UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL
ESCOLA DE ADMINISTRAÇÃO
PROGRAMA DE PÓS-GRADUAÇÃO EM ADMINISTRAÇÃO

HENRIQUE ROSSET FERREIRA

Optimization Model for Banking Asset Liability Management

Porto Alegre

2023

HENRIQUE ROSSET FERREIRA

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Dissertação apresentada à Escola de Administração da Universidade Federal do Rio Grande do Sul como requisito parcial para obtenção do grau de mestre em Administração.

Supervisor: Prof. Dr. Tiago Pascoal Filomena

Porto Alegre

2023

CIP - Catalogação na Publicação

Ferreira, Henrique Rosset
Optimization Model for Banking Asset Liability Management / Henrique Rosset Ferreira. -- 2023.
170 f.
Orientador: Tiago Pascoal Filomena.

Dissertação (Mestrado) -- Universidade Federal do Rio Grande do Sul, Escola de Administração, Programa de Pós-Graduação em Administração, Porto Alegre, BR-RS, 2023.

1. Risco de Taxa de Juros. 2. Modelo de Otimização. 3. Gestão de Ativos e Passivos. I. Filomena, Tiago Pascoal, orient. II. Título.

Elaborada pelo Sistema de Geração Automática de Ficha Catalográfica da UFRGS com os dados fornecidos pelo(a) autor(a).

Dissertação de autoria de Henrique Rosset Ferreira, sob o título **"Optimization Model for Banking Asset Liability Management"**, apresentada à Escola de Administração da Universidade Federal do Rio Grande do Sul, para obtenção do título de Mestre em Administração pelo Programa de Pós-graduação em Administração, na área de concentração Pesquisa Operacional, aprovada em 20 de outubro de 2023 pela comissão julgadora constituída pelos doutores:

Prof. Dr. Tiago Pascoal Filomena
Universidade Federal do Rio Grande do Sul
Orientador

Prof. Dr. Eduardo de Oliveira Horta
Universidade Federal do Rio Grande do Sul

Prof. Dr. Leonardo Riegel Sant'Anna
Universidade Federal do Rio Grande do Sul

Prof. Dr. Marcelo Brutti Righi
Universidade Federal do Rio Grande do Sul

Prof. Dr. Pablo Cristini Guedes
Universidade Federal do Rio Grande do Sul

Abstract

This study introduces an optimization model to assist Asset and Liability Management (ALM) departments in mitigating interest rate risk (IRR) within fixed-income portfolios. The model incorporates real-world constraints, including the cost of administering interest rate derivatives and liquidity constraints, aiming to formulate effective hedge strategies across diverse scenarios. In particular, liquidity constraints gain significance, as market limitations can hinder the execution of derivatives hedging strategies, a concern particularly relevant in emerging markets with lower trading volumes. The study provides a comprehensive backtesting of the model under various portfolio lengths, nominal values, and shapes, revealing key insights. The analysis underscores the critical role of $duration and $convexity constraints in determining hedge strategy effectiveness. Liquidity constraints emerge as a pivotal factor influencing the allocation of future contracts and strategy feasibility. The study contributes a nuanced understanding of interest rate risk management, offering practitioners a valuable decision-making framework considering real-world constraints.
Keywords: Interest Rate Risk, Hedge Strategy, Optimization Model, Asset and Liability Management, Liquidity Constraints, Fixed-Income Portfolio Optimization.

Resumo

Este estudo apresenta um modelo de otimização projetado para auxiliar os departamentos de Gestão de Ativos e Passivos (ALM) na mitigação do risco de taxa de juros (IRR) em carteiras de renda fixa. O modelo incorpora restrições práticas, incluindo o custo de administração de derivativos de taxa de juros e restrições de liquidez, com o objetivo de formular estratégias eficazes de proteção em diversos cenários. Em particular, as restrições de liquidez ganham importância, uma vez que limitações de mercado podem dificultar a execução de estratégias de proteção com derivativos, uma preocupação especialmente relevante em mercados emergentes com volumes de negociação mais baixos. O custo de negociação, nesse contexto, engloba as despesas associadas à negociação e monitoramento de múltiplos instrumentos derivativos. Notavelmente, nossa abordagem minimiza o número de contratos futuros necessários para a proteção contra o risco de taxa de juros. O estudo oferece uma análise abrangente do modelo em diversos comprimentos de carteira, valores nominais e formatos, revelando insights importantes. A análise destaca o papel das restrições de $duration e $convexity na determinação da eficácia da estratégia de proteção. Restrições de liquidez surgem como um fator fundamental que influencia a alocação de contratos futuros e a viabilidade da estratégia. O estudo contribui para uma compreensão detalhada da gestão de risco de taxa de juros, oferecendo aos profissionais um valioso framework de tomada de decisão considerando restrições práticas como liquidez e custo de administração das estratégias.
Palavras-chaves: Risco de Taxa de Juros, Estratégia de Hedge, Modelo de Otimização, Gestão de Ativos e Passivos, Restrições de Liquidez, Otimização de Carteira de Renda Fixa.

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1 Introduction

One of the responsibilities of a bank's asset and liability management (ALM) department is ensuring stable income regardless of interest rate fluctuations. The bank's profit exposure to these movements, known as interest rate risk (IRR), is a well-studied subject that is long drawing researchers' attention. From the seminal work by Redington (1952) to modern work such as Gomez *et al.* (2021), researchers are looking to develop a better understanding of IRR, which is inherent to the banking business and is the main risk that banks face in their role as financial intermediaries (FABOZZI; MANN, 2021).

Despite being a topic that researchers have explored and developed solutions for quite some time, the management of IRR, or the lack thereof, remains a significant concern. Recent studies, as cited by Jiang *et al.* (2023), reveal that the use of hedging and other interest rate derivatives was insufficient in significantly mitigating the interest rate exposure of U.S. banks during the 2022 monetary tightening. This insufficiency is evident in the \$2.2 trillion loss in the value of U.S. banks' assets, leading to the failure of major banks, including the Silicon Valley Bank.

The deployment and the development of tools to cope with IRR are crucial to guarantee the healthiness of banking activities. Without these tools, financial institutions are exposed to losses that could threaten a breach of regulatory or market requirements. As a result, these banks may have to reduce their lending activities to meet capital requirements imposed by regulators or market participants and thus forfeit profits (BEUTLER *et al.*, 2020).

Modern finance currently makes extensive use of advanced mathematical techniques. To price assets, estimate risks and build portfolios, researchers and practitioners routinely perform a number of simulations or solve differential equations and estimate statistical models. Optimization algorithms are among the most important tools used in these calculations. Works such as Hagenbjörk (2020) and Capponi and Rubtsov (2022) are only a couple examples of modern research using optimization techniques to solve financial problems.

Optimization is a well-developed field of applied mathematics and is widely employed as a management and decision-support tool in a variety of industries (CORNUÉJOLS; PEÑA; TüTüNCü, 2018). Optimization techniques are used, for instance, to allocate limited resources between multiple alternatives in order to maximize the total benefit obtained from them. This characteristic of finding optimal solutions for constrained problems suits the financial industry, especially when practitioners need to comply with government policies or trade in limited markets (e.g., liquidity issues). Recent works such as Pan and Xiao (2017), Deguest *et al.* (2018) and Vieira *et al.* (2021) are examples of modern research using optimization algorithms to add real-life constraint in financial models.

This work seeks to propose a model to assist bank ALM departments to hedge against interest rate uncertainty. Here, we use optimization concepts to add two real-life constraints to the decision-making process: the cost of monitoring and administrate interest rate derivatives and the availability (liquidity) of these instruments in the market. The liquidity constraint is relevant to the risk management process, since market limitations may prevent the execution of a derivatives hedging strategy. These liquidity constraints are especially relevant in emerging markets that present lower trading volumes (VIEIRA; FILOMENA, 2020). The cost of trading, in this work, is the cost of trading and monitoring multiple derivative instruments. In our approach, we minimize the number of futures contracts needed for interest rate risk hedging.

Our approach of using optimization models to offset the sensitivities of the fixed-income portfolio to yield curve fluctuations is relevant because it allows the possibility to attach real-life constraints to the decision process. Optimization methods are more flexible to model constraints than, for instance, econometric ones.

In that way, we believe that our work brings contributions in the following aspects. First, we provide a framework to build a hedging strategy that pursues the minimum number of different securities to be effective. The relevance lays in the fact that fewer securities are easier to control and report for and, thus, eases the job of demonstrating the hedge efficiency to regulators. Second, our framework accounts for the liquidity of the selected derivatives when building the hedging strategy. The contribution here is that the derivatives chosen by the model have a high probability of being available when the manager is in the market implementing the hedge. Besides making the execution straightforward, it also protects the efficiency of the strategy, given that optimal substitute derivatives may not exist. Third, the framework employs an approach that utilizes Key Rate Duration (KRD) buckets to account for non-parallel movements in the yield curve, showcasing its performance in the context of the Brazilian financial landscape.

The text is organized as follows. In chapter 2, we establish the theoretical framework, presenting the financial concepts underlying interest rate risk management and optimization problems. In chapter 3, we introduce an optimization model for the IRR hedging problem. This chapter also discusses the methodologies used to assess the model's effectiveness and presents a comprehensive back test schema designed to verify the model's performance across different portfolio shapes, lengths, and nominal values. In chapter 4, we analyze the model's performance, including how it constructs a hedge strategy for a single month and the dynamics of interest rate-induced fluctuations in the portfolio and the hedge. This chapter compares the model's performance across various portfolio shapes, lengths, and nominal values, providing an in-depth analysis of its effectiveness, allocation of securities, and the impact of the proposed liquidity constraints in these different scenarios. The chapter also presents a comparison of the proposed model with an established econometric methodology for constructing hedge strategies. Finally, chapter 5 offers a comprehensive summary of the study's findings and its conclusions.

2 Theoretical Framework

This chapter lays fundamental concepts of fixed income instruments to show the challenges a bank's ALM department face to ensure stable income regardless of market movements. The chapter also shows an overview of optimization methods, how they are applied as a decision-support tool to reach the best possible solution between multiple alternatives, and how constraints can be used to create closer-to-reality models.

2.1 Hedging Interest-Rate Risk

To build an optimization model to hedge against interest rate (IR) fluctuations, first we need to understand what IR is and how it can affect the value of a fixed-income portfolio. This section sheds light on how financial institutions are exposed to interest rate movements, or to interest rate risk (IRR). Throughout the section, we use bond concepts to understand how a bank's loan portfolio (fixed income portfolio) is valued and also how this portfolio value is sensitive it is to changes in the interest rates. Later in the section, we show methods used to measure the sensitivity of these portfolios to interest rate fluctuations.

2.1.1 Interest Rate

One of the core principles of finance is the well-known economic notion of the time value of money, which states that a sum of money now is worth more than an equal sum in the future due to its potential earning power. The concept that connects the value of present money to the value of future money is the interest rate (FARAHVASH, 2020).

To better comprehend a bank's activities as financial intermediaries is worthwhile to understand how a Bond works. A Bond is a financial instrument, very similar to a long-term loan, that represents a commitment between borrower (or issuer) and lender (or bondholder). This obligation states that the issuer will pay back to the bondholder the cash amount borrowed (called the *principal*), plus interests on this amount during a given period of time (MARTELLINI; PRIAULET; PRIAULET, 2003).

A second, key concept of bonds is the coupon. The coupon on a bond is the periodic interest payment made to bondholders over the life of the bond. The coupon payment, for instance, can be made in semiannual installments, that is, the bondholder will receive cash flows, or interest, every six months until the expiration of the bond (FARAHVASH, 2020).

The commitment length, or the number of years during which the borrower has promised to meet the conditions of the debt, is called maturity. The maturity of a bond is critical for several reasons. First, maturity indicates the instrument's expected life or the number of periods during which the bond holder can expect to receive the coupon interest and the period when the principal is fully paid. Second, the yield on a bond depends on its maturity (FABOZZI; MANN, 2021).

The price of these bonds is related to the bond's (expected) present value. Based on a given interest rate, the present value (PV) of a future cash flow tells us how much that cash flow is worth today. The PV is one of the most important concepts in finance. In finance, the process of calculating the present value of a future cash flow (future money) is known as discounting, and the multiplier factor used to do so is known as the discount factor (FABOZZI; MANN, 2021).

According to Cornuéjols, Peña and Tütüncü (2018), the present value PV of a zero-coupon bond can be stated by the following equation:

$$PV = \frac{F_t}{[1 + r_t]^t} \tag{1}$$

where F_t is the face value (principal) of the bond (future cash-flow), t is the maturity of the bond and r_t is the yield (interest rate that applies to money invested between now and time t) associated to the bond.

2.1.2 Interest-Rate Risk

Bond price fluctuations can be a function of a variety of risks. Among them are default risk, inflation risk, and call risk. A significant single source of risk is price fluctuations caused by interest rate changes (INGERSOLL; SKELTON; WEIL, 1978).

Interest-rate risk refers to these price fluctuations that result from an adverse change in interest rates. This risk is classified as either level risk or yield-curve risk. A typical fixed-income security's price moves in the opposite direction of interest rate changes: as interest rates rise (fall), the price of a fixed income security falls (rise).

The work of Martellini, Priaulet and Priaulet (2003) states some IRR properties as follows:

- Bond prices move in the opposite direction of interest rates. That is, bond prices move in the opposite direction of market yields. When the level of required yields demanded by investors on new issues changes, the required yields on all existing bonds change as well. The prices of these bonds must change for these yields to change. In the perspective of bank's lending activities, the bank's loan portfolio will fluctuate in the opposite direction of market yields.
- Holding maturity constant, a decrease in rates raises bond prices more than a corresponding increase in rates lowers bond prices. Bond price volatility, of course, can work both for and against investors.
- A related maturity principle is that as time to maturity increases, the percentage price change that occurs as a result of the direct relationship between a bond's maturity and its price volatility decreases.
- Another principle regarding maturity is: the percentage price change that occurs as a result of the direct relationship between a bond's maturity and its price volatility increases at a decreasing rate as time to maturity increases.
- Aside from the maturity effect, the change in the price of a bond as a result of an interest rate change is determined by the bond's coupon rate. This principle can be stated as follows: bond price fluctuations (volatility) and bond coupon rates are inversely related.

2.1.3 Fixed Income Portfolio Value

To better understand how to manage the interest rate risk in lending activities, it is necessary to understand how to calculate the value of the portfolio we aim to hedge against adverse changes in interest rates.

In this study, an ALM desk wants to hedge the value of a fixed-income portfolio composed of deterministic cash flows with a fixed coupon rate. Here, we assume that there aren't any pre-payment options in the loans. That is, all the cash flows are known in advance and the only source of price changes in time are the ones related to interest rates movements.

Cornuéjols, Peña and Tütüncü (2018) show that the present value (PV) of a fixed-income portfolio that delivers a number (m) certain cash flows F_t, at future dates

$t \in \{1, \ldots, m\}$, can be expressed as the sum of future cash flows discounted at the proper zero-coupon rate. Thus, for stream of cash flows (F_1,...,F_m), where F_t occurs at time t, the present value of the portfolio is given by:

$$PV = \sum_{t=1}^{m} \frac{F_t}{[1 + r_t]^t} \tag{2}$$

where r_t is the yield on a risk-free zero-coupon bond with maturity t, that is, the IR for a amount money invested between now and time t.

2.1.4 Dollar Duration

Before developing any hedging method against the interest-rate risk, we need to go beyond the value of the fixed income portfolio and understand the way that those portfolios are affected by changes in the yield-curve.

The sensitivity of a fixed income portfolio to IR fluctuations is a topic of research for more than 80 years now. From the introduction of the concept of bond duration by Macaulay (1938) to more recent work such as Bierwag, Corrado and Kaufman (1990) or Crack and Nawalkha (2000), duration-based metrics are being extensively employed to assess the effect of IR changes on portfolios of bonds.

The term “duration” relates to how one can interpret the metric as it was proposed by Macaulay (1938). The Macaulay duration, or simply duration, may be interpreted as a weighted average maturity for the portfolio. For instance, if one assesses that a bond paying ten annual cash flows have a duration equal to 8 then, in terms of sensitivity to interest-rate changes, the 10-year bond behaves similarly as an 8-year zero-coupon bond.

The Macaulay duration is just one of the notions of duration. Another measure of the price changes of a bond in relation of IR fluctuations is the dollar duration ($duration), also known as sensitivity. The $duration metric also measures of the volatility of a bond portfolio.

For a stream of cash flows (F_1,...,F_m), Cornuéjols, Peña and Tütüncü (2018) show that we can express the dollar duration (DD) as the first order partial derivative of of the present value of the portfolio with respect to its yield to maturity r_t:

$$DD = \frac{\partial PV}{\partial r_t} = \sum_{t=1}^{m} -\frac{tF_t}{[1 + r_t]^{t+1}} \tag{3}$$

where PV is the present value of the portfolio, F_t is a cash flow at future date t and r_t is the yield on a risk-free zero-coupon bond with maturity t.

The $duration, or sensitivity, quantifies the absolute profit (or loss) a bondholder will face for a small change of the yield to maturity of the bond.

The work of Martellini, Priaulet and Priaulet (2003) states some properties of duration measures presented bellow:

- The duration (Macaulay Duration) of a zero-coupon bond is equal to the bond maturity.
- Keeping constant the maturity and the yield-to-maturity (YTM) of a bond, the bond's duration (or $duration) is higher when the coupon rate is lower.
- Holding the YTM and the coupon rate of a bond constant, its duration increases as time to maturity increases.

2.1.5 Dollar Convexity

Following the introduction of the duration concept, researchers introduced the measure known as dollar convexity ($convexity) to improve interest rate risk measuring even further (HAGENBJÖRK, 2020).

At any point along a price/yield curve, the slope represents the change in price for a given change in yield. The price/yield curve of zero-coupon is convex (GRANTIER, 1988). Because the price of a bond as a function of yield is nonlinear, duration hedging only works for small yield changes.

As the yield on a bond changes, so does its duration. Crack and Nawalkha (2000) points that a convexity term captures the sensitivity of bond to a change in the slope of the term structure, improving the IRR approximation in large yield fluctuations.

For a stream of cash flows $(F_1,...,F_m)$, Cornuéjols, Peña and Tütüncü (2018) show that we can express the dollar convexity (DC) as the second order partial derivative of of the present value of the portfolio in respect to its yield to maturity r_t:

$$DC = \frac{\partial^2 PV}{\partial r_t^2} = \sum_{t=1}^{m} \frac{t(t+1)F_t}{[1+r_t]^{t+2}} \tag{4}$$

where PV is the present value of the portfolio, F_t is a cash flow at future date t and r_t is the yield on a risk-free zero-coupon bond with maturity t.

The properties of convexity are generally the same as those of duration. Some properties of the $convexity as shown in Martellini, Priaulet and Priaulet (2003) are:

- Holding the maturity and YTM constant, the lower the coupon rate, the higher the convexity and the lower the $convexity of the bond.
- Holding the coupon rate and YTM constant, the convexity and $convexity of a bond increase with time to maturity.
- When all other factors remain constant, the $convexity of a coupon bond increase when the yield to maturity decreases.

2.1.6 Non-paralel Shocks

Models based only in $duration and $convexity are extremely practical concepts to deploy in IRR hedging ventures. This simplicity exists mainly because of the large assumptions on which they are based.

Through the years, many researchers studied the effectiveness of hedging strategies based on duration and convexity and its limitations. The research led to view of IRR hedging that aims to take market positions that eliminate risk factors in a portfolio. Modern risk measures require not only the portfolio sensitivities to certain risk factors to be measured, but also the dynamics of these risk factors to be evaluated (HAGENBJÖRK, 2020).

This goal has motivated the development in the literature of alternative models that seek to better capture the dynamics of the yield curve so the hedging strategies can be carried out more cleverly (HAGENBJÖRK, 2020). In the seminal work, Fisher and Weil (1971) demonstrate how to construct an immunized bond portfolio by selecting a portfolio duration equal to the investment horizon. The demonstration, on the other hand, is accomplished through a constant parallel shift in the forward rate term structure. These unrealistic assumptions were later challenged by further works such as Ingersoll, Skelton and Weil (1978) which shown that duration measure cannot be used as a risk proxy unless the term structure is flat.

The works of Bierwag (1977) and Khang (1979) also challenge the efficiency achieved by single-factor models when the yield curve fluctuation doesn't move in a parallel fashion. In his study, Khang (1979) adapts the duration measure to a closer-to-reality scenario where the long-term interest rates are more volatile than the short-term ones. The issue with the approach is that the accuracy of their strategy is dependent on the specific behavior of the term structure.

Upon the recognition of additional risk factors, alternative approaches to extend the representation of the term structure were introduced to expand the IRR coverage methods. Among the approaches, researchers explored parametric fittings of the term structure and use them to derive duration vectors. Polynomial functions were used by Chambers, Carleton and McEnally (1988) and Prisman and Shores (1988) to represent the term structure and further construct duration measures. In the works of Willner (1996) and Diebold, Ji and Li (2006) the exponential functions proposed by Nelson and Siegel (1987) were employed to define duration vector models.

In the early nineties, two relevant strategies to deal with non-parallel shocks were introduced: the use of principal component analysis (PCA) by Litterman and Scheinkman (1991) and the use of Key-rate durations by Ho (1992). Litterman and Scheinkman (1991) used PCA to identify systematic risk factors for interest rate term structure. Their work show that three principal components explained up to 95% of all the historical movements of the treasury security portfolio. These three factors were referred to by them as: level, curvature (or convexity) and slope.

Key rate durations (KRD) was introduced by Ho (1992) and is a method to measure the portfolio sensitivity to changes in different key rates for the term structure. Even among criticisms regarding the arbitrarily nature of the KRDs and its necessity of monitoring, key rate durations have proven to be an effective and simple tool for yield curve risk management and hedging. Their popularity is due to the fact that they provide an intuitive way of thinking about curve risk (GOLUB; TILMAN, 1997; HAGENBJÖRK, 2020).

Eighty years after the first studies by (MACAULAY, 1938) to measure and mitigate the impact of yield curve fluctuations on portfolios, IRR Management is still a topic that draws the attention of researchers. For instance, the work of Caldeira, Moura and Santos (2016) studies the use of a duration-constrained mean-variance optimization approach to obtain optimal portfolios of fixed-income securities. Deguest *et al.* (2018) also research bond portfolio optimization with duration constraints and study out-of-sample empirical efficiency of the optimized bond portfolios. Adam *et al.* (2020), in their work, study a modelling framework for mitigation of IRR management in demand deposits.

2.2 Optimization Methods

Optimization methods constitute a well-established and extensively employed tool in the domain of Operations Research (OR). Operations Research involves the application of advanced analytical methods to make decisions grounded in scientific principles. This field relies on mathematical modeling, statistical analysis, and computational algorithms to systematically identify the most effective solutions to mathematically formulated problems. OR is a pivotal discipline spanning various industries, providing a structured approach to problem-solving and decision analysis (TAHA, 2011).

At its essence, OR is centered around optimization, which involves finding the best solution within specified constraints. Leveraging quantitative methods, OR provides a systematic framework for addressing real-world challenges and achieving optimal outcomes across diverse applications. In the context of the Interest Rate Risk Hedge problem, we employ OR tools, specifically optimization methods, to choose a set of securities that maximizes income stability or minimizes portfolio sensitivity, all while adhering to constraints such as cost and securities liquidity.

2.2.1 Optimization Problems and Finance

The interface between operations research (OR) and finance is well-established, and there is clear and current evidence of its academic relevance. The well-known Handbooks in Operations Research and Management Science contain fifteen volumes, two of which are devoted to finance: Volume 9 Jarrow, Maksimovic and Ziemba (1997) and Volume 15 Birge and Linetsky (2007). Jarrow, Maksimovic and Ziemba (1997) discuss in their preface how Harry Markowitz, Merton Miller, and William Sharpe, three Nobel Prize winners in Financial Economics, had very solid basic training in Operations Research and Management Science.

George Dantzig and Harry Markowitz, two icons, respectively, of OR and finance, shows the interface between the two fields evident already in their early work. In the middle of the last century, while Dantzig was already publishing his ideas in financial econometrics journals such as (DANTZIG, 1960), Markowitz was disseminating his research in operations research journals as in Markowitz (1957), Markowitz (1955). In a note published in Operations Research of INFORMS, Markowitz mentions the influence that Dantzig had in his seminal theory on investment portfolios (MARKOWITZ, 2002).

The intersection of these remains relevant. The most prestigious INFORMS journals, Operations Research (OR) and Management Science (MS), each have specific area editors for Financial Engineering and Finance. Recent financial engineering studies published in top OR journals include Banerjee and Feinstein (2022), Capponi and Rubtsov (2022), Edirisinghe, Chen and Jeong (2023).

Andriosopoulos *et al.* (2019) discuss portfolio optimization as one of the main areas of application of OR in finance. The mean-variance problem of Markowitz (1952) is the most fundamental portfolio optimization problem. Kolm, Tütüncü and Fabozzi (2014) provide an overview of investment portfolio developments over the last sixty years. Some of the advances discussed Kolm, Tütüncü and Fabozzi (2014) include the inclusion of transaction costs (LOBO; FAZEL; BOYD, 2007; MUTHURAMAN, 2007; FILOMENA; LEJEUNE, 2012), practical allocation constraints (SCOZZARI *et al.*, 2013; OLIVEIRA *et al.*, 2017; STRUB; TRAUTMANN, 2019), and estimation error (ZHU; FUKUSHIMA, 2009; FILOMENA; LEJEUNE, 2014).

2.2.2 Constraints in Portfolio Selection

The fit between finance and OR in many studies is related to the concept of incorporating real-life constraints in the models, such as the ability to explore scenarios with a shortage of resources or limited margins (e.g., bounds on accepted risk levels) in a thighly regulated industry as the financial industry.

Chang *et al.* (2000) looked into the addition of cardinality constraints to the portfolio selection. The work seeks to improve the diversification by limiting the number of assets in the portfolio and introducing a cap to the maximum percentage allocated in each asset.

The work of Magill and Constantinides (1976) develops constraints to explore the portfolio problem with transactional costs. Lobo, Fazel and Boyd (2007) further explores portfolio optimization using risk exposure and transactional constraints. The work proposes models to maximize the expected return of a portfolio taking into consideration fixed transactional costs, variable transactional costs and diversification constraints to limit the maximum allocation on a single asset.

Probabilistic modeling in portfolio selection is explored by Bonami and Lejeune (2009) where the stochastic nature of expected asset returns is taken into consideration for solving the mean-variance portfolio optimization proposed by Markowitz (1952). In the work, the probabilistic constraint imposes that the expected return of the constructed portfolio must exceed a specified return threshold with some confidence level.

The development of portfolio constraints remains an active area of research. Ma, Siu and Zhu (2019) delves into an optimal portfolio selection problem that incorporates return predictability and transaction costs. Kamma and Pelsser (2022) explores a dual-control method for approximating investment strategies in multidimensional financial markets with convex trading constraints. Leung and Wang (2022) investigates an optimization method for cardinality-constrained portfolio selection, addressing concerns of high-frequency traders to prevent odd lots and minimize transaction fees.

2.2.3 Liquidity Constraints

Liquidity is relevant from risk management perspectives and is a pertinent real-life constraint to acknowledge in portfolio selection problems. These constraints are relevant in portfolios in markets where assets can face low trading volumes, such as in emerging markets. In these markets, as the Brazilian case, the selected portfolio can be difficult to generate or liquidate which can ultimately affect the expected optimal risk-return (VIEIRA; FILOMENA, 2020; VIEIRA *et al.*, 2021).

Liquidity represents the capacity, or how easy it is, to quickly transact an asset in a market with little effect on the asset prices. Liquidity risk is related to the quantity of assets managed. Large market participants, such as financial institutions, will face more challenges to manage its portfolios than small investor (VIEIRA; FILOMENA, 2020; VIEIRA *et al.*, 2021).

Vieira and Filomena (2020) chooses the financial volume traded as a proxy for liquidity of the assets. The choice is based on advantages of the financial volume traded such as being readily available in financial databases. The constraint added resulted in high portfolio liquidation levels when compared to cases without the imposed constraints.

Almeida and Lund (2014) discusses liquidity in their work with econometric models for IRR management in the Brazilian market. One of the positive findings of their work is the possibility to dodge securities price distortions in low liquidity assets, showing that is important to acknowledge liquidity risk in interest rate risk management.

3 Methodology

This chapter discusses the optimization method proposed to hedge fixed-income portfolios as well as the constraints and parameters used. The chapter also displays the datasets and methods for measuring the effectiveness of the hedge employed in the present work. Finally, we discuss a test framework to evaluate the performance of the proposed optimization model against a benchmark of principal components multi-factor models.

3.1 Optimization Model

To ensure stable income from a fixed-income portfolio of bonds that delivers m certain cash flows $F_1, \ldots, F_m$, at future dates t regardless of interest rate fluctuations, or to hedge a portfolio against IRR, it is necessary to build a portfolio whose present value (PV) stays the same throughout the time.

To achieve income stability, we want to add to the portfolio securities, denoted by $x_{i,k}$, that offset the impact to the bonds in the portfolio when facing interest rate fluctuations. That is, we want to select a set of financial instruments that compensates for the portfolio loss of value when the IR goes up. In this work, we want to counterbalance portfolio value fluctuations. Thus, when the interest rates fall and the loan portfolio values go up, we expect these gains to be offset by the value loss of the hedging securities.

Besides being robust to IRR, we also want the portfolio hedge to acknowledge the managerial costs and the liquidity of these securities. To find the group of securities to meet these purposes, we propose the optimization model shown in the following equations:

$$\min_{x,\, z} \quad \sum_{i=1}^{m} z_i \tag{5a}$$

s.t.

$$\sum_{i=1}^{m} DD_i x_{i,k} \leq (1+\delta_D) DD_{P,k}, \forall k \tag{5b}$$

$$\sum_{i=1}^{m} DD_i x_{i,k} \geq (1-\delta_D) DD_{P,k}, \forall k \tag{5c}$$

$$\sum_{i=1}^{m} DC_i x_{i,k} \leq (1+\delta_C) DC_{P,k}\,, \forall k \tag{5d}$$

$$\sum_{i=1}^{m} DC_i x_{i,k} \geq (1-\delta_C) DC_{P,k} \,, \forall k \tag{5e}$$

$$\sum_{k=1}^{K} x_{i,k} \leq L_i \qquad , \forall i \tag{5f}$$

$$\sum_{k=1}^{K} x_{i,k} \leq z_i M \qquad , \forall i \tag{5g}$$

$$x_{i,k} \leq \gamma_{i,k} M \qquad , \forall k, i \tag{5h}$$

$$z_i \in \{0, 1\} \qquad , \forall i \tag{5i}$$

$$x_{i,k} \geq 0 \qquad , \forall i \tag{5j}$$

where subindices $i = 1, ..., S$ refers to the S fixed-income securities available and subindices k refers to a KRD bucket. The decision variable z_i is a binary variable that indicates if a security i was used in the hedging solution. The $x_{i,k}$ variable represents the amount of securities i used in bucket k to hedge the portfolio. DD_i and DC_i, respectively, represents the dollar duration and dollar convexity of a security i. $DD_{P,k}$ and $DC_{P,k}$ refers, respectively, to the bucket k dollar duration and dollar convexity of the portfolio. L_i refers to the available liquidity of a security i. $\gamma_{i,k}$ and M are auxiliary variables used to model the problem. M is a large number, used in Eq. 5g and 5h to avoid the necessity to use integer variables. $\gamma_{i,k}$ is a binary numbers matrix of size $i \times k$ that indicates for the model if a security t can be used to hedge a bucket k. The symbols δ_D and δ_C represent user-supplied tolerance values for duration and convexity, respectively, which are employed to loosen the constraints of equality.

A key component of this method is the designation of the variable z as a binary variable. This binary property allows us to capture the 'switch on' or 'switch off' of a particular security in the hedge strategy. Subsequently, we introduce the second facet of the strategy through constraint 5h. This constraint asserts that the cumulative sum of the securities $x_{i,k}$ purchased must not exceed the product of z_i and a predetermined large value denoted as M (in this work, set to 10^{11}).

By this formulation, if the optimization model intends to allocate a security $x_{i,k}$ within any bucket k of the hedging portfolio, the corresponding z_i must assume a value of 1. Thus, the decision variable z_i operates as a 'switch' that indicates the inclusion of a specific security for hedging purposes, providing a convenient proxy for assessing the count of distinct securities utilized in the portfolio hedging process.

3.1.1 Transactional Costs

A proxy for the transactional costs of trading, in this work, is the cost of trading and monitoring multiple derivative instruments (securities). A design assumption of the model is that it is more expensive and complex to monitor and trade several different securities than monitor and trade only one security. The assumption is relevant for financial institutions that need to periodically measure, report, proof hedging effectiveness and disclose any changes in the fair value (market value) of their portfolio and opposing hedge strategy, known as hedging accounting (PANARETOU; SHACKLETON; TAYLOR, 2013).

In light of this assumption, our approach integrates transactional costs into the objective function, as illustrated in Equation 5a. This objective function is designed to minimize the amount for different securities (z_i) for managing the portfolio's interest rate risk. By doing so, we minimize the costs associated with monitoring, trading, and reporting these activities to the regulatory authorities.

In order to ensure that the decision variable z_i serves as an indicator of a specific security's utilization within the hedging strategy, our optimization model employs an approach involving binary variables and a model constraint.

3.1.2 IRR Constraints

The portfolio exposure to IR fluctuations is acknowledged by the model through a set of dollar duration constraints, Eq. 5b and 5c, and a set of dollar convexity constraints, Eq. 5d and 5e. The constraints follow the strategy shown in Cornuéjols, Peña and Tütüncü (2018) of matching dollar duration and dollar convexity to trade an amount $x_{i,k}$ of securities with the same sensitivity to changes in IR as the loan portfolio.

Although hedging with \$duration and \$convexity constraints offers protection against parallel shocks within the term structure of a one-factor interest risk model, it's imperative to acknowledge it is limitations, as detailed in chapter 2. To comprehensively address risks beyond the scope of parallel shocks, we embrace a strategic approach rooted in key-rate duration (KRD) methods. These methods, pioneered by Ho (1992), provide a robust framework to mitigate the challenges posed by non-parallel shock scenarios.

The KRD approach involves partitioning the term structure into N distinct segments or buckets. These segments are delineated based on the term structure historical response to market-induced fluctuations. The underlying assumption is that cash flows within each bucket exhibit similar variations when subjected to changes in the interest rate curve.

Essentially, this approach treats the interest rate curve as though it experiences only parallel shocks within these specific buckets. In adopting this premise, we leverage the $duration and $convexity constraints to safeguard the portfolio against parallel shocks occurring within these segmented buckets.

Term Structure Partitioning

As previously mentioned, the KRD strategy divides the term structure into K segments. In this study, we've chosen to partition the term structure into nine distinct segments for analysis. The segments are delineated in Table 1 below:

Table 1 – Segmentation of the Term Structure (KRD Buckets)

Bucket	**Start Day**	**End Day**
1st	0	137
2nd	138	228
3rd	229	320
4th	321	457
5th	458	639
6th	640	913
7th	914	1461
8th	1462	2556
9th	2557	(ongoing)

It is important to note that the ninth and final bucket starts on day 2557 and includes all subsequent cash flows beyond that day. To comprehend the functioning of the buckets, let us consider a few scenarios. How cash flows are categorized becomes evident through these examples:

- In the scenario where a portfolio comprises only one cash flow due in 2 years from day 0, the \$duration and the \$convexity of this cash flow will be assessed within a single bucket. In this case, it aligns with the sixth bucket.
- In contrast, a portfolio with cash flows spread over 36 months will be distributed across seven distinct buckets, each determined by its respective due date.
- Similarly, portfolios spanning 60 and 84 months will be partitioned into a maximum of eight buckets, reflecting the due dates of their cash flows.
- For portfolios with a duration of 120 months, the cash flows will be classified among nine buckets.

To guarantee that the model only uses securities within a certain range of the same key rate of the key-rate duration it is trying to hedge, we use an auxiliary matrix $\gamma_{i,k}$. The matrix, of size $i \times k$, contains binary numbers that indicates for the model if a security i can be used to hedge a bucket k. That is, if $\gamma_{i,k} = 1$ then the security i will be in the pool of securities that the model considers to hedge the key rate duration k.

In Appendix A, we delve deeper into the methodology of partitioning the term structure into the KRD buckets outlined in Table 1, providing additional insights and details.

Measurement of \$duration and \$convexity

Within the framework, we aggregate the \$duration (DD) and \$convexity (DC) values for individual cash flows within a specific bucket. These aggregated values, denoted as $DD_{P,k}$ and $DC_{P,k}$, are then input to the model.

The model employs the constraints of dollar duration and dollar convexity, as represented by Equations 5b, 5c, 5d and 5e. It performs a search for a subset of securities (i) within each bucket (k), where the combined \$duration (DD_i) and \$convexity (DC_i) of these securities equate to the corresponding \$duration and \$convexity values of the portfolio within that bucket.

Let us illustrate this through some scenarios to help solidify the concept. Consider a scenario where the portfolio consists of a single cash flow due in 2 years from day 0, placing it within the 6th bucket. The model seeks a security or a combination of securities from the sixth bucket's allowable set.

For a more intricate example, when the portfolio involves multiple cash flows spread across various buckets—let's say 36 cash flows distributed over 36 consecutive months—a two-step process unfolds. Firstly, the $duration and $convexity values for each bucket are computed by summing the corresponding values of individual cash flows within the bucket. These computed values ($DD_{P,k}$ and $DC_{P,k}$) serve as inputs for the model. Secondly, the model explores combinations of securities for each bucket, adhering to the permissible sets for each bucket. It seeks solutions that fulfill the constraints. In this particular instance, the model endeavors to identify seven distinct combinations — one for each of the seven buckets that segment the 36-month period.

3.1.3 Liquidity Constraints

To include liquidity constraints, we use the individual constraint on assets strategy as discussed by Vieira and Filomena (2020). In Eq. 5f, we are defining a bound for the maximum amount allocated in a given security. The cap is related to its available liquidity. We choose the financial volume as the liquidity proxy of the security. One of the reasons for the choice is how intuitive the measure is for practitioners. In that way, is easy to see that we are saying for the model that it can't exceed a maximum liquidation amount L_i for each security i. For the sake of simplicity, L_i is expressed as the number of securities traded instead of the financial volume traded.

Within the scope of this work, when the discussion references the utilization of a 100% cap (Li) in the imposed constraints, it signifies that the optimization model is granted the autonomy to select a quantity of securities equal to the volume traded over the course of 21 business days, effectively encompassing an entire month's worth of trading activity.

For illustration purposes, consider a security that maintained a daily average of 10,000 traded contracts over the past year. Employing a $L_i = 10\%$ as an example, we grant the model the ability to trade approximately 2 days' worth of contracts on average, equating to roughly 20,000 contracts. A reduction to $L_i = 5\%$ would correspondingly permit the model to engage in trades representing about 1 day of contract activity, amounting to approximately 10,000 contracts.

In this work, we subject the constraints to testing across four distinct cap values: **100%**, **10%**, **5%**, and **1%**.

3.2 Datasets

To conduct empirical tests for analyzing and evaluating the optimization model, a fixed-income portfolio designated for hedging and broad trading information on securities is required. In our research, we opted to simulate the fixed-income portfolios and chose to focous on Brazilian financial securities.

Specifically, for this study, the focus is on interest rate futures contracts, which the model utilizes to devise the hedging strategy. This section elucidates the futures contracts under consideration and offers a comprehensive outline of the methodology employed in constructing these fixed-income portfolios.

3.2.1 Fixed-Income Portifolios' Dataset

The primary motivation behind simulating the fixed-income portfolios stems from the flexibility this approach affords. By constructing these portfolios, we gain the ability to define crucial parameters, such as their nominal value, the time horizon of the portfolio (in months) and the portfolio's structure encompassing the distribution of cashflows.

Our study utilized five distinct nominal values for the portfolios, and these values correspond to the cumulative sum of future cashflows F_t that compose the portfolio over the period of m months, denoted as $\sum_{t=1}^{m} F_t$. The nominal values explored in the study were: **100 million** BRL, **300 million** BRL, **500 million** BRL, **1 billion** BRL, and **5 billion** BRL. The goal behind these five different amounts is to explore how the model's decision is affected as the portfolio's size increases.

The study also examined four distinct time horizons for the portfolios. We constructed portfolios spanning **three years** ($m = 36$), **five years** ($m = 60$), **seven years** ($m = 84$), and **ten years** ($m = 120$). These four different time horizons allow us to explore how the model's decision is affected when the duration of the portfolio increases. Additionally, these varying timeframes enable us to examine different sets of Key Rate Durations (KRD) buckets in the analysis.

In terms of the cash flow distribution within the portfolio, this study delved into three distinct structures:

- **Uniform Distribution**: In this scenario, the cash flows maintain a consistent nominal value throughout the entire time horizon of the portfolio.
- **Bell-Shaped Distribution**: This structure involves a distribution where the cash flows are more concentrated in the intermediate phase of the portfolio's time span.
- **Inverted Bell Distribution**: This configuration sees the cash flows being more pronounced at both the beginning and the end of the portfolio's time horizon.

Figure 1, below, provides a visual representation of these three portfolio structures.

The figure illustrates the three different portfolio shapes explored in this study. In our testing efforts, we explored the interplay of these three cash flow shapes with the five nominal values and four time horizons commented on early in this subsection. When we combine these portfolio parameters — five nominal values, four time horizons, and three cash flow shapes — we uncover 60 distinct configurations for each model parameter we wish to analyze.

