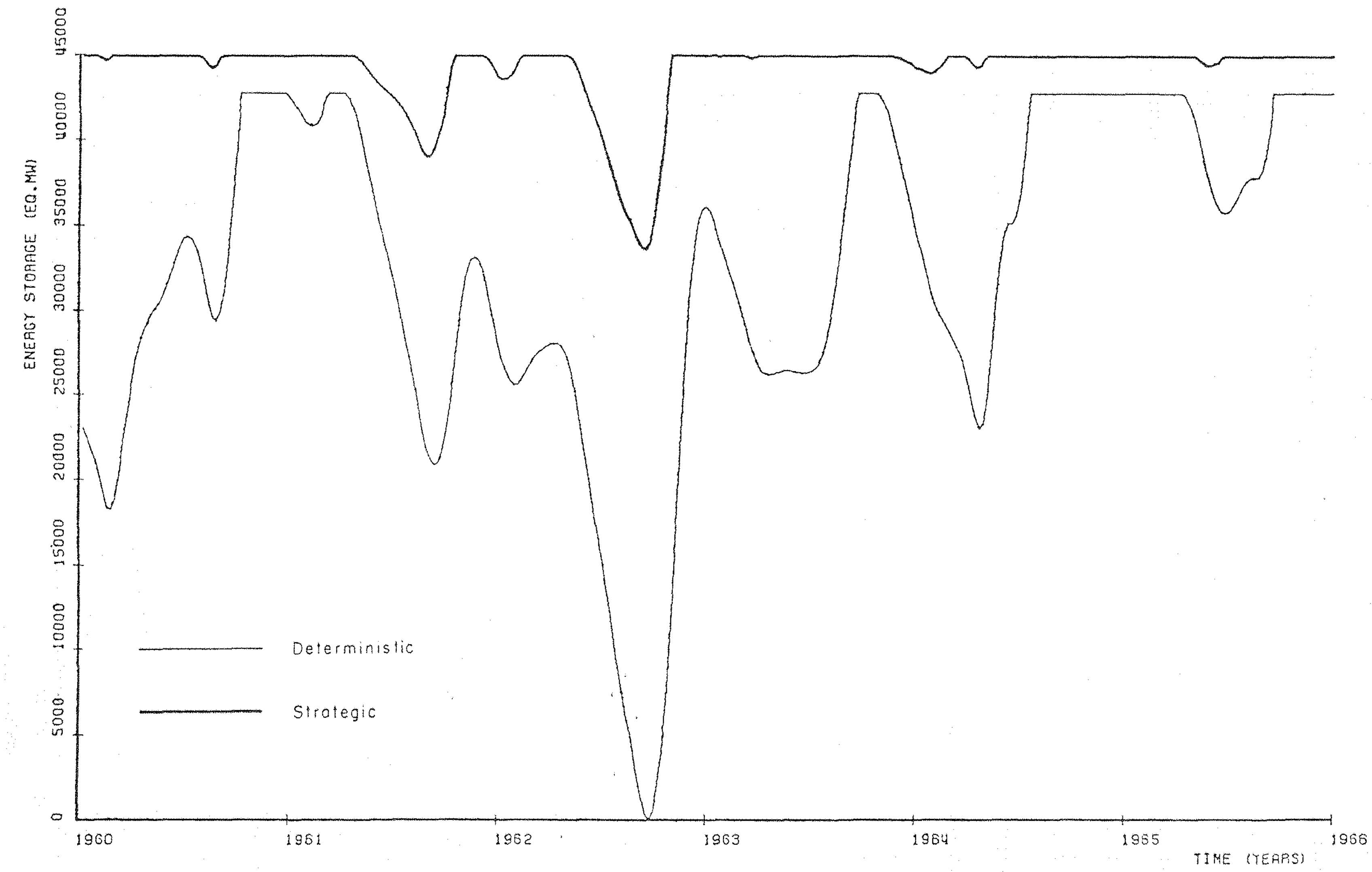


Figure 26.f. Energy Storage Trajectories, Deterministic and Strategic Operations, 1961 to 1966

considered a disaster level shortage. About 40 percent of the time the thermal production was minimum. It contrasts with the 58 percent value obtained in the tactical operation.

The resultant variable cost of operation for the entire period of operation was 5,682 million dollars (U.S.). The corresponding average annual cost is 157.8 million dollars which implies an average annual cost for uncertainty of 119.1 million dollars. This allows appraisal of the importance of information concerning future inflows in the strategic operation of reservoirs.