3.2.2 Securities' Dataset

For the hedge instruments, we collected a sample of data between January 2007 and December 2021 of the one-day Interbank Deposit Futures (DI contract), a standardized interest rate futures contract based on the DI rate which is traded on the exchange market operated by B3, the main financial exchange in Brazil. The contract has a face value of R$100,000 on the maturity date, and the value on the trading date (PU) is equal to the value of R$100,000 discounted by the negotiated rate. As the position is updated daily by the DI Rate through the dynamics of updating the PU by the correction factor, the investor who carries the position to maturity receives daily adjustments that added together will equal the difference between the contracted interest rate and the realized one, on the financial amount of the operation. Daily closing prices and daily financial traded volume of DI contracts were used in our work.

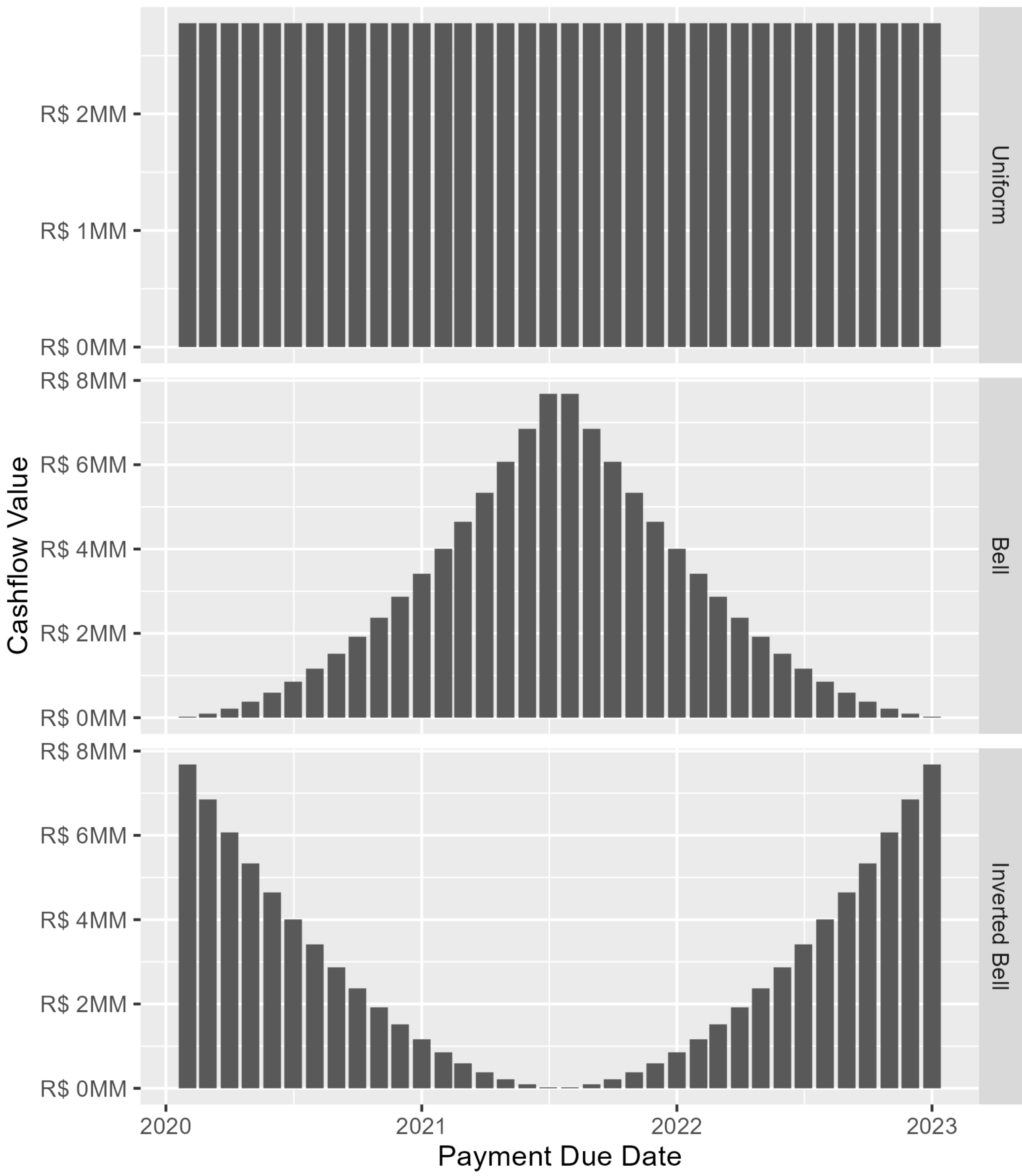

Figure 1 – Comparison of Cash Flow Distribution Patterns for example portfolios of 100MM BRL nominal value over a time horizon of three years ($m = 36$): Uniform, Bell-Shaped, and Inverted Bell Distributions.

3.3 *Hedge Effectiveness*

The banking industry is heavily regulated and practitioners are required to constantly measure and disclosure the effectiveness of the hedge to prove its compliance with risk management policies. In the Brazilian case, the Central Bank of Brazil establishes a few criteria[1] for recording and accounting valuation of derivative financial operations. It states that, to prove the effectiveness of the hedge operation, banks need to indicate that

[1] Circular 3.082 of January 30th, 2002

the variations in the market value or in the cash flow of the hedge instrument offset the variations in the market value or cash flow of the hedged item in a range between 80% (eighty percent) and 125% (one hundred and twenty-five percent).

For the sake of simplicity, we create a few assumptions to evaluate the hedge's effectiveness. First, we assume that the length of the hedge operation is composed of one-month windows, starting at the beginning of the month. Second, we assume that the future cash flows (loan payments) occur at the first trading day of the following month, coinciding with the date of expiration of the derivative contracts.

Thus, to obtain the hedge effectiveness during a period, we sum the cashflows generated/demanded by the derivative contracts (that is, the fluctuation of the Present Value of the derivatives) and compare it with the fluctuations of the Present Value (PV) of the loan portfolio. If the cash flow of the derivatives offsets the fluctuation of the portfolio's PV in a range between 80% and 125%, then the hedge is deemed effective.

Another assumption considered in evaluating hedge effectiveness was to treat cases from the analysis where there wasn't sufficient variation during the month. To illustrate the reasoning behind this assumption, let's delve into an example. Imagine we're hedging a portfolio with a present value of 1 million BRL. At the end of the month, the present value of the portfolio fluctuated by just one cent, while the hedge portfolio fluctuated by four cents. Suppose we apply the hedge effectiveness criteria and compare the fluctuations of the present value of the portfolio to those of the hedge. In that case, we'd find that the hedge operation exhibited a variation of 400%. This result might lead us to label the hedging ineffective, but that would be an overstatement.

To prevent instances of minimal variation from affecting the analysis, we have implemented a criterion that assumes as effective the cases where both the monthly variation of the portfolio and the monthly variation of the hedge are **below 0.5% of their respective present values**. For a analysis of the cut-off's sensitivity, please refer to Appendix B.

As demonstrated in Meirelles and Fernandes (2018), an alternative method for evaluating the effectiveness of the hedge involves employing linear regressions to quantify the extent to which the returns of the hedge portfolio explain the returns of the original portfolio. The equation illustrates this relationship:

$$Ret_t^{Portfolio} = \alpha_0 + \alpha_1 Ret_t^{Hedge} + \epsilon_t \tag{6}$$

Here, α_0 represents the intercept, α_1 is the slope coefficient, and ϵ_t denotes the error term. The coefficient of determination (R^2) resulting from this regression provides a valuable metric for assessing the fit of the hedge. In an ideal scenario, where the hedge perfectly offsets the risk exposure of the original portfolio, R^2 approaches unity.

3.4 Backtesting

A robust analysis of the proposed liquidity-constrained model performance demands several empirical tests to stress-test the model under different term structure shapes. The hedge needs to prove itself reliable to counterbalance the portfolio's value fluctuations under different scenarios of the yield curve.

The dataset comprising 15-year DI futures trading data collected for this study enables stress-testing the model across periods characterized by interest rate tightening and easing cycles. Additionally, the sixty distinct combinations of portfolio nominal amount, period, and cash flow distribution offer insights into how the model's solution varies when applied to different scenarios. These combinations enable us to explore the behavior of the optimization model in various contexts. For instance, we can observe how the model responds when the need arises to hedge portfolios heavily concentrated in distant maturities—a situation that, in the Brazilian context, involves a limited number of contracts with liquidity.

As detailed in the hedge effectiveness section, we have assumed that the hedge operation's duration consists of one-month intervals. Consequently, utilizing the 15-year trading data collected, we subjected our model to testing across 180 months for every combination of model parameters and portfolio characteristics. Within each month, we constructed one of three portfolio shapes, for a specified nominal amount and time range.

Equipped with the portfolio, we employed the optimization model to select a set of i securities that form the hedging strategy. With the constituent derivatives of the hedging strategy determined, we assessed the fluctuations in the present value of both the hedging strategy and the portfolio throughout each trading day of the month. At the end of the month, we aggregated these variations, subsequently computing the changes in the portfolio's Net Present Value (NPV) and the hedge NPV. The illustration in Figure 2 below visually depicts these variations over the month.

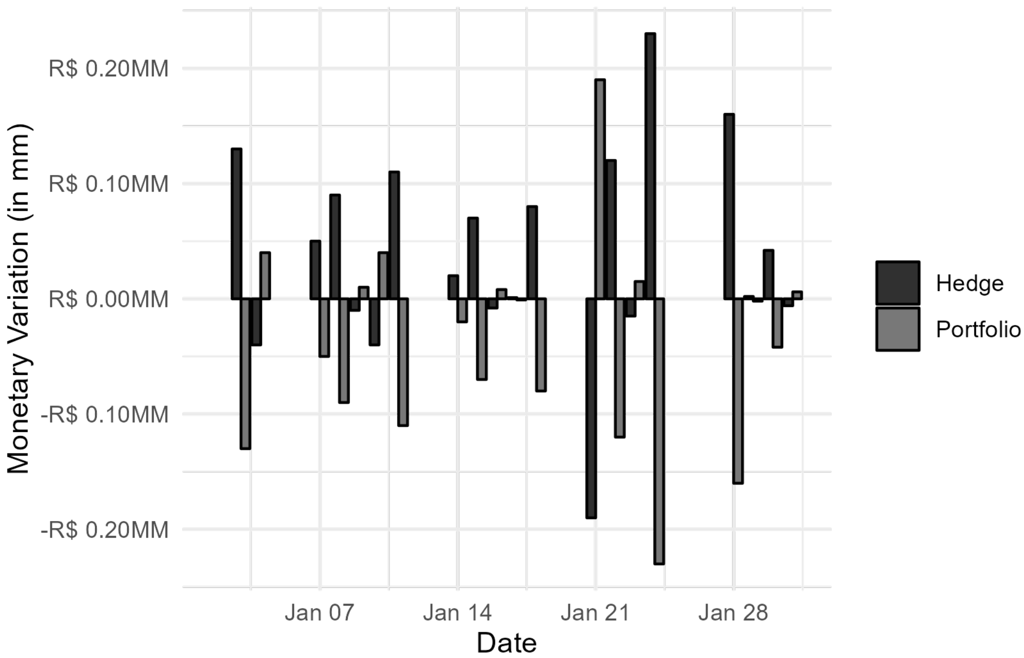

Figure 2 – Portfolio and Hedge Net Present Value (NPV) Fluctuations in a Hypothetical 100% Effective Scenario

The figure illustrates the fluctuations in the portfolio Net Present Value (NPV) and the hedge NPV for each trading date throughout the month. In the example presented, the hedge demonstrates 100% effectiveness, as an equal-magnitude fluctuation in the hedge strategy precisely counteracts each fluctuation in the portfolio.

To conclude this section, it's important to recapitulate that we're conducting 180 effectiveness evaluations for each combination of portfolio and model parameters. With 60 distinct portfolios under examination and four different model parameters being tested for liquidity caps, the scope of our study entails the creation of 43,200 hedge strategies. This comprehensive approach enables us to rigorously assess our model across portfolio conditions, liquidity scenarios, and interest-rate cycles.

3.4.1 Optimization Model Implementation

It's important to highlight that a simplification is applied in the Brazilian context during back-testing evaluation. In our study, we allow $x_{i,k}$ to take real values. However, DI future contracts are traded in unit quantities, meaning fractional quantities of contracts cannot be acquired. This assumption is in line with other works in the literature, as observed in Meirelles and Fernandes (2018), where it is noted that this assumption doesn't significantly impact the results, provided the portfolio is sufficiently large to dilute the rounding impact. This indeed holds true for the portfolios used in this study.

We employed the Gurobi Optimizer solver (Gurobi Optimization, LLC, 2023) to solve the proposed optimization model. The assumptions made to convert the optimization problem into a Mixed-Integer Programming (MIP) formulation help address the computational challenges associated with solving this problem. The optimization instances required selecting between S distinct securities, with the value of S varying between 27 and 39 for the instances evaluated during the back-tested period.

The Gurobi solver allowed us to efficiently solve the optimization instances, without imposing any significant computational time constraints on the study. Handling scenarios, managing the datasets, and interacting with the solver were all carried out through the Python programming language.

3.5 Benchmark

For benchmarking purposes, we turn to the econometric methodology of hedging based on Principal Component Analysis (PCA). This approach extends beyond parallel movements and accounts for slope and curvature within yield curves. Within the PCA framework, the objective is to identify a reduced set of financial securities that effectively captures the primary sources of interest rate risk (MARTELLINI; PRIAULET; PRIAULET, 2003). Litterman and Scheinkman (1991) propose that merely three securities can explain a substantial portion of the variance in interest rate variations.

To compute this benchmarking strategy, we adhere to the methodology outlined by Meirelles and Fernandes (2018) and Kunzler (2019), who applied Principal Component Analysis (PCA) to the Brazilian financial landscape. We systematically evaluated the econometric methodology using a 100 MM BRL 120-month uniform portfolio. We subjected the PCA-based hedge to the same comprehensive backtesting approach to ensure a robust comparison, simulating its performance against 15-year DI futures trading data.

4 Results

In this chapter, we present the outcomes of an extensive and comprehensive testing regime designed to assess the model performance and the effectiveness of interest rate risk management strategies. The study spans a wide array of scenarios, including different portfolio lengths, nominal values, and shapes, each subjected to various liquidity constraints. Our aim was to conduct a meticulous exploration, in order to unravel the nuanced dynamics of interest rate risk mitigation.

Before delving into the specific results, it is crucial to underscore the depth and breadth of our testing framework. We systematically examined a multitude of scenarios, performing exhaustive backtesting across diverse parameterizations. The testing regimen covers portfolios of varying lengths, ranging from 36 to 120 months, each subjected to liquidity caps at different levels. Additionally, we explored different portfolio shapes, including uniform, bell, and inverted bell structures.

Before delving into the detailed results, let us offer an overview of some key insights garnered from the extensive testing:

- The $duration and $convexity constraints emerged as the primary drivers of hedge strategy effectiveness.
- Liquidity constraints played a pivotal role in influencing the allocation of future contracts and the feasibility of formulated strategies.
- Different portfolio shapes introduced unique challenges, yet the core principles of effective risk management remained rooted in navigating $duration and $convexity constraints.
- The $duration and $convexity constraints were calibrated to a level where the model either identified strategies that were effective or came remarkably close to effectiveness, or, alternatively, the model refrained from formulating a strategy altogether.

Now, let us explore the detailed results across these dimensions to gain a comprehensive understanding of how these insights manifest in our extensive testing scenarios. The discussion commences with thoroughly examining how the model succeeds in ensuring stable income from a fixed-income portfolio of bonds that delivers m cash flows F_t, at future dates t. To achieve this, we will delve into the backtesting results of a uniform portfolio with a nominal value of 100 MM and a 36-month time horizon. Subsequently, we will scrutinize the observed outcomes as the nominal value increases incrementally, ranging from 100 million BRL to 5000 million BRL.

Our exploration will then consider results derived from these five distinct portfolio values as we expand the time range from 36 months to 120 months. Once we have thoroughly covered these aspects of the uniform portfolio, the chapter will discuss these findings within the context of bell and inverted-bell portfolios, designed to examine how the model performs when cash flows are concentrated predominantly at extremes and in the middle of the portfolio time length.

The chapter seeks to present the analyses incrementally, following a structured sequence from a single portfolio to portfolios of increasing nominal values and different time lengths and then to portfolios of different shapes. The idea behind the structured sequence is to allow us to show the different dimensions our comprehensive backtesting of 43,200 hedge strategies was designed to uncover.

4.1 Examining the Model's Behavior For a 36-Month, 100 Million BRL Uniform Portfolio

To initiate the analysis, we will begin by presenting a table featuring five key metrics. These metrics will serve as a foundation for our discussion and will be subsequently presented in the text for other tested portfolios. The column labeled **%Feasibles** delineates the percentage, over 180 tested months, at which the model successfully identified a solution—a portfolio that concurrently adhered to the constraints of \$duration, \$convexity, and liquidity. The column labeled **%Effectives** reflects the proportion of test cases wherein an effective strategy emerged. An effective strategy denotes that the fluctuations induced by the interest rate within the portfolio were counterbalanced within the stipulated range by the implemented hedge strategy. Further columns, namely **Avg Eff.**, **Std Eff.**, **Min Eff.**, and **Max Eff.**, present the statistical measurements of effectiveness. **Avg Eff.** and **Std Eff.** corresponds to the average effectiveness and standard deviation observed across the tested months, shedding light on the overall performance trend. **Min Eff.** and **Max Eff.**, on the other hand, respectively signify the lowest and highest recorded levels of effectiveness throughout the testing period.

These metrics, for the uniform Portfolio with a Nominal Value of 100 MM BRL over 36-Month Time Span are presented in Table 2 below.

Table 2 – Feasibility and Effectiveness Summary for the uniform Portfolio with a Nominal Value of 100 MM BRL over 36-Month Time Span

L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
			Avg	**Std**	**Min**	**Max**
100.0 %	100.0 %	99.4 %	101.4 %	4.2 %	79.7 %	117.0 %
10.0 %	100.0 %	99.4 %	101.6 %	4.2 %	79.7 %	117.0 %
5.0 %	100.0 %	99.4 %	101.9 %	4.2 %	79.7 %	117.0 %
2.5 %	100.0 %	99.4 %	101.5 %	4.3 %	79.7 %	122.2 %

The outcomes in Table 2 exhibit similar performance metrics for the four assessed liquidity caps. This similarity aligns with expectations, primarily due to the relatively modest scale of the uniform 100MM BRL portfolio when compared with the financial volume associated with future contracts. Given the similarity in these results, our forthcoming discussion will focus on the outcomes associated with the 100% liquidity cap. It's important to highlight, though, that the interpretation of these findings would be the similar for the other liquidity caps.

The model consistently identified a viable securities strategy for each month under examination, and it achieved a 100% success rate in finding a feasible solution for the optimization problem. Notably, these strategies exhibited a high level of effectiveness, prevailing in 99.4% of the instances and yielding an average effectiveness rating of 101.4% with a standard deviation of 4.2%. The average effectiveness over 100% signifies that the hedge strategies displayed, on average, comparatively lower fluctuations in terms of present value when contrasted with the portfolios. In other words, on average, the portfolios tended to gain or lose more value than what was offset by the hedging strategy.

4.1.1 Effectiveness of the Strategies

Let us delve deeper into the day-to-day dynamics of the portfolio and the hedge's present value. This exploration will focus on the context of monthly fluctuations within the interest rate curve. Figure 3 serves as a visual aid, depicting the varying present values associated with the portfolio and the hedge for October 2018.

The figure illustrates that the present values of both the Hedge and the Portfolio exhibit similar fluctuations but in opposing directions. This observed behavior directly corresponds to our intended objective, serving as a clear indicator that the implementation

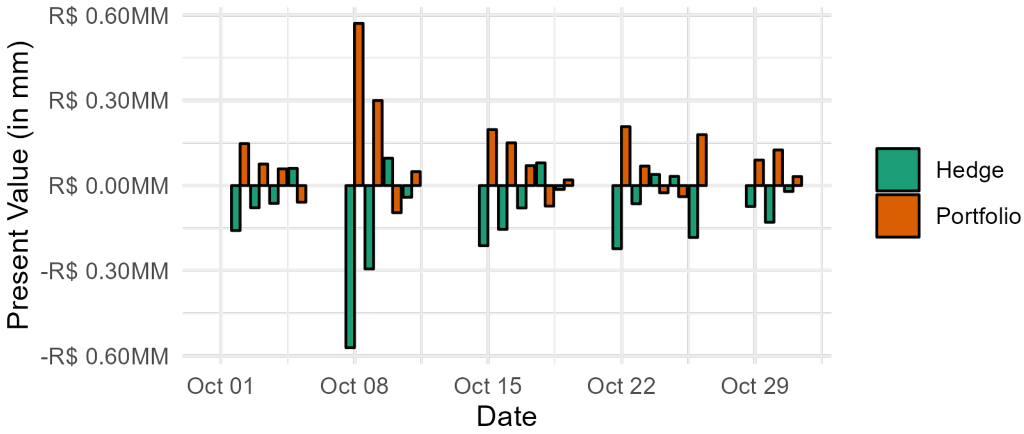

Figure 3 – Day-to-Day Fluctuations in Present Value (PV) for a 36-Month Uniform Portfolio of 100MM BRL and its Hedge ($L_i = 100\%$) during October 2018.

of $duration and $convexity constraints, coupled with the strategic utilization of Key Rate Durations (KRDs), is yielding the intended effect of stabilizing the present value of the portfolio. Figure 4 shows the total present value fluctation along the month.

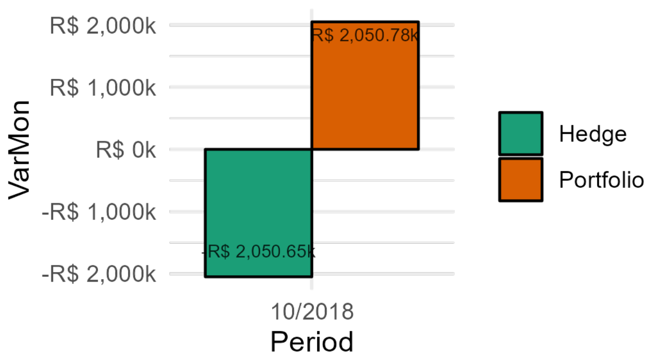

Figure 4 – Cummulative Fluctuation in Present Value (PV) for a 36-Month Uniform Portfolio of 100MM BRL and its Hedge ($L_i = 100\%$) for October 2018.

Figure 4 shows the fluctuations of a scenario deemed effective since the increase of 2.05 million BRL in the portfolio PV, was offset by the cash requirements of the hedging strategy, amounting to 2.05 million BRL. Now, let us examine a scenario where the hedge was considered ineffective. Figure 5 illustrates the day-to-day dynamics of the same portfolio, but this time in conjunction with a strategy constructed for March 2020.

Figure 5 illustrates a scenario where both the Hedge and the Portfolio continue to display fluctuations in opposing directions. However, in this case, there are notable distinctions in the magnitudes of these fluctuations. By the end of the month, these discrepancies accumulate, resulting in a situation where the portfolio's fluctuation fails to

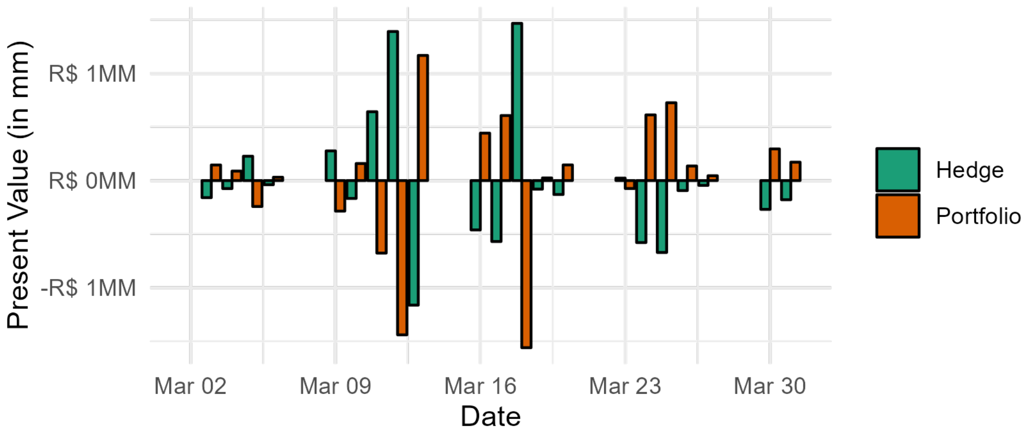

Figure 5 – Day-to-Day Fluctuations in Present Value (PV) for a 36-Month Uniform Portfolio of 100MM BRL and its Hedge ($L_i = 100\%$) during March 2020.

counterbalance within the specified range of 80% to 120%. Figure 6 further elucidates this by presenting the cumulative variations at the end of the month.

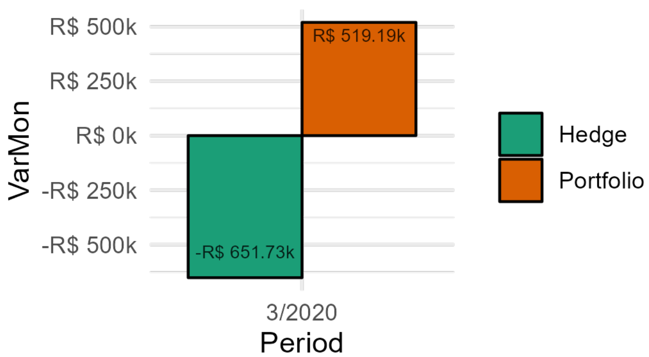

Figure 6 – Cummulative Fluctuation in Present Value (PV) for a 36-Month Uniform Portfolio of 100MM BRL and its Hedge ($L_i = 100\%$) for March 2020.

The scenario depicted in Figure 6 illustrates a situation where the portfolio's present value increased by 519.19 thousand BRL. However, the hedging strategy required 651.73 thousand BRL, resulting in an effectiveness outcome of 79.6%. This effectiveness falls below the established threshold of 80%, categorizing it as ineffective, as it exceededs the bounds specified by the effectiveness rule.

4.1.2 Portfolio Present Value (PV) vs Hedge Strategy PV

An aspect of our model, as described in Equation 5, is that it does not rely on the approach of equalizing the Net Present Value (NPV) of the portfolio and the hedging strategy, commonly known as cash flow matching. This approach enabled the model to formulate hedge strategies such as the one depicted in Figure 7, where the Hedge's present value is lower than that of the portfolio.

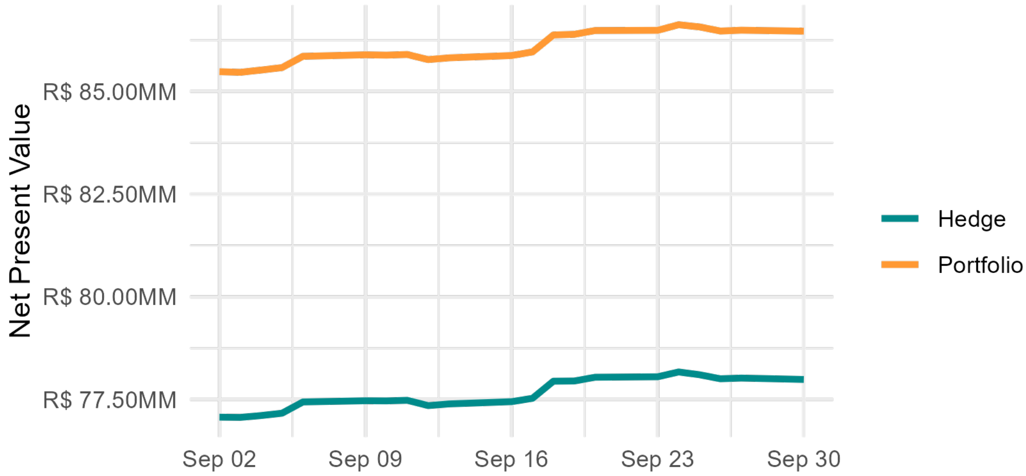

Figure 7 – Present Value (PV) for a 36-Month Uniform Portfolio of 100MM BRL and its Hedge ($L_i = 100\%$) for September 2013.

Figure 7 reveals that the Hedge strategy displays a present value of approximately 7.5 million BRL lower than the portfolio's. This observation is particularly interesting because it allows the hedge agent to immobilize less capital in hedge activities compared to strategies like cash flow matching, where the goal is to match the net present value of the portfolio with the hedge.

Cash flow matching strategies seek hedge formations with the same net present value as the portfolio. Thus, from an economic perspective, it would be advantageous if our model consistently identifies hedge strategies with lower net present values.

To further investigate this phenomenon, let us compare the present values of all the scenarios tested for the 36-month Uniform Portfolio of 100MM BRL. Table 3 below provides a comprehensive set of statistics comparing the Net Present Value (NPV) of the Hedge and that of the portfolio throughout the 180-month backtest.

The comparison reveals that the property of finding hedge strategies with lower present values compared to the portfolios is not a consistent occurrence across all tested scenarios. For example, in the scenario with $L_i = 100\%$, approximately 65% of the cases exhibited greater portfolio NPV than the hedges. In these cases, the hedge required, on average, approximately 4.5% less in cash than the amount in the portfolio. Conversely, in

Table 3 – Comparison of Portfolio and Hedge PV for a 36-Month Uniform Portfolio of 100MM BRL and its Hedge. Both PV_p and PV_h are measured in the first day of the month.

Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
		% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
100.0	100.0 %	65.0 %	4.5 %	3.0 %	35.0 %	2.7 %	1.8 %
100.0	10.0 %	66.1 %	4.4 %	3.0 %	33.9 %	2.5 %	1.9 %
100.0	5.0 %	67.2 %	4.5 %	3.2 %	32.8 %	2.8 %	2.0 %
100.0	2.5 %	60.0 %	4.6 %	3.3 %	40.0 %	2.7 %	1.9 %

the remaining 35% of the cases, the hedge demanded, on average, approximately 2.5% BRL more than the portfolio's NPV. The results for L_i values of 10%, 5%, and 2.5% closely resemble those demonstrated for the 100% liquidity cap scenario.

4.1.3 Allocation of securities within Key Rate Durations (KRD) buckets

So far, the analysis has demonstrated that the \$duration and \$convexity constraints, represented by equations 5b and 5d, within the proposed optimization model achieve their intended purpose. This purpose involves the creation of hedge strategies with a similar sensitivity to interest rate movements. This assertion is supported by the 99.4% effectiveness rate observed in the tested scenarios for the 36-month Uniform Portfolio of 100MM BRL observed in 2.

These \$duration and \$convexity constraints also relate to the KRD segmentation strategy utilized in the optimization model. As outlined in the methodology chapter, the model is constrained to exclusively consider securities within the same KRD segment as the cash flow that needs to be hedged. Consequently, the model must allocate at least one security for each KRD segment to which the portfolio has been segmented.

The objective function of our model, as expressed in Equation 5a, is designed to minimize the number of distinct securities that meet the specified constraints. Consequently, the model strives to identify the smallest set of unique securities required for hedging. In our case, this number of distinct securities will be equal to the number of different KRDs to which the portfolio has been segmented.

Let us examine two hedging strategy for the 36-month Uniform Portfolio of 100MM BRL. This portfolio spans over 1080 days, and referring back to Table 1, we can observe that this portfolio encompasses seven KRD buckets, with the seventh bucket starting on the 914th day of the portfolio. Consequently, our hedge strategy will require at least seven distinct future contracts in its formulation. Table 4 provides an overview of the unique contracts employed in constructing the hedging strategy for the backtest conducted in March 2011.

Table 4 – Overview of Distinct Contracts Allocated for Building a Hedging Strategy for a 36-Month Uniform Portfolio of 100 Million BRL during the Month of March 2011 ($L_i = 100\%$).

Eff	**Contracts**	**KRD**						
		1	**2**	**3**	**4**	**5**	**6**	**7**
100.2 %	7	N11	V11	F12	J12	N12	F13	F14

Table 4 provides an overview of the hedging allocation for March 2011. The column **Eff** illustrates the effectiveness achieved by the strategy, while the column **Contracts** displays the total number of distinct contracts assigned by the model. It is important to note that this quantity is what the objective function aims to minimize. The columns under **KRD** also indicate the code for the purchased DI futures. We can observe a scenario where the model assigned a number of contracts to the hedging strategy that matched the number of KRD segments over which the portfolio spans. Specifically, with only one contract allocated per KRD segment, the model successfully satisfied all the constraints imposed by the optimization model.

To gain a more profound understanding of the liquidity, \$duration, and \$convexity constraints in the scenario depicted for March 2011, let's delve deeper into the model's allocation. Our objective is to illustrate how the assigned contracts align with the liquidity limits and the duration ranges imposed by the constraints. We will commence by closely examining the liquidity constraints, as presented in 5 for the mentioned case in March 2011.

Table 5 – Contract purchased for the hedging strategy of a 36-Month Uniform Portfolio of 100 Million BRL during the Month of March 2011 (Li = 100%)

KRD	**Contract Code**	**Purchased Amount**	**Purchase Cap**	**% Cap**
1	N11	68.8	1557983.3	0.0 %
2	V11	70.5	77933.8	0.09 %
3	F12	73.9	3959114.2	0.0 %
4	J12	104.7	30410.4	0.34 %
5	N12	174.9	828560.1	0.02 %
6	F13	270.3	1764426.2	0.02 %
7	F14	185.5	396883.3	0.05 %

As observed in Table 5, the model is significantly distant from the liquidity constraint in the analyzed case with $L_i = 100\%$. The closest KRD segment to the purchase limit is the fourth KRD, where the contract used has the lowest liquidity value of the chosen set of securities.

We have gained insight into the impact of the liquidity constraint on the model, or the lack thereof. Let us then further investigate the \$duration and \$convexity constraints in the scenario presented for January 2021. Table 6, provides details about these constraints.

Table 6 – \$Duration and \$Convexity Overview for a 36-Month Uniform Portfolio of 100 Million BRL and Its Hedging Strategy during the Month of January 2021.

KRD	$DD_{H,k}$	$DD_{P,k}$	$\Delta_{D,k}$	$DC_{H,k}$	$DC_{P,k}$	$\Delta_{C,k}$
1	1.98	2.03	-0.025	2.36	2.28	0.038
2	3.43	3.52	-0.025	4.85	4.75	0.021
3	4.99	5.12	-0.025	8.17	8.02	0.018
4	8.89	9.12	-0.025	16.49	16.64	-0.009
5	17.71	18.17	-0.025	36.76	39.92	-0.079
6	35.52	36.11	-0.016	89.62	99.57	-0.1
7	33.44	34.3	-0.025	114.1	114.64	-0.005

The columns $DD_{H,k}$ and $DD_{P,k}$ in Table 6 display the values of \$duration for the hedging strategy and the portfolio, respectively, measured in MM BRL. Similarly, the columns $DC_{H,k}$ and $DC_{P,k}$ show the values of \$convexity for the hedging strategy and the portfolio, respectively, measured in MM BRL. The columns $\Delta_{D,k}$ and $\Delta_{C,k}$ indicate the distance between the portfolio and the hedge in terms of \$duration and \$convexity, respectively. This distance is subject to the tolerance δ_D and δ_C, as defined in Equation 5b and 5d. For this work, the duration tolerance assume the value of 0.025 and the convexity tolerance assume the value of 0.1, signifying that the hedge's \$duration can remain within 97.5% to 102.5% of the portfolio's values and \$convexity can remain within 90% to 110% of the portfolio's values.

As observed in Table 6, the model's allocation is very close to the imposed limits for the fourth bucket. However, it's noteworthy that the model was able to identify a single contract that met both the \$duration and \$convexity constraints. This scenario, where all constraints are satisfied with only one contract, is not a universal rule. To illustrate this, Table 7 offers an overview of the contracts employed in the hedging strategy for a 36-Month uniform 100 MM portfolio for August 2008.

Table 7 – Overview of Distinct Contracts Allocated for Building a Hedging Strategy for a 36-Month Uniform Portfolio of 100 Million BRL during the Month of August 2008 ($L_i = 2.5\%$).

Eff	**Contracts**	**KRD**						
		1	**2**	**3**	**4**	**5**	**6**	**7**
104.4 %	8	X08	F09	J09	V09	J10	N10	F12 and N11

As shown in Table 7, the model effectively employed only one contract for the first six KRD segments but needed to allocate two contracts in the seventh KRD to fulfill all constraints. To gain a deeper understanding of why the model could not use just one contract in the seventh KRD, we need to examine the constraints. Table 8 provides an overview of the liquidity constraints in the depicted case.

Table 8 – Contract purchased for the hedging strategy of a 36-Month Uniform Portfolio of 100 Million BRL during the Month of August 2008 ($Li = 2.5\%$)

KRD	Contract Code	Purchased Amount	Purchase Cap	% Cap
1	X08	90.3	4965.1	1.82 %
2	F09	95.9	114131.0	0.08 %
3	J09	90.2	1186.0	7.61 %
4	V09	133.8	1378.9	9.7 %
5	J10	152.8	394.9	38.69 %
6	N10	234.0	2075.4	11.27 %
7	F12	103.5	18994.6	0.54 %
7	N11	67.9	158.5	42.84 %

Table 8 reveals that liquidity is not the factor preventing the model from allocating just one contract type to hedge the seventh bucket. The F12 contract, used by the model in the seventh krd, is highly liquid, and the purchased amounts is below the purchase limit, approximately 43% of the allowed cap. Therefore, it is safe to assume that the model required the utilization of these two contracts to achieve a linear combination of \$duration and \$convexity that would meet the specified tolerances $\delta_D = 0.025$ and $\delta_C = 0.1$, in Equation 5b and 5d.

A facet that is important to comprehend is how frequently the hedging strategy requires more than one contract for a KRD bucket. Table 9, below, presents summarized metrics for the number of distinct contracts employed in the hedging strategies for a 36-month Uniform Portfolio of 100 Million BRL under liquidity upper bounds of 100%, 10%, 5% and 2.5%.

Table 9 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy of a 100 Million BRL uniform Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0 %	1	100.0 %	100.0 %	1
100.0 %	2	100.0 %	100.0 %	1
100.0 %	3	100.0 %	100.0 %	1

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L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	4	100.0 %	100.0 %	1
100.0 %	5	100.0 %	100.0 %	1
100.0 %	6	100.0 %	100.0 %	1
100.0 %	7	100.0 %	100.0 %	1
100.0 %	8	100.0 %	0.0 %	0
100.0 %	9	100.0 %	0.0 %	0
10.0 %	1	100.0 %	100.0 %	1
10.0 %	2	100.0 %	100.0 %	1
10.0 %	3	100.0 %	100.0 %	1
10.0 %	4	100.0 %	100.0 %	1
10.0 %	5	100.0 %	100.0 %	1
10.0 %	6	100.0 %	100.0 %	1
10.0 %	7	100.0 %	100.0 %	1
10.0 %	8	100.0 %	0.0 %	0
10.0 %	9	100.0 %	0.0 %	0
5.0 %	1	100.0 %	100.0 %	1
5.0 %	2	100.0 %	100.0 %	1
5.0 %	3	100.0 %	100.0 %	1
5.0 %	4	100.0 %	100.0 %	1
5.0 %	5	100.0 %	100.0 %	1
5.0 %	6	100.0 %	100.0 %	1
5.0 %	7	100.0 %	100.0 %	1
5.0 %	8	100.0 %	0.0 %	0
5.0 %	9	100.0 %	0.0 %	0
2.5 %	1	100.0 %	100.0 %	1
2.5 %	2	100.0 %	100.0 %	1
2.5 %	3	100.0 %	100.0 %	1
2.5 %	4	100.0 %	100.0 %	1
2.5 %	5	100.0 %	100.0 %	1
2.5 %	6	100.0 %	99.4 %	2
2.5 %	7	100.0 %	99.4 %	2
2.5 %	8	100.0 %	0.0 %	0
2.5 %	9	100.0 %	0.0 %	0

Table 9 demonstrates that finding a single contract for each bucket is not an absolute rule but is often the outcome. In the case of the 36-Month Hedging Strategy the model was unable to narrow down to a single contract for each bucket in only 2 out of the 720 months during back-testing, necessitating the use of 8 instead of 7 contracts to meet the constraints in these cases. Table 9 illustrates that, much like the scenario with $L_i = 100\%$, the model consistently identified a single contract for every month during back-testing, even under liquidity caps of 10% and 5%. The exception only emerged under the $L_i = 2.5\%$ cap, which aligns with our expectations.

4.2 Examining the Model's Behavior For a 36-Month Uniform Portfolio Across Increasing Nominal Values: From 100 Millions to 5000 MM BRL.

Thus far, our analysis has encompassed the assessment of strategy effectiveness, comparisons of portfolio and hedge present values, and an exploration of security allocation within each Key Rate Duration (KRD) for a specific portfolio configuration—the 36-month Uniform Portfolio with a 100 Million BRL nominal value. Consistent with the chapter's philosophy, we now embark on a new dimension of inquiry. This section evaluates how the model's responses evolve when confronted with ascending nominal values, spanning from 100 MM BRL to 5 Billion BRL.

Up until now, we have observed the model's ability to identify feasible solutions for every month during backtesting, even under various liquidity caps. Furthermore, it has consistently achieved highly effective solutions, with a 99.4% effectiveness across the 180 months in the backtest for the four liquidity cap tested. Notably, the model has predominantly relied on a single contract per KRD bucket for most months.

Our current analysis focuses on determining whether this performance holds as nominal portfolio values increase. To do this, let us examine Table 10 below, which provides an aggregated overview of the model's performance as nominal values escalate.

Table 10 – Feasibility and Effectiveness Summary for the uniform Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 36-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	99.4 %	101.4 %	4.2 %	79.7 %	117.0 %
100.0	10.0 %	100.0 %	99.4 %	101.6 %	4.2 %	79.7 %	117.0 %
100.0	5.0 %	100.0 %	99.4 %	101.9 %	4.2 %	79.7 %	117.0 %
100.0	2.5 %	100.0 %	99.4 %	101.5 %	4.3 %	79.7 %	122.2 %
300.0	100.0 %	100.0 %	100.0 %	102.9 %	4.6 %	86.1 %	122.9 %
300.0	10.0 %	100.0 %	100.0 %	103.1 %	4.6 %	94.8 %	123.7 %
300.0	5.0 %	100.0 %	100.0 %	102.2 %	4.4 %	85.6 %	122.2 %
300.0	2.5 %	96.7 %	100.0 %	101.5 %	4.4 %	85.6 %	117.6 %
500.0	100.0 %	100.0 %	100.0 %	103.2 %	4.9 %	85.6 %	122.9 %
500.0	10.0 %	100.0 %	100.0 %	102.8 %	4.8 %	85.6 %	122.2 %
500.0	5.0 %	98.3 %	100.0 %	101.8 %	4.5 %	85.6 %	117.7 %
500.0	2.5 %	92.2 %	100.0 %	101.7 %	5.1 %	85.0 %	122.2 %

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Value	L_i	%Feasibles	%Effectives	Effectiveness			
				Avg	Std	Min	Max
1000.0	100.0 %	100.0 %	100.0 %	102.1 %	4.1 %	82.2 %	117.7 %
1000.0	10.0 %	98.3 %	100.0 %	101.3 %	4.1 %	82.2 %	118.7 %
1000.0	5.0 %	92.2 %	100.0 %	101.4 %	4.9 %	82.7 %	117.6 %
1000.0	2.5 %	86.7 %	99.4 %	101.7 %	5.0 %	80.0 %	118.7 %
5000.0	100.0 %	100.0 %	100.0 %	101.4 %	5.0 %	80.5 %	122.2 %
5000.0	10.0 %	79.4 %	99.3 %	101.3 %	4.9 %	79.9 %	118.6 %
5000.0	5.0 %	60.6 %	100.0 %	101.8 %	5.1 %	81.0 %	118.3 %
5000.0	2.5 %	43.3 %	100.0 %	101.3 %	4.5 %	81.4 %	122.2 %

Table 10 shows that as the nominal value of the portfolios increases, the model struggles to find feasible solutions for each month backtested. Even with the 300 MM BRL portfolio, the model encountered difficulties identifying feasible solutions for 100% of the backtested months. This trend persisted as the portfolio values continued to rise.

This observed behavior aligns with our expectations, since we are imposing more restrictions on the model, particularly in selecting contracts that align with the \$duration and \$convexity constraints. It is worth noting that the model consistently found feasible solutions for every month backtested in scenarios with a liquidity cap (L_i) set at 100%. Therefore, the increase in the portfolio's nominal value - while maintaining the same time horizon of 36 months - primarily affects the model's ability to allocate contracts that match the \$duration and \$convexity constraints to the solution.

When examining the average effectiveness and the standard deviation of the formulated hedge strategies, we observed a striking similarity across various nominal values and liquidity caps. The strategies yielded an average effectiveness ranging from 101.3% to 103.2%. Additionally, the standard deviation for different scenarios remained consistently around 4.5%. These results are highly positive, as they indicate that nearly every hedge strategy constructed was effective. Exceptions occurred in less than 1% of the strategies, primarily within the 100 MM portfolios and the 1000 MM portfolios under a 2.5% liquidity cap, as well as the 5000 MM portfolios under a 10% liquidity cap.

4.2.1 Portfolio PV vs Hedge PV

In the previous section, we highlighted the model's remarkable ability to identify hedge strategies that do not necessarily require matching the portfolio PV with the hedge PV. This economically advantageous property allows the hedge agent to immobilize less capital for portfolio hedging. However, it is paramount to note that this characteristic is only sometimes observed throughout the backtesting process.

Now, let us delve deeper into this phenomenon and explore how it behaves as we increase the nominal value of the portfolios. Table 3 compares the present values of the hedge and portfolio as the nominal values rise.

Table 11 – Comparison of Portfolio and Hedge PV for a 36-Month uniform Portfolio of nominal values of 100MM BRL to 5000MM BRL and its Hedge. Both PV_p and PV_h are measured in the first day of the month.

Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
		% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
100.0	100.0 %	65.0 %	4.5 %	3.0 %	35.0 %	2.7 %	1.8 %
100.0	10.0 %	66.1 %	4.4 %	3.0 %	33.9 %	2.5 %	1.9 %
100.0	5.0 %	67.2 %	4.5 %	3.2 %	32.8 %	2.8 %	2.0 %
100.0	2.5 %	60.0 %	4.6 %	3.3 %	40.0 %	2.7 %	1.9 %
300.0	100.0 %	73.9 %	6.4 %	4.0 %	26.1 %	2.8 %	1.9 %
300.0	10.0 %	73.9 %	6.5 %	4.2 %	26.1 %	2.8 %	1.7 %
300.0	5.0 %	67.2 %	5.8 %	3.6 %	32.8 %	3.4 %	1.9 %
300.0	2.5 %	61.5 %	5.7 %	3.8 %	38.5 %	3.3 %	1.9 %
500.0	100.0 %	76.7 %	6.2 %	3.9 %	23.3 %	3.0 %	2.0 %
500.0	10.0 %	73.9 %	5.8 %	3.8 %	26.1 %	3.2 %	1.8 %
500.0	5.0 %	65.5 %	5.5 %	3.6 %	34.5 %	3.3 %	1.9 %
500.0	2.5 %	65.1 %	5.3 %	3.9 %	34.9 %	3.7 %	2.4 %
1000.0	100.0 %	75.6 %	4.8 %	3.4 %	24.4 %	3.1 %	1.9 %
1000.0	10.0 %	65.5 %	4.5 %	3.3 %	34.5 %	3.4 %	2.1 %
1000.0	5.0 %	61.4 %	4.8 %	3.6 %	38.6 %	3.8 %	2.4 %
1000.0	2.5 %	64.1 %	5.2 %	3.8 %	35.9 %	3.8 %	2.9 %
5000.0	100.0 %	55.6 %	5.1 %	3.0 %	44.4 %	4.5 %	3.2 %
5000.0	10.0 %	60.8 %	5.2 %	4.0 %	39.2 %	5.2 %	3.6 %
5000.0	5.0 %	67.0 %	5.8 %	4.1 %	33.0 %	5.6 %	3.7 %
5000.0	2.5 %	59.0 %	6.4 %	3.8 %	41.0 %	4.7 %	3.7 %

Table 3 demonstrates a consistent pattern as the portfolio value increases. In the majority of cases, the hedge strategy immobilizes approximately 5% less capital than the portfolio. However, in the remaining 30-40% of cases, it tends to immobilize around 3-4% more capital than the portfolio.

We recommend that future research explores the feasibility of exclusively identifying hedge strategies with lower present values (PV) than the corresponding portfolios. Such investigations could provide valuable insights into optimizing capital allocation in risk management strategies.

4.2.2 Allocation of Securities within KRD Buckets

In subsection 4.1.3, we displayed that for the 100 MM BRL 36-Month Uniform Portfolio, the model consistently found feasible solutions using only one contract for almost every month during back-testing. Exceptions to this trend were minimal, accounting for less than 1% of the cases, as shown in 9.

Now, we introduce a new dimension to our analysis by increasing the nominal values of the portfolios. This addition places additional stress on liquidity constraints. As we tighten these restrictions, moving from 100% to 2.5%, we expect the model to face challenges in identifying feasible scenarios with a single contract. Consequently, it may need more distinct contracts to create a combination with similar interest rate sensitivity. If the model cannot discover a linear combination among the available contracts, it fails to construct a viable hedging strategy.

The back-testing covered five portfolio nominal values: 100 MM, 300 MM, 500 MM, 1000 MM, and 5000 MM. While we detailed the 100 MM portfolio in subsection 4.1.3, we will omit the results for the 300 MM, 500 MM, and 1000 MM portfolios here. These portfolios, when scaled, did not significantly strain the liquidity constraints. The complete results are displayed in Table 39 of Appendix D.

Now, let us focus on Table 12, which provides a summary of metrics regarding the number of distinct contracts used in hedging strategies for the 5000 Million BRL 36-Month Uniform Portfolio. We consider different liquidity upper bounds: 100%, 10%, 5%, and 2.5%.

Table 12 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy of a 5000 Million BRL uniform Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	1	100.0 %	100.0 %	1
100.0 %	2	100.0 %	100.0 %	1
100.0 %	3	100.0 %	100.0 %	1

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L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	4	100.0 %	100.0 %	1
100.0 %	5	100.0 %	99.4 %	2
100.0 %	6	100.0 %	99.4 %	2
100.0 %	7	100.0 %	98.3 %	2
100.0 %	8	100.0 %	0.0 %	0
100.0 %	9	100.0 %	0.0 %	0
10.0 %	1	79.4 %	100.0 %	1
10.0 %	2	79.4 %	100.0 %	1
10.0 %	3	79.4 %	100.0 %	1
10.0 %	4	79.4 %	99.3 %	2
10.0 %	5	79.4 %	99.3 %	2
10.0 %	6	79.4 %	92.3 %	3
10.0 %	7	79.4 %	93.7 %	5
10.0 %	8	79.4 %	0.0 %	0
10.0 %	9	79.4 %	0.0 %	0
5.0 %	1	60.6 %	100.0 %	1
5.0 %	2	60.6 %	100.0 %	1
5.0 %	3	60.6 %	100.0 %	1
5.0 %	4	60.6 %	100.0 %	1
5.0 %	5	60.6 %	96.3 %	2
5.0 %	6	60.6 %	94.5 %	2
5.0 %	7	60.6 %	94.5 %	3
5.0 %	8	60.6 %	0.0 %	0
5.0 %	9	60.6 %	0.0 %	0
2.5 %	1	43.3 %	98.7 %	2
2.5 %	2	43.3 %	100.0 %	1
2.5 %	3	43.3 %	100.0 %	1
2.5 %	4	43.3 %	98.7 %	2
2.5 %	5	43.3 %	78.2 %	2
2.5 %	6	43.3 %	97.4 %	2
2.5 %	7	43.3 %	87.2 %	6
2.5 %	8	43.3 %	0.0 %	0
2.5 %	9	43.3 %	0.0 %	0

In Table 12, we can observe that as the liquidity constraints tighten for the 5000 MM BRL portfolio, the model requires a greater number of distinct contracts per bucket. For instance, in the fifth bucket, nearly 20% of the back-tested months demanded using two distinct contracts. In the case of the seventh bucket, one back-tested months even required six distinct contracts to formulate a hedging strategy.

While there are instances where a hedging strategy requires up to six distinct contracts, scenarios demanding more than two contracts are quite rare. In the example of the 36-Month 5000 MM uniform portfolio under $L_i = 2.5\%$, out of the 10 cases that aren't single-contract solutions, the situation requiring six distinct contracts is the only one that needed more than two contracts.

Moreover, in most cases, only one bucket fails to achieve a feasible solution with a single contract. In the scenario we are analyzing, there is just one occurrence where more than one KRD bucket fails to form a single-contract solution, necessitating two contracts for the first and fourth KRD buckets.

4.3 60-Month Uniform Portfolio Across Increasing Nominal Values

This chapter was structured to present results methodically and incrementally, allowing for an in-depth exploration of the various aspects of the comprehensive back-test aimed to study. So far, the chapter have provided a detailed analysis of the model's dynamics and its hedging strategies for a 36-month 100 MM BRL portfolio. Subsequently, it expanded the scope by increasing the nominal portfolio values from 100 MM to 5000 MM BRL to investigate how this dimension affects the model. The results indicated that the model encountered more significant challenges in finding feasible solutions as the nominal value increased and liquidity constraints (L_i) tightened. For a singular hedge cases, it had to increase the number of distinct contracts to six contracts in a single KRD bucket to identify viable combinations of future contracts.

This section will delve into a new dimension by evaluating the model's response to an extended portfolio duration, specifically transitioning from a 36-month to a 60-month portfolio. To explore this aspect, let us examine Table 13 below, which offers a consolidated overview of the model's performance for the 60-month scenario.

Table 13 – Feasibility and Effectiveness Summary for the uniform Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 60-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	100.0 %	101.1 %	3.3 %	89.8 %	113.2 %
100.0	10.0 %	100.0 %	100.0 %	101.4 %	3.3 %	89.8 %	112.7 %
100.0	5.0 %	100.0 %	100.0 %	101.4 %	3.5 %	86.9 %	114.6 %
100.0	2.5 %	100.0 %	100.0 %	101.8 %	3.6 %	88.8 %	116.0 %
300.0	100.0 %	100.0 %	100.0 %	101.2 %	2.9 %	90.5 %	108.8 %

Continued on next page

Value	L_i	%Feasibles	%Effectives	Effectiveness			
				Avg	Std	Min	Max
300.0	10.0 %	100.0 %	100.0 %	101.6 %	3.3 %	88.8 %	115.1 %
300.0	5.0 %	98.9 %	100.0 %	101.6 %	3.5 %	88.8 %	115.1 %
300.0	2.5 %	98.3 %	100.0 %	101.5 %	3.3 %	88.8 %	112.7 %
500.0	100.0 %	100.0 %	100.0 %	101.4 %	3.3 %	90.4 %	113.2 %
500.0	10.0 %	100.0 %	100.0 %	101.9 %	3.5 %	88.8 %	118.5 %
500.0	5.0 %	98.3 %	100.0 %	101.8 %	3.5 %	88.8 %	114.9 %
500.0	2.5 %	91.1 %	100.0 %	101.7 %	4.1 %	83.9 %	117.1 %
1000.0	100.0 %	100.0 %	100.0 %	101.5 %	3.2 %	90.5 %	112.1 %
1000.0	10.0 %	98.3 %	100.0 %	101.7 %	3.4 %	88.8 %	113.5 %
1000.0	5.0 %	91.1 %	100.0 %	101.4 %	3.4 %	83.9 %	111.7 %
1000.0	2.5 %	75.6 %	100.0 %	101.7 %	3.8 %	88.8 %	114.9 %
5000.0	100.0 %	100.0 %	100.0 %	101.5 %	4.1 %	85.0 %	115.1 %
5000.0	10.0 %	68.9 %	100.0 %	101.5 %	4.4 %	80.0 %	115.4 %
5000.0	5.0 %	47.8 %	100.0 %	101.5 %	4.3 %	82.2 %	112.5 %
5000.0	2.5 %	37.2 %	100.0 %	100.8 %	4.7 %	85.8 %	113.4 %

Table 13 illustrates that for the 60-month uniform portfolio, the model exhibited slightly improved performance, with 100% of the formulated strategies achieving effectiveness. The average effectiveness achieved for the 60-month portfolios closely aligns with the values shown in 10, but there is an improvement in the standard deviation. It has reduced from a range of 4-5% to a range of 3-4%.

The model's performance in finding feasible strategies for the 60-month portfolios is comparable to that observed for the 36-month portfolios (Table 10). However, as nominal values increase, the model encounters slightly more difficulty in identifying feasible solutions, a trend exemplified in the cases of the 1000 MM and 5000 MM portfolios when subject to a liquidity constraint of $L_i = 2.5\%$. In the 36-month scenario, the model finds feasible solutions for 86.7% and 43.3% of the backtested months for the 1000 MM and 5000 MM portfolios, respectively. In the 60-month scenario, these percentages drop to 75.6% and 37.2% for the same nominal values.

4.3.1 Portfolio PV vs Hedge PV

The previous chapter demonstrated the model's capability to identify hedge strategies that do not strictly require matching the portfolio PV with the hedge PV. It also indicated that this occurrence can be advantageous, where the Hedge PV is lower than the portfolio's PV, but not always. Now, let's delve into Table 14 to investigate whether this characteristic persists when analyzing the 60-month portfolio.

Table 14 – Comparison of Portfolio and Hedge PV for a 60-Month uniform Portfolio of nominal values of 100MM BRL to 5000MM BRL and its Hedge. Both PV_p and PV_h are measured in the first day of the month.

Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
		% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
100.0	100.0 %	63.3 %	3.7 %	2.5 %	36.7 %	2.5 %	1.9 %
100.0	10.0 %	62.2 %	3.7 %	2.8 %	37.8 %	2.4 %	1.9 %
100.0	5.0 %	64.4 %	3.7 %	2.5 %	35.6 %	2.5 %	2.0 %
100.0	2.5 %	67.2 %	4.2 %	2.7 %	32.8 %	2.2 %	1.7 %
300.0	100.0 %	68.3 %	3.4 %	2.5 %	31.7 %	2.1 %	1.7 %
300.0	10.0 %	72.2 %	3.9 %	2.9 %	27.8 %	1.8 %	1.4 %
300.0	5.0 %	73.0 %	4.0 %	2.9 %	27.0 %	1.9 %	1.3 %
300.0	2.5 %	65.0 %	4.2 %	2.9 %	35.0 %	2.4 %	1.9 %
500.0	100.0 %	70.0 %	4.9 %	3.0 %	30.0 %	2.9 %	2.5 %
500.0	10.0 %	71.7 %	5.4 %	3.2 %	28.3 %	2.4 %	1.9 %
500.0	5.0 %	67.2 %	5.1 %	3.1 %	32.8 %	2.7 %	1.8 %
500.0	2.5 %	64.6 %	5.0 %	3.3 %	35.4 %	3.4 %	2.1 %
1000.0	100.0 %	67.2 %	3.7 %	2.5 %	32.8 %	2.4 %	1.9 %
1000.0	10.0 %	67.2 %	3.9 %	2.6 %	32.8 %	2.5 %	1.6 %
1000.0	5.0 %	62.8 %	3.9 %	2.7 %	37.2 %	2.9 %	1.9 %
1000.0	2.5 %	62.5 %	4.8 %	3.3 %	37.5 %	2.9 %	2.0 %
5000.0	100.0 %	62.8 %	3.8 %	2.5 %	37.2 %	3.7 %	2.5 %
5000.0	10.0 %	57.3 %	5.1 %	3.6 %	42.7 %	4.0 %	3.0 %
5000.0	5.0 %	68.6 %	4.5 %	3.3 %	31.4 %	5.0 %	3.2 %
5000.0	2.5 %	59.7 %	4.3 %	3.2 %	40.3 %	4.7 %	3.3 %

Table 14 illustrates that the characteristics observed for the 36-month portfolios, as shown in 11, also hold true for the 60-month portfolios. In the majority of cases, we do not have perfect cash flow matching. Additionally, the instances where this property is advantageous - $PV_p > PV_h$ - are not predominant enough for us to consider this property an advantage of the model.

4.3.2 Allocation of Securities within KRD Buckets

We explored, in previous chapter, how the increase in the nominal value influenced the allocation whithin the KRD buckets as the liquidity contraints were tighned. In this section we are exploring 60-month portfolios and thus we can analyze how increasing the portfolio length influences in the allocation.

Here, the results for the 100 MM BRL to the 1000 MM BRL portfolios are omitted. The complete results can be found in Table 40 of Appendix D. The results in Table 40 (for the 60-month) exhibit a strong similarity to those in Table 39 (for the 36-month). As the portfolio size increases and liquidity constraints tighten, the percentage of feasible solutions decreases, and the model starts to utilize more contracts within these feasible solutions.

A notable difference is that, for the 60-month portfolios, the eighth bucket begins to receive allocated contracts. Let us delve into the analysis of Table 15, which offers an overview of metrics related to the number of distinct contracts employed in the hedging strategies for the 5000 Million BRL 60-Month Uniform Portfolio.

Table 15 – Summary of Unique Contracts Acquired for the 60-Month Hedging Strategy of a 5000 Million BRL uniform Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	1	100.0 %	100.0 %	1
100.0 %	2	100.0 %	100.0 %	1
100.0 %	3	100.0 %	100.0 %	1
100.0 %	4	100.0 %	100.0 %	1
100.0 %	5	100.0 %	100.0 %	1
100.0 %	6	100.0 %	100.0 %	1
100.0 %	7	100.0 %	100.0 %	1
100.0 %	8	100.0 %	96.7 %	5
100.0 %	9	100.0 %	0.0 %	0
10.0 %	1	68.9 %	100.0 %	1
10.0 %	2	68.9 %	100.0 %	1
10.0 %	3	68.9 %	100.0 %	1
10.0 %	4	68.9 %	100.0 %	1
10.0 %	5	68.9 %	97.6 %	2
10.0 %	6	68.9 %	93.5 %	2
10.0 %	7	68.9 %	98.4 %	2

Continued on next page

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
10.0 %	8	68.9 %	66.1 %	7
10.0 %	9	68.9 %	0.0 %	0
5.0 %	1	47.8 %	100.0 %	1
5.0 %	2	47.8 %	100.0 %	1
5.0 %	3	47.8 %	100.0 %	1
5.0 %	4	47.8 %	97.7 %	2
5.0 %	5	47.8 %	100.0 %	1
5.0 %	6	47.8 %	96.5 %	3
5.0 %	7	47.8 %	94.2 %	4
5.0 %	8	47.8 %	75.6 %	6
5.0 %	9	47.8 %	0.0 %	0
2.5 %	1	37.2 %	100.0 %	1
2.5 %	2	37.2 %	100.0 %	1
2.5 %	3	37.2 %	100.0 %	1
2.5 %	4	37.2 %	97.0 %	3
2.5 %	5	37.2 %	100.0 %	1
2.5 %	6	37.2 %	98.5 %	2
2.5 %	7	37.2 %	77.6 %	3
2.5 %	8	37.2 %	82.1 %	5
2.5 %	9	37.2 %	0.0 %	0

When we compare the results in Table 15 with those in Table 12 for the 36-month portfolios, an interesting trend becomes apparent. As we increase the time horizon for the portfolio from 36 months to 60 months, the model encounters more difficulties in finding feasible solutions. For instance, in the case of $L_i = 10\%$, the 36-month 5000 MM BRL portfolio presented feasible solutions for 79.4% of the cases, whereas for the 60-month portfolio, the model found feasible solutions for 68.9%.

A noteworthy trend evident in Table 15 is that, in some instances, the model utilized an increasing number of contracts to formulate the hedge strategy. This behavior aligns with the extended time horizon of the portfolio since a larger portion of the portfolio now extends into the later buckets, which, in the Brazilian context, have fewer contracts with high liquidity.

For instance, in the $L_i = 10\%$ scenario, the model failed to find single-contract solutions for the eighth bucket in 34% of the feasible cases. Among these cases, there was an isolated occurrence with seven distinct contracts in the eighth bucket, while the majority of the 34% required 3 or 2 distinct contracts.

Another noteworthy example is an instance of the liquidity constraint at $L_i = 10\%$, where the model necessitated the use of 12 distinct contracts. In this scenario, four buckets failed to achieve single-contract solutions. To gain deeper insights into the specifics of this allocation, let us explore the details provided in Table 16.

Table 16 – Contract purchased for the hedging strategy of a 60-Month Uniform Portfolio of 5000 Million BRL during the Month of May 2013 (Li = 10%).

KRD	**Contract Code**	**Purchased Amount**	**Purchase Cap**	**% Cap**
1	N13	4064.9	293516.1	1.38 %
2	V13	2904.8	41896.7	6.93 %
3	F14	2718.5	772003.3	0.35 %
4	J14	4976.0	39833.6	12.49 %
5	F15	3895.6	319273.8	1.22 %
6	J15	7512.1	7512.1	100.0 %
6	V15	1437.7	1437.7	100.0 %
7	F17	13529.2	214155.0	6.32 %
8	F18	6089.1	6089.1	100.0 %
8	F19	1510.4	1510.4	100.0 %
8	F20	1024.5	1024.5	100.0 %
8	N17	839.4	839.4	100.0 %

Table 16 reveals that, in both the sixth and eighth buckets, the model allocated more than one contract and had to utilize the maximum purchase limit for each contract. These additional contracts were necessary to ensure effective adherence to the $duration and $convexity constraints.

Notably, for tighter liquidity constraints, such as $L_i = 5\%$ and $L_i = 2.5\%$, the maximum number of distinct contracts employed decreases to 6 and 5, respectively. This suggests that under these constraints, the model was unable to allocate the required amounts to certain contracts, resulting in infeasible solutions. This is further supported by the fact that these two liquidity constraints exhibit fewer cases of feasible instances.

4.4 84-Month Uniform Portfolio Across Increasing Nominal Values

In the previous sections, we systematically explored various dimensions of the comprehensive back testing methodology. We've delved into the model's behavior concerning different portfolio sizes, liquidity constraints, and portfolio lengths. As we move forward, we'll continue to investigate how changing the length of the portfolio impacts the model's results. In this section, our focus will be on 84-month uniform portfolios. Let's begin our exploration by examining Table 17, which provides a summarized view of the model's performance for the 84-month scenario.

Table 17 – Feasibility and Effectiveness Summary for the uniform Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 84-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	99.4 %	101.2 %	4.2 %	89.3 %	125.8 %
100.0	10.0 %	100.0 %	99.4 %	101.4 %	4.2 %	89.3 %	125.8 %
100.0	5.0 %	100.0 %	99.4 %	101.4 %	4.2 %	89.3 %	126.8 %
100.0	2.5 %	98.3 %	98.9 %	101.3 %	4.7 %	79.3 %	126.8 %
300.0	100.0 %	100.0 %	100.0 %	101.2 %	3.8 %	89.3 %	117.3 %
300.0	10.0 %	99.4 %	100.0 %	101.5 %	3.8 %	89.4 %	117.9 %
300.0	5.0 %	97.2 %	99.4 %	101.1 %	4.3 %	80.0 %	117.9 %
300.0	2.5 %	88.9 %	99.4 %	101.3 %	4.5 %	79.3 %	117.9 %
500.0	100.0 %	100.0 %	100.0 %	101.3 %	4.3 %	87.9 %	123.2 %
500.0	10.0 %	97.8 %	99.4 %	101.4 %	4.8 %	78.1 %	124.1 %
500.0	5.0 %	95.0 %	99.4 %	101.6 %	4.9 %	78.1 %	124.1 %
500.0	2.5 %	73.9 %	99.2 %	101.6 %	5.0 %	78.1 %	124.1 %
1000.0	100.0 %	100.0 %	100.0 %	101.3 %	3.6 %	89.1 %	115.2 %
1000.0	10.0 %	95.0 %	99.4 %	101.3 %	4.7 %	77.9 %	117.5 %
1000.0	5.0 %	73.9 %	99.2 %	101.5 %	4.9 %	77.9 %	117.1 %
1000.0	2.5 %	63.3 %	99.1 %	101.5 %	5.0 %	77.9 %	123.9 %
5000.0	100.0 %	97.8 %	98.9 %	101.2 %	4.7 %	77.3 %	126.2 %
5000.0	10.0 %	62.2 %	99.1 %	101.2 %	4.8 %	77.7 %	117.4 %
5000.0	5.0 %	58.9 %	98.1 %	100.9 %	5.1 %	77.9 %	126.0 %
5000.0	2.5 %	48.3 %	98.9 %	100.5 %	5.1 %	77.7 %	123.1 %

Table 17 demonstrates results similar to those observed for the 60-month portfolios in Table 13. In both cases, almost every feasible solution was deemed effective. However, there is an difference in the 84-month portfolios. For the 100 MM BRL portfolios under the $L_i = 2.5\%$ constraint, the model was unable to formulate feasible solutions for 100% of the instances. Similarly, for the 5000 MM BRL portfolios, the model couldn't find feasible hedge strategies for every month when subjected to the $L_i = 100\%$ constraint.

The table also reveals that for the 84-month scenarios, the average effectiveness and standard deviations are very similar to the 60-month cases, with a slightly higher standard deviation. This variance is reflected in the percentage of cases with effective hedge (%Effectives), which is slightly worse in the 84-month scenarios compared to the 60-month ones.

4.4.1 Portfolio PV vs Hedge PV

The comparison of PVs for the 84-month portfolios follows a pattern similar to what was observed in the 60-month and 36-month portfolios. Therefore, this comparison will be omitted in this section, as well as in the subsequent section covering the 120-month case. The comprehensive results are depicted in Table 36 within Appendix C.

4.4.2 Allocation of Securities within the KRD Buckets

Incorporating 84-month portfolios into our analysis has introduced us to cash flows that require hedging across all nine different KRD buckets. It's worth noting that in the 84-month case, only a small percentage of the portfolio, approximately 2%, falls into the ninth bucket. This condition will be further stressed in the next section when we delve into the 120-month uniform portfolio.

To understand how this new KRD bucket, even with a small percentage of the portfolio value, impacts our model's performance, let's delve into the outcomes associated with the largest nominal value, which amounts to 5000 million Brazilian Reais (MM BRL). For a comprehensive overview of the results, please refer to Table 42 in Appendix D. Let us proceed to Table 18 to evaluate the model's performance in the 84-month 5000 MM BRL scenario.

Table 18 – Summary of Unique Contracts Acquired for the 84-Month Hedging Strategy of a 5000 Million BRL uniform Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	1	97.8 %	100.0 %	1.0
100.0 %	2	97.8 %	100.0 %	1.0
100.0 %	3	97.8 %	100.0 %	1.0
100.0 %	4	97.8 %	100.0 %	1.0
100.0 %	5	97.8 %	100.0 %	1.0
100.0 %	6	97.8 %	100.0 %	1.0
100.0 %	7	97.8 %	100.0 %	1.0
100.0 %	8	97.8 %	83.0 %	3.0
100.0 %	9	97.8 %	94.8 %	2.0
10.0 %	1	62.2 %	100.0 %	1.0

Continued on next page

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
10.0 %	2	62.2 %	100.0 %	1.0
10.0 %	3	62.2 %	100.0 %	1.0
10.0 %	4	62.2 %	98.2 %	2.0
10.0 %	5	62.2 %	100.0 %	1.0
10.0 %	6	62.2 %	99.1 %	2.0
10.0 %	7	62.2 %	100.0 %	1.0
10.0 %	8	62.2 %	77.7 %	5.0
10.0 %	9	62.2 %	100.0 %	1.0
5.0 %	1	58.9 %	100.0 %	1.0
5.0 %	2	58.9 %	100.0 %	1.0
5.0 %	3	58.9 %	100.0 %	1.0
5.0 %	4	58.9 %	99.1 %	3.0
5.0 %	5	58.9 %	99.1 %	2.0
5.0 %	6	58.9 %	96.2 %	2.0
5.0 %	7	58.9 %	95.3 %	2.0
5.0 %	8	58.9 %	74.5 %	3.0
5.0 %	9	58.9 %	100.0 %	1.0
2.5 %	1	48.3 %	100.0 %	1.0
2.5 %	2	48.3 %	100.0 %	1.0
2.5 %	3	48.3 %	100.0 %	1.0
2.5 %	4	48.3 %	97.7 %	3.0
2.5 %	5	48.3 %	100.0 %	1.0
2.5 %	6	48.3 %	96.6 %	2.0
2.5 %	7	48.3 %	85.1 %	6.0
2.5 %	8	48.3 %	71.3 %	5.0
2.5 %	9	48.3 %	97.4 %	2.0

In Table 18, we observe a similar allocation pattern as in the results shown in Table 15. The model encounters more significant challenges finding single-contract solutions for later KRDs than the first KRDs. Notably, the ninth bucket begins to be hedged with almost exclusively single-solution hedges for most backtested cases.

The table also highlights that the model faced increased challenges when dealing with the $L_i = 10\%$ case. However, for $L_i = 5\%$ and $L_i = 2.5\%$, the model found it slightly easier to identify feasible solutions. Specifically, the success rate improved from 47.8% in the 60-month case to 58.9% and from 37.2% in the 60-month case to 48.3%, respectively. One plausible explanation for this phenomenon is that the sixth bucket is less overwhelmed in the 84-month case, and this redistribution may alleviate the liquidity constraints associated with contracts in the sixth bucket.

4.5 120-Month Uniform Portfolio Across Increasing Nominal

Up to this point, we have analyzed uniform-shaped portfolios with increasing nominal values, ranging from 100 MM BRL to 5000 MM BRL. We have also explored the impact of extending the time spans, progressing from 36-month portfolios to 84-month ones. In this section, we will assess the model's performance in the context of 120-month portfolios.

As we extend the time span covered by the portfolio, we add a larger portion of the portfolio in the later buckets, at the same time that we decrease the amount of money allocated to the earlier buckets. In the previous section, with the 84-month portfolio, we began to introduce cash flows into the ninth KRD bucket. With the 120-month portfolios, we are further increasing the percentage of the portfolio allocated to this bucket. For an examination of the model's performance in this new scenario, let us examine Table 19.

Table 19 – Feasibility and Effectiveness Summary for the uniform Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 120-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	100.0 %	101.5 %	4.3 %	86.0 %	119.8 %
100.0	10.0 %	97.8 %	99.4 %	101.4 %	4.1 %	77.5 %	113.6 %
100.0	5.0 %	96.1 %	99.4 %	101.3 %	4.1 %	77.5 %	115.2 %
100.0	2.5 %	92.8 %	99.4 %	101.3 %	4.3 %	75.8 %	113.6 %
300.0	100.0 %	98.9 %	99.4 %	101.1 %	3.9 %	79.3 %	111.9 %
300.0	10.0 %	95.0 %	99.4 %	101.5 %	3.9 %	77.4 %	110.8 %
300.0	5.0 %	91.1 %	100.0 %	101.8 %	3.8 %	86.2 %	122.9 %
300.0	2.5 %	88.9 %	100.0 %	101.6 %	3.7 %	87.9 %	121.5 %
500.0	100.0 %	98.9 %	99.4 %	101.1 %	4.3 %	77.3 %	113.6 %
500.0	10.0 %	91.7 %	100.0 %	101.8 %	3.7 %	86.2 %	113.6 %
500.0	5.0 %	88.9 %	100.0 %	101.4 %	3.4 %	86.9 %	113.7 %
500.0	2.5 %	76.1 %	100.0 %	101.6 %	3.2 %	91.2 %	110.5 %
1000.0	100.0 %	97.8 %	99.4 %	101.4 %	3.9 %	79.4 %	113.1 %
1000.0	10.0 %	88.9 %	100.0 %	101.6 %	3.9 %	86.9 %	122.9 %
1000.0	5.0 %	76.1 %	100.0 %	101.6 %	3.4 %	91.2 %	116.6 %
1000.0	2.5 %	66.1 %	100.0 %	101.6 %	3.8 %	89.0 %	122.1 %
5000.0	100.0 %	91.7 %	100.0 %	101.8 %	3.4 %	86.0 %	113.8 %
5000.0	10.0 %	57.8 %	100.0 %	101.5 %	3.3 %	87.1 %	108.8 %
5000.0	5.0 %	46.7 %	100.0 %	100.5 %	3.1 %	86.8 %	107.7 %

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Value	L_i	%Feasibles	%Effectives	Effectiveness			
				Avg	Std	Min	Max
5000.0	2.5 %	23.3 %	100.0 %	99.7 %	3.7 %	87.0 %	105.3 %

Table 19 presents similar average effectiveness and standard deviation results. Up to this point, we can assume that, for the same portfolio shape (in this case, uniform), the model consistently achieves similar levels of effectiveness regardless of the nominal value of the portfolio and its length. Furthermore, it is evident that liquidity constraints primarily impact the model's ability to find feasible results rather than significantly affecting the effectiveness of the hedge strategies. These findings suggest that the constraints governing the effectiveness of the hedge strategies are predominantly related to the $duration and $convexity constraints. We will revisit and delve deeper into this discussion later in the text.

We can also observe that the model encountered more significant challenges in identifying feasible solutions for the back-tested scenarios. This challenge aligns with expectations, as for the 120-month uniform portfolios, most of the cashflows are concentrated in the eighth and ninth KRD buckets, representing approximately 60% of the portfolio's nominal value. This concentration overwhelms these two buckets, and as the liquidity constraints tighten, the model needs to find feasible solutions more frequently.

4.5.1 Allocation of Securities within the KRD Buckets

Let us delve into Table 20 to gain insights into how the increasing allocation of cash flows to later buckets, particularly the ninth bucket, impacts the distribution of securities used in the formulated hedge strategies. The focus here is on the most extreme case, represented by the 5000 MM BRL portfolio nominal value. For a comprehensive overview of the results, please refer to Table 42 in Appendix D.

Table 20 – Summary of Unique Contracts Acquired for the 120-Month Hedging Strategy of a 5000 Million BRL uniform Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0 %	1	91.7 %	100.0 %	1.0

Continued on next page

L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0 %	2	91.7 %	100.0 %	1.0
100.0 %	3	91.7 %	100.0 %	1.0
100.0 %	4	91.7 %	100.0 %	1.0
100.0 %	5	91.7 %	100.0 %	1.0
100.0 %	6	91.7 %	100.0 %	1.0
100.0 %	7	91.7 %	100.0 %	1.0
100.0 %	8	91.7 %	94.5 %	3.0
100.0 %	9	91.7 %	97.0 %	3.0
10.0 %	1	57.8 %	100.0 %	1.0
10.0 %	2	57.8 %	100.0 %	1.0
10.0 %	3	57.8 %	100.0 %	1.0
10.0 %	4	57.8 %	100.0 %	1.0
10.0 %	5	57.8 %	100.0 %	1.0
10.0 %	6	57.8 %	100.0 %	1.0
10.0 %	7	57.8 %	100.0 %	1.0
10.0 %	8	57.8 %	77.9 %	2.0
10.0 %	9	57.8 %	85.6 %	3.0
5.0 %	1	46.7 %	100.0 %	1.0
5.0 %	2	46.7 %	100.0 %	1.0
5.0 %	3	46.7 %	100.0 %	1.0
5.0 %	4	46.7 %	100.0 %	1.0
5.0 %	5	46.7 %	100.0 %	1.0
5.0 %	6	46.7 %	96.4 %	2.0
5.0 %	7	46.7 %	100.0 %	1.0
5.0 %	8	46.7 %	77.4 %	2.0
5.0 %	9	46.7 %	79.8 %	3.0
2.5 %	1	23.3 %	100.0 %	1.0
2.5 %	2	23.3 %	100.0 %	1.0
2.5 %	3	23.3 %	100.0 %	1.0
2.5 %	4	23.3 %	100.0 %	1.0
2.5 %	5	23.3 %	100.0 %	1.0
2.5 %	6	23.3 %	100.0 %	1.0
2.5 %	7	23.3 %	100.0 %	1.0
2.5 %	8	23.3 %	78.6 %	2.0
2.5 %	9	23.3 %	40.5 %	3.0

Table 20 reveals that the model faced more challenges locating feasible solutions as cashflows moved to the later contracts, particularly in the eighth and ninth buckets. Another noteworthy occurrence to emphasize is the ninth bucket. In the 84-month scenario, approximately 2% of the nominal value ended up in the ninth bucket, while in the 120-month scenario, this percentage increased to around 30% of the nominal value. This shift is apparent in the percentage of single-contract solutions. In the 84-month case, most scenarios yielded single-contract solutions, but in the 120-month scenario, achieving this became more demanding. The challenge imposed by having a more significant percentage of the nominal value in the sixth is particularly evident in the $L_i = 2.5\%$ case, where more than half of the feasible solutions failed to identify single-contract solutions.

4.6 36-Month Bell Portfolio Across Increasing Nominal Values

In our previous sections, we focused on analyzing uniform portfolios, where cashflows remained consistent in nominal value throughout each month of the portfolio. Now, we are shifting our attention to a different category of 'bell-shaped portfolios.' These portfolios introduce a new dimension of the analysis by adding an uneven distribution of cashflows, with a significant concentration occurring in the middle of the portfolio's length.

With this transition, we aim to understand how our model performs when faced with uneven cash flow distributions, as they often mirror real-world financial scenarios. Specifically, we want to explore how our model adapts when a substantial portion of the portfolio's cashflows is concentrated in the middle buckets. To achieve this, we will commence our analysis by delving into a case study spanning 36 months. This case study will provide us with valuable insights into the dynamics of bell-shaped portfolios. Let us introduce Table 21, which offers a consolidated overview of the model's performance for the 36-month bell portfolio case.

Table 21 – Feasibility and Effectiveness Summary for the bell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 36-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	100.0 %	101.6 %	4.1 %	80.6 %	124.9 %
100.0	10.0 %	100.0 %	100.0 %	101.7 %	4.0 %	80.6 %	124.9 %
100.0	5.0 %	100.0 %	100.0 %	101.6 %	3.9 %	91.3 %	124.9 %
100.0	2.5 %	98.9 %	100.0 %	101.0 %	4.6 %	80.6 %	124.9 %
300.0	100.0 %	100.0 %	100.0 %	102.0 %	3.5 %	89.2 %	115.7 %
300.0	10.0 %	100.0 %	100.0 %	101.5 %	3.9 %	84.2 %	116.1 %
300.0	5.0 %	97.8 %	99.4 %	100.7 %	4.8 %	72.4 %	114.9 %
300.0	2.5 %	92.2 %	99.4 %	100.4 %	4.3 %	71.1 %	114.9 %
500.0	100.0 %	100.0 %	100.0 %	101.4 %	4.6 %	82.0 %	116.2 %

Continued on next page

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
500.0	10.0 %	98.3 %	99.4 %	100.5 %	5.1 %	72.4 %	114.9 %
500.0	5.0 %	95.6 %	99.4 %	100.1 %	4.7 %	71.1 %	114.9 %
500.0	2.5 %	90.0 %	100.0 %	100.3 %	4.8 %	80.0 %	114.9 %
1000.0	100.0 %	100.0 %	99.4 %	100.7 %	5.7 %	72.4 %	124.9 %
1000.0	10.0 %	95.6 %	99.4 %	99.7 %	5.3 %	71.1 %	116.6 %
1000.0	5.0 %	90.0 %	100.0 %	100.3 %	5.0 %	80.0 %	124.9 %
1000.0	2.5 %	77.8 %	100.0 %	100.8 %	4.8 %	82.6 %	117.7 %
5000.0	100.0 %	98.3 %	100.0 %	100.1 %	5.0 %	80.6 %	122.8 %
5000.0	10.0 %	75.0 %	100.0 %	100.2 %	4.5 %	82.7 %	116.4 %
5000.0	5.0 %	48.9 %	100.0 %	100.8 %	5.0 %	82.7 %	124.1 %
5000.0	2.5 %	29.4 %	100.0 %	99.8 %	5.1 %	81.3 %	124.9 %

Table 21 presents results comparable to those of the 36-month uniform portfolio, as displayed in Table 10, regarding average effectiveness and standard deviation. The effectiveness of the bell-shaped 36-month portfolio is on par with all uniform portfolio ranges and nominal values that were backtested.