Chapter 7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The convexity (or quasi-convexity) of the formulated problem allowed the application of the Kuhn-Tucker theorem of Mathematical Programming to the deterministic operation. The mathematical formulations of the optimality conditions did not allow much insight into the nature of the optimal operation. The logic of the optimal decisions was identified through an analytical approach which involved two concepts: the marginal economic meaning of Lagrange multipliers and the concept of optimal temporal and spatial water transfer in reservoir operation. The characteristics of the optimal operation were expressed in five optimality conditions in terms of marginal values.

The first optimality condition states that the marginal instantaneous benefit of the releases equals the marginal future benefit of the remaining storage in each reservoir and at each time interval. When a given reservoir is serially-linked to a downstream reservoir another component exists in the benefit of the releases derived from the spatial transfer of water from the upstream to the downstream reservoir or the benefit of water transfer.

When the optimal transfer of water in time and between reservoirs is not tightened by any storage constraint the marginal future benefit of the storage is constant for each reservoir. Its value will increase in a reservoir when this reservoir's maximum storage constraint tightens the optimal operation. When the minimum storage constraint tightens the optimal operation of a reservoir the marginal future benefit of its storage will decrease. These are the second, third and fourth optimality conditions.

The variations of the marginal future benefit of the storage and of the marginal benefit of the water transfer between reservoirs implies the occurrence of a chain of extreme storage. In serially-linked reservoirs the upstream reservoir reaches an extreme (maximum or minimum) storage before the downstream reservoir does. The anticipation equals the travel time of the water between the reservoirs. Moreover, the upstream reservoir again reaches the same extreme storage in the same time interval the downstream reservoir does. This occurrence is a result of the optimization of temporal and spatial water transfer in serially-linked reservoirs.

The existence of a period of operation when the maximum release tactic is optimal was demonstrated in the fifth optimality condition. These periods may occur in an isolated reservoir or in the system as a whole. The optimal operation in this period will start with the reservoir empty and will end with the reservoir full.

The objective function in a interconnected hydropower system is related to its overall energy production. This implies the equality of the marginal instantaneous benefit of the energy production in each reservoir along the operation. As a consequence, an aggregated formulation of the problem was suggested for definition of its optimal operation. It had the basic advantage of decreasing the programming and the computational efforts in the solution. The price was derivation of an optimal solution in only an approximate sense. Nevertheless, it was considered acceptable for screening purposes of the deterministic operation study.

An algorithm was developed to identify the periods where the maximum release tactic is optimal. Its low computational requirements made the algorithm extremely efficient in the search of the optimal solution.

The optimization of the operation of aggregated formulation of the problem was accomplished by a new optimization technique named Constrained Marginal Analysis. It was developed based on the characteristics of the trajectories of the reservoir storage under optimal variation of the marginal instantaneous benefit of the releases. This technique is akin to Massé's Constrained Calculus of Variations. It was shown to be a generalization of a previously proposed algorithm which was addressed to the optimal operation of a seasonal alpine reservoir (Laufer, 1977 and Laufer and Morel-Seytoux, 1979). The Constrained Marginal Analysis showed a remarkable improvement in computational and memory requirements when compared with Dynamic Programming. Its application is restricted to convex problems, however.

A criterion of disaggregation was presented to define the operation of each reservoir given the optimal operation of the aggregated system. It was based on the enforcement of a heuristic distribution of storage among the system's reservoirs with the objective of avoiding water spills. At the same time, the disaggregation procedure was developed in such a way that it was applicable to the strategic operation.

This aggregation-optimization-disaggregation procedure was able to define a near optimal operation for the multireservoir system. A characteristic of the operation referred to the partitioning of energy storage and inflow among the reservoirs of the system. It was observed that these partitions presented a significant deterministic trend in the dominant reservoirs and rivers of the system.

The optimal strategy of operation is obtained when the marginal instantaneous benefit of releases equals the marginal expectation of the remaining storage. Massé's Theory of Marginal Expectations defined the basis of a computational procedure used in the derivation of marginal expectations of the storages in the optimal operation. This computational procedure was referred to by Massé as the Chain of Marginal Expectations. It is regarded as an improved version of the Explicit Stochastic Approach of Dynamic Programming.

Computational limitations implied formulation of the problem in its aggregated representation. The correction of the resultant relaxed strategy was performed with the consideration of the partition of energy inflow and storage among reservoirs observed in the deterministic operation. A statistical analysis was applied in the definition of the partition models. The Chain of Marginal Expectations was adapted to include these models. It allowed the spills not accounted in the aggregated formulation to be considered in the computation of the strategy of operation.