Notably, the model encountered more significant challenges in identifying feasible solutions for bell-shaped portfolios than uniform portfolios. This challenge becomes evident in scenarios like the 5000 MM BRL $L_i = 2.5\%$, where the model found feasible solutions for 43.3% of the backtested months in the uniform portfolio but only 29.4% in the case of bell-shaped portfolios. This difference highlights the model's sensitivity to the distribution of cash flows within a portfolio, which aligns with expectations, as a different distribution may concentrate the nominal value and add stress to a specific bucket. When comparing the uniform and bell-shaped 36-month cases, the fifth KRD bucket is more stressed in the bell case, increasing from approximately 16% of the nominal value to hosting approximately 41% of the portfolio nominal value.

4.6.1 Portfolio PV vs Hedge PV

The comparison of present values (PVs) for the 36-month portfolios and the remaining periods exhibits a pattern similar to that observed in the uniform portfolios. We will omit this comparison in the current section and subsequent sections dedicated to bell-shaped portfolios to avoid redundancy. For comprehensive results, please refer to Table 37 located in Appendix C.

4.6.2 Allocation of Securities within the KRD Buckets

The analysis demonstrates a notable challenge encountered during the transition from uniform to bell-shaped portfolios—the difficulty in finding feasible solutions. This difficulty arises because the middle key rate duration (KRD) buckets now account for a more substantial portion of the portfolio's nominal value, making them more susceptible to liquidity constraints. This subsection will closely examine the model's allocation for the 5000 MM BRL 36-month bell portfolio. Please note that the results for nominal values ranging from 100 MM BRL to 1000 MM BRL will not be discussed here; please refer to Table 43 within Appendix D for a comprehensive overview of those results.

Let us delve into Table 22 to gain insights into how the change in the portfolio shape has influenced the allocation of securities to hedge the portfolio.

Table 22 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy of a 5000 Million BRL bell Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0 %	1	98.3 %	100.0 %	1
100.0 %	2	98.3 %	100.0 %	1
100.0 %	3	98.3 %	100.0 %	1
100.0 %	4	98.3 %	100.0 %	1
100.0 %	5	98.3 %	100.0 %	1
100.0 %	6	98.3 %	98.9 %	2
100.0 %	7	98.3 %	99.4 %	2
100.0 %	8	98.3 %	0.0 %	0
100.0 %	9	98.3 %	0.0 %	0
10.0 %	1	75.0 %	100.0 %	1
10.0 %	2	75.0 %	100.0 %	1
10.0 %	3	75.0 %	100.0 %	1
10.0 %	4	75.0 %	100.0 %	1
10.0 %	5	75.0 %	96.3 %	2
10.0 %	6	75.0 %	93.3 %	3
10.0 %	7	75.0 %	95.6 %	3
10.0 %	8	75.0 %	0.0 %	0
10.0 %	9	75.0 %	0.0 %	0
5.0 %	1	48.9 %	100.0 %	1
5.0 %	2	48.9 %	100.0 %	1
5.0 %	3	48.9 %	100.0 %	1

Continued on next page

L_i	KRD	%Feasibles	% Single Contract	Max Contracts
5.0 %	4	48.9 %	98.9 %	2
5.0 %	5	48.9 %	80.7 %	2
5.0 %	6	48.9 %	90.9 %	3
5.0 %	7	48.9 %	92.0 %	3
5.0 %	8	48.9 %	0.0 %	0
5.0 %	9	48.9 %	0.0 %	0
2.5 %	1	29.4 %	100.0 %	1
2.5 %	2	29.4 %	100.0 %	1
2.5 %	3	29.4 %	100.0 %	1
2.5 %	4	29.4 %	94.3 %	2
2.5 %	5	29.4 %	81.1 %	2
2.5 %	6	29.4 %	86.8 %	2
2.5 %	7	29.4 %	96.2 %	4
2.5 %	8	29.4 %	0.0 %	0
2.5 %	9	29.4 %	0.0 %	0

Table 22 ilustrate that apart from the increased difficulty in finding feasible portfolios when compared to uniform 36-month portfolios, the model also showed increased struggle to find single contract strategies in the middle KRD buckets, specially in the fifth KRD bucket. For instance, when $L_i = 100\%$, the model fails to find solutions for single contracts in the 5th KRD bucket for 100% of the backtested months. This occurrence is absent in the uniform 36-month case, as depicted in Table 12. Besides these minor changes, the allocation statistics are similar to what we saw in the uniform case, with the main difference being the focus on the middle buckets.

4.7 60-Month Bell Portfolio Across Increasing Nominal Values

Similar to our exploration of the uniform portfolio shape, we will now extend the length of the bell-shaped portfolios to observe their behavior. Thus far, the model has successfully created hedge strategies for the bell portfolio with effectiveness levels comparable to those of the uniform portfolio. To examine how the increase in portfolio length influences the model's performance, let us turn our attention to Table 23.

Table 23 – Feasibility and Effectiveness Summary for the bell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 60-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	98.9 %	100.6 %	5.7 %	76.2 %	126.9 %
100.0	10.0 %	100.0 %	98.9 %	100.7 %	5.7 %	76.2 %	126.9 %
100.0	5.0 %	100.0 %	98.9 %	101.0 %	6.1 %	76.2 %	126.6 %
100.0	2.5 %	100.0 %	99.4 %	100.7 %	5.7 %	76.2 %	122.6 %
300.0	100.0 %	100.0 %	98.9 %	100.8 %	5.3 %	79.4 %	126.9 %
300.0	10.0 %	100.0 %	99.4 %	100.9 %	5.8 %	79.4 %	123.1 %
300.0	5.0 %	100.0 %	98.9 %	101.0 %	6.0 %	79.4 %	125.4 %
300.0	2.5 %	98.3 %	100.0 %	100.6 %	6.1 %	83.8 %	124.4 %
500.0	100.0 %	100.0 %	98.9 %	100.6 %	5.0 %	79.4 %	126.9 %
500.0	10.0 %	100.0 %	98.9 %	101.1 %	5.6 %	79.4 %	132.7 %
500.0	5.0 %	98.3 %	98.3 %	100.7 %	5.8 %	79.4 %	132.7 %
500.0	2.5 %	95.0 %	99.4 %	100.6 %	5.7 %	75.4 %	123.1 %
1000.0	100.0 %	100.0 %	98.3 %	100.8 %	5.9 %	78.7 %	135.2 %
1000.0	10.0 %	98.3 %	98.9 %	101.1 %	5.8 %	79.4 %	127.7 %
1000.0	5.0 %	95.0 %	99.4 %	100.7 %	5.6 %	75.4 %	123.1 %
1000.0	2.5 %	85.6 %	99.4 %	101.0 %	6.2 %	73.5 %	121.1 %
5000.0	100.0 %	100.0 %	99.4 %	100.9 %	5.7 %	75.5 %	123.1 %
5000.0	10.0 %	79.4 %	99.3 %	100.8 %	5.7 %	74.3 %	120.0 %
5000.0	5.0 %	59.4 %	100.0 %	100.6 %	5.3 %	86.7 %	120.0 %
5000.0	2.5 %	40.6 %	100.0 %	99.7 %	5.1 %	82.5 %	113.7 %

Table 23 reveals that the bell-shaped 60-month portfolio exhibits average effectiveness values slightly closer to 100% compared to the 60-month uniform portfolio, as shown in Table 13, and other portfolios presented thus far. Another distinguishing factor is the standard deviation, with the 60-month bell-shaped case displaying slightly higher volatility compared to the other cases observed thus far.

Regarding feasibility, the model exhibited a slightly improved performance as the liquidity restrictions tightened. This is an interesting occurrence because, in the 60-month uniform portfolio, the sixth and seventh buckets accounted for approximately 45% of the portfolio's nominal value. However, in the case of the bell-shaped portfolio, these buckets now represent approximately 80% of the portfolio's nominal value. This concentration might be expected to increase the challenge of finding feasible solutions. On the contrary, this concentration also alleviates the restrictions of the other KRD buckets, especially the eighth bucket and the first three buckets.

4.7.1 Allocation of Securities within the KRD Buckets

The allocation for the 60-month bell-shaped portfolio is similar to that shown for the 60-month uniform portfolio. While the change in portfolio shape altered the percentage of feasible solutions encountered by the model for portfolios of the same length but different shapes, the allocation remains relatively consistent. The results indicate that the buckets with a more significant percentage of the portfolio's nominal value are where the model encounters more challenges in finding single-contract solutions. For a comprehensive overview of the complete results, including nominal values ranging from 100 MM BRL to 5000 MM BRL, please refer to Table 44 within Appendix D.

4.8 84-Month and 120-Month Bell Portfolio Across Increasing Nominal Values

Up to this point, the comprehensive backtesting has revealed that the effectiveness achieved by the model remains remarkably consistent across different portfolio lengths, nominal values, and shapes. Thus far, the change in portfolio shape has primarily influenced which buckets pose more significant challenges for the model to find single-portfolio solutions. At the same time, the tightening of liquidity constraints has impacted the number of feasible solutions the model can find.

In this section, we will delve into the results for the 84-month and 120-month portfolios, which exhibit a similar behavior regarding effectiveness statistics compared to the other cases in our backtesting. Let us begin by examining Table 24, which summarizes the feasibility and effectiveness of the 84-month bell-shaped portfolio.

Table 24 – Feasibility and Effectiveness Summary for the bell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 84-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	100.0 %	100.6 %	4.4 %	84.5 %	115.6 %
100.0	10.0 %	100.0 %	100.0 %	100.8 %	4.0 %	88.4 %	115.6 %
100.0	5.0 %	100.0 %	100.0 %	101.0 %	4.2 %	88.4 %	120.7 %
100.0	2.5 %	100.0 %	100.0 %	101.2 %	4.5 %	88.4 %	115.7 %
300.0	100.0 %	100.0 %	100.0 %	100.7 %	4.1 %	87.4 %	116.7 %
300.0	10.0 %	100.0 %	100.0 %	101.1 %	4.4 %	87.4 %	120.7 %
300.0	5.0 %	98.9 %	100.0 %	101.0 %	4.4 %	87.4 %	117.4 %
300.0	2.5 %	97.8 %	100.0 %	101.2 %	4.9 %	83.0 %	119.2 %
500.0	100.0 %	100.0 %	100.0 %	100.7 %	4.0 %	87.1 %	116.6 %
500.0	10.0 %	98.9 %	100.0 %	100.9 %	4.3 %	87.1 %	116.6 %
500.0	5.0 %	97.8 %	100.0 %	100.9 %	4.3 %	84.4 %	122.0 %
500.0	2.5 %	86.7 %	100.0 %	101.3 %	4.3 %	87.2 %	116.8 %
1000.0	100.0 %	100.0 %	100.0 %	100.8 %	3.8 %	87.9 %	116.5 %
1000.0	10.0 %	97.8 %	100.0 %	101.2 %	4.7 %	84.9 %	122.0 %
1000.0	5.0 %	86.7 %	100.0 %	101.4 %	4.4 %	88.3 %	116.8 %
1000.0	2.5 %	73.3 %	100.0 %	100.6 %	4.7 %	88.3 %	116.6 %
5000.0	100.0 %	98.9 %	100.0 %	100.8 %	4.3 %	87.8 %	115.8 %
5000.0	10.0 %	67.2 %	100.0 %	101.0 %	4.7 %	88.3 %	116.6 %
5000.0	5.0 %	53.9 %	99.0 %	101.4 %	5.5 %	88.3 %	134.3 %
5000.0	2.5 %	39.4 %	98.6 %	100.4 %	5.9 %	79.4 %	120.7 %

Table 24 offers insights into the effectiveness averages and standard deviations, presenting results that align with the trends observed in our previous portfolio discussions. Compared to the 84-month uniform portfolio, the bell-shaped portfolio demonstrates a slightly better performance in finding feasible solutions.

As we shift our focus to Table 25, let us examine whether this observed pattern of effectiveness persists in the context of the 120-month portfolio.

Table 25 – Feasibility and Effectiveness Summary for the bell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 120-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	98.9 %	101.1 %	4.9 %	86.1 %	125.7 %
100.0	10.0 %	98.3 %	99.4 %	101.6 %	4.5 %	89.2 %	125.7 %
100.0	5.0 %	96.1 %	99.4 %	101.6 %	4.7 %	86.1 %	125.7 %
100.0	2.5 %	88.9 %	100.0 %	101.2 %	4.4 %	86.1 %	119.0 %
300.0	100.0 %	98.9 %	100.0 %	100.9 %	4.5 %	86.1 %	121.1 %
300.0	10.0 %	92.2 %	100.0 %	101.1 %	4.1 %	86.1 %	114.5 %
300.0	5.0 %	86.1 %	100.0 %	101.3 %	4.1 %	88.6 %	114.5 %
300.0	2.5 %	72.8 %	100.0 %	101.5 %	4.5 %	88.3 %	119.0 %
500.0	100.0 %	98.9 %	99.4 %	101.4 %	4.6 %	86.1 %	125.7 %
500.0	10.0 %	87.2 %	100.0 %	101.2 %	4.5 %	86.1 %	119.3 %
500.0	5.0 %	78.9 %	100.0 %	101.5 %	4.8 %	88.3 %	119.0 %
500.0	2.5 %	65.6 %	100.0 %	101.8 %	4.4 %	89.7 %	114.5 %
1000.0	100.0 %	98.3 %	100.0 %	101.5 %	4.4 %	89.2 %	121.3 %
1000.0	10.0 %	78.9 %	100.0 %	101.3 %	4.7 %	88.3 %	116.5 %
1000.0	5.0 %	65.6 %	100.0 %	101.8 %	4.6 %	91.2 %	119.0 %
1000.0	2.5 %	60.6 %	100.0 %	101.8 %	4.6 %	89.9 %	119.1 %
5000.0	100.0 %	87.2 %	100.0 %	101.0 %	4.7 %	86.0 %	119.3 %
5000.0	10.0 %	59.4 %	100.0 %	101.6 %	4.4 %	89.9 %	118.9 %
5000.0	5.0 %	57.2 %	100.0 %	101.0 %	4.1 %	87.4 %	114.5 %
5000.0	2.5 %	45.6 %	100.0 %	100.5 %	4.8 %	85.1 %	114.3 %

As anticipated, Table 25 reveals effectiveness statistics that closely mirror those observed in our previous backtested cases. This consistent pattern across diverse scenarios further strengthens the hypothesis that the primary determinants of hedge strategy effectiveness are the $duration and $convexity constraints.

These results reaffirm that factors such as portfolio length, nominal values, and shape present challenges to finding the minimum set of distinct contracts to build the hedging strategy rather than impose difficulties in making effective strategies.

4.8.1 Allocation of Securities within the KRD Buckets

The behavior of contracts allocated in the hedge strategies for the 84-month and 120-month bell-shaped portfolios closely resembles that observed in the uniform portfolio.

For a comprehensive overview of the complete results, including nominal values ranging from 100 MM BRL to 5000 MM BRL, please refer to Table 45 for the 84-month backtest and Table 46 for the 120-month backtest within Appendix D.

4.9 36-Month Inverted Bell Portfolio Across Increasing Nominal Values

So far, this chapter has presented the results of a comprehensive backtesting effort designed to stress-test the optimization model proposed under various scenarios, including different nominal values, liquidity caps, portfolio lengths, and shapes. Remarkably, the effectiveness statistics achieved by the hedge strategies under these diverse scenarios have demonstrated remarkable similarities. The primary distinction among the scenarios has been the number of distinct contracts required to construct the hedge strategies, a quantity that the model aims to minimize.

Now, we introduce the inverted bell portfolio shape, which exhibits more significant concentrations of nominal value at the extremes of the portfolio. Our initial focus is on the 36-month inverted bell portfolio, following the structured approach outlined in this chapter.

Among all the backtested scenarios, the 36-month inverted bell portfolio places the most stress on the first and second KRD buckets, with the first bucket hosting approximately 25% of the nominal value and the second bucket accommodating approximately 12%—the highest nominal values allocated to these buckets in our tests. To explore these findings in detail, let us focus on Table 26, which comprehensively summarizes the feasibility and effectiveness of the 36-month inverted bell case.

Table 26 – Feasibility and Effectiveness Summary for the invertedbell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 36-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	97.8 %	104.7 %	8.4 %	86.5 %	134.0 %
100.0	10.0 %	100.0 %	96.7 %	104.7 %	8.4 %	86.9 %	141.5 %
100.0	5.0 %	100.0 %	96.1 %	104.5 %	8.5 %	86.9 %	143.9 %
100.0	2.5 %	100.0 %	96.1 %	104.7 %	9.2 %	86.9 %	146.5 %
300.0	100.0 %	100.0 %	97.2 %	104.5 %	8.3 %	86.5 %	134.0 %

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Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
300.0	10.0 %	100.0 %	96.7 %	104.6 %	9.0 %	86.9 %	149.1 %
300.0	5.0 %	100.0 %	95.6 %	104.7 %	9.4 %	86.5 %	147.8 %
300.0	2.5 %	100.0 %	96.1 %	104.3 %	9.5 %	83.7 %	146.7 %
500.0	100.0 %	100.0 %	97.2 %	104.7 %	8.8 %	82.9 %	141.6 %
500.0	10.0 %	100.0 %	95.0 %	105.2 %	9.7 %	85.2 %	149.1 %
500.0	5.0 %	100.0 %	96.1 %	104.7 %	9.4 %	83.7 %	147.8 %
500.0	2.5 %	97.8 %	97.2 %	104.2 %	9.0 %	83.7 %	148.1 %
1000.0	100.0 %	100.0 %	95.6 %	104.9 %	8.8 %	86.9 %	141.1 %
1000.0	10.0 %	100.0 %	96.1 %	104.4 %	9.4 %	83.7 %	147.3 %
1000.0	5.0 %	97.8 %	97.2 %	104.1 %	9.0 %	83.7 %	147.6 %
1000.0	2.5 %	93.3 %	97.6 %	103.8 %	8.8 %	83.7 %	134.5 %
5000.0	100.0 %	100.0 %	95.0 %	104.6 %	9.5 %	86.5 %	149.1 %
5000.0	10.0 %	90.0 %	96.3 %	104.0 %	9.3 %	83.0 %	149.1 %
5000.0	5.0 %	68.3 %	95.9 %	104.1 %	9.0 %	75.2 %	134.5 %
5000.0	2.5 %	50.6 %	98.9 %	104.0 %	7.7 %	83.8 %	134.6 %

Table 26 depicts average and standard deviation values for the effectiveness different from what we have seen. The average effectiveness is around 104.5%, and the standard deviation is much higher than the 3-4% shown so far, with values of 9%. This scenario of more volatility is related to the first two buckets being with a more significant percentage of the portfolio, and the following sections will show that the values decrease closer to the one showed for the bell and uniform portfolios as the lengths increase and the first two portfolios are with less portions of the nominal value.

Regarding feasibility, the 36-month inverted bell portfolio presents more challenges in forming hedge strategies than the uniform portfolios, as depicted in Table 10. For the uniform case, the model found feasible solutions for 100% of the $L_i = 2.5\%$ cap, while it achieved just 50.6% for the inverted bell. The main difference between these portfolios is that the inverted bell's first and seventh KRD buckets concentrate a significant portion of the nominal value. In this case, the model faces challenges with the seventh KRD bucket, which holds 34% of the portfolio's nominal value. The first KRD bucket, with approximately 26% of the portfolio, does not pose a liquidity problem because the contracts in the first KRD are the most liquid in the Brazilian market.

Additionally, compared to the bell-shaped case, the 36-month inverted bell shape gave the model an easier time finding feasible solutions. In the bell-shaped case, the model had approximately 41% of the portfolio's nominal value in the fourth bucket, which proved more restrictive than when this percentage was distributed across the KRDs closer to the extremes.

4.9.1 Portfolio PV vs Hedge PV

The comparison of present values (PVs) for the 36-month inverted bell portfolios and the remaining periods exhibits a pattern similar to that observed in the uniform portfolios. We will omit this comparison in the current section and subsequent sections dedicated to inverted bell-shaped portfolios to avoid redundancy as we already did in the bell-shaped sections. For comprehensive results, please refer to Table 38 located in Appendix C.

4.9.2 Allocation of Securities within the KRD Buckets

The 36-month inverted bell portfolio shape has introduced a scenario where the portfolio nominal value is more concentrated in the first and seventh buckets. Let us examine Table 27 to see how this concentration affects the allocation.

Table 27 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy of a 5000 Million BRL invertedbell Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	1	100.0 %	100.0 %	1
100.0 %	2	100.0 %	100.0 %	1
100.0 %	3	100.0 %	100.0 %	1
100.0 %	4	100.0 %	100.0 %	1
100.0 %	5	100.0 %	100.0 %	1
100.0 %	6	100.0 %	100.0 %	1
100.0 %	7	100.0 %	93.9 %	2
100.0 %	8	100.0 %	0.0 %	0
100.0 %	9	100.0 %	0.0 %	0
10.0 %	1	90.0 %	100.0 %	1
10.0 %	2	90.0 %	100.0 %	1
10.0 %	3	90.0 %	100.0 %	1
10.0 %	4	90.0 %	100.0 %	1
10.0 %	5	90.0 %	100.0 %	1
10.0 %	6	90.0 %	90.7 %	3

Continued on next page

L_i	KRD	%Feasibles	% Single Contract	Max Contracts
10.0 %	7	90.0 %	90.1 %	5
10.0 %	8	90.0 %	0.0 %	0
10.0 %	9	90.0 %	0.0 %	0
5.0 %	1	68.3 %	100.0 %	1
5.0 %	2	68.3 %	100.0 %	1
5.0 %	3	68.3 %	100.0 %	1
5.0 %	4	68.3 %	99.2 %	2
5.0 %	5	68.3 %	100.0 %	1
5.0 %	6	68.3 %	90.2 %	3
5.0 %	7	68.3 %	89.4 %	4
5.0 %	8	68.3 %	0.0 %	0
5.0 %	9	68.3 %	0.0 %	0
2.5 %	1	50.6 %	94.5 %	2
2.5 %	2	50.6 %	96.7 %	3
2.5 %	3	50.6 %	100.0 %	1
2.5 %	4	50.6 %	97.8 %	2
2.5 %	5	50.6 %	100.0 %	1
2.5 %	6	50.6 %	91.2 %	2
2.5 %	7	50.6 %	76.9 %	6
2.5 %	8	50.6 %	0.0 %	0
2.5 %	9	50.6 %	0.0 %	0

Table 27 reveals that the model encountered increased challenges in finding single solutions for the seventh KRD as the liquidity cap increased, a result consistent with our expectations. This observation aligns with other portfolios where a significant portion of the nominal value resided in the seventh KRD, such as the uniform 60-month portfolio depicted in Table 15. Interestingly, for the first KRD, despite hosting a substantial portion of the nominal value, the model successfully identified single-contract solutions for almost every feasible strategy, reflecting the high liquidity of these contracts.

4.10 60-Month and 84-Month Inverted Bell Portfolio Across Increasing Nominal Values

Continuing with the chapter's approach, let us delve into the 60-month and the 84-month inverted bell portfolios.

First, let us discuss the 60-month case. Similar to the 36-month case, the first and second buckets still carry a significant portion of the nominal value. However, in this instance, their combined share has reduced from nearly 38% of the portfolio's nominal value to approximately 27%. To explore these findings in detail, please refer to Table 28, which summarizes the feasibility and effectiveness of the 60-month inverted bell case.

Table 28 – Feasibility and Effectiveness Summary for the invertedbell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 60-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	99.4 %	102.7 %	6.0 %	87.1 %	125.3 %
100.0	10.0 %	100.0 %	100.0 %	102.7 %	5.9 %	85.7 %	122.3 %
100.0	5.0 %	100.0 %	100.0 %	102.6 %	5.8 %	85.7 %	119.2 %
100.0	2.5 %	98.9 %	100.0 %	102.5 %	5.9 %	82.9 %	117.9 %
300.0	100.0 %	100.0 %	99.4 %	102.5 %	5.6 %	87.1 %	127.9 %
300.0	10.0 %	99.4 %	100.0 %	102.4 %	5.7 %	81.6 %	117.9 %
300.0	5.0 %	98.3 %	100.0 %	102.5 %	5.8 %	82.6 %	119.2 %
300.0	2.5 %	93.9 %	100.0 %	102.9 %	5.9 %	83.4 %	121.0 %
500.0	100.0 %	100.0 %	99.4 %	102.7 %	5.9 %	85.7 %	127.9 %
500.0	10.0 %	98.3 %	100.0 %	102.4 %	5.9 %	82.0 %	120.1 %
500.0	5.0 %	95.6 %	100.0 %	102.6 %	5.8 %	82.6 %	121.0 %
500.0	2.5 %	78.3 %	100.0 %	103.0 %	6.0 %	87.7 %	121.0 %
1000.0	100.0 %	100.0 %	100.0 %	102.8 %	5.9 %	85.6 %	122.3 %
1000.0	10.0 %	95.6 %	100.0 %	102.7 %	6.1 %	82.6 %	120.4 %
1000.0	5.0 %	78.3 %	100.0 %	103.1 %	5.8 %	89.2 %	120.4 %
1000.0	2.5 %	60.0 %	100.0 %	102.5 %	6.0 %	82.9 %	117.0 %
5000.0	100.0 %	98.3 %	100.0 %	102.2 %	5.7 %	83.7 %	119.2 %
5000.0	10.0 %	56.1 %	100.0 %	102.8 %	6.4 %	82.9 %	117.6 %
5000.0	5.0 %	46.7 %	100.0 %	102.6 %	6.3 %	85.9 %	117.3 %
5000.0	2.5 %	39.4 %	100.0 %	102.3 %	6.3 %	82.0 %	120.0 %

Table 28 presents average and standard deviation values for effectiveness that closely resemble the cases observed in the uniform and bell-shaped portfolios. The decrease in volatility aligns with the reduction in the portion of the nominal value in the first two buckets, consistent with our expectations. As the length increases, we anticipate that the average effectiveness and standard deviation will converge toward the values observed in previous backtests. Let us examine the statistics for the 84-month case in Table 29.

Table 29 – Feasibility and Effectiveness Summary for the invertedbell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 84-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	100.0 %	101.6 %	4.7 %	87.3 %	116.2 %
100.0	10.0 %	99.4 %	100.0 %	101.8 %	4.6 %	87.3 %	116.2 %
100.0	5.0 %	98.3 %	100.0 %	101.5 %	4.6 %	86.4 %	116.2 %
100.0	2.5 %	92.2 %	100.0 %	100.9 %	5.2 %	85.5 %	116.2 %
300.0	100.0 %	100.0 %	100.0 %	101.9 %	4.8 %	85.8 %	116.2 %
300.0	10.0 %	95.0 %	100.0 %	101.4 %	5.2 %	86.8 %	116.2 %
300.0	5.0 %	87.8 %	100.0 %	101.3 %	5.2 %	86.0 %	116.2 %
300.0	2.5 %	72.2 %	100.0 %	101.3 %	5.0 %	86.4 %	116.2 %
500.0	100.0 %	100.0 %	100.0 %	101.7 %	4.6 %	88.4 %	116.2 %
500.0	10.0 %	90.6 %	100.0 %	101.0 %	5.2 %	86.4 %	116.2 %
500.0	5.0 %	76.1 %	100.0 %	101.5 %	5.4 %	86.3 %	123.1 %
500.0	2.5 %	62.8 %	100.0 %	101.8 %	5.0 %	90.5 %	116.2 %
1000.0	100.0 %	99.4 %	100.0 %	101.9 %	5.1 %	88.4 %	116.2 %
1000.0	10.0 %	76.1 %	100.0 %	101.6 %	5.5 %	86.7 %	123.2 %
1000.0	5.0 %	62.8 %	100.0 %	101.9 %	5.0 %	90.5 %	116.2 %
1000.0	2.5 %	61.1 %	100.0 %	101.7 %	5.1 %	89.2 %	116.2 %
5000.0	100.0 %	90.6 %	100.0 %	101.0 %	5.5 %	84.1 %	115.4 %
5000.0	10.0 %	60.6 %	100.0 %	101.3 %	5.5 %	84.9 %	116.2 %
5000.0	5.0 %	56.1 %	100.0 %	101.1 %	5.6 %	81.2 %	116.2 %
5000.0	2.5 %	35.6 %	100.0 %	100.4 %	5.5 %	83.4 %	114.6 %

Table 29 affirms our expectation that the average effectiveness and standard deviation would approach the values in the previously discussed scenarios as the portfolio length increased. Notably, while with the standard deviation slightly higher than the uniform and bell-shaped portfolios, the inverted bell portfolios still resulted in effective hedge strategies for every month of backtested.

4.10.1 Allocation of Securities within the KRD Buckets

The allocation of contracts in the hedge strategies for the 60-month and 84-month inverted bell-shaped portfolios closely mirrors that observed in the uniform and bell portfolios. The model tends to use more contracts in the buckets with a more significant portion of the portfolio's nominal value. For a comprehensive overview of the complete results, including nominal values ranging from 100 MM BRL to 5000 MM BRL, please refer to Table 48 for the 60-month backtest and Table 49 for the 84-month backtest in Appendix D.

4.11 120-Month Inverted Bell Portfolio Across Increasing Nominal Values

The 120-Month inverted bell presents a unique scenario where the ninth bucket receives its most significant allocation compared to all the combinations tested thus far. In this case, almost half of the portfolio's total value concentrates within this ninth KRD bucket. Now, let us delve into the results for this specific scenario, which constitutes the final one among the twelve portfolio shape and length combinations. We will examine Table 30, presenting summarized effectiveness statistics for the inverted 120-month portfolio with nominal values ranging from 100 to 5000 million BRL, all subject to four different liquidity caps. These results represent the culmination of the comprehensive backtest that this study conducted to thoroughly exercise and stress the optimization model, involving 240 combinations of portfolio shape, length, nominal value and liquidity caps.

Table 30 – Feasibility and Effectiveness Summary for the invertedbell Portfolio with a Nominal Values from 100 MM to 5000 MM BRL over 120-Month Time Span (L_i of 100%, 10%, 5% and 2.5%).

Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
100.0	100.0 %	100.0 %	99.4 %	101.7 %	6.4 %	81.5 %	161.0 %
100.0	10.0 %	97.8 %	99.4 %	102.0 %	4.8 %	83.9 %	129.4 %
100.0	5.0 %	93.3 %	99.4 %	101.7 %	5.1 %	84.2 %	129.4 %
100.0	2.5 %	92.2 %	98.8 %	101.8 %	5.0 %	84.2 %	130.5 %
300.0	100.0 %	98.9 %	100.0 %	101.5 %	4.7 %	80.9 %	115.3 %
300.0	10.0 %	92.8 %	100.0 %	101.9 %	4.5 %	83.0 %	117.5 %
300.0	5.0 %	91.7 %	100.0 %	101.7 %	4.7 %	85.0 %	117.5 %
300.0	2.5 %	80.6 %	100.0 %	101.9 %	4.6 %	83.3 %	115.5 %
500.0	100.0 %	98.9 %	100.0 %	101.5 %	4.9 %	83.9 %	121.1 %
500.0	10.0 %	91.7 %	98.8 %	101.8 %	5.3 %	83.6 %	130.5 %
500.0	5.0 %	85.6 %	99.4 %	101.9 %	5.5 %	82.8 %	130.5 %
500.0	2.5 %	77.2 %	100.0 %	101.6 %	4.8 %	81.2 %	116.0 %
1000.0	100.0 %	97.8 %	100.0 %	102.2 %	4.7 %	82.5 %	116.3 %

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Value	L_i	**%Feasibles**	**%Effectives**	**Effectiveness**			
				Avg	**Std**	**Min**	**Max**
1000.0	10.0 %	85.6 %	99.4 %	101.7 %	5.2 %	72.1 %	116.0 %
1000.0	5.0 %	77.2 %	100.0 %	101.5 %	5.0 %	80.9 %	115.3 %
1000.0	2.5 %	60.0 %	100.0 %	100.6 %	5.3 %	82.5 %	113.0 %
5000.0	100.0 %	91.7 %	99.4 %	101.7 %	5.1 %	78.6 %	117.5 %
5000.0	10.0 %	55.0 %	100.0 %	100.1 %	5.5 %	80.0 %	114.7 %
5000.0	5.0 %	32.8 %	100.0 %	99.3 %	5.3 %	80.5 %	118.1 %
5000.0	2.5 %	5.0 %	100.0 %	97.4 %	4.3 %	87.8 %	102.9 %

Table 30 further confirms our expectation that the average effectiveness and standard deviation would approach the values observed in the previously discussed scenarios as the portfolio length increased. It demonstrates that the average effectiveness closely aligns with the values presented thus far, with a slightly lower standard deviation than the other inverted bell-shaped portfolios.

Regarding the model's ability to form hedge strategies, the 120-month inverted bell case represents the scenario where the model encountered the most challenges in finding feasible solutions. This difficulty is exemplified in the case of the 5000 MM portfolio under $L_i = 2.5\%$, where the model found feasible solutions for only 5% of the backtested months. Let's take a closer look at Table 31 to gain a more detailed understanding of the contract allocation for the 120-month case.

Table 31 – Summary of Unique Contracts Acquired for the 120-Month Hedging Strategy of a 5000 Million BRL invertedbell Portfolio.(L_i of 10%, 5% and 2.5%)

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0 %	1	91.7 %	100.0 %	1.0
100.0 %	2	91.7 %	100.0 %	1.0
100.0 %	3	91.7 %	100.0 %	1.0
100.0 %	4	91.7 %	100.0 %	1.0
100.0 %	5	91.7 %	100.0 %	1.0
100.0 %	6	91.7 %	100.0 %	1.0
100.0 %	7	91.7 %	100.0 %	1.0
100.0 %	8	91.7 %	100.0 %	1.0
100.0 %	9	91.7 %	97.0 %	2.0
10.0 %	1	55.0 %	100.0 %	1.0
10.0 %	2	55.0 %	100.0 %	1.0
10.0 %	3	55.0 %	100.0 %	1.0

Continued on next page

L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
10.0 %	4	55.0 %	100.0 %	1.0
10.0 %	5	55.0 %	100.0 %	1.0
10.0 %	6	55.0 %	100.0 %	1.0
10.0 %	7	55.0 %	100.0 %	1.0
10.0 %	8	55.0 %	98.0 %	2.0
10.0 %	9	55.0 %	72.7 %	2.0
5.0 %	1	32.8 %	100.0 %	1.0
5.0 %	2	32.8 %	100.0 %	1.0
5.0 %	3	32.8 %	100.0 %	1.0
5.0 %	4	32.8 %	100.0 %	1.0
5.0 %	5	32.8 %	100.0 %	1.0
5.0 %	6	32.8 %	94.9 %	2.0
5.0 %	7	32.8 %	100.0 %	1.0
5.0 %	8	32.8 %	100.0 %	1.0
5.0 %	9	32.8 %	37.3 %	2.0
2.5 %	1	5.0 %	100.0 %	1.0
2.5 %	2	5.0 %	100.0 %	1.0
2.5 %	3	5.0 %	100.0 %	1.0
2.5 %	4	5.0 %	100.0 %	1.0
2.5 %	5	5.0 %	100.0 %	1.0
2.5 %	6	5.0 %	100.0 %	1.0
2.5 %	7	5.0 %	100.0 %	1.0
2.5 %	8	5.0 %	100.0 %	1.0
2.5 %	9	5.0 %	0.0 %	3.0

Table 31 illustrates the model's struggle to find single-contract solutions for the ninth bucket. Interestingly, the $L_i = 2.5\%$ scenario exhibits a unique result among the cases tested so far, where the model failed to find single-contract solutions for all feasible hedges formulated.

4.12 Backtesting Summary

The preceding sections offered a comprehensive overview of how the model performed under a diverse set of conditions. The backtesting rigorously tested the model across different interest rate cycles, spanning over 15 years of future contract negotiations. Additionally, the backtesting evaluated the model's adaptability to various portfolios, assessing its performance with portfolios of different nominal values, shapes, and time horizons. The following are the key findings:

- The effectiveness achieved by the model was slightly affected by variations in the portfolio shape, nominal value, and time horizon. The results, as depicted in Tables 10, 13, 17, 19, 21, 23, 24, 25, 28, 29, and 30, consistently show similar effectiveness averages and standard deviations. The case where the most notable difference was observed compared to the others was the inverted bell 36-month case, illustrated in Table 26, where a significant portion of the portfolio is concentrated in the first bucket, leading to hedging with more volatile contracts. However, even in the inverted bell 36-month case the effectiveness is still close to the results achieved in the other cases.
- The effectiveness exhibited stable averages and standard deviations when facing distinct liquidity constraints. This stability is particularly evident in cases where the model was able to find feasible hedge strategies for at least 80% of the backtested scenarios. This suggests that variations in effectiveness statistics under different liquidity constraints were a byproduct of the model's ability or inability to formulate strategies rather than by the liquidity constraints themselves.
- The findings outlined above underscore the pivotal role played by the set of dollar duration constraints (Eq. 5b and 5c), the set of dollar convexity constraints (Eq. 5d and 5e), and the KRD constraint (Eq. 5g) in determining the effectiveness of the model. These constraints were tightened to a degree where the model faced a binary choice: formulate a strategy that was, in most cases, effective, or refrain from formulating a strategy altogether.
- The liquidity constraints mainly influenced on the allocation of future contracts to hedge each KRD and the feasibility of the strategies. As the nominal value increased and liquidity caps tightened, the model sought larger financial instruments to adhere to the liquidity constraints. When it became impossible to find such instruments, the model chose not to formulate any strategy. This aligns with our expectations for the constraint behavior.
- The outcomes achieved by the model provide valuable insights for the agent responsible for formulating the selection of securities to protect a portfolio from interest rate risk. When the model indicates that it is not possible to formulate a strategy adhering to the specified parameters, it signals to the agent that they may need to accept trading a larger portion of intraday liquidity. This decision carries poten-

tial consequences, including increased price volatility and, potentially, less effective portfolios.

4.13 \$Duration and \$Convexity Constraints Variations

The backtesting analysis has highlighted that the effectiveness of the strategies is primarily influenced by the \$duration and \$convexity constraints. To gain a deeper understanding of these constraints, additional tests were conducted with variations in the dollar duration constraints and the dollar convexity constraints.

Our research explored four variations of the proposed \$duration and \$convexity constraints, outlined as follows:

1. Only \$Duration without δ_d tolerance (\$Duration Matching).
2. Only \$Duration constraint with tightened δ_d tolerance ($\delta_d = 0.01$).
3. \$Duration and \$Convexity constraints with high tolerances for both $\delta_d = 0.10$ and $\delta_c = 0.10$.
4. \$Duration and \$Convexity constraints with a high tolerance of $\delta_d = 0.10$ and stricter $\delta_c = 0.05$ tolerances.

In the first variation (\$Duration Matching), we replaced the \$duration and \$convexity inequalities (Eq. 5b, 5c, Eq. 5d, and 5e) with a single \$duration equality constraint. This modification requires the model to formulate a hedge strategy with the same \$duration as the portfolio. In the second variation, we removed the \$convexity constraints (Eq. 5d and 5e) and thighted the δ_d tolerance. For the third and fourth variations, we adjusted the δ_d and δ_c tolerances, which initially assumed values of $\delta_d = 0.025$ and $\delta_c = 0.10$.

It is essential to note that an equal number of tests, totaling 43,200 for each variation, were conducted, mirroring the approach taken for the previously presented backtested results. Therefore, a total of 216,000 portfolio selections were constructed in the current study. Table 32 presents the statistics for the tested variations.

Table 32 illustrates that tighter values of \$duration tolerances result in strategies with average effectiveness closer to 100%. This trend is evident in both the \$Duration Matching and $\delta_d = 0.01$ variations. However, it is noteworthy that these results exhibit higher standard deviations compared to the $\delta_d = 0.025$ and $\delta_c = 0.10$ variation.

Table 32 – Effectiveness and Feasibility Comparison of Different Variations of \$Duration and \$Convexity Constraints

Variation	%Feasibles	%Effectives	Effectiveness Avg	Effectiveness Std
\$Duration Matching	90.2 %	97.9 %	99.8 %	7.3 %
$\delta_d = 0.01$	90.3 %	98.0 %	100.0 %	7.3 %
$\delta_d = 0.025$ and $\delta_c = 0.10$	87.4 %	99.4 %	101.7 %	5.4 %
$\delta_d = 0.10$ and $\delta_c = 0.05$	88.5 %	98.1 %	102.5 %	8.1 %
$\delta_d = 0.10$ and $\delta_c = 0.10$	89.8 %	97.8 %	103.5 %	8.6 %

The inclusion of the \$convexity constraint proves beneficial in reducing the volatility of the results. This is reflected in average effectiveness values that are slightly further from 100%, but with a greater percentage of effective strategies. On the other hand, strategies with more lenient δ_d tolerances yielded poorer results, with average effectiveness values further from 100% and higher standard deviations.

It's important to mention that we did not include a variation with strict tolerances for both δ_d and δ_c simultaneously. The model faced significant challenges in formulating feasible strategies under these conditions. The variation we employed in our study, characterized by $\delta_d = 0.025$ and $\delta_c = 0.10$, is the case that, among the five variations detailed in Table 32, encounters more challenges in formulating portfolios. In our study, there exists a clear trade-off between standard deviation and %Feasibles. We opted not to formulate numerous portfolios in order to achieve better effectiveness results.

4.14 Hedge Effectiveness: Linear Regression Analysis

To facilitate a comprehensive comparison with existing literature and econometric methodologies, we employed a linear regression analysis to evaluate the effectiveness of our proposed hedge strategies. This method provides a perspective on how well the returns of the hedge portfolio elucidate the returns of the original portfolio by quantifying this relationship through the coefficient of determination (R^2).

In the forthcoming Table 33, we present the R^2 values for the 36-month uniform portfolios as a representative illustration. The detailed results for uniform portfolios, bell-shaped portfolios, and inverted bell portfolios are provided in Tables 51, 52 and 53, respectively, within Appendix E for thorough examination and comparison.

Table 33 – Comparison of Hedge Effectiveness: R^2 Values for 36-Month uniform Portfolios (L_i of 100%, 10%, 5% and 2.5%).

Months	**Value**	L_i	R^2
36	100	100.0 %	0.996
36	100	10.0 %	0.997
36	100	5.0 %	0.997
36	100	2.5 %	0.996
36	300	100.0 %	0.997
36	300	10.0 %	0.997
36	300	5.0 %	0.996
36	300	2.5 %	0.996
36	500	100.0 %	0.996
36	500	10.0 %	0.996
36	500	5.0 %	0.996
36	500	2.5 %	0.995
36	1000	100.0 %	0.997
36	1000	10.0 %	0.997
36	1000	5.0 %	0.996
36	1000	2.5 %	0.995
36	5000	100.0 %	0.995
36	5000	10.0 %	0.995
36	5000	5.0 %	0.994
36	5000	2.5 %	0.996

The R^2 values presented in Table 33 and the detailed results in Appendix E consistently demonstrate a remarkable level of adherence between the returns of the hedge portfolios and the original portfolios across diverse scenarios. In most cases, the R^2 values hover around 0.99, indicating a high degree of explanatory power and, thus, revealing high adherence to the hedge strategies. The closeness to unity underscores our hedge strategies' effectiveness in capturing the portfolios' underlying risk factors. Notably, even in the challenging scenarios represented by the inverted bell-shaped portfolios, the R^2 values remain robust, although slightly lower, typically in the 0.98 range. This consistent pattern reaffirms the ability of our proposed hedging model to provide effective risk mitigation across various portfolio shapes and scenarios.

4.15 Benchmark Analysis: Comparative Evaluation with Econometric Models

In this section, we present the results of a benchmark analysis to place the results obtained from our proposed optimization model in perspective. While our model represents a novel approach to interest rate risk management, it is crucial to compare its effectiveness against established hedge econometric models found in the literature, in an effort to provide a comprehensive assessment of the relative performance and practical applicability of our proposed model in real-world scenarios.

We adhered to the methodology proposed by Kunzler (2019) and Meirelles and Fernandes (2018) to construct Principal Component Analysis (PCA) models. Following their approach, we focused on a 120-month uniform portfolio, exhibiting a duration approximately ranging from 4.3 to 4.5 years across the 180 tested months. For each month in the dataset, we systematically developed PCA-based hedge strategies. Subsequently, we subjected these PCA models to the rigorous backtesting process as employed for the hedge strategies derived from our optimization model. This involved evaluating the effectiveness of the PCA-based hedge strategies against 15-year DI futures trading data, offering a comparative assessment with the results obtained from our optimization model.

For the reduced set of three financial securities aimed at hedging against the principal risk factors, we adopted a methodology inspired by Kunzler (2019). The selection process involved contracts with due dates corresponding to January plus one year, January plus five years (closely aligning with the portfolio's duration), and another contract around January plus nine years. This approach mirrors the procedure outlined by Kunzler (2019), where the securities were chosen to include one with a duration close to that of the portfolio and two others with equidistant durations from the central security. Table 34 shows the results for the benchmark analysis performed for the 100 MM 120-Month Uniform Portfolio.

Table 34 – Comparison of PCA Methodology and Optimization Model for a 100 MM 120-Month Uniform Portfolio

Method	R^2	**%Effectives**	**Effectiveness**			
			Avg	**Std**	**Min**	**Max**
ACP	0.982	88.9 %	98.6 %	15.5 %	54.7 %	186.5 %
Optimization Model	0.999	100.0 %	101.5 %	4.3 %	86.0 %	119.8 %

The results indicate that the PCA method yielded an R^2 of 0.982. In comparison, the study conducted by Kunzler (2019) achieved an R^2 of 0.999, while Meirelles and Fernandes (2018) obtained R^2 values of 0.994 and 0.974 for two different hedge strategies using a similar method. It is essential to recognize that these studies employed other portfolios and backtested with a smaller dataset, potentially accounting for the variance in results.

The performance exhibited by the hedge constructed with the proposed optimization model surpasses that of the PCA method regarding the percentage of effective strategies over 180 months. Moreover, it demonstrates a smaller standard deviation and maintains an average effectiveness closer to 100

Although the PCA method offers administrative simplicity with fewer contracts (three compared to at least nine for the 120-month case), the trade-off is reflected in its inferior performance. Consequently, managing more contracts for a less volatile and more effective hedge strategy may be justified, particularly considering the optimization model's superior results.

A noteworthy distinction between the PCA method and the proposed optimization model lies in how liquidity is addressed. In the PCA method, practitioners must pre-process securities by selecting those with higher liquidity before executing the methodology. On the other hand, the optimization model integrates liquidity into the decision-making process, considering all available contracts without excluding any. This approach expands the available liquidity for the practitioner since the model can choose to utilize more contracts, thus complementing liquidity while still accounting for risk factors. This flexibility can be advantageous when maintaining sufficient liquidity is crucial for effective risk management, such as scenarios of high nominal value portfolios.

5 Conclusion

This study proposed a robust model to assist bank ALM departments in navigating interest rate uncertainty. The models contributes with the literature by incorporating real-life constraints, including the cost of monitoring and administering interest rate derivatives, and accounting for the liquidity of these instruments in the market, into the allocation (decision-making) process. This analysis explored the model's performance with a comprehensive backtesting across a variety of scenarios, stressing its robustness under different yield curve cycles and portfolio shapes.

The key findings highlighted the crucial role of the \$duration and \$convexity constraints in determining the effectiveness of hedge strategies. Backtesting revealed that strategies with tighter \$duration tolerances contributed to keep the effectiveness of the strategies around 100%. Notably, the inclusion of the \$convexity constraint reduced result volatility of the strategies.

Furthermore, the study highlighted the significance of liquidity constraints as a critical factor influencing the allocation of future contracts and the feasibility of formulated strategies. As nominal values increased and liquidity caps tightened, the model sought for a larger set of financial instruments to adhere to liquidity constraints, addressing real-world challenges in executing effective strategies while managing intraday liquidity trade-offs.

The model stressing extended to diverse portfolio shapes, including uniform, bell, and inverted bell structures across varying time spans. While these shape variations introduced unique challenges, the overarching principles governing effective risk management remained rooted in navigating the constraints of \$duration and \$convexity.

The study illuminated that, despite the diverse scenarios tested, the core foundations of effective risk management lie in the model's ability to adhere to \$duration and \$convexity constraints. Nominal values, liquidity caps, and portfolio shapes played roles in presenting unique challenges to formulate a feasible strategy with a small set of securities but did not influence the overall effectiveness of the strategies built.

In conclusion, this study successfully utilized optimization models as a tool to establish a comprehensive framework for interest rate risk management. It offers valuable insights for practitioners navigating the complex landscape of financial risk, providing a decision-making framework that considers real-world constraints such as liquidity and the cost of administering fixed-income portfolios.

The proposed model demonstrates its adaptability and effectiveness across a spectrum of scenarios, marking a significant step forward in the field of interest rate risk management. The meticulous backtesting, coupled with the benchmarking against the widely used PCA methodology, further validates the model's practical utility. The flexible approach to liquidity management positions this model as a valuable tool for financial institutions seeking robust risk mitigation strategies.

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Appendix

APPENDIX A – Term Structure Partitioning into Key Rate Duration (KRD) Buckets

In this appendix, we provide an in-depth explanation of the methodology employed to partition the term structure into Key Rate Durations (KRDs). Our approach draws inspiration from the concept introduced by Ho (1992), which posits that the yield curve can be effectively decomposed into segments. Within each segment, the yield curve is assumed to fluctuate in a parallel manner, specifically in terms of level. This assumption enhances the precision of applying $duration and $convexity concepts to these distinct segments.

To operationalize this concept, we calculated the covariance between DI future contracts. This statistical measure allows us to identify segments where the yield curve tends to move in a similar and parallel fashion, thus justifying the application of duration and convexity calculations with greater precision.

Furthermore, to augment our methodology and ensure its relevance to real-world applications, we sought expert input. We conducted interviews with two experienced fixed-income portfolio managers to gain insights into how they traditionally partition the term structure in their practices. Their perspectives enriched our approach, aligning it with industry practices and ensuring its practicality.

In this study, a strategic choice has been made to partition the term structure into nine specific segments for detailed analysis, as outlined in Table 1. Notably, the segmentation allocates a greater number of partitions to the initial years of the curve, representing the short-term. This decision is grounded in the recognition that the short-term segment tends to exhibit higher volatility. Utilizing smaller sets of futures in this region enables a more precise capture of nuanced fluctuations within the short-term domain. Conversely, as we extend into the time domain and consider contracts with greater maturities, the segmentation becomes more expansive. This adjustment is informed by the understanding that longer-term contracts generally exhibit lower volatility.

Figure 8 ilustrates the correlation between DI instruments for the period between 01/01/2020 and 01/01/2022.

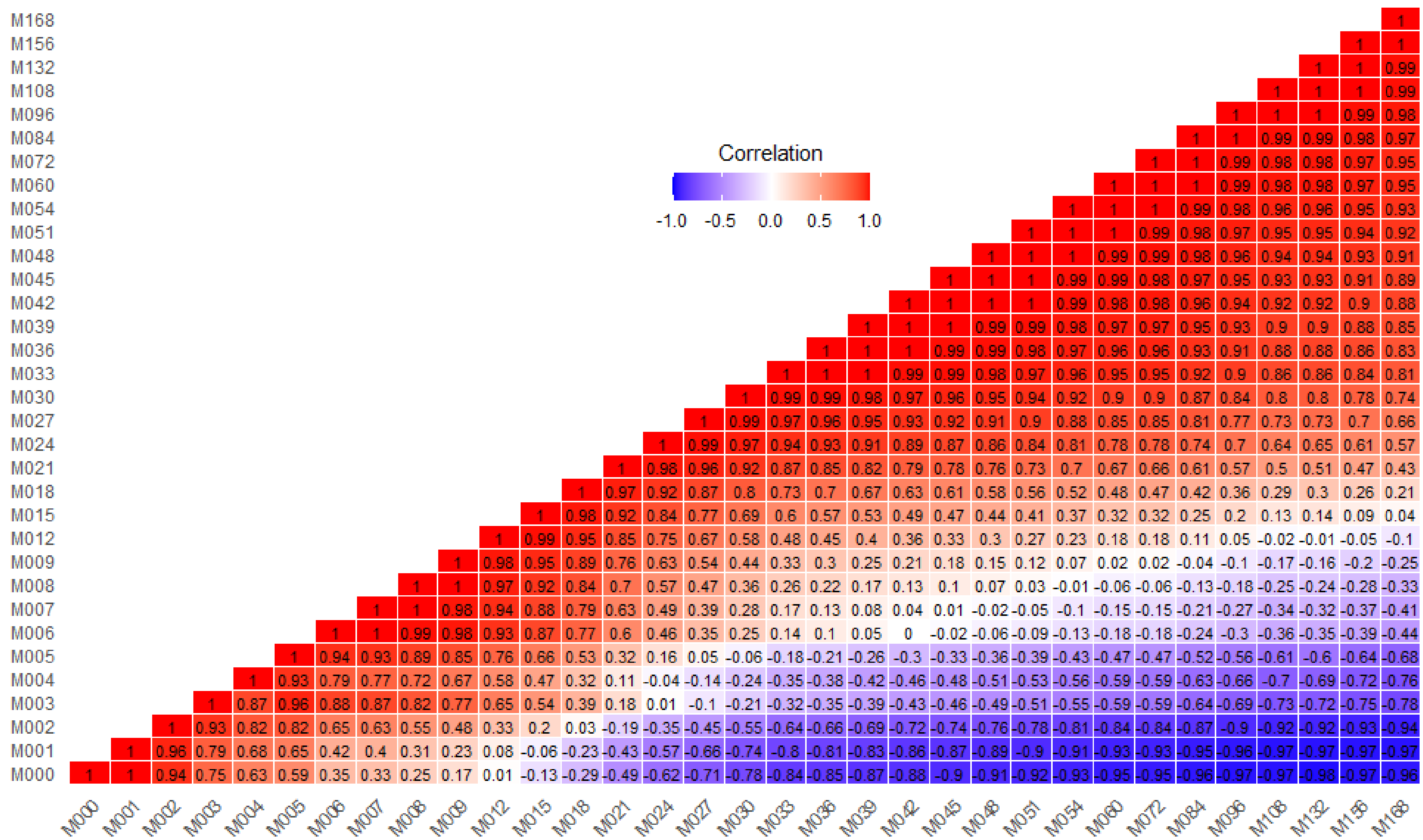

Figure 8 – Correlation of DI instruments between 01/01/2020 and 01/01/2022, where MXXX represents a security with a maturity of XXX months; for example, M030 represents a security with a maturity of 30 months.

As illustrated in the accompanying Figure 8, the segmentation proposed in Table 1 aligns with the correlation patterns within the DI contracts. Notably, the adherence of the segmentation becomes particularly insightful when examining longer maturities, where correlations exhibit notable trends.

The figure clearly depicts that for contracts with maturities ranging from M021 to M030, corresponding to the 7th bucket, correlations consistently surpass the 0.90 threshold. Moving into the subsequent period, from M30 to M85 (8th bucket), correlations persistently exceed 0.90, indicating a sustained relationship among these maturities. Further along the maturity spectrum, specifically from M85 to M168 (9th bucket), correlations escalate significantly, surpassing the 0.97 mark.

APPENDIX B – Sensitivity Analysis on Variability Threshold: Exploring the Impact of Different Cut-off Values on Hedge Effectiveness Evaluation

This study relies on a critical assumption in the evaluation of hedge effectiveness, as outlined in section 3.3. We implemented a criterion that deems cases effective when both the monthly variation of the portfolio and the monthly variation of the hedge are below 0.5% of their respective present values. This criterion aims to prevent instances of minimal variation from unduly influencing the analysis.

The practical implications of this assumption become evident when examining scenarios with minimal fluctuations in both the portfolio and the hedge. In such instances, the choice of the cut-off value significantly shapes the perceived effectiveness of the hedging strategy.

Recognizing the arbitrary nature of the 0.5% cut-off, we conducted a sensitivity analysis, exploring alternative cut-off values and even considering no cut-off. This exploration aims to illuminate how different cut-off values may impact the results. Table 35, below, presents the average and standard deviation of effectiveness for cut-offs of 0.5% (the value used in the analysis), 0.1%, 0.05%, and no cut-off at all.

Table 35 – Effectiveness Evaluation Sensitivity Analysis: Impact of Different Cut-off Values

Cut-off	**%Effectives**	**Effectiveness**			
		Avg	**Std**	**Min**	**Max**
0.5%	99.4 %	101.7 %	5.4 %	71.1 %	161.0 %
0.1%	94.4 %	102.6 %	11.7 %	32.5 %	231.5 %
0.05%	92.1 %	102.6 %	17.2 %	−195.7 %	428.4 %
0.01%	90.3 %	102.1 %	36.6 %	−1469.5 %	1451.5 %
No cut	89.5 %	98.9 %	208.8 %	−17 698.7 %	8867.6 %

Table 35 underscores the importance of assigning a cut-off value. It reveals that the percentage of effective cases varies across different cut-off values but remains above 89.5% even when no cut-off is applied. The average, too, demonstrates remarkable stability, hovering close to 100% even without the imposed threshold. However, the standard deviation sees a significant increase, indicating that instances with minimal variation do impact the analysis. Thus, the use of a threshold becomes imperative to ensure the robustness of the evaluation.

APPENDIX C – Complete Results for the Comparison Between Portfolio and Hedge Present Values

C.1 Uniform Portfolios

Table 36 – Comparison of Portfolio and Hedge PV for 36, 60, 84 and 120-months uniform Portfolio of nominal values of 100MM BRL to 5000MM BRL and its Hedge. Both PV_p and PV_h are measured in the first day of the month.

Months	**Value**	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
36	100.0	100.0 %	65.0 %	4.5 %	3.0 %	35.0 %	2.7 %	1.8 %
36	100.0	10.0 %	66.1 %	4.4 %	3.0 %	33.9 %	2.5 %	1.9 %
36	100.0	5.0 %	67.2 %	4.5 %	3.2 %	32.8 %	2.8 %	2.0 %
36	100.0	2.5 %	60.0 %	4.6 %	3.3 %	40.0 %	2.7 %	1.9 %
36	300.0	100.0 %	73.9 %	6.4 %	4.0 %	26.1 %	2.8 %	1.9 %
36	300.0	10.0 %	73.9 %	6.5 %	4.2 %	26.1 %	2.8 %	1.7 %
36	300.0	5.0 %	67.2 %	5.8 %	3.6 %	32.8 %	3.4 %	1.9 %
36	300.0	2.5 %	61.5 %	5.7 %	3.8 %	38.5 %	3.3 %	1.9 %
36	500.0	100.0 %	76.7 %	6.2 %	3.9 %	23.3 %	3.0 %	2.0 %
36	500.0	10.0 %	73.9 %	5.8 %	3.8 %	26.1 %	3.2 %	1.8 %
36	500.0	5.0 %	65.5 %	5.5 %	3.6 %	34.5 %	3.3 %	1.9 %
36	500.0	2.5 %	65.1 %	5.3 %	3.9 %	34.9 %	3.7 %	2.4 %
36	1000.0	100.0 %	75.6 %	4.8 %	3.4 %	24.4 %	3.1 %	1.9 %
36	1000.0	10.0 %	65.5 %	4.5 %	3.3 %	34.5 %	3.4 %	2.1 %
36	1000.0	5.0 %	61.4 %	4.8 %	3.6 %	38.6 %	3.8 %	2.4 %
36	1000.0	2.5 %	64.1 %	5.2 %	3.8 %	35.9 %	3.8 %	2.9 %
36	5000.0	100.0 %	55.6 %	5.1 %	3.0 %	44.4 %	4.5 %	3.2 %
36	5000.0	10.0 %	60.8 %	5.2 %	4.0 %	39.2 %	5.2 %	3.6 %
36	5000.0	5.0 %	67.0 %	5.8 %	4.1 %	33.0 %	5.6 %	3.7 %
36	5000.0	2.5 %	59.0 %	6.4 %	3.8 %	41.0 %	4.7 %	3.7 %
60	100.0	100.0 %	63.3 %	3.7 %	2.5 %	36.7 %	2.5 %	1.9 %
60	100.0	10.0 %	62.2 %	3.7 %	2.8 %	37.8 %	2.4 %	1.9 %
60	100.0	5.0 %	64.4 %	3.7 %	2.5 %	35.6 %	2.5 %	2.0 %
60	100.0	2.5 %	67.2 %	4.2 %	2.7 %	32.8 %	2.2 %	1.7 %
60	300.0	100.0 %	68.3 %	3.4 %	2.5 %	31.7 %	2.1 %	1.7 %
60	300.0	10.0 %	72.2 %	3.9 %	2.9 %	27.8 %	1.8 %	1.4 %
60	300.0	5.0 %	73.0 %	4.0 %	2.9 %	27.0 %	1.9 %	1.3 %
60	300.0	2.5 %	65.0 %	4.2 %	2.9 %	35.0 %	2.4 %	1.9 %
60	500.0	100.0 %	70.0 %	4.9 %	3.0 %	30.0 %	2.9 %	2.5 %
60	500.0	10.0 %	71.7 %	5.4 %	3.2 %	28.3 %	2.4 %	1.9 %

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Months	Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
60	500.0	5.0 %	67.2 %	5.1 %	3.1 %	32.8 %	2.7 %	1.8 %
60	500.0	2.5 %	64.6 %	5.0 %	3.3 %	35.4 %	3.4 %	2.1 %
60	1000.0	100.0 %	67.2 %	3.7 %	2.5 %	32.8 %	2.4 %	1.9 %
60	1000.0	10.0 %	67.2 %	3.9 %	2.6 %	32.8 %	2.5 %	1.6 %
60	1000.0	5.0 %	62.8 %	3.9 %	2.7 %	37.2 %	2.9 %	1.9 %
60	1000.0	2.5 %	62.5 %	4.8 %	3.3 %	37.5 %	2.9 %	2.0 %
60	5000.0	100.0 %	62.8 %	3.8 %	2.5 %	37.2 %	3.7 %	2.5 %
60	5000.0	10.0 %	57.3 %	5.1 %	3.6 %	42.7 %	4.0 %	3.0 %
60	5000.0	5.0 %	68.6 %	4.5 %	3.3 %	31.4 %	5.0 %	3.2 %
60	5000.0	2.5 %	59.7 %	4.3 %	3.2 %	40.3 %	4.7 %	3.3 %
84	100.0	100.0 %	67.2 %	3.6 %	2.4 %	32.8 %	3.2 %	2.8 %
84	100.0	10.0 %	65.0 %	3.8 %	2.4 %	35.0 %	3.1 %	2.6 %
84	100.0	5.0 %	67.2 %	3.5 %	2.1 %	32.8 %	3.0 %	2.6 %
84	100.0	2.5 %	63.3 %	3.6 %	2.4 %	36.7 %	3.3 %	2.6 %
84	300.0	100.0 %	68.9 %	3.9 %	2.6 %	31.1 %	3.0 %	2.4 %
84	300.0	10.0 %	68.2 %	4.2 %	2.8 %	31.8 %	3.0 %	2.3 %
84	300.0	5.0 %	58.9 %	4.8 %	3.0 %	41.1 %	3.2 %	2.3 %
84	300.0	2.5 %	60.6 %	4.4 %	3.1 %	39.4 %	3.6 %	2.5 %
84	500.0	100.0 %	68.9 %	4.3 %	2.8 %	31.1 %	3.0 %	2.7 %
84	500.0	10.0 %	66.5 %	4.6 %	3.1 %	33.5 %	3.5 %	2.7 %
84	500.0	5.0 %	65.5 %	4.8 %	3.2 %	34.5 %	3.2 %	2.7 %
84	500.0	2.5 %	62.4 %	4.5 %	3.1 %	37.6 %	3.6 %	2.9 %
84	1000.0	100.0 %	68.3 %	4.6 %	3.0 %	31.7 %	2.9 %	2.3 %
84	1000.0	10.0 %	64.3 %	4.8 %	3.5 %	35.7 %	3.4 %	2.6 %
84	1000.0	5.0 %	60.2 %	4.7 %	3.3 %	39.8 %	3.6 %	2.7 %
84	1000.0	2.5 %	65.8 %	4.6 %	3.2 %	34.2 %	3.9 %	2.7 %
84	5000.0	100.0 %	54.0 %	4.0 %	2.7 %	46.0 %	3.9 %	3.0 %
84	5000.0	10.0 %	55.4 %	4.8 %	3.6 %	44.6 %	3.7 %	3.0 %
84	5000.0	5.0 %	55.7 %	4.5 %	3.7 %	44.3 %	4.3 %	3.3 %
84	5000.0	2.5 %	57.5 %	4.9 %	3.3 %	42.5 %	4.9 %	3.6 %
120	100.0	100.0 %	69.4 %	3.6 %	2.4 %	30.6 %	2.4 %	1.8 %
120	100.0	10.0 %	65.9 %	3.5 %	2.5 %	34.1 %	2.6 %	1.8 %
120	100.0	5.0 %	69.4 %	3.4 %	2.3 %	30.6 %	2.7 %	2.0 %
120	100.0	2.5 %	64.7 %	3.4 %	2.0 %	35.3 %	3.0 %	2.3 %
120	300.0	100.0 %	63.5 %	3.5 %	2.4 %	36.5 %	2.4 %	1.8 %
120	300.0	10.0 %	66.7 %	3.4 %	2.3 %	33.3 %	2.8 %	2.2 %
120	300.0	5.0 %	66.5 %	3.7 %	2.4 %	33.5 %	3.1 %	2.3 %
120	300.0	2.5 %	60.0 %	3.6 %	2.4 %	40.0 %	3.1 %	2.3 %
120	500.0	100.0 %	64.0 %	3.3 %	2.2 %	36.0 %	2.8 %	2.0 %
120	500.0	10.0 %	67.3 %	3.5 %	2.3 %	32.7 %	2.7 %	2.2 %
120	500.0	5.0 %	60.6 %	3.8 %	2.6 %	39.4 %	2.6 %	2.0 %
120	500.0	2.5 %	65.7 %	3.7 %	2.7 %	34.3 %	2.8 %	2.1 %
120	1000.0	100.0 %	68.2 %	4.5 %	3.0 %	31.8 %	2.6 %	2.0 %

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Months	Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
120	1000.0	10.0 %	63.7 %	4.4 %	3.1 %	36.2 %	2.7 %	2.2 %
120	1000.0	5.0 %	62.0 %	4.7 %	3.1 %	38.0 %	2.5 %	2.2 %
120	1000.0	2.5 %	63.0 %	3.9 %	2.9 %	37.0 %	2.8 %	2.3 %
120	5000.0	100.0 %	65.5 %	3.8 %	2.6 %	34.5 %	2.9 %	2.3 %
120	5000.0	10.0 %	66.3 %	3.8 %	2.6 %	33.7 %	2.9 %	2.1 %
120	5000.0	5.0 %	54.8 %	3.2 %	2.7 %	45.2 %	3.5 %	2.2 %
120	5000.0	2.5 %	45.2 %	3.5 %	2.6 %	54.8 %	3.3 %	1.9 %

C.2 Bell Portfolios

Table 37 – Comparison of Portfolio and Hedge PV for 36, 60, 84 and 120-months bell Portfolio of nominal values of 100MM BRL to 5000MM BRL and its Hedge. Both PV_p and PV_h are measured in the first day of the month.

Months	Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
36	100.0	100.0 %	58.3 %	4.1 %	2.5 %	41.7 %	2.5 %	1.9 %
36	100.0	10.0 %	61.1 %	4.1 %	2.6 %	38.9 %	2.4 %	1.9 %
36	100.0	5.0 %	58.9 %	4.1 %	2.7 %	41.1 %	2.7 %	2.3 %
36	100.0	2.5 %	53.4 %	4.3 %	2.9 %	46.6 %	3.6 %	3.0 %
36	300.0	100.0 %	80.0 %	4.3 %	2.3 %	20.0 %	1.9 %	2.1 %
36	300.0	10.0 %	69.4 %	4.4 %	2.5 %	30.6 %	2.8 %	2.4 %
36	300.0	5.0 %	63.6 %	3.8 %	2.6 %	36.4 %	4.2 %	3.4 %
36	300.0	2.5 %	56.0 %	4.1 %	3.0 %	44.0 %	4.1 %	3.3 %
36	500.0	100.0 %	61.1 %	4.5 %	2.5 %	38.9 %	4.3 %	3.3 %
36	500.0	10.0 %	56.5 %	4.2 %	2.9 %	43.5 %	5.2 %	3.4 %
36	500.0	5.0 %	54.1 %	4.1 %	2.8 %	45.9 %	5.1 %	3.7 %
36	500.0	2.5 %	54.9 %	4.6 %	3.0 %	45.1 %	5.5 %	3.6 %
36	1000.0	100.0 %	55.6 %	5.7 %	3.5 %	44.4 %	5.9 %	3.7 %
36	1000.0	10.0 %	52.3 %	4.7 %	3.6 %	47.7 %	6.1 %	3.8 %
36	1000.0	5.0 %	54.9 %	4.7 %	3.1 %	45.1 %	6.0 %	3.6 %
36	1000.0	2.5 %	55.7 %	5.0 %	3.4 %	44.3 %	5.4 %	3.3 %
36	5000.0	100.0 %	46.3 %	3.7 %	2.4 %	53.7 %	4.9 %	3.5 %
36	5000.0	10.0 %	45.9 %	4.8 %	3.4 %	54.1 %	5.4 %	3.7 %
36	5000.0	5.0 %	45.5 %	4.8 %	3.8 %	54.5 %	3.7 %	2.8 %
36	5000.0	2.5 %	50.9 %	5.2 %	3.9 %	49.1 %	5.4 %	3.7 %
60	100.0	100.0 %	43.3 %	4.8 %	3.2 %	56.7 %	5.1 %	3.6 %
60	100.0	10.0 %	46.1 %	5.2 %	3.4 %	53.9 %	4.7 %	3.7 %

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Months	**Value**	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
60	100.0	5.0 %	44.4 %	5.3 %	3.3 %	55.6 %	4.9 %	3.6 %
60	100.0	2.5 %	42.8 %	4.7 %	3.4 %	57.2 %	5.3 %	3.7 %
60	300.0	100.0 %	43.3 %	4.3 %	3.0 %	56.7 %	4.9 %	3.2 %
60	300.0	10.0 %	42.8 %	4.4 %	3.0 %	57.2 %	5.0 %	3.5 %
60	300.0	5.0 %	37.8 %	4.2 %	2.5 %	62.2 %	5.5 %	3.5 %
60	300.0	2.5 %	33.9 %	4.1 %	2.5 %	66.1 %	5.8 %	3.6 %
60	500.0	100.0 %	52.2 %	4.0 %	3.2 %	47.8 %	4.4 %	3.3 %
60	500.0	10.0 %	47.2 %	4.0 %	3.3 %	52.8 %	4.7 %	3.6 %
60	500.0	5.0 %	38.4 %	4.0 %	2.8 %	61.6 %	4.9 %	3.5 %
60	500.0	2.5 %	43.9 %	4.4 %	3.2 %	56.1 %	5.3 %	3.5 %
60	1000.0	100.0 %	53.3 %	5.4 %	3.5 %	46.7 %	4.6 %	3.3 %
60	1000.0	10.0 %	42.9 %	4.5 %	3.3 %	57.1 %	4.9 %	3.3 %
60	1000.0	5.0 %	46.2 %	4.9 %	3.5 %	53.8 %	5.1 %	3.4 %
60	1000.0	2.5 %	49.4 %	5.6 %	3.5 %	50.6 %	5.1 %	3.5 %
60	5000.0	100.0 %	44.4 %	5.2 %	2.8 %	55.6 %	4.6 %	3.5 %
60	5000.0	10.0 %	46.9 %	5.0 %	3.1 %	53.1 %	5.5 %	3.8 %
60	5000.0	5.0 %	39.3 %	5.2 %	2.6 %	60.7 %	5.5 %	3.4 %
60	5000.0	2.5 %	56.2 %	4.5 %	2.8 %	43.8 %	7.0 %	4.1 %
84	100.0	100.0 %	45.6 %	4.1 %	3.0 %	54.4 %	4.7 %	4.0 %
84	100.0	10.0 %	48.9 %	4.5 %	3.1 %	51.1 %	4.5 %	3.7 %
84	100.0	5.0 %	51.1 %	4.8 %	3.1 %	48.9 %	4.0 %	3.6 %
84	100.0	2.5 %	52.8 %	5.5 %	3.7 %	47.2 %	4.0 %	3.6 %
84	300.0	100.0 %	50.0 %	4.6 %	3.3 %	50.0 %	4.3 %	3.5 %
84	300.0	10.0 %	52.8 %	5.6 %	3.7 %	47.2 %	3.7 %	3.1 %
84	300.0	5.0 %	49.4 %	5.6 %	3.5 %	50.6 %	3.6 %	2.6 %
84	300.0	2.5 %	46.0 %	5.7 %	3.9 %	54.0 %	4.9 %	3.5 %
84	500.0	100.0 %	38.9 %	4.3 %	3.1 %	61.1 %	4.7 %	3.7 %
84	500.0	10.0 %	46.1 %	5.7 %	3.5 %	53.9 %	4.2 %	3.1 %
84	500.0	5.0 %	44.3 %	5.5 %	3.6 %	55.7 %	4.4 %	3.4 %
84	500.0	2.5 %	48.1 %	6.0 %	4.1 %	51.9 %	5.0 %	2.9 %
84	1000.0	100.0 %	46.7 %	4.2 %	2.9 %	53.3 %	4.5 %	3.6 %
84	1000.0	10.0 %	42.6 %	5.5 %	3.8 %	57.4 %	4.7 %	3.8 %
84	1000.0	5.0 %	48.1 %	5.9 %	3.7 %	51.9 %	4.9 %	3.4 %
84	1000.0	2.5 %	47.7 %	5.6 %	4.1 %	52.3 %	5.5 %	3.6 %
84	5000.0	100.0 %	44.9 %	4.8 %	3.9 %	55.1 %	4.4 %	3.4 %
84	5000.0	10.0 %	51.2 %	6.3 %	4.5 %	48.8 %	5.5 %	3.6 %
84	5000.0	5.0 %	48.5 %	5.5 %	4.5 %	51.5 %	6.6 %	3.9 %
84	5000.0	2.5 %	31.0 %	6.7 %	4.3 %	69.0 %	6.9 %	3.9 %
120	100.0	100.0 %	68.3 %	5.4 %	3.5 %	31.7 %	4.8 %	3.0 %
120	100.0	10.0 %	68.9 %	5.7 %	3.9 %	31.1 %	4.1 %	2.7 %
120	100.0	5.0 %	66.5 %	5.6 %	3.7 %	33.5 %	4.9 %	3.2 %
120	100.0	2.5 %	49.4 %	5.8 %	4.0 %	50.6 %	5.3 %	3.0 %
120	300.0	100.0 %	64.0 %	5.1 %	3.5 %	36.0 %	4.7 %	3.1 %

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Months	Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
120	300.0	10.0 %	61.4 %	5.2 %	3.7 %	38.6 %	5.2 %	3.5 %
120	300.0	5.0 %	49.0 %	6.3 %	4.4 %	51.0 %	5.1 %	3.2 %
120	300.0	2.5 %	54.2 %	6.7 %	4.0 %	45.8 %	5.3 %	3.8 %
120	500.0	100.0 %	65.7 %	5.0 %	3.3 %	34.3 %	4.7 %	3.3 %
120	500.0	10.0 %	47.1 %	6.0 %	4.2 %	52.9 %	5.0 %	3.0 %
120	500.0	5.0 %	54.9 %	6.4 %	4.4 %	45.1 %	5.4 %	3.7 %
120	500.0	2.5 %	62.7 %	6.7 %	4.4 %	37.3 %	4.7 %	2.9 %
120	1000.0	100.0 %	66.1 %	5.6 %	3.9 %	33.9 %	3.8 %	3.0 %
120	1000.0	10.0 %	53.5 %	7.2 %	4.3 %	46.5 %	5.2 %	3.9 %
120	1000.0	5.0 %	56.8 %	7.7 %	4.3 %	43.2 %	4.5 %	3.2 %
120	1000.0	2.5 %	53.2 %	7.3 %	4.0 %	46.8 %	4.1 %	2.8 %
120	5000.0	100.0 %	41.4 %	5.8 %	4.3 %	58.6 %	5.2 %	3.1 %
120	5000.0	10.0 %	56.1 %	7.2 %	4.4 %	43.9 %	4.3 %	2.9 %
120	5000.0	5.0 %	55.3 %	6.7 %	4.0 %	44.7 %	6.1 %	4.3 %
120	5000.0	2.5 %	35.4 %	6.3 %	4.1 %	64.6 %	8.4 %	4.6 %

C.3 Inverted Bell Portfolios

Table 38 – Comparison of Portfolio and Hedge PV for 36, 60, 84 and 120-months invertedbell Portfolio of nominal values of 100MM BRL to 5000MM BRL and its Hedge. Both PV_p and PV_h are measured in the first day of the month.