The application of the algorithm to the case study showed that the state of the system, upon which the strategic decisions are made, can be represented by aggregated variables. The resultant strategy of operation is defined as a function of the overall values of energy storage and previous energy inflow into the system. It explicitly considered the autocorrelation in the overall energy inflow variables. The strategy also implicitly considered cross correlation among the energy inflow and storage variables in each reservoir. Finally, a substantial part of the autocorrelation in the energy inflow and storage variables in each reservoir is indirectly considered in the derivation of the operating strategy.

The aggregation-optimization-disaggregation procedure has derived a near optimal solution. The aggregation part of the procedure underestimated the energy inflows to the serially-linked reservoirs. The definition of improved aggregation techniques in this case may probably be obtained. One suggestion is the introduction of the relative size of the storage capacities of each serially-linked reservoir into the estimate of the energy inflows. Notice that this problem does not exist in the strategic operation of the system. There, the energy storages and inflows to each reservoir are updated each time interval previous to the decision making. Eventual errors in the estimate will not be accumulated.

The derived methods are considered attractive both from the point of view of theoretical consistency and computational effort. The viability of extension of the methods to other multireservoir hydropower systems is a matter of discussion. It is verified by the practice that no problem is general in management. The accomplishments of each method are related to how well it may approach the particularities of each problem. However, it cannot be left without mention that some aspects of the presented methods seem to be of general applicability.

In the disaggregation part of the procedure the chain of extreme reservoir storages was ignored. It was possible due to the short travel time between serially-linked reservoirs in the studied system. When travel time increases the spatial transfer of water may be an important factor in the optimization. It may be argued, however that travel time is increased due to the increase of distance between reservoirs, among other less significant factors. In this case, the natural water contribution between reservoirs may force a decrease in importance of the spatial water transfer in the optimal solution. In the Brazilian hydropower systems this is true except for possibly the São Francisco River basin. In this case, the mean course of the river goes through a semi-arid region where the natural water contributions may be somewhat small compared with the spatial water transfer. In this situation, some criteria must be defined to enforce the chain of extreme reservoir storages.

The disaggregation procedure had significant computational requirements. These requirements tend to increase as the system increases in quantity of reservoirs, particularly serially-linked. In Brazil this may result in cumbersome computational requirements for the hydropower system of the Paraná River and tributaries. In this system more than 20 reservoirs are projected having multiple serial linkages. Utilization of simpler disaggregation criteria may be required in such cases with a possible degradation of the quality of results.

The correction of the relaxed strategy for the operation under uncertainty of the future inflows was derived from the observed regularity of the partition of the system's energy inflow and storage. Occurrence of such a regularity in larger systems is not assured. In these cases the random component of the partition models may increase the significance. If this occurs the state of the system may not be suitable to be represented by aggregated values of storage and inflow. One possible solution is the aggregation of the system on a regional basis. This will increase exponentially the computational requirements of the Chain of Marginal Expectations which are already extensive. One alternative is to optimize the operation of each regional system according to the regional energy demand only. After that, analysis of the marginal values is used to define optimal transfer of energy among regions. Transfer from the region A to the region B is optimal when the marginal benefit of the transferred energy to region B is greater than the marginal expectation of the remaining storage in region A. This results in a kind of Overlapping Decomposition Approach (Haimes, 1977, p. 70) to the operation optimization of the regionally aggregated systems.

The extension of the methodology to evolutive or non-stationary systems has the problem of the test of the convergence of the Chain of Marginal Expectations. One possible approach is based on the observation that generally 4 to 5 iterations are required to attain convergence. If the computations start 4 to 5 years in the future, it may be expected the convergence will be obtained in the studied year. No modification in the method is required in this multi-annual operation except when a reservoir is introduced in the system, with serial linkage to a downstream reservoir. Then, in the phase of filling the reservoir's dead storage a second decision must be made concerning that part of the reservoir's inflow which must be transferred to the downstream reservoir and that part that must be kept to fill the dead storage. In this case, the optimal decision occurs when the marginal benefit of the water transfer equals the marginal benefit related to the use of water to fill the dead storage. This benefit is derived from the shortening of the transient period that preceeds the start of the reservoir's energy production. The variables identifying the state of the system must include the reservoir's storage. This will obviously increase the computational requirements during this phase. This transient phase has a possible known beginning but the end is unknown. This creates difficulties in the study of the deterministic operation and an indefinition of the partition models of energy storage and inflow during this period.

In conclusion, the presented method may derive reliable solutions for the operation of the studied system under certainty and uncertainty of future inflow conditions without being constrained by burdensome computational requirements. Extension of the methods to larger, multi-regional systems may bring some theoretical and computational difficulties that do not appear in the study. Nevertheless, exploitation of the marginal economic meaning of optimal decisions seems to be a promising approach to handling such difficulties.

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