Months	Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
36	100.0	100.0 %	67.2 %	13.9 %	6.5 %	32.8 %	6.3 %	4.8 %
36	100.0	10.0 %	66.7 %	13.4 %	6.0 %	33.3 %	5.8 %	4.3 %
36	100.0	5.0 %	67.2 %	12.9 %	6.1 %	32.8 %	6.2 %	4.4 %
36	100.0	2.5 %	66.7 %	13.2 %	6.3 %	33.3 %	6.0 %	4.3 %
36	300.0	100.0 %	67.2 %	13.3 %	6.1 %	32.8 %	6.7 %	4.9 %
36	300.0	10.0 %	66.1 %	12.8 %	5.7 %	33.9 %	6.4 %	4.4 %
36	300.0	5.0 %	66.7 %	13.2 %	6.5 %	33.3 %	6.5 %	4.6 %
36	300.0	2.5 %	67.2 %	12.4 %	7.0 %	32.8 %	6.9 %	4.5 %
36	500.0	100.0 %	68.3 %	14.6 %	6.2 %	31.7 %	5.8 %	2.5 %
36	500.0	10.0 %	67.8 %	15.0 %	6.6 %	32.2 %	5.8 %	2.6 %
36	500.0	5.0 %	67.8 %	14.1 %	6.8 %	32.2 %	6.2 %	2.9 %
36	500.0	2.5 %	68.2 %	13.3 %	7.0 %	31.8 %	6.5 %	2.6 %
36	1000.0	100.0 %	66.7 %	13.5 %	5.8 %	33.3 %	6.4 %	4.4 %
36	1000.0	10.0 %	67.2 %	12.5 %	6.8 %	32.8 %	6.7 %	4.7 %

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Months	Value	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
36	1000.0	5.0 %	67.0 %	12.6 %	6.9 %	33.0 %	6.8 %	5.1 %
36	1000.0	2.5 %	66.1 %	13.7 %	6.9 %	33.9 %	7.7 %	5.0 %
36	5000.0	100.0 %	66.7 %	12.9 %	6.5 %	33.3 %	6.8 %	4.4 %
36	5000.0	10.0 %	67.3 %	14.0 %	7.1 %	32.7 %	7.7 %	5.2 %
36	5000.0	5.0 %	64.2 %	14.6 %	7.1 %	35.8 %	6.9 %	5.3 %
36	5000.0	2.5 %	63.7 %	15.6 %	7.2 %	36.3 %	6.6 %	6.2 %
60	100.0	100.0 %	68.9 %	9.7 %	5.2 %	31.1 %	6.6 %	2.7 %
60	100.0	10.0 %	67.8 %	9.5 %	5.0 %	32.2 %	6.5 %	2.5 %
60	100.0	5.0 %	68.3 %	9.8 %	4.9 %	31.7 %	6.3 %	2.3 %
60	100.0	2.5 %	68.0 %	10.0 %	5.0 %	32.0 %	6.5 %	2.8 %
60	300.0	100.0 %	67.2 %	9.4 %	4.9 %	32.8 %	6.7 %	2.7 %
60	300.0	10.0 %	67.6 %	9.6 %	5.0 %	32.4 %	6.2 %	2.7 %
60	300.0	5.0 %	66.7 %	10.0 %	4.8 %	33.3 %	6.0 %	2.8 %
60	300.0	2.5 %	65.1 %	10.6 %	5.4 %	34.9 %	5.7 %	3.0 %
60	500.0	100.0 %	64.4 %	9.9 %	5.2 %	35.6 %	4.7 %	2.7 %
60	500.0	10.0 %	64.4 %	10.4 %	5.3 %	35.6 %	4.6 %	2.8 %
60	500.0	5.0 %	64.5 %	10.0 %	5.7 %	35.5 %	4.5 %	2.8 %
60	500.0	2.5 %	66.0 %	10.3 %	6.1 %	34.0 %	4.9 %	2.6 %
60	1000.0	100.0 %	67.8 %	10.4 %	5.0 %	32.2 %	4.4 %	2.6 %
60	1000.0	10.0 %	68.6 %	10.4 %	5.3 %	31.4 %	4.6 %	2.7 %
60	1000.0	5.0 %	68.8 %	10.4 %	5.5 %	31.2 %	4.9 %	2.8 %
60	1000.0	2.5 %	65.7 %	11.2 %	5.9 %	34.3 %	4.2 %	2.7 %
60	5000.0	100.0 %	66.7 %	9.7 %	4.8 %	33.3 %	6.9 %	3.0 %
60	5000.0	10.0 %	63.4 %	11.4 %	5.5 %	36.6 %	5.9 %	3.1 %
60	5000.0	5.0 %	61.9 %	10.9 %	5.5 %	38.1 %	6.3 %	3.1 %
60	5000.0	2.5 %	62.0 %	10.6 %	5.0 %	38.0 %	7.0 %	3.9 %
84	100.0	100.0 %	64.4 %	7.4 %	4.0 %	35.6 %	3.8 %	2.7 %
84	100.0	10.0 %	62.6 %	6.8 %	4.0 %	37.4 %	4.0 %	2.9 %
84	100.0	5.0 %	59.3 %	6.5 %	4.1 %	40.7 %	4.1 %	3.2 %
84	100.0	2.5 %	55.4 %	6.7 %	4.1 %	44.6 %	4.9 %	3.7 %
84	300.0	100.0 %	65.0 %	7.2 %	4.4 %	35.0 %	4.0 %	2.7 %
84	300.0	10.0 %	57.9 %	7.1 %	4.5 %	42.1 %	4.9 %	3.2 %
84	300.0	5.0 %	57.0 %	7.5 %	4.5 %	43.0 %	4.8 %	3.5 %
84	300.0	2.5 %	58.5 %	7.4 %	4.4 %	41.5 %	4.6 %	3.4 %
84	500.0	100.0 %	65.6 %	7.4 %	4.1 %	34.4 %	3.9 %	2.5 %
84	500.0	10.0 %	61.3 %	7.3 %	4.7 %	38.7 %	4.8 %	3.4 %
84	500.0	5.0 %	65.0 %	7.4 %	4.8 %	35.0 %	4.7 %	3.3 %
84	500.0	2.5 %	63.7 %	8.0 %	4.6 %	36.3 %	4.2 %	3.3 %
84	1000.0	100.0 %	67.6 %	8.3 %	4.4 %	32.4 %	4.6 %	2.9 %
84	1000.0	10.0 %	67.9 %	8.2 %	5.1 %	32.1 %	4.6 %	3.4 %
84	1000.0	5.0 %	68.1 %	8.6 %	5.1 %	31.9 %	4.2 %	3.2 %
84	1000.0	2.5 %	63.6 %	8.5 %	4.6 %	36.4 %	4.7 %	3.6 %
84	5000.0	100.0 %	58.3 %	7.3 %	4.4 %	41.7 %	6.5 %	3.8 %

Continued on next page

Months	**Value**	L_i	$PV_p > PV_h$			$PV_p < PV_h$		
			% Cases	**Avg Δ**	**Std Δ**	**% Cases**	**Avg Δ**	**Std Δ**
84	5000.0	10.0 %	61.5 %	7.7 %	4.4 %	38.5 %	6.2 %	3.5 %
84	5000.0	5.0 %	58.4 %	8.1 %	4.5 %	41.6 %	6.3 %	4.2 %
84	5000.0	2.5 %	53.1 %	6.8 %	4.4 %	46.9 %	7.0 %	5.2 %
120	100.0	100.0 %	58.9 %	5.0 %	3.5 %	41.1 %	3.6 %	2.1 %
120	100.0	10.0 %	55.7 %	4.8 %	3.1 %	44.3 %	3.5 %	2.2 %
120	100.0	5.0 %	56.5 %	4.7 %	3.0 %	43.5 %	3.5 %	2.3 %
120	100.0	2.5 %	60.8 %	5.1 %	3.1 %	39.2 %	3.5 %	2.4 %
120	300.0	100.0 %	61.8 %	5.1 %	3.3 %	38.2 %	4.0 %	2.2 %
120	300.0	10.0 %	64.1 %	5.6 %	3.5 %	35.9 %	3.5 %	2.2 %
120	300.0	5.0 %	64.8 %	5.6 %	3.4 %	35.2 %	3.6 %	2.3 %
120	300.0	2.5 %	60.7 %	5.9 %	3.3 %	39.3 %	3.6 %	2.6 %
120	500.0	100.0 %	64.0 %	6.4 %	3.8 %	36.0 %	3.6 %	2.2 %
120	500.0	10.0 %	65.5 %	6.7 %	3.8 %	34.5 %	3.3 %	2.5 %
120	500.0	5.0 %	66.9 %	6.6 %	3.8 %	33.1 %	3.4 %	2.6 %
120	500.0	2.5 %	63.3 %	5.7 %	3.5 %	36.7 %	4.1 %	2.9 %
120	1000.0	100.0 %	62.5 %	6.0 %	3.6 %	37.5 %	3.5 %	2.0 %
120	1000.0	10.0 %	63.6 %	6.6 %	3.7 %	36.4 %	3.4 %	2.4 %
120	1000.0	5.0 %	61.2 %	5.4 %	3.4 %	38.8 %	4.1 %	2.7 %
120	1000.0	2.5 %	55.6 %	5.2 %	3.8 %	44.4 %	5.1 %	3.0 %
120	5000.0	100.0 %	62.4 %	6.8 %	3.2 %	37.6 %	5.0 %	3.1 %
120	5000.0	10.0 %	56.6 %	4.4 %	3.1 %	43.4 %	6.5 %	3.4 %
120	5000.0	5.0 %	47.5 %	4.3 %	2.5 %	52.5 %	6.6 %	3.6 %
120	5000.0	2.5 %	66.7 %	4.1 %	2.1 %	33.3 %	6.2 %	2.5 %

APPENDIX D – Complete Results of Allocation of securities whithin Key Rate Duration Buckets

D.1 Uniform Portfolios

Table 39 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy uniform Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	100.0 %	1	100.0 %	100.0 %	1
100.0	100.0 %	2	100.0 %	100.0 %	1
100.0	100.0 %	3	100.0 %	100.0 %	1
100.0	100.0 %	4	100.0 %	100.0 %	1
100.0	100.0 %	5	100.0 %	100.0 %	1
100.0	100.0 %	6	100.0 %	100.0 %	1
100.0	100.0 %	7	100.0 %	100.0 %	1
100.0	100.0 %	8	100.0 %	0.0 %	0
100.0	100.0 %	9	100.0 %	0.0 %	0
100.0	10.0 %	1	100.0 %	100.0 %	1
100.0	10.0 %	2	100.0 %	100.0 %	1
100.0	10.0 %	3	100.0 %	100.0 %	1
100.0	10.0 %	4	100.0 %	100.0 %	1
100.0	10.0 %	5	100.0 %	100.0 %	1
100.0	10.0 %	6	100.0 %	100.0 %	1
100.0	10.0 %	7	100.0 %	100.0 %	1
100.0	10.0 %	8	100.0 %	0.0 %	0
100.0	10.0 %	9	100.0 %	0.0 %	0
100.0	5.0 %	1	100.0 %	100.0 %	1
100.0	5.0 %	2	100.0 %	100.0 %	1
100.0	5.0 %	3	100.0 %	100.0 %	1
100.0	5.0 %	4	100.0 %	100.0 %	1
100.0	5.0 %	5	100.0 %	100.0 %	1
100.0	5.0 %	6	100.0 %	100.0 %	1
100.0	5.0 %	7	100.0 %	100.0 %	1
100.0	5.0 %	8	100.0 %	0.0 %	0
100.0	5.0 %	9	100.0 %	0.0 %	0
100.0	2.5 %	1	100.0 %	100.0 %	1
100.0	2.5 %	2	100.0 %	100.0 %	1
100.0	2.5 %	3	100.0 %	100.0 %	1
100.0	2.5 %	4	100.0 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	2.5 %	5	100.0 %	100.0 %	1
100.0	2.5 %	6	100.0 %	99.4 %	2
100.0	2.5 %	7	100.0 %	99.4 %	2
100.0	2.5 %	8	100.0 %	0.0 %	0
100.0	2.5 %	9	100.0 %	0.0 %	0
300.0	100.0 %	1	100.0 %	100.0 %	1
300.0	100.0 %	2	100.0 %	100.0 %	1
300.0	100.0 %	3	100.0 %	100.0 %	1
300.0	100.0 %	4	100.0 %	99.4 %	2
300.0	100.0 %	5	100.0 %	100.0 %	1
300.0	100.0 %	6	100.0 %	100.0 %	1
300.0	100.0 %	7	100.0 %	100.0 %	1
300.0	100.0 %	8	100.0 %	0.0 %	0
300.0	100.0 %	9	100.0 %	0.0 %	0
300.0	10.0 %	1	100.0 %	100.0 %	1
300.0	10.0 %	2	100.0 %	100.0 %	1
300.0	10.0 %	3	100.0 %	100.0 %	1
300.0	10.0 %	4	100.0 %	98.3 %	2
300.0	10.0 %	5	100.0 %	100.0 %	1
300.0	10.0 %	6	100.0 %	100.0 %	1
300.0	10.0 %	7	100.0 %	100.0 %	1
300.0	10.0 %	8	100.0 %	0.0 %	0
300.0	10.0 %	9	100.0 %	0.0 %	0
300.0	5.0 %	1	100.0 %	100.0 %	1
300.0	5.0 %	2	100.0 %	100.0 %	1
300.0	5.0 %	3	100.0 %	100.0 %	1
300.0	5.0 %	4	100.0 %	98.3 %	2
300.0	5.0 %	5	100.0 %	100.0 %	1
300.0	5.0 %	6	100.0 %	98.3 %	2
300.0	5.0 %	7	100.0 %	96.7 %	2
300.0	5.0 %	8	100.0 %	0.0 %	0
300.0	5.0 %	9	100.0 %	0.0 %	0
300.0	2.5 %	1	96.7 %	100.0 %	1
300.0	2.5 %	2	96.7 %	100.0 %	1
300.0	2.5 %	3	96.7 %	100.0 %	1
300.0	2.5 %	4	96.7 %	98.9 %	2
300.0	2.5 %	5	96.7 %	100.0 %	1
300.0	2.5 %	6	96.7 %	100.0 %	1
300.0	2.5 %	7	96.7 %	93.7 %	3
300.0	2.5 %	8	96.7 %	0.0 %	0
300.0	2.5 %	9	96.7 %	0.0 %	0
500.0	100.0 %	1	100.0 %	100.0 %	1
500.0	100.0 %	2	100.0 %	100.0 %	1
500.0	100.0 %	3	100.0 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
500.0	100.0 %	4	100.0 %	99.4 %	2
500.0	100.0 %	5	100.0 %	100.0 %	1
500.0	100.0 %	6	100.0 %	100.0 %	1
500.0	100.0 %	7	100.0 %	100.0 %	1
500.0	100.0 %	8	100.0 %	0.0 %	0
500.0	100.0 %	9	100.0 %	0.0 %	0
500.0	10.0 %	1	100.0 %	100.0 %	1
500.0	10.0 %	2	100.0 %	100.0 %	1
500.0	10.0 %	3	100.0 %	100.0 %	1
500.0	10.0 %	4	100.0 %	99.4 %	2
500.0	10.0 %	5	100.0 %	100.0 %	1
500.0	10.0 %	6	100.0 %	99.4 %	2
500.0	10.0 %	7	100.0 %	98.3 %	2
500.0	10.0 %	8	100.0 %	0.0 %	0
500.0	10.0 %	9	100.0 %	0.0 %	0
500.0	5.0 %	1	98.3 %	100.0 %	1
500.0	5.0 %	2	98.3 %	100.0 %	1
500.0	5.0 %	3	98.3 %	100.0 %	1
500.0	5.0 %	4	98.3 %	98.3 %	2
500.0	5.0 %	5	98.3 %	100.0 %	1
500.0	5.0 %	6	98.3 %	98.3 %	3
500.0	5.0 %	7	98.3 %	93.8 %	3
500.0	5.0 %	8	98.3 %	0.0 %	0
500.0	5.0 %	9	98.3 %	0.0 %	0
500.0	2.5 %	1	92.2 %	100.0 %	1
500.0	2.5 %	2	92.2 %	100.0 %	1
500.0	2.5 %	3	92.2 %	100.0 %	1
500.0	2.5 %	4	92.2 %	99.4 %	2
500.0	2.5 %	5	92.2 %	99.4 %	2
500.0	2.5 %	6	92.2 %	97.6 %	2
500.0	2.5 %	7	92.2 %	95.2 %	3
500.0	2.5 %	8	92.2 %	0.0 %	0
500.0	2.5 %	9	92.2 %	0.0 %	0
1000.0	100.0 %	1	100.0 %	100.0 %	1
1000.0	100.0 %	2	100.0 %	100.0 %	1
1000.0	100.0 %	3	100.0 %	100.0 %	1
1000.0	100.0 %	4	100.0 %	100.0 %	1
1000.0	100.0 %	5	100.0 %	100.0 %	1
1000.0	100.0 %	6	100.0 %	100.0 %	1
1000.0	100.0 %	7	100.0 %	100.0 %	1
1000.0	100.0 %	8	100.0 %	0.0 %	0
1000.0	100.0 %	9	100.0 %	0.0 %	0
1000.0	10.0 %	1	98.3 %	100.0 %	1
1000.0	10.0 %	2	98.3 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	10.0 %	3	98.3 %	100.0 %	1
1000.0	10.0 %	4	98.3 %	98.9 %	2
1000.0	10.0 %	5	98.3 %	100.0 %	1
1000.0	10.0 %	6	98.3 %	98.3 %	3
1000.0	10.0 %	7	98.3 %	93.8 %	3
1000.0	10.0 %	8	98.3 %	0.0 %	0
1000.0	10.0 %	9	98.3 %	0.0 %	0
1000.0	5.0 %	1	92.2 %	100.0 %	1
1000.0	5.0 %	2	92.2 %	100.0 %	1
1000.0	5.0 %	3	92.2 %	100.0 %	1
1000.0	5.0 %	4	92.2 %	100.0 %	1
1000.0	5.0 %	5	92.2 %	99.4 %	2
1000.0	5.0 %	6	92.2 %	97.6 %	2
1000.0	5.0 %	7	92.2 %	95.2 %	3
1000.0	5.0 %	8	92.2 %	0.0 %	0
1000.0	5.0 %	9	92.2 %	0.0 %	0
1000.0	2.5 %	1	86.7 %	100.0 %	1
1000.0	2.5 %	2	86.7 %	100.0 %	1
1000.0	2.5 %	3	86.7 %	100.0 %	1
1000.0	2.5 %	4	86.7 %	99.4 %	2
1000.0	2.5 %	5	86.7 %	98.1 %	2
1000.0	2.5 %	6	86.7 %	95.5 %	3
1000.0	2.5 %	7	86.7 %	94.9 %	3
1000.0	2.5 %	8	86.7 %	0.0 %	0
1000.0	2.5 %	9	86.7 %	0.0 %	0
5000.0	100.0 %	1	100.0 %	100.0 %	1
5000.0	100.0 %	2	100.0 %	100.0 %	1
5000.0	100.0 %	3	100.0 %	100.0 %	1
5000.0	100.0 %	4	100.0 %	100.0 %	1
5000.0	100.0 %	5	100.0 %	99.4 %	2
5000.0	100.0 %	6	100.0 %	99.4 %	2
5000.0	100.0 %	7	100.0 %	98.3 %	2
5000.0	100.0 %	8	100.0 %	0.0 %	0
5000.0	100.0 %	9	100.0 %	0.0 %	0
5000.0	10.0 %	1	79.4 %	100.0 %	1
5000.0	10.0 %	2	79.4 %	100.0 %	1
5000.0	10.0 %	3	79.4 %	100.0 %	1
5000.0	10.0 %	4	79.4 %	99.3 %	2
5000.0	10.0 %	5	79.4 %	99.3 %	2
5000.0	10.0 %	6	79.4 %	92.3 %	3
5000.0	10.0 %	7	79.4 %	93.7 %	5
5000.0	10.0 %	8	79.4 %	0.0 %	0
5000.0	10.0 %	9	79.4 %	0.0 %	0
5000.0	5.0 %	1	60.6 %	100.0 %	1

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
5000.0	5.0 %	2	60.6 %	100.0 %	1
5000.0	5.0 %	3	60.6 %	100.0 %	1
5000.0	5.0 %	4	60.6 %	100.0 %	1
5000.0	5.0 %	5	60.6 %	96.3 %	2
5000.0	5.0 %	6	60.6 %	94.5 %	2
5000.0	5.0 %	7	60.6 %	94.5 %	3
5000.0	5.0 %	8	60.6 %	0.0 %	0
5000.0	5.0 %	9	60.6 %	0.0 %	0
5000.0	2.5 %	1	43.3 %	98.7 %	2
5000.0	2.5 %	2	43.3 %	100.0 %	1
5000.0	2.5 %	3	43.3 %	100.0 %	1
5000.0	2.5 %	4	43.3 %	98.7 %	2
5000.0	2.5 %	5	43.3 %	78.2 %	2
5000.0	2.5 %	6	43.3 %	97.4 %	2
5000.0	2.5 %	7	43.3 %	87.2 %	6
5000.0	2.5 %	8	43.3 %	0.0 %	0
5000.0	2.5 %	9	43.3 %	0.0 %	0

Table 40 – Summary of Unique Contracts Acquired for the 60-Month Hedging Strategy uniform Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	100.0 %	1	100.0 %	100.0 %	1
100.0	100.0 %	2	100.0 %	100.0 %	1
100.0	100.0 %	3	100.0 %	100.0 %	1
100.0	100.0 %	4	100.0 %	100.0 %	1
100.0	100.0 %	5	100.0 %	100.0 %	1
100.0	100.0 %	6	100.0 %	100.0 %	1
100.0	100.0 %	7	100.0 %	100.0 %	1
100.0	100.0 %	8	100.0 %	98.9 %	2
100.0	100.0 %	9	100.0 %	0.0 %	0
100.0	10.0 %	1	100.0 %	100.0 %	1
100.0	10.0 %	2	100.0 %	100.0 %	1
100.0	10.0 %	3	100.0 %	100.0 %	1
100.0	10.0 %	4	100.0 %	100.0 %	1
100.0	10.0 %	5	100.0 %	100.0 %	1
100.0	10.0 %	6	100.0 %	100.0 %	1
100.0	10.0 %	7	100.0 %	100.0 %	1
100.0	10.0 %	8	100.0 %	100.0 %	1

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	10.0 %	9	100.0 %	0.0 %	0
100.0	5.0 %	1	100.0 %	100.0 %	1
100.0	5.0 %	2	100.0 %	100.0 %	1
100.0	5.0 %	3	100.0 %	100.0 %	1
100.0	5.0 %	4	100.0 %	100.0 %	1
100.0	5.0 %	5	100.0 %	100.0 %	1
100.0	5.0 %	6	100.0 %	100.0 %	1
100.0	5.0 %	7	100.0 %	100.0 %	1
100.0	5.0 %	8	100.0 %	100.0 %	1
100.0	5.0 %	9	100.0 %	0.0 %	0
100.0	2.5 %	1	100.0 %	100.0 %	1
100.0	2.5 %	2	100.0 %	100.0 %	1
100.0	2.5 %	3	100.0 %	100.0 %	1
100.0	2.5 %	4	100.0 %	100.0 %	1
100.0	2.5 %	5	100.0 %	100.0 %	1
100.0	2.5 %	6	100.0 %	100.0 %	1
100.0	2.5 %	7	100.0 %	100.0 %	1
100.0	2.5 %	8	100.0 %	97.8 %	3
100.0	2.5 %	9	100.0 %	0.0 %	0
300.0	100.0 %	1	100.0 %	100.0 %	1
300.0	100.0 %	2	100.0 %	100.0 %	1
300.0	100.0 %	3	100.0 %	100.0 %	1
300.0	100.0 %	4	100.0 %	95.6 %	2
300.0	100.0 %	5	100.0 %	100.0 %	1
300.0	100.0 %	6	100.0 %	100.0 %	1
300.0	100.0 %	7	100.0 %	100.0 %	1
300.0	100.0 %	8	100.0 %	100.0 %	1
300.0	100.0 %	9	100.0 %	0.0 %	0
300.0	10.0 %	1	100.0 %	100.0 %	1
300.0	10.0 %	2	100.0 %	100.0 %	1
300.0	10.0 %	3	100.0 %	100.0 %	1
300.0	10.0 %	4	100.0 %	98.3 %	2
300.0	10.0 %	5	100.0 %	100.0 %	1
300.0	10.0 %	6	100.0 %	100.0 %	1
300.0	10.0 %	7	100.0 %	100.0 %	1
300.0	10.0 %	8	100.0 %	100.0 %	1
300.0	10.0 %	9	100.0 %	0.0 %	0
300.0	5.0 %	1	98.9 %	100.0 %	1
300.0	5.0 %	2	98.9 %	100.0 %	1
300.0	5.0 %	3	98.9 %	100.0 %	1
300.0	5.0 %	4	98.9 %	98.9 %	2
300.0	5.0 %	5	98.9 %	100.0 %	1
300.0	5.0 %	6	98.9 %	99.4 %	2
300.0	5.0 %	7	98.9 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	5.0 %	8	98.9 %	97.2 %	4
300.0	5.0 %	9	98.9 %	0.0 %	0
300.0	2.5 %	1	98.3 %	100.0 %	1
300.0	2.5 %	2	98.3 %	100.0 %	1
300.0	2.5 %	3	98.3 %	100.0 %	1
300.0	2.5 %	4	98.3 %	97.2 %	2
300.0	2.5 %	5	98.3 %	100.0 %	1
300.0	2.5 %	6	98.3 %	97.7 %	2
300.0	2.5 %	7	98.3 %	98.9 %	2
300.0	2.5 %	8	98.3 %	88.7 %	7
300.0	2.5 %	9	98.3 %	0.0 %	0
500.0	100.0 %	1	100.0 %	100.0 %	1
500.0	100.0 %	2	100.0 %	100.0 %	1
500.0	100.0 %	3	100.0 %	100.0 %	1
500.0	100.0 %	4	100.0 %	99.4 %	2
500.0	100.0 %	5	100.0 %	100.0 %	1
500.0	100.0 %	6	100.0 %	100.0 %	1
500.0	100.0 %	7	100.0 %	100.0 %	1
500.0	100.0 %	8	100.0 %	100.0 %	1
500.0	100.0 %	9	100.0 %	0.0 %	0
500.0	10.0 %	1	100.0 %	100.0 %	1
500.0	10.0 %	2	100.0 %	100.0 %	1
500.0	10.0 %	3	100.0 %	100.0 %	1
500.0	10.0 %	4	100.0 %	98.9 %	2
500.0	10.0 %	5	100.0 %	100.0 %	1
500.0	10.0 %	6	100.0 %	100.0 %	1
500.0	10.0 %	7	100.0 %	100.0 %	1
500.0	10.0 %	8	100.0 %	96.7 %	5
500.0	10.0 %	9	100.0 %	0.0 %	0
500.0	5.0 %	1	98.3 %	100.0 %	1
500.0	5.0 %	2	98.3 %	100.0 %	1
500.0	5.0 %	3	98.3 %	100.0 %	1
500.0	5.0 %	4	98.3 %	99.4 %	2
500.0	5.0 %	5	98.3 %	99.4 %	2
500.0	5.0 %	6	98.3 %	98.3 %	2
500.0	5.0 %	7	98.3 %	100.0 %	1
500.0	5.0 %	8	98.3 %	90.4 %	5
500.0	5.0 %	9	98.3 %	0.0 %	0
500.0	2.5 %	1	91.1 %	100.0 %	1
500.0	2.5 %	2	91.1 %	100.0 %	1
500.0	2.5 %	3	91.1 %	100.0 %	1
500.0	2.5 %	4	91.1 %	98.8 %	2
500.0	2.5 %	5	91.1 %	100.0 %	1
500.0	2.5 %	6	91.1 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
500.0	2.5 %	7	91.1 %	98.2 %	3
500.0	2.5 %	8	91.1 %	79.3 %	4
500.0	2.5 %	9	91.1 %	0.0 %	0
1000.0	100.0 %	1	100.0 %	100.0 %	1
1000.0	100.0 %	2	100.0 %	100.0 %	1
1000.0	100.0 %	3	100.0 %	100.0 %	1
1000.0	100.0 %	4	100.0 %	99.4 %	2
1000.0	100.0 %	5	100.0 %	100.0 %	1
1000.0	100.0 %	6	100.0 %	100.0 %	1
1000.0	100.0 %	7	100.0 %	100.0 %	1
1000.0	100.0 %	8	100.0 %	100.0 %	1
1000.0	100.0 %	9	100.0 %	0.0 %	0
1000.0	10.0 %	1	98.3 %	100.0 %	1
1000.0	10.0 %	2	98.3 %	100.0 %	1
1000.0	10.0 %	3	98.3 %	100.0 %	1
1000.0	10.0 %	4	98.3 %	98.9 %	2
1000.0	10.0 %	5	98.3 %	99.4 %	2
1000.0	10.0 %	6	98.3 %	98.3 %	2
1000.0	10.0 %	7	98.3 %	100.0 %	1
1000.0	10.0 %	8	98.3 %	90.4 %	5
1000.0	10.0 %	9	98.3 %	0.0 %	0
1000.0	5.0 %	1	91.1 %	100.0 %	1
1000.0	5.0 %	2	91.1 %	100.0 %	1
1000.0	5.0 %	3	91.1 %	100.0 %	1
1000.0	5.0 %	4	91.1 %	99.4 %	2
1000.0	5.0 %	5	91.1 %	100.0 %	1
1000.0	5.0 %	6	91.1 %	100.0 %	1
1000.0	5.0 %	7	91.1 %	98.2 %	3
1000.0	5.0 %	8	91.1 %	79.3 %	4
1000.0	5.0 %	9	91.1 %	0.0 %	0
1000.0	2.5 %	1	75.6 %	100.0 %	1
1000.0	2.5 %	2	75.6 %	100.0 %	1
1000.0	2.5 %	3	75.6 %	100.0 %	1
1000.0	2.5 %	4	75.6 %	100.0 %	1
1000.0	2.5 %	5	75.6 %	98.5 %	2
1000.0	2.5 %	6	75.6 %	97.1 %	2
1000.0	2.5 %	7	75.6 %	100.0 %	1
1000.0	2.5 %	8	75.6 %	64.0 %	8
1000.0	2.5 %	9	75.6 %	0.0 %	0
5000.0	100.0 %	1	100.0 %	100.0 %	1
5000.0	100.0 %	2	100.0 %	100.0 %	1
5000.0	100.0 %	3	100.0 %	100.0 %	1
5000.0	100.0 %	4	100.0 %	100.0 %	1
5000.0	100.0 %	5	100.0 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
5000.0	100.0 %	6	100.0 %	100.0 %	1
5000.0	100.0 %	7	100.0 %	100.0 %	1
5000.0	100.0 %	8	100.0 %	96.7 %	5
5000.0	100.0 %	9	100.0 %	0.0 %	0
5000.0	10.0 %	1	68.9 %	100.0 %	1
5000.0	10.0 %	2	68.9 %	100.0 %	1
5000.0	10.0 %	3	68.9 %	100.0 %	1
5000.0	10.0 %	4	68.9 %	100.0 %	1
5000.0	10.0 %	5	68.9 %	97.6 %	2
5000.0	10.0 %	6	68.9 %	93.5 %	2
5000.0	10.0 %	7	68.9 %	98.4 %	2
5000.0	10.0 %	8	68.9 %	66.1 %	7
5000.0	10.0 %	9	68.9 %	0.0 %	0
5000.0	5.0 %	1	47.8 %	100.0 %	1
5000.0	5.0 %	2	47.8 %	100.0 %	1
5000.0	5.0 %	3	47.8 %	100.0 %	1
5000.0	5.0 %	4	47.8 %	97.7 %	2
5000.0	5.0 %	5	47.8 %	100.0 %	1
5000.0	5.0 %	6	47.8 %	96.5 %	3
5000.0	5.0 %	7	47.8 %	94.2 %	4
5000.0	5.0 %	8	47.8 %	75.6 %	6
5000.0	5.0 %	9	47.8 %	0.0 %	0
5000.0	2.5 %	1	37.2 %	100.0 %	1
5000.0	2.5 %	2	37.2 %	100.0 %	1
5000.0	2.5 %	3	37.2 %	100.0 %	1
5000.0	2.5 %	4	37.2 %	97.0 %	3
5000.0	2.5 %	5	37.2 %	100.0 %	1
5000.0	2.5 %	6	37.2 %	98.5 %	2
5000.0	2.5 %	7	37.2 %	77.6 %	3
5000.0	2.5 %	8	37.2 %	82.1 %	5
5000.0	2.5 %	9	37.2 %	0.0 %	0

Table 41 – Summary of Unique Contracts Acquired for the 84-Month Hedging Strategy uniform Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	100.0 %	1.0
100.0	100.0 %	2	100.0 %	100.0 %	1.0
100.0	100.0 %	3	100.0 %	100.0 %	1.0
100.0	100.0 %	4	100.0 %	100.0 %	1.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	100.0 %	5	100.0 %	100.0 %	1.0
100.0	100.0 %	6	100.0 %	100.0 %	1.0
100.0	100.0 %	7	100.0 %	100.0 %	1.0
100.0	100.0 %	8	100.0 %	100.0 %	1.0
100.0	100.0 %	9	100.0 %	100.0 %	1.0
100.0	10.0 %	1	100.0 %	100.0 %	1.0
100.0	10.0 %	2	100.0 %	100.0 %	1.0
100.0	10.0 %	3	100.0 %	100.0 %	1.0
100.0	10.0 %	4	100.0 %	100.0 %	1.0
100.0	10.0 %	5	100.0 %	100.0 %	1.0
100.0	10.0 %	6	100.0 %	100.0 %	1.0
100.0	10.0 %	7	100.0 %	100.0 %	1.0
100.0	10.0 %	8	100.0 %	98.3 %	2.0
100.0	10.0 %	9	100.0 %	100.0 %	1.0
100.0	5.0 %	1	100.0 %	100.0 %	1.0
100.0	5.0 %	2	100.0 %	100.0 %	1.0
100.0	5.0 %	3	100.0 %	100.0 %	1.0
100.0	5.0 %	4	100.0 %	100.0 %	1.0
100.0	5.0 %	5	100.0 %	100.0 %	1.0
100.0	5.0 %	6	100.0 %	100.0 %	1.0
100.0	5.0 %	7	100.0 %	100.0 %	1.0
100.0	5.0 %	8	100.0 %	91.7 %	2.0
100.0	5.0 %	9	100.0 %	98.8 %	2.0
100.0	2.5 %	1	98.3 %	100.0 %	1.0
100.0	2.5 %	2	98.3 %	100.0 %	1.0
100.0	2.5 %	3	98.3 %	100.0 %	1.0
100.0	2.5 %	4	98.3 %	100.0 %	1.0
100.0	2.5 %	5	98.3 %	100.0 %	1.0
100.0	2.5 %	6	98.3 %	100.0 %	1.0
100.0	2.5 %	7	98.3 %	100.0 %	1.0
100.0	2.5 %	8	98.3 %	88.7 %	3.0
100.0	2.5 %	9	98.3 %	97.4 %	2.0
300.0	100.0 %	1	100.0 %	100.0 %	1.0
300.0	100.0 %	2	100.0 %	100.0 %	1.0
300.0	100.0 %	3	100.0 %	100.0 %	1.0
300.0	100.0 %	4	100.0 %	98.3 %	2.0
300.0	100.0 %	5	100.0 %	100.0 %	1.0
300.0	100.0 %	6	100.0 %	100.0 %	1.0
300.0	100.0 %	7	100.0 %	100.0 %	1.0
300.0	100.0 %	8	100.0 %	100.0 %	1.0
300.0	100.0 %	9	100.0 %	100.0 %	1.0
300.0	10.0 %	1	99.4 %	100.0 %	1.0
300.0	10.0 %	2	99.4 %	100.0 %	1.0
300.0	10.0 %	3	99.4 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	10.0 %	4	99.4 %	98.3 %	2.0
300.0	10.0 %	5	99.4 %	100.0 %	1.0
300.0	10.0 %	6	99.4 %	100.0 %	1.0
300.0	10.0 %	7	99.4 %	100.0 %	1.0
300.0	10.0 %	8	99.4 %	91.1 %	3.0
300.0	10.0 %	9	99.4 %	100.0 %	1.0
300.0	5.0 %	1	97.2 %	100.0 %	1.0
300.0	5.0 %	2	97.2 %	100.0 %	1.0
300.0	5.0 %	3	97.2 %	100.0 %	1.0
300.0	5.0 %	4	97.2 %	98.9 %	2.0
300.0	5.0 %	5	97.2 %	100.0 %	1.0
300.0	5.0 %	6	97.2 %	100.0 %	1.0
300.0	5.0 %	7	97.2 %	100.0 %	1.0
300.0	5.0 %	8	97.2 %	78.9 %	4.0
300.0	5.0 %	9	97.2 %	90.8 %	3.0
300.0	2.5 %	1	88.9 %	100.0 %	1.0
300.0	2.5 %	2	88.9 %	100.0 %	1.0
300.0	2.5 %	3	88.9 %	100.0 %	1.0
300.0	2.5 %	4	88.9 %	98.8 %	2.0
300.0	2.5 %	5	88.9 %	100.0 %	1.0
300.0	2.5 %	6	88.9 %	100.0 %	1.0
300.0	2.5 %	7	88.9 %	100.0 %	1.0
300.0	2.5 %	8	88.9 %	61.9 %	5.0
300.0	2.5 %	9	88.9 %	83.6 %	3.0
500.0	100.0 %	1	100.0 %	100.0 %	1.0
500.0	100.0 %	2	100.0 %	100.0 %	1.0
500.0	100.0 %	3	100.0 %	100.0 %	1.0
500.0	100.0 %	4	100.0 %	97.2 %	2.0
500.0	100.0 %	5	100.0 %	100.0 %	1.0
500.0	100.0 %	6	100.0 %	100.0 %	1.0
500.0	100.0 %	7	100.0 %	100.0 %	1.0
500.0	100.0 %	8	100.0 %	100.0 %	1.0
500.0	100.0 %	9	100.0 %	100.0 %	1.0
500.0	10.0 %	1	97.8 %	100.0 %	1.0
500.0	10.0 %	2	97.8 %	100.0 %	1.0
500.0	10.0 %	3	97.8 %	100.0 %	1.0
500.0	10.0 %	4	97.8 %	96.6 %	2.0
500.0	10.0 %	5	97.8 %	100.0 %	1.0
500.0	10.0 %	6	97.8 %	100.0 %	1.0
500.0	10.0 %	7	97.8 %	100.0 %	1.0
500.0	10.0 %	8	97.8 %	83.0 %	3.0
500.0	10.0 %	9	97.8 %	94.8 %	2.0
500.0	5.0 %	1	95.0 %	100.0 %	1.0
500.0	5.0 %	2	95.0 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
500.0	5.0 %	3	95.0 %	100.0 %	1.0
500.0	5.0 %	4	95.0 %	98.8 %	2.0
500.0	5.0 %	5	95.0 %	100.0 %	1.0
500.0	5.0 %	6	95.0 %	99.4 %	2.0
500.0	5.0 %	7	95.0 %	100.0 %	1.0
500.0	5.0 %	8	95.0 %	60.8 %	4.0
500.0	5.0 %	9	95.0 %	79.7 %	3.0
500.0	2.5 %	1	73.9 %	100.0 %	1.0
500.0	2.5 %	2	73.9 %	100.0 %	1.0
500.0	2.5 %	3	73.9 %	100.0 %	1.0
500.0	2.5 %	4	73.9 %	96.2 %	2.0
500.0	2.5 %	5	73.9 %	100.0 %	1.0
500.0	2.5 %	6	73.9 %	100.0 %	1.0
500.0	2.5 %	7	73.9 %	100.0 %	1.0
500.0	2.5 %	8	73.9 %	70.7 %	4.0
500.0	2.5 %	9	73.9 %	89.7 %	3.0
1000.0	100.0 %	1	100.0 %	100.0 %	1.0
1000.0	100.0 %	2	100.0 %	100.0 %	1.0
1000.0	100.0 %	3	100.0 %	100.0 %	1.0
1000.0	100.0 %	4	100.0 %	97.8 %	2.0
1000.0	100.0 %	5	100.0 %	100.0 %	1.0
1000.0	100.0 %	6	100.0 %	100.0 %	1.0
1000.0	100.0 %	7	100.0 %	100.0 %	1.0
1000.0	100.0 %	8	100.0 %	98.3 %	2.0
1000.0	100.0 %	9	100.0 %	100.0 %	1.0
1000.0	10.0 %	1	95.0 %	100.0 %	1.0
1000.0	10.0 %	2	95.0 %	100.0 %	1.0
1000.0	10.0 %	3	95.0 %	100.0 %	1.0
1000.0	10.0 %	4	95.0 %	98.2 %	2.0
1000.0	10.0 %	5	95.0 %	100.0 %	1.0
1000.0	10.0 %	6	95.0 %	99.4 %	2.0
1000.0	10.0 %	7	95.0 %	100.0 %	1.0
1000.0	10.0 %	8	95.0 %	60.8 %	4.0
1000.0	10.0 %	9	95.0 %	79.7 %	3.0
1000.0	5.0 %	1	73.9 %	100.0 %	1.0
1000.0	5.0 %	2	73.9 %	100.0 %	1.0
1000.0	5.0 %	3	73.9 %	100.0 %	1.0
1000.0	5.0 %	4	73.9 %	95.5 %	2.0
1000.0	5.0 %	5	73.9 %	100.0 %	1.0
1000.0	5.0 %	6	73.9 %	100.0 %	1.0
1000.0	5.0 %	7	73.9 %	100.0 %	1.0
1000.0	5.0 %	8	73.9 %	70.7 %	4.0
1000.0	5.0 %	9	73.9 %	89.7 %	3.0
1000.0	2.5 %	1	63.3 %	100.0 %	1.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
1000.0	2.5 %	2	63.3 %	100.0 %	1.0
1000.0	2.5 %	3	63.3 %	100.0 %	1.0
1000.0	2.5 %	4	63.3 %	99.1 %	2.0
1000.0	2.5 %	5	63.3 %	100.0 %	1.0
1000.0	2.5 %	6	63.3 %	99.1 %	2.0
1000.0	2.5 %	7	63.3 %	100.0 %	1.0
1000.0	2.5 %	8	63.3 %	76.3 %	4.0
1000.0	2.5 %	9	63.3 %	100.0 %	1.0
5000.0	100.0 %	1	97.8 %	100.0 %	1.0
5000.0	100.0 %	2	97.8 %	100.0 %	1.0
5000.0	100.0 %	3	97.8 %	100.0 %	1.0
5000.0	100.0 %	4	97.8 %	100.0 %	1.0
5000.0	100.0 %	5	97.8 %	100.0 %	1.0
5000.0	100.0 %	6	97.8 %	100.0 %	1.0
5000.0	100.0 %	7	97.8 %	100.0 %	1.0
5000.0	100.0 %	8	97.8 %	83.0 %	3.0
5000.0	100.0 %	9	97.8 %	94.8 %	2.0
5000.0	10.0 %	1	62.2 %	100.0 %	1.0
5000.0	10.0 %	2	62.2 %	100.0 %	1.0
5000.0	10.0 %	3	62.2 %	100.0 %	1.0
5000.0	10.0 %	4	62.2 %	98.2 %	2.0
5000.0	10.0 %	5	62.2 %	100.0 %	1.0
5000.0	10.0 %	6	62.2 %	99.1 %	2.0
5000.0	10.0 %	7	62.2 %	100.0 %	1.0
5000.0	10.0 %	8	62.2 %	77.7 %	5.0
5000.0	10.0 %	9	62.2 %	100.0 %	1.0
5000.0	5.0 %	1	58.9 %	100.0 %	1.0
5000.0	5.0 %	2	58.9 %	100.0 %	1.0
5000.0	5.0 %	3	58.9 %	100.0 %	1.0
5000.0	5.0 %	4	58.9 %	99.1 %	3.0
5000.0	5.0 %	5	58.9 %	99.1 %	2.0
5000.0	5.0 %	6	58.9 %	96.2 %	2.0
5000.0	5.0 %	7	58.9 %	95.3 %	2.0
5000.0	5.0 %	8	58.9 %	74.5 %	3.0
5000.0	5.0 %	9	58.9 %	100.0 %	1.0
5000.0	2.5 %	1	48.3 %	100.0 %	1.0
5000.0	2.5 %	2	48.3 %	100.0 %	1.0
5000.0	2.5 %	3	48.3 %	100.0 %	1.0
5000.0	2.5 %	4	48.3 %	97.7 %	3.0
5000.0	2.5 %	5	48.3 %	100.0 %	1.0
5000.0	2.5 %	6	48.3 %	96.6 %	2.0
5000.0	2.5 %	7	48.3 %	85.1 %	6.0
5000.0	2.5 %	8	48.3 %	71.3 %	5.0
5000.0	2.5 %	9	48.3 %	97.4 %	2.0

Table 42 – Summary of Unique Contracts Acquired for the 120-Month Hedging Strategy uniform Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	100.0 %	1.0
100.0	100.0 %	2	100.0 %	100.0 %	1.0
100.0	100.0 %	3	100.0 %	100.0 %	1.0
100.0	100.0 %	4	100.0 %	100.0 %	1.0
100.0	100.0 %	5	100.0 %	100.0 %	1.0
100.0	100.0 %	6	100.0 %	100.0 %	1.0
100.0	100.0 %	7	100.0 %	100.0 %	1.0
100.0	100.0 %	8	100.0 %	100.0 %	1.0
100.0	100.0 %	9	100.0 %	96.7 %	3.0
100.0	10.0 %	1	97.8 %	100.0 %	1.0
100.0	10.0 %	2	97.8 %	100.0 %	1.0
100.0	10.0 %	3	97.8 %	100.0 %	1.0
100.0	10.0 %	4	97.8 %	100.0 %	1.0
100.0	10.0 %	5	97.8 %	100.0 %	1.0
100.0	10.0 %	6	97.8 %	100.0 %	1.0
100.0	10.0 %	7	97.8 %	100.0 %	1.0
100.0	10.0 %	8	97.8 %	98.9 %	2.0
100.0	10.0 %	9	97.8 %	93.2 %	2.0
100.0	5.0 %	1	96.1 %	100.0 %	1.0
100.0	5.0 %	2	96.1 %	100.0 %	1.0
100.0	5.0 %	3	96.1 %	100.0 %	1.0
100.0	5.0 %	4	96.1 %	100.0 %	1.0
100.0	5.0 %	5	96.1 %	100.0 %	1.0
100.0	5.0 %	6	96.1 %	100.0 %	1.0
100.0	5.0 %	7	96.1 %	100.0 %	1.0
100.0	5.0 %	8	96.1 %	99.4 %	2.0
100.0	5.0 %	9	96.1 %	94.8 %	3.0
100.0	2.5 %	1	92.8 %	100.0 %	1.0
100.0	2.5 %	2	92.8 %	100.0 %	1.0
100.0	2.5 %	3	92.8 %	100.0 %	1.0
100.0	2.5 %	4	92.8 %	100.0 %	1.0
100.0	2.5 %	5	92.8 %	100.0 %	1.0
100.0	2.5 %	6	92.8 %	100.0 %	1.0
100.0	2.5 %	7	92.8 %	100.0 %	1.0
100.0	2.5 %	8	92.8 %	95.2 %	3.0
100.0	2.5 %	9	92.8 %	96.4 %	3.0
300.0	100.0 %	1	98.9 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	100.0 %	2	98.9 %	100.0 %	1.0
300.0	100.0 %	3	98.9 %	100.0 %	1.0
300.0	100.0 %	4	98.9 %	100.0 %	1.0
300.0	100.0 %	5	98.9 %	100.0 %	1.0
300.0	100.0 %	6	98.9 %	100.0 %	1.0
300.0	100.0 %	7	98.9 %	100.0 %	1.0
300.0	100.0 %	8	98.9 %	100.0 %	1.0
300.0	100.0 %	9	98.9 %	92.1 %	2.0
300.0	10.0 %	1	95.0 %	100.0 %	1.0
300.0	10.0 %	2	95.0 %	100.0 %	1.0
300.0	10.0 %	3	95.0 %	100.0 %	1.0
300.0	10.0 %	4	95.0 %	100.0 %	1.0
300.0	10.0 %	5	95.0 %	100.0 %	1.0
300.0	10.0 %	6	95.0 %	100.0 %	1.0
300.0	10.0 %	7	95.0 %	100.0 %	1.0
300.0	10.0 %	8	95.0 %	94.7 %	2.0
300.0	10.0 %	9	95.0 %	94.7 %	4.0
300.0	5.0 %	1	91.1 %	100.0 %	1.0
300.0	5.0 %	2	91.1 %	100.0 %	1.0
300.0	5.0 %	3	91.1 %	100.0 %	1.0
300.0	5.0 %	4	91.1 %	100.0 %	1.0
300.0	5.0 %	5	91.1 %	100.0 %	1.0
300.0	5.0 %	6	91.1 %	100.0 %	1.0
300.0	5.0 %	7	91.1 %	100.0 %	1.0
300.0	5.0 %	8	91.1 %	90.9 %	3.0
300.0	5.0 %	9	91.1 %	96.3 %	2.0
300.0	2.5 %	1	88.9 %	100.0 %	1.0
300.0	2.5 %	2	88.9 %	100.0 %	1.0
300.0	2.5 %	3	88.9 %	100.0 %	1.0
300.0	2.5 %	4	88.9 %	100.0 %	1.0
300.0	2.5 %	5	88.9 %	100.0 %	1.0
300.0	2.5 %	6	88.9 %	99.4 %	2.0
300.0	2.5 %	7	88.9 %	100.0 %	1.0
300.0	2.5 %	8	88.9 %	70.0 %	3.0
300.0	2.5 %	9	88.9 %	98.8 %	2.0
500.0	100.0 %	1	98.9 %	100.0 %	1.0
500.0	100.0 %	2	98.9 %	100.0 %	1.0
500.0	100.0 %	3	98.9 %	100.0 %	1.0
500.0	100.0 %	4	98.9 %	99.4 %	2.0
500.0	100.0 %	5	98.9 %	100.0 %	1.0
500.0	100.0 %	6	98.9 %	100.0 %	1.0
500.0	100.0 %	7	98.9 %	100.0 %	1.0
500.0	100.0 %	8	98.9 %	100.0 %	1.0
500.0	100.0 %	9	98.9 %	91.6 %	2.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
500.0	10.0 %	1	91.7 %	100.0 %	1.0
500.0	10.0 %	2	91.7 %	100.0 %	1.0
500.0	10.0 %	3	91.7 %	100.0 %	1.0
500.0	10.0 %	4	91.7 %	98.8 %	2.0
500.0	10.0 %	5	91.7 %	100.0 %	1.0
500.0	10.0 %	6	91.7 %	100.0 %	1.0
500.0	10.0 %	7	91.7 %	100.0 %	1.0
500.0	10.0 %	8	91.7 %	94.5 %	3.0
500.0	10.0 %	9	91.7 %	97.6 %	3.0
500.0	5.0 %	1	88.9 %	100.0 %	1.0
500.0	5.0 %	2	88.9 %	100.0 %	1.0
500.0	5.0 %	3	88.9 %	100.0 %	1.0
500.0	5.0 %	4	88.9 %	97.5 %	2.0
500.0	5.0 %	5	88.9 %	100.0 %	1.0
500.0	5.0 %	6	88.9 %	100.0 %	1.0
500.0	5.0 %	7	88.9 %	100.0 %	1.0
500.0	5.0 %	8	88.9 %	77.5 %	2.0
500.0	5.0 %	9	88.9 %	98.8 %	2.0
500.0	2.5 %	1	76.1 %	100.0 %	1.0
500.0	2.5 %	2	76.1 %	100.0 %	1.0
500.0	2.5 %	3	76.1 %	100.0 %	1.0
500.0	2.5 %	4	76.1 %	99.3 %	2.0
500.0	2.5 %	5	76.1 %	100.0 %	1.0
500.0	2.5 %	6	76.1 %	100.0 %	1.0
500.0	2.5 %	7	76.1 %	100.0 %	1.0
500.0	2.5 %	8	76.1 %	68.6 %	6.0
500.0	2.5 %	9	76.1 %	94.9 %	3.0
1000.0	100.0 %	1	97.8 %	100.0 %	1.0
1000.0	100.0 %	2	97.8 %	100.0 %	1.0
1000.0	100.0 %	3	97.8 %	100.0 %	1.0
1000.0	100.0 %	4	97.8 %	98.9 %	2.0
1000.0	100.0 %	5	97.8 %	100.0 %	1.0
1000.0	100.0 %	6	97.8 %	100.0 %	1.0
1000.0	100.0 %	7	97.8 %	100.0 %	1.0
1000.0	100.0 %	8	97.8 %	98.9 %	2.0
1000.0	100.0 %	9	97.8 %	93.2 %	2.0
1000.0	10.0 %	1	88.9 %	100.0 %	1.0
1000.0	10.0 %	2	88.9 %	100.0 %	1.0
1000.0	10.0 %	3	88.9 %	100.0 %	1.0
1000.0	10.0 %	4	88.9 %	98.8 %	2.0
1000.0	10.0 %	5	88.9 %	100.0 %	1.0
1000.0	10.0 %	6	88.9 %	100.0 %	1.0
1000.0	10.0 %	7	88.9 %	100.0 %	1.0
1000.0	10.0 %	8	88.9 %	77.5 %	2.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
1000.0	10.0 %	9	88.9 %	96.9 %	2.0
1000.0	5.0 %	1	76.1 %	100.0 %	1.0
1000.0	5.0 %	2	76.1 %	100.0 %	1.0
1000.0	5.0 %	3	76.1 %	100.0 %	1.0
1000.0	5.0 %	4	76.1 %	98.5 %	2.0
1000.0	5.0 %	5	76.1 %	100.0 %	1.0
1000.0	5.0 %	6	76.1 %	100.0 %	1.0
1000.0	5.0 %	7	76.1 %	100.0 %	1.0
1000.0	5.0 %	8	76.1 %	68.6 %	6.0
1000.0	5.0 %	9	76.1 %	97.1 %	3.0
1000.0	2.5 %	1	66.1 %	100.0 %	1.0
1000.0	2.5 %	2	66.1 %	100.0 %	1.0
1000.0	2.5 %	3	66.1 %	100.0 %	1.0
1000.0	2.5 %	4	66.1 %	97.5 %	2.0
1000.0	2.5 %	5	66.1 %	100.0 %	1.0
1000.0	2.5 %	6	66.1 %	100.0 %	1.0
1000.0	2.5 %	7	66.1 %	100.0 %	1.0
1000.0	2.5 %	8	66.1 %	73.9 %	10.0
1000.0	2.5 %	9	66.1 %	92.4 %	4.0
5000.0	100.0 %	1	91.7 %	100.0 %	1.0
5000.0	100.0 %	2	91.7 %	100.0 %	1.0
5000.0	100.0 %	3	91.7 %	100.0 %	1.0
5000.0	100.0 %	4	91.7 %	100.0 %	1.0
5000.0	100.0 %	5	91.7 %	100.0 %	1.0
5000.0	100.0 %	6	91.7 %	100.0 %	1.0
5000.0	100.0 %	7	91.7 %	100.0 %	1.0
5000.0	100.0 %	8	91.7 %	94.5 %	3.0
5000.0	100.0 %	9	91.7 %	97.0 %	3.0
5000.0	10.0 %	1	57.8 %	100.0 %	1.0
5000.0	10.0 %	2	57.8 %	100.0 %	1.0
5000.0	10.0 %	3	57.8 %	100.0 %	1.0
5000.0	10.0 %	4	57.8 %	100.0 %	1.0
5000.0	10.0 %	5	57.8 %	100.0 %	1.0
5000.0	10.0 %	6	57.8 %	100.0 %	1.0
5000.0	10.0 %	7	57.8 %	100.0 %	1.0
5000.0	10.0 %	8	57.8 %	77.9 %	2.0
5000.0	10.0 %	9	57.8 %	85.6 %	3.0
5000.0	5.0 %	1	46.7 %	100.0 %	1.0
5000.0	5.0 %	2	46.7 %	100.0 %	1.0
5000.0	5.0 %	3	46.7 %	100.0 %	1.0
5000.0	5.0 %	4	46.7 %	100.0 %	1.0
5000.0	5.0 %	5	46.7 %	100.0 %	1.0
5000.0	5.0 %	6	46.7 %	96.4 %	2.0
5000.0	5.0 %	7	46.7 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
5000.0	5.0 %	8	46.7 %	77.4 %	2.0
5000.0	5.0 %	9	46.7 %	79.8 %	3.0
5000.0	2.5 %	1	23.3 %	100.0 %	1.0
5000.0	2.5 %	2	23.3 %	100.0 %	1.0
5000.0	2.5 %	3	23.3 %	100.0 %	1.0
5000.0	2.5 %	4	23.3 %	100.0 %	1.0
5000.0	2.5 %	5	23.3 %	100.0 %	1.0
5000.0	2.5 %	6	23.3 %	100.0 %	1.0
5000.0	2.5 %	7	23.3 %	100.0 %	1.0
5000.0	2.5 %	8	23.3 %	78.6 %	2.0
5000.0	2.5 %	9	23.3 %	40.5 %	3.0

D.2 Bell Portfolios

Table 43 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy bell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	87.2 %	2
100.0	100.0 %	2	100.0 %	100.0 %	1
100.0	100.0 %	3	100.0 %	100.0 %	1
100.0	100.0 %	4	100.0 %	98.9 %	2
100.0	100.0 %	5	100.0 %	100.0 %	1
100.0	100.0 %	6	100.0 %	100.0 %	1
100.0	100.0 %	7	100.0 %	93.3 %	2
100.0	100.0 %	8	100.0 %	0.0 %	0
100.0	100.0 %	9	100.0 %	0.0 %	0
100.0	10.0 %	1	100.0 %	100.0 %	1
100.0	10.0 %	2	100.0 %	100.0 %	1
100.0	10.0 %	3	100.0 %	100.0 %	1
100.0	10.0 %	4	100.0 %	97.8 %	2
100.0	10.0 %	5	100.0 %	99.4 %	2
100.0	10.0 %	6	100.0 %	100.0 %	1
100.0	10.0 %	7	100.0 %	98.3 %	2
100.0	10.0 %	8	100.0 %	0.0 %	0
100.0	10.0 %	9	100.0 %	0.0 %	0
100.0	5.0 %	1	100.0 %	100.0 %	1
100.0	5.0 %	2	100.0 %	100.0 %	1
100.0	5.0 %	3	100.0 %	100.0 %	1
100.0	5.0 %	4	100.0 %	98.9 %	2

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	5.0 %	5	100.0 %	100.0 %	1
100.0	5.0 %	6	100.0 %	100.0 %	1
100.0	5.0 %	7	100.0 %	98.9 %	2
100.0	5.0 %	8	100.0 %	0.0 %	0
100.0	5.0 %	9	100.0 %	0.0 %	0
100.0	2.5 %	1	98.9 %	100.0 %	1
100.0	2.5 %	2	98.9 %	100.0 %	1
100.0	2.5 %	3	98.9 %	100.0 %	1
100.0	2.5 %	4	98.9 %	98.3 %	2
100.0	2.5 %	5	98.9 %	100.0 %	1
100.0	2.5 %	6	98.9 %	100.0 %	1
100.0	2.5 %	7	98.9 %	99.4 %	2
100.0	2.5 %	8	98.9 %	0.0 %	0
100.0	2.5 %	9	98.9 %	0.0 %	0
300.0	100.0 %	1	100.0 %	97.8 %	2
300.0	100.0 %	2	100.0 %	100.0 %	1
300.0	100.0 %	3	100.0 %	100.0 %	1
300.0	100.0 %	4	100.0 %	98.9 %	2
300.0	100.0 %	5	100.0 %	100.0 %	1
300.0	100.0 %	6	100.0 %	100.0 %	1
300.0	100.0 %	7	100.0 %	96.7 %	2
300.0	100.0 %	8	100.0 %	0.0 %	0
300.0	100.0 %	9	100.0 %	0.0 %	0
300.0	10.0 %	1	100.0 %	100.0 %	1
300.0	10.0 %	2	100.0 %	100.0 %	1
300.0	10.0 %	3	100.0 %	100.0 %	1
300.0	10.0 %	4	100.0 %	98.9 %	2
300.0	10.0 %	5	100.0 %	100.0 %	1
300.0	10.0 %	6	100.0 %	100.0 %	1
300.0	10.0 %	7	100.0 %	99.4 %	2
300.0	10.0 %	8	100.0 %	0.0 %	0
300.0	10.0 %	9	100.0 %	0.0 %	0
300.0	5.0 %	1	97.8 %	100.0 %	1
300.0	5.0 %	2	97.8 %	100.0 %	1
300.0	5.0 %	3	97.8 %	100.0 %	1
300.0	5.0 %	4	97.8 %	98.3 %	2
300.0	5.0 %	5	97.8 %	99.4 %	2
300.0	5.0 %	6	97.8 %	98.3 %	2
300.0	5.0 %	7	97.8 %	99.4 %	2
300.0	5.0 %	8	97.8 %	0.0 %	0
300.0	5.0 %	9	97.8 %	0.0 %	0
300.0	2.5 %	1	92.2 %	100.0 %	1
300.0	2.5 %	2	92.2 %	100.0 %	1
300.0	2.5 %	3	92.2 %	100.0 %	1

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
300.0	2.5 %	4	92.2 %	98.8 %	2
300.0	2.5 %	5	92.2 %	98.2 %	2
300.0	2.5 %	6	92.2 %	99.4 %	2
300.0	2.5 %	7	92.2 %	99.4 %	2
300.0	2.5 %	8	92.2 %	0.0 %	0
300.0	2.5 %	9	92.2 %	0.0 %	0
500.0	100.0 %	1	100.0 %	100.0 %	1
500.0	100.0 %	2	100.0 %	100.0 %	1
500.0	100.0 %	3	100.0 %	100.0 %	1
500.0	100.0 %	4	100.0 %	97.8 %	2
500.0	100.0 %	5	100.0 %	100.0 %	1
500.0	100.0 %	6	100.0 %	100.0 %	1
500.0	100.0 %	7	100.0 %	98.3 %	2
500.0	100.0 %	8	100.0 %	0.0 %	0
500.0	100.0 %	9	100.0 %	0.0 %	0
500.0	10.0 %	1	98.3 %	100.0 %	1
500.0	10.0 %	2	98.3 %	100.0 %	1
500.0	10.0 %	3	98.3 %	100.0 %	1
500.0	10.0 %	4	98.3 %	100.0 %	1
500.0	10.0 %	5	98.3 %	100.0 %	1
500.0	10.0 %	6	98.3 %	98.9 %	2
500.0	10.0 %	7	98.3 %	99.4 %	2
500.0	10.0 %	8	98.3 %	0.0 %	0
500.0	10.0 %	9	98.3 %	0.0 %	0
500.0	5.0 %	1	95.6 %	100.0 %	1
500.0	5.0 %	2	95.6 %	100.0 %	1
500.0	5.0 %	3	95.6 %	100.0 %	1
500.0	5.0 %	4	95.6 %	100.0 %	1
500.0	5.0 %	5	95.6 %	97.7 %	2
500.0	5.0 %	6	95.6 %	98.3 %	3
500.0	5.0 %	7	95.6 %	99.4 %	2
500.0	5.0 %	8	95.6 %	0.0 %	0
500.0	5.0 %	9	95.6 %	0.0 %	0
500.0	2.5 %	1	90.0 %	100.0 %	1
500.0	2.5 %	2	90.0 %	100.0 %	1
500.0	2.5 %	3	90.0 %	100.0 %	1
500.0	2.5 %	4	90.0 %	99.4 %	2
500.0	2.5 %	5	90.0 %	98.8 %	2
500.0	2.5 %	6	90.0 %	96.9 %	2
500.0	2.5 %	7	90.0 %	100.0 %	1
500.0	2.5 %	8	90.0 %	0.0 %	0
500.0	2.5 %	9	90.0 %	0.0 %	0
1000.0	100.0 %	1	100.0 %	100.0 %	1
1000.0	100.0 %	2	100.0 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	100.0 %	3	100.0 %	100.0 %	1
1000.0	100.0 %	4	100.0 %	100.0 %	1
1000.0	100.0 %	5	100.0 %	100.0 %	1
1000.0	100.0 %	6	100.0 %	100.0 %	1
1000.0	100.0 %	7	100.0 %	98.3 %	2
1000.0	100.0 %	8	100.0 %	0.0 %	0
1000.0	100.0 %	9	100.0 %	0.0 %	0
1000.0	10.0 %	1	95.6 %	100.0 %	1
1000.0	10.0 %	2	95.6 %	100.0 %	1
1000.0	10.0 %	3	95.6 %	100.0 %	1
1000.0	10.0 %	4	95.6 %	100.0 %	1
1000.0	10.0 %	5	95.6 %	97.7 %	2
1000.0	10.0 %	6	95.6 %	97.7 %	3
1000.0	10.0 %	7	95.6 %	99.4 %	2
1000.0	10.0 %	8	95.6 %	0.0 %	0
1000.0	10.0 %	9	95.6 %	0.0 %	0
1000.0	5.0 %	1	90.0 %	100.0 %	1
1000.0	5.0 %	2	90.0 %	100.0 %	1
1000.0	5.0 %	3	90.0 %	100.0 %	1
1000.0	5.0 %	4	90.0 %	100.0 %	1
1000.0	5.0 %	5	90.0 %	98.8 %	2
1000.0	5.0 %	6	90.0 %	97.5 %	2
1000.0	5.0 %	7	90.0 %	100.0 %	1
1000.0	5.0 %	8	90.0 %	0.0 %	0
1000.0	5.0 %	9	90.0 %	0.0 %	0
1000.0	2.5 %	1	77.8 %	100.0 %	1
1000.0	2.5 %	2	77.8 %	100.0 %	1
1000.0	2.5 %	3	77.8 %	100.0 %	1
1000.0	2.5 %	4	77.8 %	100.0 %	1
1000.0	2.5 %	5	77.8 %	97.1 %	2
1000.0	2.5 %	6	77.8 %	94.3 %	3
1000.0	2.5 %	7	77.8 %	96.4 %	3
1000.0	2.5 %	8	77.8 %	0.0 %	0
1000.0	2.5 %	9	77.8 %	0.0 %	0
5000.0	100.0 %	1	98.3 %	100.0 %	1
5000.0	100.0 %	2	98.3 %	100.0 %	1
5000.0	100.0 %	3	98.3 %	100.0 %	1
5000.0	100.0 %	4	98.3 %	100.0 %	1
5000.0	100.0 %	5	98.3 %	100.0 %	1
5000.0	100.0 %	6	98.3 %	98.9 %	2
5000.0	100.0 %	7	98.3 %	99.4 %	2
5000.0	100.0 %	8	98.3 %	0.0 %	0
5000.0	100.0 %	9	98.3 %	0.0 %	0
5000.0	10.0 %	1	75.0 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
5000.0	10.0 %	2	75.0 %	100.0 %	1
5000.0	10.0 %	3	75.0 %	100.0 %	1
5000.0	10.0 %	4	75.0 %	100.0 %	1
5000.0	10.0 %	5	75.0 %	96.3 %	2
5000.0	10.0 %	6	75.0 %	93.3 %	3
5000.0	10.0 %	7	75.0 %	95.6 %	3
5000.0	10.0 %	8	75.0 %	0.0 %	0
5000.0	10.0 %	9	75.0 %	0.0 %	0
5000.0	5.0 %	1	48.9 %	100.0 %	1
5000.0	5.0 %	2	48.9 %	100.0 %	1
5000.0	5.0 %	3	48.9 %	100.0 %	1
5000.0	5.0 %	4	48.9 %	98.9 %	2
5000.0	5.0 %	5	48.9 %	80.7 %	2
5000.0	5.0 %	6	48.9 %	90.9 %	3
5000.0	5.0 %	7	48.9 %	92.0 %	3
5000.0	5.0 %	8	48.9 %	0.0 %	0
5000.0	5.0 %	9	48.9 %	0.0 %	0
5000.0	2.5 %	1	29.4 %	100.0 %	1
5000.0	2.5 %	2	29.4 %	100.0 %	1
5000.0	2.5 %	3	29.4 %	100.0 %	1
5000.0	2.5 %	4	29.4 %	94.3 %	2
5000.0	2.5 %	5	29.4 %	81.1 %	2
5000.0	2.5 %	6	29.4 %	86.8 %	2
5000.0	2.5 %	7	29.4 %	96.2 %	4
5000.0	2.5 %	8	29.4 %	0.0 %	0
5000.0	2.5 %	9	29.4 %	0.0 %	0

Table 44 – Summary of Unique Contracts Acquired for the 60-Month Hedging Strategy bell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	83.9 %	2
100.0	100.0 %	2	100.0 %	100.0 %	1
100.0	100.0 %	3	100.0 %	100.0 %	1
100.0	100.0 %	4	100.0 %	100.0 %	1
100.0	100.0 %	5	100.0 %	100.0 %	1
100.0	100.0 %	6	100.0 %	100.0 %	1
100.0	100.0 %	7	100.0 %	100.0 %	1
100.0	100.0 %	8	100.0 %	99.4 %	2
100.0	100.0 %	9	100.0 %	0.0 %	0
100.0	10.0 %	1	100.0 %	95.6 %	2

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	10.0 %	2	100.0 %	100.0 %	1
100.0	10.0 %	3	100.0 %	100.0 %	1
100.0	10.0 %	4	100.0 %	100.0 %	1
100.0	10.0 %	5	100.0 %	100.0 %	1
100.0	10.0 %	6	100.0 %	100.0 %	1
100.0	10.0 %	7	100.0 %	100.0 %	1
100.0	10.0 %	8	100.0 %	100.0 %	1
100.0	10.0 %	9	100.0 %	0.0 %	0
100.0	5.0 %	1	100.0 %	100.0 %	1
100.0	5.0 %	2	100.0 %	100.0 %	1
100.0	5.0 %	3	100.0 %	100.0 %	1
100.0	5.0 %	4	100.0 %	100.0 %	1
100.0	5.0 %	5	100.0 %	100.0 %	1
100.0	5.0 %	6	100.0 %	100.0 %	1
100.0	5.0 %	7	100.0 %	100.0 %	1
100.0	5.0 %	8	100.0 %	100.0 %	1
100.0	5.0 %	9	100.0 %	0.0 %	0
100.0	2.5 %	1	100.0 %	100.0 %	1
100.0	2.5 %	2	100.0 %	100.0 %	1
100.0	2.5 %	3	100.0 %	100.0 %	1
100.0	2.5 %	4	100.0 %	100.0 %	1
100.0	2.5 %	5	100.0 %	100.0 %	1
100.0	2.5 %	6	100.0 %	97.8 %	2
100.0	2.5 %	7	100.0 %	100.0 %	1
100.0	2.5 %	8	100.0 %	100.0 %	1
100.0	2.5 %	9	100.0 %	0.0 %	0
300.0	100.0 %	1	100.0 %	85.0 %	2
300.0	100.0 %	2	100.0 %	100.0 %	1
300.0	100.0 %	3	100.0 %	100.0 %	1
300.0	100.0 %	4	100.0 %	100.0 %	1
300.0	100.0 %	5	100.0 %	100.0 %	1
300.0	100.0 %	6	100.0 %	100.0 %	1
300.0	100.0 %	7	100.0 %	99.4 %	2
300.0	100.0 %	8	100.0 %	99.4 %	2
300.0	100.0 %	9	100.0 %	0.0 %	0
300.0	10.0 %	1	100.0 %	100.0 %	1
300.0	10.0 %	2	100.0 %	100.0 %	1
300.0	10.0 %	3	100.0 %	100.0 %	1
300.0	10.0 %	4	100.0 %	100.0 %	1
300.0	10.0 %	5	100.0 %	100.0 %	1
300.0	10.0 %	6	100.0 %	99.4 %	2
300.0	10.0 %	7	100.0 %	100.0 %	1
300.0	10.0 %	8	100.0 %	100.0 %	1
300.0	10.0 %	9	100.0 %	0.0 %	0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	5.0 %	1	100.0 %	100.0 %	1
300.0	5.0 %	2	100.0 %	100.0 %	1
300.0	5.0 %	3	100.0 %	100.0 %	1
300.0	5.0 %	4	100.0 %	100.0 %	1
300.0	5.0 %	5	100.0 %	100.0 %	1
300.0	5.0 %	6	100.0 %	96.1 %	2
300.0	5.0 %	7	100.0 %	100.0 %	1
300.0	5.0 %	8	100.0 %	100.0 %	1
300.0	5.0 %	9	100.0 %	0.0 %	0
300.0	2.5 %	1	98.3 %	100.0 %	1
300.0	2.5 %	2	98.3 %	100.0 %	1
300.0	2.5 %	3	98.3 %	100.0 %	1
300.0	2.5 %	4	98.3 %	100.0 %	1
300.0	2.5 %	5	98.3 %	100.0 %	1
300.0	2.5 %	6	98.3 %	96.6 %	2
300.0	2.5 %	7	98.3 %	98.9 %	2
300.0	2.5 %	8	98.3 %	100.0 %	1
300.0	2.5 %	9	98.3 %	0.0 %	0
500.0	100.0 %	1	100.0 %	88.3 %	2
500.0	100.0 %	2	100.0 %	100.0 %	1
500.0	100.0 %	3	100.0 %	100.0 %	1
500.0	100.0 %	4	100.0 %	98.9 %	2
500.0	100.0 %	5	100.0 %	100.0 %	1
500.0	100.0 %	6	100.0 %	100.0 %	1
500.0	100.0 %	7	100.0 %	100.0 %	1
500.0	100.0 %	8	100.0 %	99.4 %	2
500.0	100.0 %	9	100.0 %	0.0 %	0
500.0	10.0 %	1	100.0 %	100.0 %	1
500.0	10.0 %	2	100.0 %	100.0 %	1
500.0	10.0 %	3	100.0 %	100.0 %	1
500.0	10.0 %	4	100.0 %	98.9 %	2
500.0	10.0 %	5	100.0 %	100.0 %	1
500.0	10.0 %	6	100.0 %	96.1 %	2
500.0	10.0 %	7	100.0 %	100.0 %	1
500.0	10.0 %	8	100.0 %	100.0 %	1
500.0	10.0 %	9	100.0 %	0.0 %	0
500.0	5.0 %	1	98.3 %	100.0 %	1
500.0	5.0 %	2	98.3 %	100.0 %	1
500.0	5.0 %	3	98.3 %	100.0 %	1
500.0	5.0 %	4	98.3 %	96.0 %	2
500.0	5.0 %	5	98.3 %	100.0 %	1
500.0	5.0 %	6	98.3 %	98.3 %	2
500.0	5.0 %	7	98.3 %	100.0 %	1
500.0	5.0 %	8	98.3 %	100.0 %	1

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
500.0	5.0 %	9	98.3 %	0.0 %	0
500.0	2.5 %	1	95.0 %	100.0 %	1
500.0	2.5 %	2	95.0 %	100.0 %	1
500.0	2.5 %	3	95.0 %	100.0 %	1
500.0	2.5 %	4	95.0 %	98.8 %	2
500.0	2.5 %	5	95.0 %	100.0 %	1
500.0	2.5 %	6	95.0 %	93.6 %	2
500.0	2.5 %	7	95.0 %	95.3 %	4
500.0	2.5 %	8	95.0 %	98.8 %	2
500.0	2.5 %	9	95.0 %	0.0 %	0
1000.0	100.0 %	1	100.0 %	95.6 %	2
1000.0	100.0 %	2	100.0 %	100.0 %	1
1000.0	100.0 %	3	100.0 %	100.0 %	1
1000.0	100.0 %	4	100.0 %	98.9 %	2
1000.0	100.0 %	5	100.0 %	100.0 %	1
1000.0	100.0 %	6	100.0 %	100.0 %	1
1000.0	100.0 %	7	100.0 %	100.0 %	1
1000.0	100.0 %	8	100.0 %	100.0 %	1
1000.0	100.0 %	9	100.0 %	0.0 %	0
1000.0	10.0 %	1	98.3 %	100.0 %	1
1000.0	10.0 %	2	98.3 %	100.0 %	1
1000.0	10.0 %	3	98.3 %	100.0 %	1
1000.0	10.0 %	4	98.3 %	98.3 %	2
1000.0	10.0 %	5	98.3 %	100.0 %	1
1000.0	10.0 %	6	98.3 %	99.4 %	2
1000.0	10.0 %	7	98.3 %	100.0 %	1
1000.0	10.0 %	8	98.3 %	100.0 %	1
1000.0	10.0 %	9	98.3 %	0.0 %	0
1000.0	5.0 %	1	95.0 %	100.0 %	1
1000.0	5.0 %	2	95.0 %	100.0 %	1
1000.0	5.0 %	3	95.0 %	100.0 %	1
1000.0	5.0 %	4	95.0 %	98.8 %	2
1000.0	5.0 %	5	95.0 %	100.0 %	1
1000.0	5.0 %	6	95.0 %	92.4 %	2
1000.0	5.0 %	7	95.0 %	95.3 %	4
1000.0	5.0 %	8	95.0 %	98.8 %	2
1000.0	5.0 %	9	95.0 %	0.0 %	0
1000.0	2.5 %	1	85.6 %	100.0 %	1
1000.0	2.5 %	2	85.6 %	100.0 %	1
1000.0	2.5 %	3	85.6 %	100.0 %	1
1000.0	2.5 %	4	85.6 %	99.4 %	2
1000.0	2.5 %	5	85.6 %	99.4 %	2
1000.0	2.5 %	6	85.6 %	90.9 %	3
1000.0	2.5 %	7	85.6 %	93.5 %	6

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
1000.0	2.5 %	8	85.6 %	94.8 %	6
1000.0	2.5 %	9	85.6 %	0.0 %	0
5000.0	100.0 %	1	100.0 %	100.0 %	1
5000.0	100.0 %	2	100.0 %	100.0 %	1
5000.0	100.0 %	3	100.0 %	100.0 %	1
5000.0	100.0 %	4	100.0 %	100.0 %	1
5000.0	100.0 %	5	100.0 %	100.0 %	1
5000.0	100.0 %	6	100.0 %	96.7 %	2
5000.0	100.0 %	7	100.0 %	100.0 %	1
5000.0	100.0 %	8	100.0 %	100.0 %	1
5000.0	100.0 %	9	100.0 %	0.0 %	0
5000.0	10.0 %	1	79.4 %	100.0 %	1
5000.0	10.0 %	2	79.4 %	100.0 %	1
5000.0	10.0 %	3	79.4 %	100.0 %	1
5000.0	10.0 %	4	79.4 %	100.0 %	1
5000.0	10.0 %	5	79.4 %	99.3 %	2
5000.0	10.0 %	6	79.4 %	92.3 %	3
5000.0	10.0 %	7	79.4 %	93.0 %	3
5000.0	10.0 %	8	79.4 %	94.4 %	3
5000.0	10.0 %	9	79.4 %	0.0 %	0
5000.0	5.0 %	1	59.4 %	100.0 %	1
5000.0	5.0 %	2	59.4 %	100.0 %	1
5000.0	5.0 %	3	59.4 %	100.0 %	1
5000.0	5.0 %	4	59.4 %	99.1 %	2
5000.0	5.0 %	5	59.4 %	98.1 %	2
5000.0	5.0 %	6	59.4 %	95.3 %	2
5000.0	5.0 %	7	59.4 %	91.6 %	6
5000.0	5.0 %	8	59.4 %	98.1 %	4
5000.0	5.0 %	9	59.4 %	0.0 %	0
5000.0	2.5 %	1	40.6 %	100.0 %	1
5000.0	2.5 %	2	40.6 %	100.0 %	1
5000.0	2.5 %	3	40.6 %	100.0 %	1
5000.0	2.5 %	4	40.6 %	100.0 %	1
5000.0	2.5 %	5	40.6 %	94.5 %	2
5000.0	2.5 %	6	40.6 %	94.5 %	3
5000.0	2.5 %	7	40.6 %	78.1 %	3
5000.0	2.5 %	8	40.6 %	84.9 %	5
5000.0	2.5 %	9	40.6 %	0.0 %	0

Table 45 – Summary of Unique Contracts Acquired for the 84-Month Hedging Strategy bell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	83.9 %	2.0
100.0	100.0 %	2	100.0 %	100.0 %	1.0
100.0	100.0 %	3	100.0 %	100.0 %	1.0
100.0	100.0 %	4	100.0 %	100.0 %	1.0
100.0	100.0 %	5	100.0 %	100.0 %	1.0
100.0	100.0 %	6	100.0 %	100.0 %	1.0
100.0	100.0 %	7	100.0 %	100.0 %	1.0
100.0	100.0 %	8	100.0 %	100.0 %	1.0
100.0	100.0 %	9	100.0 %	95.0 %	2.0
100.0	10.0 %	1	100.0 %	85.6 %	2.0
100.0	10.0 %	2	100.0 %	100.0 %	1.0
100.0	10.0 %	3	100.0 %	100.0 %	1.0
100.0	10.0 %	4	100.0 %	100.0 %	1.0
100.0	10.0 %	5	100.0 %	100.0 %	1.0
100.0	10.0 %	6	100.0 %	100.0 %	1.0
100.0	10.0 %	7	100.0 %	100.0 %	1.0
100.0	10.0 %	8	100.0 %	100.0 %	1.0
100.0	10.0 %	9	100.0 %	100.0 %	1.0
100.0	5.0 %	1	100.0 %	91.1 %	2.0
100.0	5.0 %	2	100.0 %	100.0 %	1.0
100.0	5.0 %	3	100.0 %	100.0 %	1.0
100.0	5.0 %	4	100.0 %	100.0 %	1.0
100.0	5.0 %	5	100.0 %	100.0 %	1.0
100.0	5.0 %	6	100.0 %	100.0 %	1.0
100.0	5.0 %	7	100.0 %	100.0 %	1.0
100.0	5.0 %	8	100.0 %	98.9 %	2.0
100.0	5.0 %	9	100.0 %	100.0 %	1.0
100.0	2.5 %	1	100.0 %	98.9 %	2.0
100.0	2.5 %	2	100.0 %	100.0 %	1.0
100.0	2.5 %	3	100.0 %	100.0 %	1.0
100.0	2.5 %	4	100.0 %	100.0 %	1.0
100.0	2.5 %	5	100.0 %	100.0 %	1.0
100.0	2.5 %	6	100.0 %	100.0 %	1.0
100.0	2.5 %	7	100.0 %	100.0 %	1.0
100.0	2.5 %	8	100.0 %	94.4 %	6.0
100.0	2.5 %	9	100.0 %	100.0 %	1.0
300.0	100.0 %	1	100.0 %	83.9 %	2.0
300.0	100.0 %	2	100.0 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	100.0 %	3	100.0 %	100.0 %	1.0
300.0	100.0 %	4	100.0 %	100.0 %	1.0
300.0	100.0 %	5	100.0 %	100.0 %	1.0
300.0	100.0 %	6	100.0 %	100.0 %	1.0
300.0	100.0 %	7	100.0 %	100.0 %	1.0
300.0	100.0 %	8	100.0 %	100.0 %	1.0
300.0	100.0 %	9	100.0 %	96.3 %	2.0
300.0	10.0 %	1	100.0 %	96.7 %	2.0
300.0	10.0 %	2	100.0 %	100.0 %	1.0
300.0	10.0 %	3	100.0 %	100.0 %	1.0
300.0	10.0 %	4	100.0 %	100.0 %	1.0
300.0	10.0 %	5	100.0 %	100.0 %	1.0
300.0	10.0 %	6	100.0 %	100.0 %	1.0
300.0	10.0 %	7	100.0 %	100.0 %	1.0
300.0	10.0 %	8	100.0 %	95.6 %	3.0
300.0	10.0 %	9	100.0 %	100.0 %	1.0
300.0	5.0 %	1	98.9 %	100.0 %	1.0
300.0	5.0 %	2	98.9 %	100.0 %	1.0
300.0	5.0 %	3	98.9 %	100.0 %	1.0
300.0	5.0 %	4	98.9 %	100.0 %	1.0
300.0	5.0 %	5	98.9 %	100.0 %	1.0
300.0	5.0 %	6	98.9 %	100.0 %	1.0
300.0	5.0 %	7	98.9 %	99.4 %	2.0
300.0	5.0 %	8	98.9 %	91.6 %	6.0
300.0	5.0 %	9	98.9 %	100.0 %	1.0
300.0	2.5 %	1	97.8 %	100.0 %	1.0
300.0	2.5 %	2	97.8 %	100.0 %	1.0
300.0	2.5 %	3	97.8 %	100.0 %	1.0
300.0	2.5 %	4	97.8 %	100.0 %	1.0
300.0	2.5 %	5	97.8 %	100.0 %	1.0
300.0	2.5 %	6	97.8 %	97.7 %	2.0
300.0	2.5 %	7	97.8 %	96.0 %	3.0
300.0	2.5 %	8	97.8 %	76.7 %	7.0
300.0	2.5 %	9	97.8 %	100.0 %	1.0
500.0	100.0 %	1	100.0 %	83.9 %	2.0
500.0	100.0 %	2	100.0 %	100.0 %	1.0
500.0	100.0 %	3	100.0 %	100.0 %	1.0
500.0	100.0 %	4	100.0 %	100.0 %	1.0
500.0	100.0 %	5	100.0 %	100.0 %	1.0
500.0	100.0 %	6	100.0 %	100.0 %	1.0
500.0	100.0 %	7	100.0 %	100.0 %	1.0
500.0	100.0 %	8	100.0 %	100.0 %	1.0
500.0	100.0 %	9	100.0 %	96.3 %	2.0
500.0	10.0 %	1	98.9 %	99.4 %	2.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
500.0	10.0 %	2	98.9 %	100.0 %	1.0
500.0	10.0 %	3	98.9 %	100.0 %	1.0
500.0	10.0 %	4	98.9 %	100.0 %	1.0
500.0	10.0 %	5	98.9 %	100.0 %	1.0
500.0	10.0 %	6	98.9 %	100.0 %	1.0
500.0	10.0 %	7	98.9 %	100.0 %	1.0
500.0	10.0 %	8	98.9 %	92.7 %	4.0
500.0	10.0 %	9	98.9 %	100.0 %	1.0
500.0	5.0 %	1	97.8 %	100.0 %	1.0
500.0	5.0 %	2	97.8 %	100.0 %	1.0
500.0	5.0 %	3	97.8 %	100.0 %	1.0
500.0	5.0 %	4	97.8 %	100.0 %	1.0
500.0	5.0 %	5	97.8 %	100.0 %	1.0
500.0	5.0 %	6	97.8 %	99.4 %	2.0
500.0	5.0 %	7	97.8 %	97.2 %	2.0
500.0	5.0 %	8	97.8 %	87.5 %	5.0
500.0	5.0 %	9	97.8 %	100.0 %	1.0
500.0	2.5 %	1	86.7 %	100.0 %	1.0
500.0	2.5 %	2	86.7 %	100.0 %	1.0
500.0	2.5 %	3	86.7 %	100.0 %	1.0
500.0	2.5 %	4	86.7 %	100.0 %	1.0
500.0	2.5 %	5	86.7 %	100.0 %	1.0
500.0	2.5 %	6	86.7 %	98.7 %	2.0
500.0	2.5 %	7	86.7 %	100.0 %	1.0
500.0	2.5 %	8	86.7 %	63.5 %	6.0
500.0	2.5 %	9	86.7 %	100.0 %	1.0
1000.0	100.0 %	1	100.0 %	85.6 %	2.0
1000.0	100.0 %	2	100.0 %	100.0 %	1.0
1000.0	100.0 %	3	100.0 %	100.0 %	1.0
1000.0	100.0 %	4	100.0 %	100.0 %	1.0
1000.0	100.0 %	5	100.0 %	100.0 %	1.0
1000.0	100.0 %	6	100.0 %	100.0 %	1.0
1000.0	100.0 %	7	100.0 %	100.0 %	1.0
1000.0	100.0 %	8	100.0 %	100.0 %	1.0
1000.0	100.0 %	9	100.0 %	100.0 %	1.0
1000.0	10.0 %	1	97.8 %	100.0 %	1.0
1000.0	10.0 %	2	97.8 %	100.0 %	1.0
1000.0	10.0 %	3	97.8 %	100.0 %	1.0
1000.0	10.0 %	4	97.8 %	99.4 %	2.0
1000.0	10.0 %	5	97.8 %	100.0 %	1.0
1000.0	10.0 %	6	97.8 %	99.4 %	2.0
1000.0	10.0 %	7	97.8 %	97.2 %	2.0
1000.0	10.0 %	8	97.8 %	87.5 %	5.0
1000.0	10.0 %	9	97.8 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	5.0 %	1	86.7 %	100.0 %	1.0
1000.0	5.0 %	2	86.7 %	100.0 %	1.0
1000.0	5.0 %	3	86.7 %	100.0 %	1.0
1000.0	5.0 %	4	86.7 %	100.0 %	1.0
1000.0	5.0 %	5	86.7 %	100.0 %	1.0
1000.0	5.0 %	6	86.7 %	98.7 %	2.0
1000.0	5.0 %	7	86.7 %	100.0 %	1.0
1000.0	5.0 %	8	86.7 %	63.5 %	6.0
1000.0	5.0 %	9	86.7 %	100.0 %	1.0
1000.0	2.5 %	1	73.3 %	100.0 %	1.0
1000.0	2.5 %	2	73.3 %	100.0 %	1.0
1000.0	2.5 %	3	73.3 %	100.0 %	1.0
1000.0	2.5 %	4	73.3 %	99.2 %	2.0
1000.0	2.5 %	5	73.3 %	99.2 %	2.0
1000.0	2.5 %	6	73.3 %	99.2 %	2.0
1000.0	2.5 %	7	73.3 %	92.4 %	2.0
1000.0	2.5 %	8	73.3 %	66.7 %	7.0
1000.0	2.5 %	9	73.3 %	100.0 %	1.0
5000.0	100.0 %	1	98.9 %	99.4 %	2.0
5000.0	100.0 %	2	98.9 %	100.0 %	1.0
5000.0	100.0 %	3	98.9 %	100.0 %	1.0
5000.0	100.0 %	4	98.9 %	98.9 %	2.0
5000.0	100.0 %	5	98.9 %	100.0 %	1.0
5000.0	100.0 %	6	98.9 %	100.0 %	1.0
5000.0	100.0 %	7	98.9 %	100.0 %	1.0
5000.0	100.0 %	8	98.9 %	92.7 %	4.0
5000.0	100.0 %	9	98.9 %	100.0 %	1.0
5000.0	10.0 %	1	67.2 %	100.0 %	1.0
5000.0	10.0 %	2	67.2 %	100.0 %	1.0
5000.0	10.0 %	3	67.2 %	100.0 %	1.0
5000.0	10.0 %	4	67.2 %	98.3 %	2.0
5000.0	10.0 %	5	67.2 %	100.0 %	1.0
5000.0	10.0 %	6	67.2 %	96.7 %	2.0
5000.0	10.0 %	7	67.2 %	90.9 %	2.0
5000.0	10.0 %	8	67.2 %	73.6 %	5.0
5000.0	10.0 %	9	67.2 %	98.2 %	2.0
5000.0	5.0 %	1	53.9 %	100.0 %	1.0
5000.0	5.0 %	2	53.9 %	100.0 %	1.0
5000.0	5.0 %	3	53.9 %	100.0 %	1.0
5000.0	5.0 %	4	53.9 %	97.9 %	2.0
5000.0	5.0 %	5	53.9 %	100.0 %	1.0
5000.0	5.0 %	6	53.9 %	93.8 %	3.0
5000.0	5.0 %	7	53.9 %	72.2 %	3.0
5000.0	5.0 %	8	53.9 %	79.4 %	5.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
5000.0	5.0 %	9	53.9 %	100.0 %	1.0
5000.0	2.5 %	1	39.4 %	100.0 %	1.0
5000.0	2.5 %	2	39.4 %	100.0 %	1.0
5000.0	2.5 %	3	39.4 %	100.0 %	1.0
5000.0	2.5 %	4	39.4 %	98.6 %	2.0
5000.0	2.5 %	5	39.4 %	100.0 %	1.0
5000.0	2.5 %	6	39.4 %	91.5 %	2.0
5000.0	2.5 %	7	39.4 %	54.9 %	4.0
5000.0	2.5 %	8	39.4 %	87.3 %	2.0
5000.0	2.5 %	9	39.4 %	100.0 %	1.0

Table 46 – Summary of Unique Contracts Acquired for the 120-Month Hedging Strategy bell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	100.0 %	1	100.0 %	83.9 %	2.0
100.0	100.0 %	2	100.0 %	100.0 %	1.0
100.0	100.0 %	3	100.0 %	100.0 %	1.0
100.0	100.0 %	4	100.0 %	100.0 %	1.0
100.0	100.0 %	5	100.0 %	100.0 %	1.0
100.0	100.0 %	6	100.0 %	100.0 %	1.0
100.0	100.0 %	7	100.0 %	100.0 %	1.0
100.0	100.0 %	8	100.0 %	100.0 %	1.0
100.0	100.0 %	9	100.0 %	100.0 %	1.0
100.0	10.0 %	1	98.3 %	83.6 %	2.0
100.0	10.0 %	2	98.3 %	100.0 %	1.0
100.0	10.0 %	3	98.3 %	100.0 %	1.0
100.0	10.0 %	4	98.3 %	100.0 %	1.0
100.0	10.0 %	5	98.3 %	100.0 %	1.0
100.0	10.0 %	6	98.3 %	100.0 %	1.0
100.0	10.0 %	7	98.3 %	100.0 %	1.0
100.0	10.0 %	8	98.3 %	97.2 %	2.0
100.0	10.0 %	9	98.3 %	91.5 %	2.0
100.0	5.0 %	1	96.1 %	83.8 %	2.0
100.0	5.0 %	2	96.1 %	100.0 %	1.0
100.0	5.0 %	3	96.1 %	100.0 %	1.0
100.0	5.0 %	4	96.1 %	100.0 %	1.0
100.0	5.0 %	5	96.1 %	100.0 %	1.0
100.0	5.0 %	6	96.1 %	100.0 %	1.0
100.0	5.0 %	7	96.1 %	100.0 %	1.0
100.0	5.0 %	8	96.1 %	93.6 %	3.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	5.0 %	9	96.1 %	88.4 %	3.0
100.0	2.5 %	1	88.9 %	88.1 %	2.0
100.0	2.5 %	2	88.9 %	100.0 %	1.0
100.0	2.5 %	3	88.9 %	100.0 %	1.0
100.0	2.5 %	4	88.9 %	100.0 %	1.0
100.0	2.5 %	5	88.9 %	100.0 %	1.0
100.0	2.5 %	6	88.9 %	100.0 %	1.0
100.0	2.5 %	7	88.9 %	100.0 %	1.0
100.0	2.5 %	8	88.9 %	84.4 %	3.0
100.0	2.5 %	9	88.9 %	95.6 %	3.0
300.0	100.0 %	1	98.9 %	83.7 %	2.0
300.0	100.0 %	2	98.9 %	100.0 %	1.0
300.0	100.0 %	3	98.9 %	100.0 %	1.0
300.0	100.0 %	4	98.9 %	100.0 %	1.0
300.0	100.0 %	5	98.9 %	100.0 %	1.0
300.0	100.0 %	6	98.9 %	100.0 %	1.0
300.0	100.0 %	7	98.9 %	100.0 %	1.0
300.0	100.0 %	8	98.9 %	100.0 %	1.0
300.0	100.0 %	9	98.9 %	100.0 %	1.0
300.0	10.0 %	1	92.2 %	84.9 %	2.0
300.0	10.0 %	2	92.2 %	100.0 %	1.0
300.0	10.0 %	3	92.2 %	100.0 %	1.0
300.0	10.0 %	4	92.2 %	100.0 %	1.0
300.0	10.0 %	5	92.2 %	100.0 %	1.0
300.0	10.0 %	6	92.2 %	100.0 %	1.0
300.0	10.0 %	7	92.2 %	100.0 %	1.0
300.0	10.0 %	8	92.2 %	92.2 %	3.0
300.0	10.0 %	9	92.2 %	91.6 %	4.0
300.0	5.0 %	1	86.1 %	91.0 %	2.0
300.0	5.0 %	2	86.1 %	100.0 %	1.0
300.0	5.0 %	3	86.1 %	100.0 %	1.0
300.0	5.0 %	4	86.1 %	100.0 %	1.0
300.0	5.0 %	5	86.1 %	100.0 %	1.0
300.0	5.0 %	6	86.1 %	100.0 %	1.0
300.0	5.0 %	7	86.1 %	100.0 %	1.0
300.0	5.0 %	8	86.1 %	66.5 %	3.0
300.0	5.0 %	9	86.1 %	96.1 %	3.0
300.0	2.5 %	1	72.8 %	98.5 %	2.0
300.0	2.5 %	2	72.8 %	100.0 %	1.0
300.0	2.5 %	3	72.8 %	100.0 %	1.0
300.0	2.5 %	4	72.8 %	100.0 %	1.0
300.0	2.5 %	5	72.8 %	100.0 %	1.0
300.0	2.5 %	6	72.8 %	100.0 %	1.0
300.0	2.5 %	7	72.8 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	2.5 %	8	72.8 %	66.4 %	7.0
300.0	2.5 %	9	72.8 %	95.4 %	3.0
500.0	100.0 %	1	98.9 %	83.7 %	2.0
500.0	100.0 %	2	98.9 %	100.0 %	1.0
500.0	100.0 %	3	98.9 %	100.0 %	1.0
500.0	100.0 %	4	98.9 %	100.0 %	1.0
500.0	100.0 %	5	98.9 %	100.0 %	1.0
500.0	100.0 %	6	98.9 %	100.0 %	1.0
500.0	100.0 %	7	98.9 %	100.0 %	1.0
500.0	100.0 %	8	98.9 %	98.9 %	2.0
500.0	100.0 %	9	98.9 %	97.8 %	2.0
500.0	10.0 %	1	87.2 %	89.2 %	2.0
500.0	10.0 %	2	87.2 %	100.0 %	1.0
500.0	10.0 %	3	87.2 %	100.0 %	1.0
500.0	10.0 %	4	87.2 %	100.0 %	1.0
500.0	10.0 %	5	87.2 %	100.0 %	1.0
500.0	10.0 %	6	87.2 %	100.0 %	1.0
500.0	10.0 %	7	87.2 %	100.0 %	1.0
500.0	10.0 %	8	87.2 %	79.6 %	3.0
500.0	10.0 %	9	87.2 %	95.5 %	3.0
500.0	5.0 %	1	78.9 %	97.2 %	2.0
500.0	5.0 %	2	78.9 %	100.0 %	1.0
500.0	5.0 %	3	78.9 %	100.0 %	1.0
500.0	5.0 %	4	78.9 %	100.0 %	1.0
500.0	5.0 %	5	78.9 %	100.0 %	1.0
500.0	5.0 %	6	78.9 %	100.0 %	1.0
500.0	5.0 %	7	78.9 %	100.0 %	1.0
500.0	5.0 %	8	78.9 %	62.7 %	4.0
500.0	5.0 %	9	78.9 %	96.5 %	3.0
500.0	2.5 %	1	65.6 %	100.0 %	1.0
500.0	2.5 %	2	65.6 %	100.0 %	1.0
500.0	2.5 %	3	65.6 %	100.0 %	1.0
500.0	2.5 %	4	65.6 %	100.0 %	1.0
500.0	2.5 %	5	65.6 %	100.0 %	1.0
500.0	2.5 %	6	65.6 %	100.0 %	1.0
500.0	2.5 %	7	65.6 %	100.0 %	1.0
500.0	2.5 %	8	65.6 %	72.9 %	5.0
500.0	2.5 %	9	65.6 %	90.7 %	2.0
1000.0	100.0 %	1	98.3 %	83.6 %	2.0
1000.0	100.0 %	2	98.3 %	100.0 %	1.0
1000.0	100.0 %	3	98.3 %	100.0 %	1.0
1000.0	100.0 %	4	98.3 %	100.0 %	1.0
1000.0	100.0 %	5	98.3 %	100.0 %	1.0
1000.0	100.0 %	6	98.3 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	100.0 %	7	98.3 %	100.0 %	1.0
1000.0	100.0 %	8	98.3 %	97.2 %	2.0
1000.0	100.0 %	9	98.3 %	91.0 %	2.0
1000.0	10.0 %	1	78.9 %	97.2 %	2.0
1000.0	10.0 %	2	78.9 %	100.0 %	1.0
1000.0	10.0 %	3	78.9 %	100.0 %	1.0
1000.0	10.0 %	4	78.9 %	100.0 %	1.0
1000.0	10.0 %	5	78.9 %	100.0 %	1.0
1000.0	10.0 %	6	78.9 %	100.0 %	1.0
1000.0	10.0 %	7	78.9 %	100.0 %	1.0
1000.0	10.0 %	8	78.9 %	62.7 %	4.0
1000.0	10.0 %	9	78.9 %	97.2 %	3.0
1000.0	5.0 %	1	65.6 %	100.0 %	1.0
1000.0	5.0 %	2	65.6 %	100.0 %	1.0
1000.0	5.0 %	3	65.6 %	100.0 %	1.0
1000.0	5.0 %	4	65.6 %	100.0 %	1.0
1000.0	5.0 %	5	65.6 %	100.0 %	1.0
1000.0	5.0 %	6	65.6 %	100.0 %	1.0
1000.0	5.0 %	7	65.6 %	100.0 %	1.0
1000.0	5.0 %	8	65.6 %	72.9 %	5.0
1000.0	5.0 %	9	65.6 %	89.8 %	2.0
1000.0	2.5 %	1	60.6 %	100.0 %	1.0
1000.0	2.5 %	2	60.6 %	100.0 %	1.0
1000.0	2.5 %	3	60.6 %	100.0 %	1.0
1000.0	2.5 %	4	60.6 %	100.0 %	1.0
1000.0	2.5 %	5	60.6 %	100.0 %	1.0
1000.0	2.5 %	6	60.6 %	99.1 %	2.0
1000.0	2.5 %	7	60.6 %	100.0 %	1.0
1000.0	2.5 %	8	60.6 %	78.0 %	3.0
1000.0	2.5 %	9	60.6 %	96.3 %	2.0
5000.0	100.0 %	1	87.2 %	89.2 %	2.0
5000.0	100.0 %	2	87.2 %	100.0 %	1.0
5000.0	100.0 %	3	87.2 %	100.0 %	1.0
5000.0	100.0 %	4	87.2 %	98.7 %	2.0
5000.0	100.0 %	5	87.2 %	100.0 %	1.0
5000.0	100.0 %	6	87.2 %	100.0 %	1.0
5000.0	100.0 %	7	87.2 %	100.0 %	1.0
5000.0	100.0 %	8	87.2 %	79.0 %	3.0
5000.0	100.0 %	9	87.2 %	95.5 %	3.0
5000.0	10.0 %	1	59.4 %	100.0 %	1.0
5000.0	10.0 %	2	59.4 %	100.0 %	1.0
5000.0	10.0 %	3	59.4 %	100.0 %	1.0
5000.0	10.0 %	4	59.4 %	99.1 %	2.0
5000.0	10.0 %	5	59.4 %	100.0 %	1.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
5000.0	10.0 %	6	59.4 %	100.0 %	1.0
5000.0	10.0 %	7	59.4 %	99.1 %	2.0
5000.0	10.0 %	8	59.4 %	76.6 %	3.0
5000.0	10.0 %	9	59.4 %	97.2 %	2.0
5000.0	5.0 %	1	57.2 %	100.0 %	1.0
5000.0	5.0 %	2	57.2 %	100.0 %	1.0
5000.0	5.0 %	3	57.2 %	100.0 %	1.0
5000.0	5.0 %	4	57.2 %	99.0 %	2.0
5000.0	5.0 %	5	57.2 %	100.0 %	1.0
5000.0	5.0 %	6	57.2 %	99.0 %	2.0
5000.0	5.0 %	7	57.2 %	95.1 %	2.0
5000.0	5.0 %	8	57.2 %	74.8 %	6.0
5000.0	5.0 %	9	57.2 %	98.1 %	2.0
5000.0	2.5 %	1	45.6 %	100.0 %	1.0
5000.0	2.5 %	2	45.6 %	100.0 %	1.0
5000.0	2.5 %	3	45.6 %	100.0 %	1.0
5000.0	2.5 %	4	45.6 %	98.8 %	2.0
5000.0	2.5 %	5	45.6 %	100.0 %	1.0
5000.0	2.5 %	6	45.6 %	98.8 %	2.0
5000.0	2.5 %	7	45.6 %	78.0 %	2.0
5000.0	2.5 %	8	45.6 %	56.1 %	3.0
5000.0	2.5 %	9	45.6 %	82.9 %	4.0

D.3 Inverted Bell Portfolios

Table 47 – Summary of Unique Contracts Acquired for the 36-Month Hedging Strategy invertedbell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	100.0 %	1	100.0 %	100.0 %	1
100.0	100.0 %	2	100.0 %	100.0 %	1
100.0	100.0 %	3	100.0 %	100.0 %	1
100.0	100.0 %	4	100.0 %	100.0 %	1
100.0	100.0 %	5	100.0 %	100.0 %	1
100.0	100.0 %	6	100.0 %	100.0 %	1
100.0	100.0 %	7	100.0 %	98.3 %	2
100.0	100.0 %	8	100.0 %	0.0 %	0
100.0	100.0 %	9	100.0 %	0.0 %	0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	10.0 %	1	100.0 %	100.0 %	1
100.0	10.0 %	2	100.0 %	100.0 %	1
100.0	10.0 %	3	100.0 %	100.0 %	1
100.0	10.0 %	4	100.0 %	100.0 %	1
100.0	10.0 %	5	100.0 %	100.0 %	1
100.0	10.0 %	6	100.0 %	100.0 %	1
100.0	10.0 %	7	100.0 %	100.0 %	1
100.0	10.0 %	8	100.0 %	0.0 %	0
100.0	10.0 %	9	100.0 %	0.0 %	0
100.0	5.0 %	1	100.0 %	100.0 %	1
100.0	5.0 %	2	100.0 %	100.0 %	1
100.0	5.0 %	3	100.0 %	100.0 %	1
100.0	5.0 %	4	100.0 %	100.0 %	1
100.0	5.0 %	5	100.0 %	100.0 %	1
100.0	5.0 %	6	100.0 %	100.0 %	1
100.0	5.0 %	7	100.0 %	98.9 %	2
100.0	5.0 %	8	100.0 %	0.0 %	0
100.0	5.0 %	9	100.0 %	0.0 %	0
100.0	2.5 %	1	100.0 %	100.0 %	1
100.0	2.5 %	2	100.0 %	100.0 %	1
100.0	2.5 %	3	100.0 %	100.0 %	1
100.0	2.5 %	4	100.0 %	100.0 %	1
100.0	2.5 %	5	100.0 %	100.0 %	1
100.0	2.5 %	6	100.0 %	100.0 %	1
100.0	2.5 %	7	100.0 %	94.4 %	2
100.0	2.5 %	8	100.0 %	0.0 %	0
100.0	2.5 %	9	100.0 %	0.0 %	0
300.0	100.0 %	1	100.0 %	100.0 %	1
300.0	100.0 %	2	100.0 %	100.0 %	1
300.0	100.0 %	3	100.0 %	100.0 %	1
300.0	100.0 %	4	100.0 %	100.0 %	1
300.0	100.0 %	5	100.0 %	100.0 %	1
300.0	100.0 %	6	100.0 %	100.0 %	1
300.0	100.0 %	7	100.0 %	99.4 %	2
300.0	100.0 %	8	100.0 %	0.0 %	0
300.0	100.0 %	9	100.0 %	0.0 %	0
300.0	10.0 %	1	100.0 %	100.0 %	1
300.0	10.0 %	2	100.0 %	100.0 %	1
300.0	10.0 %	3	100.0 %	100.0 %	1
300.0	10.0 %	4	100.0 %	100.0 %	1
300.0	10.0 %	5	100.0 %	100.0 %	1
300.0	10.0 %	6	100.0 %	100.0 %	1
300.0	10.0 %	7	100.0 %	96.7 %	2
300.0	10.0 %	8	100.0 %	0.0 %	0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
300.0	10.0 %	9	100.0 %	0.0 %	0
300.0	5.0 %	1	100.0 %	100.0 %	1
300.0	5.0 %	2	100.0 %	100.0 %	1
300.0	5.0 %	3	100.0 %	100.0 %	1
300.0	5.0 %	4	100.0 %	100.0 %	1
300.0	5.0 %	5	100.0 %	100.0 %	1
300.0	5.0 %	6	100.0 %	98.9 %	2
300.0	5.0 %	7	100.0 %	93.9 %	2
300.0	5.0 %	8	100.0 %	0.0 %	0
300.0	5.0 %	9	100.0 %	0.0 %	0
300.0	2.5 %	1	100.0 %	100.0 %	1
300.0	2.5 %	2	100.0 %	100.0 %	1
300.0	2.5 %	3	100.0 %	100.0 %	1
300.0	2.5 %	4	100.0 %	100.0 %	1
300.0	2.5 %	5	100.0 %	100.0 %	1
300.0	2.5 %	6	100.0 %	96.7 %	2
300.0	2.5 %	7	100.0 %	93.9 %	4
300.0	2.5 %	8	100.0 %	0.0 %	0
300.0	2.5 %	9	100.0 %	0.0 %	0
500.0	100.0 %	1	100.0 %	100.0 %	1
500.0	100.0 %	2	100.0 %	100.0 %	1
500.0	100.0 %	3	100.0 %	100.0 %	1
500.0	100.0 %	4	100.0 %	100.0 %	1
500.0	100.0 %	5	100.0 %	100.0 %	1
500.0	100.0 %	6	100.0 %	100.0 %	1
500.0	100.0 %	7	100.0 %	99.4 %	2
500.0	100.0 %	8	100.0 %	0.0 %	0
500.0	100.0 %	9	100.0 %	0.0 %	0
500.0	10.0 %	1	100.0 %	100.0 %	1
500.0	10.0 %	2	100.0 %	100.0 %	1
500.0	10.0 %	3	100.0 %	100.0 %	1
500.0	10.0 %	4	100.0 %	100.0 %	1
500.0	10.0 %	5	100.0 %	100.0 %	1
500.0	10.0 %	6	100.0 %	100.0 %	1
500.0	10.0 %	7	100.0 %	93.9 %	2
500.0	10.0 %	8	100.0 %	0.0 %	0
500.0	10.0 %	9	100.0 %	0.0 %	0
500.0	5.0 %	1	100.0 %	100.0 %	1
500.0	5.0 %	2	100.0 %	100.0 %	1
500.0	5.0 %	3	100.0 %	100.0 %	1
500.0	5.0 %	4	100.0 %	100.0 %	1
500.0	5.0 %	5	100.0 %	100.0 %	1
500.0	5.0 %	6	100.0 %	97.8 %	2
500.0	5.0 %	7	100.0 %	94.4 %	3

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
500.0	5.0 %	8	100.0 %	0.0 %	0
500.0	5.0 %	9	100.0 %	0.0 %	0
500.0	2.5 %	1	97.8 %	100.0 %	1
500.0	2.5 %	2	97.8 %	100.0 %	1
500.0	2.5 %	3	97.8 %	100.0 %	1
500.0	2.5 %	4	97.8 %	100.0 %	1
500.0	2.5 %	5	97.8 %	100.0 %	1
500.0	2.5 %	6	97.8 %	97.7 %	2
500.0	2.5 %	7	97.8 %	93.2 %	3
500.0	2.5 %	8	97.8 %	0.0 %	0
500.0	2.5 %	9	97.8 %	0.0 %	0
1000.0	100.0 %	1	100.0 %	100.0 %	1
1000.0	100.0 %	2	100.0 %	100.0 %	1
1000.0	100.0 %	3	100.0 %	100.0 %	1
1000.0	100.0 %	4	100.0 %	100.0 %	1
1000.0	100.0 %	5	100.0 %	100.0 %	1
1000.0	100.0 %	6	100.0 %	100.0 %	1
1000.0	100.0 %	7	100.0 %	100.0 %	1
1000.0	100.0 %	8	100.0 %	0.0 %	0
1000.0	100.0 %	9	100.0 %	0.0 %	0
1000.0	10.0 %	1	100.0 %	100.0 %	1
1000.0	10.0 %	2	100.0 %	100.0 %	1
1000.0	10.0 %	3	100.0 %	100.0 %	1
1000.0	10.0 %	4	100.0 %	100.0 %	1
1000.0	10.0 %	5	100.0 %	100.0 %	1
1000.0	10.0 %	6	100.0 %	97.8 %	2
1000.0	10.0 %	7	100.0 %	94.4 %	3
1000.0	10.0 %	8	100.0 %	0.0 %	0
1000.0	10.0 %	9	100.0 %	0.0 %	0
1000.0	5.0 %	1	97.8 %	100.0 %	1
1000.0	5.0 %	2	97.8 %	100.0 %	1
1000.0	5.0 %	3	97.8 %	100.0 %	1
1000.0	5.0 %	4	97.8 %	100.0 %	1
1000.0	5.0 %	5	97.8 %	100.0 %	1
1000.0	5.0 %	6	97.8 %	97.7 %	2
1000.0	5.0 %	7	97.8 %	93.2 %	3
1000.0	5.0 %	8	97.8 %	0.0 %	0
1000.0	5.0 %	9	97.8 %	0.0 %	0
1000.0	2.5 %	1	93.3 %	100.0 %	1
1000.0	2.5 %	2	93.3 %	100.0 %	1
1000.0	2.5 %	3	93.3 %	100.0 %	1
1000.0	2.5 %	4	93.3 %	100.0 %	1
1000.0	2.5 %	5	93.3 %	100.0 %	1
1000.0	2.5 %	6	93.3 %	95.2 %	3

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	2.5 %	7	93.3 %	91.1 %	4
1000.0	2.5 %	8	93.3 %	0.0 %	0
1000.0	2.5 %	9	93.3 %	0.0 %	0
5000.0	100.0 %	1	100.0 %	100.0 %	1
5000.0	100.0 %	2	100.0 %	100.0 %	1
5000.0	100.0 %	3	100.0 %	100.0 %	1
5000.0	100.0 %	4	100.0 %	100.0 %	1
5000.0	100.0 %	5	100.0 %	100.0 %	1
5000.0	100.0 %	6	100.0 %	100.0 %	1
5000.0	100.0 %	7	100.0 %	93.9 %	2
5000.0	100.0 %	8	100.0 %	0.0 %	0
5000.0	100.0 %	9	100.0 %	0.0 %	0
5000.0	10.0 %	1	90.0 %	100.0 %	1
5000.0	10.0 %	2	90.0 %	100.0 %	1
5000.0	10.0 %	3	90.0 %	100.0 %	1
5000.0	10.0 %	4	90.0 %	100.0 %	1
5000.0	10.0 %	5	90.0 %	100.0 %	1
5000.0	10.0 %	6	90.0 %	90.7 %	3
5000.0	10.0 %	7	90.0 %	90.1 %	5
5000.0	10.0 %	8	90.0 %	0.0 %	0
5000.0	10.0 %	9	90.0 %	0.0 %	0
5000.0	5.0 %	1	68.3 %	100.0 %	1
5000.0	5.0 %	2	68.3 %	100.0 %	1
5000.0	5.0 %	3	68.3 %	100.0 %	1
5000.0	5.0 %	4	68.3 %	99.2 %	2
5000.0	5.0 %	5	68.3 %	100.0 %	1
5000.0	5.0 %	6	68.3 %	90.2 %	3
5000.0	5.0 %	7	68.3 %	89.4 %	4
5000.0	5.0 %	8	68.3 %	0.0 %	0
5000.0	5.0 %	9	68.3 %	0.0 %	0
5000.0	2.5 %	1	50.6 %	94.5 %	2
5000.0	2.5 %	2	50.6 %	96.7 %	3
5000.0	2.5 %	3	50.6 %	100.0 %	1
5000.0	2.5 %	4	50.6 %	97.8 %	2
5000.0	2.5 %	5	50.6 %	100.0 %	1
5000.0	2.5 %	6	50.6 %	91.2 %	2
5000.0	2.5 %	7	50.6 %	76.9 %	6
5000.0	2.5 %	8	50.6 %	0.0 %	0
5000.0	2.5 %	9	50.6 %	0.0 %	0

Table 48 – Summary of Unique Contracts Acquired for the 60-Month Hedging Strategy invertedbell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	100.0 %	1
100.0	100.0 %	2	100.0 %	100.0 %	1
100.0	100.0 %	3	100.0 %	100.0 %	1
100.0	100.0 %	4	100.0 %	100.0 %	1
100.0	100.0 %	5	100.0 %	100.0 %	1
100.0	100.0 %	6	100.0 %	100.0 %	1
100.0	100.0 %	7	100.0 %	100.0 %	1
100.0	100.0 %	8	100.0 %	98.9 %	2
100.0	100.0 %	9	100.0 %	0.0 %	0
100.0	10.0 %	1	100.0 %	100.0 %	1
100.0	10.0 %	2	100.0 %	100.0 %	1
100.0	10.0 %	3	100.0 %	100.0 %	1
100.0	10.0 %	4	100.0 %	100.0 %	1
100.0	10.0 %	5	100.0 %	100.0 %	1
100.0	10.0 %	6	100.0 %	100.0 %	1
100.0	10.0 %	7	100.0 %	100.0 %	1
100.0	10.0 %	8	100.0 %	100.0 %	1
100.0	10.0 %	9	100.0 %	0.0 %	0
100.0	5.0 %	1	100.0 %	100.0 %	1
100.0	5.0 %	2	100.0 %	100.0 %	1
100.0	5.0 %	3	100.0 %	100.0 %	1
100.0	5.0 %	4	100.0 %	100.0 %	1
100.0	5.0 %	5	100.0 %	100.0 %	1
100.0	5.0 %	6	100.0 %	100.0 %	1
100.0	5.0 %	7	100.0 %	100.0 %	1
100.0	5.0 %	8	100.0 %	97.8 %	2
100.0	5.0 %	9	100.0 %	0.0 %	0
100.0	2.5 %	1	98.9 %	100.0 %	1
100.0	2.5 %	2	98.9 %	100.0 %	1
100.0	2.5 %	3	98.9 %	100.0 %	1
100.0	2.5 %	4	98.9 %	100.0 %	1
100.0	2.5 %	5	98.9 %	100.0 %	1
100.0	2.5 %	6	98.9 %	100.0 %	1
100.0	2.5 %	7	98.9 %	100.0 %	1
100.0	2.5 %	8	98.9 %	93.3 %	5
100.0	2.5 %	9	98.9 %	0.0 %	0
300.0	100.0 %	1	100.0 %	100.0 %	1

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
300.0	100.0 %	2	100.0 %	100.0 %	1
300.0	100.0 %	3	100.0 %	100.0 %	1
300.0	100.0 %	4	100.0 %	100.0 %	1
300.0	100.0 %	5	100.0 %	100.0 %	1
300.0	100.0 %	6	100.0 %	100.0 %	1
300.0	100.0 %	7	100.0 %	100.0 %	1
300.0	100.0 %	8	100.0 %	100.0 %	1
300.0	100.0 %	9	100.0 %	0.0 %	0
300.0	10.0 %	1	99.4 %	100.0 %	1
300.0	10.0 %	2	99.4 %	100.0 %	1
300.0	10.0 %	3	99.4 %	100.0 %	1
300.0	10.0 %	4	99.4 %	99.4 %	2
300.0	10.0 %	5	99.4 %	100.0 %	1
300.0	10.0 %	6	99.4 %	100.0 %	1
300.0	10.0 %	7	99.4 %	100.0 %	1
300.0	10.0 %	8	99.4 %	96.1 %	6
300.0	10.0 %	9	99.4 %	0.0 %	0
300.0	5.0 %	1	98.3 %	100.0 %	1
300.0	5.0 %	2	98.3 %	100.0 %	1
300.0	5.0 %	3	98.3 %	100.0 %	1
300.0	5.0 %	4	98.3 %	100.0 %	1
300.0	5.0 %	5	98.3 %	100.0 %	1
300.0	5.0 %	6	98.3 %	100.0 %	1
300.0	5.0 %	7	98.3 %	99.4 %	2
300.0	5.0 %	8	98.3 %	89.8 %	6
300.0	5.0 %	9	98.3 %	0.0 %	0
300.0	2.5 %	1	93.9 %	100.0 %	1
300.0	2.5 %	2	93.9 %	100.0 %	1
300.0	2.5 %	3	93.9 %	100.0 %	1
300.0	2.5 %	4	93.9 %	100.0 %	1
300.0	2.5 %	5	93.9 %	100.0 %	1
300.0	2.5 %	6	93.9 %	100.0 %	1
300.0	2.5 %	7	93.9 %	99.4 %	2
300.0	2.5 %	8	93.9 %	67.5 %	4
300.0	2.5 %	9	93.9 %	0.0 %	0
500.0	100.0 %	1	100.0 %	100.0 %	1
500.0	100.0 %	2	100.0 %	100.0 %	1
500.0	100.0 %	3	100.0 %	100.0 %	1
500.0	100.0 %	4	100.0 %	100.0 %	1
500.0	100.0 %	5	100.0 %	100.0 %	1
500.0	100.0 %	6	100.0 %	100.0 %	1
500.0	100.0 %	7	100.0 %	100.0 %	1
500.0	100.0 %	8	100.0 %	100.0 %	1
500.0	100.0 %	9	100.0 %	0.0 %	0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
500.0	10.0 %	1	98.3 %	100.0 %	1
500.0	10.0 %	2	98.3 %	100.0 %	1
500.0	10.0 %	3	98.3 %	100.0 %	1
500.0	10.0 %	4	98.3 %	100.0 %	1
500.0	10.0 %	5	98.3 %	100.0 %	1
500.0	10.0 %	6	98.3 %	100.0 %	1
500.0	10.0 %	7	98.3 %	99.4 %	2
500.0	10.0 %	8	98.3 %	91.5 %	4
500.0	10.0 %	9	98.3 %	0.0 %	0
500.0	5.0 %	1	95.6 %	100.0 %	1
500.0	5.0 %	2	95.6 %	100.0 %	1
500.0	5.0 %	3	95.6 %	100.0 %	1
500.0	5.0 %	4	95.6 %	100.0 %	1
500.0	5.0 %	5	95.6 %	100.0 %	1
500.0	5.0 %	6	95.6 %	100.0 %	1
500.0	5.0 %	7	95.6 %	99.4 %	2
500.0	5.0 %	8	95.6 %	79.1 %	5
500.0	5.0 %	9	95.6 %	0.0 %	0
500.0	2.5 %	1	78.3 %	100.0 %	1
500.0	2.5 %	2	78.3 %	100.0 %	1
500.0	2.5 %	3	78.3 %	100.0 %	1
500.0	2.5 %	4	78.3 %	100.0 %	1
500.0	2.5 %	5	78.3 %	100.0 %	1
500.0	2.5 %	6	78.3 %	100.0 %	1
500.0	2.5 %	7	78.3 %	100.0 %	1
500.0	2.5 %	8	78.3 %	64.5 %	5
500.0	2.5 %	9	78.3 %	0.0 %	0
1000.0	100.0 %	1	100.0 %	100.0 %	1
1000.0	100.0 %	2	100.0 %	100.0 %	1
1000.0	100.0 %	3	100.0 %	100.0 %	1
1000.0	100.0 %	4	100.0 %	99.4 %	2
1000.0	100.0 %	5	100.0 %	100.0 %	1
1000.0	100.0 %	6	100.0 %	100.0 %	1
1000.0	100.0 %	7	100.0 %	100.0 %	1
1000.0	100.0 %	8	100.0 %	100.0 %	1
1000.0	100.0 %	9	100.0 %	0.0 %	0
1000.0	10.0 %	1	95.6 %	100.0 %	1
1000.0	10.0 %	2	95.6 %	100.0 %	1
1000.0	10.0 %	3	95.6 %	100.0 %	1
1000.0	10.0 %	4	95.6 %	99.4 %	2
1000.0	10.0 %	5	95.6 %	100.0 %	1
1000.0	10.0 %	6	95.6 %	100.0 %	1
1000.0	10.0 %	7	95.6 %	99.4 %	2
1000.0	10.0 %	8	95.6 %	79.1 %	5

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	10.0 %	9	95.6 %	0.0 %	0
1000.0	5.0 %	1	78.3 %	100.0 %	1
1000.0	5.0 %	2	78.3 %	100.0 %	1
1000.0	5.0 %	3	78.3 %	100.0 %	1
1000.0	5.0 %	4	78.3 %	100.0 %	1
1000.0	5.0 %	5	78.3 %	100.0 %	1
1000.0	5.0 %	6	78.3 %	100.0 %	1
1000.0	5.0 %	7	78.3 %	100.0 %	1
1000.0	5.0 %	8	78.3 %	64.5 %	5
1000.0	5.0 %	9	78.3 %	0.0 %	0
1000.0	2.5 %	1	60.0 %	100.0 %	1
1000.0	2.5 %	2	60.0 %	100.0 %	1
1000.0	2.5 %	3	60.0 %	100.0 %	1
1000.0	2.5 %	4	60.0 %	98.1 %	2
1000.0	2.5 %	5	60.0 %	100.0 %	1
1000.0	2.5 %	6	60.0 %	100.0 %	1
1000.0	2.5 %	7	60.0 %	98.1 %	2
1000.0	2.5 %	8	60.0 %	73.1 %	6
1000.0	2.5 %	9	60.0 %	0.0 %	0
5000.0	100.0 %	1	98.3 %	100.0 %	1
5000.0	100.0 %	2	98.3 %	100.0 %	1
5000.0	100.0 %	3	98.3 %	100.0 %	1
5000.0	100.0 %	4	98.3 %	100.0 %	1
5000.0	100.0 %	5	98.3 %	100.0 %	1
5000.0	100.0 %	6	98.3 %	100.0 %	1
5000.0	100.0 %	7	98.3 %	99.4 %	2
5000.0	100.0 %	8	98.3 %	91.5 %	4
5000.0	100.0 %	9	98.3 %	0.0 %	0
5000.0	10.0 %	1	56.1 %	100.0 %	1
5000.0	10.0 %	2	56.1 %	100.0 %	1
5000.0	10.0 %	3	56.1 %	100.0 %	1
5000.0	10.0 %	4	56.1 %	99.0 %	2
5000.0	10.0 %	5	56.1 %	100.0 %	1
5000.0	10.0 %	6	56.1 %	100.0 %	1
5000.0	10.0 %	7	56.1 %	98.0 %	2
5000.0	10.0 %	8	56.1 %	77.2 %	6
5000.0	10.0 %	9	56.1 %	0.0 %	0
5000.0	5.0 %	1	46.7 %	100.0 %	1
5000.0	5.0 %	2	46.7 %	100.0 %	1
5000.0	5.0 %	3	46.7 %	100.0 %	1
5000.0	5.0 %	4	46.7 %	97.6 %	2
5000.0	5.0 %	5	46.7 %	98.8 %	2
5000.0	5.0 %	6	46.7 %	100.0 %	1
5000.0	5.0 %	7	46.7 %	98.8 %	2

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
5000.0	5.0 %	8	46.7 %	84.5 %	4
5000.0	5.0 %	9	46.7 %	0.0 %	0
5000.0	2.5 %	1	39.4 %	95.8 %	2
5000.0	2.5 %	2	39.4 %	100.0 %	1
5000.0	2.5 %	3	39.4 %	98.6 %	3
5000.0	2.5 %	4	39.4 %	95.8 %	4
5000.0	2.5 %	5	39.4 %	100.0 %	1
5000.0	2.5 %	6	39.4 %	100.0 %	1
5000.0	2.5 %	7	39.4 %	84.5 %	2
5000.0	2.5 %	8	39.4 %	90.1 %	4
5000.0	2.5 %	9	39.4 %	0.0 %	0

Table 49 – Summary of Unique Contracts Acquired for the 84-Month Hedging Strategy invertedbell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	100.0 %	1	100.0 %	100.0 %	1.0
100.0	100.0 %	2	100.0 %	100.0 %	1.0
100.0	100.0 %	3	100.0 %	100.0 %	1.0
100.0	100.0 %	4	100.0 %	100.0 %	1.0
100.0	100.0 %	5	100.0 %	100.0 %	1.0
100.0	100.0 %	6	100.0 %	100.0 %	1.0
100.0	100.0 %	7	100.0 %	94.4 %	2.0
100.0	100.0 %	8	100.0 %	100.0 %	1.0
100.0	100.0 %	9	100.0 %	100.0 %	1.0
100.0	10.0 %	1	99.4 %	100.0 %	1.0
100.0	10.0 %	2	99.4 %	100.0 %	1.0
100.0	10.0 %	3	99.4 %	100.0 %	1.0
100.0	10.0 %	4	99.4 %	100.0 %	1.0
100.0	10.0 %	5	99.4 %	100.0 %	1.0
100.0	10.0 %	6	99.4 %	100.0 %	1.0
100.0	10.0 %	7	99.4 %	99.4 %	2.0
100.0	10.0 %	8	99.4 %	100.0 %	1.0
100.0	10.0 %	9	99.4 %	100.0 %	1.0
100.0	5.0 %	1	98.3 %	100.0 %	1.0
100.0	5.0 %	2	98.3 %	100.0 %	1.0
100.0	5.0 %	3	98.3 %	100.0 %	1.0
100.0	5.0 %	4	98.3 %	100.0 %	1.0
100.0	5.0 %	5	98.3 %	100.0 %	1.0
100.0	5.0 %	6	98.3 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	5.0 %	7	98.3 %	100.0 %	1.0
100.0	5.0 %	8	98.3 %	91.0 %	4.0
100.0	5.0 %	9	98.3 %	92.3 %	2.0
100.0	2.5 %	1	92.2 %	100.0 %	1.0
100.0	2.5 %	2	92.2 %	100.0 %	1.0
100.0	2.5 %	3	92.2 %	100.0 %	1.0
100.0	2.5 %	4	92.2 %	100.0 %	1.0
100.0	2.5 %	5	92.2 %	100.0 %	1.0
100.0	2.5 %	6	92.2 %	100.0 %	1.0
100.0	2.5 %	7	92.2 %	100.0 %	1.0
100.0	2.5 %	8	92.2 %	87.3 %	3.0
100.0	2.5 %	9	92.2 %	82.6 %	3.0
300.0	100.0 %	1	100.0 %	100.0 %	1.0
300.0	100.0 %	2	100.0 %	100.0 %	1.0
300.0	100.0 %	3	100.0 %	100.0 %	1.0
300.0	100.0 %	4	100.0 %	97.8 %	2.0
300.0	100.0 %	5	100.0 %	99.4 %	2.0
300.0	100.0 %	6	100.0 %	100.0 %	1.0
300.0	100.0 %	7	100.0 %	99.4 %	2.0
300.0	100.0 %	8	100.0 %	100.0 %	1.0
300.0	100.0 %	9	100.0 %	100.0 %	1.0
300.0	10.0 %	1	95.0 %	100.0 %	1.0
300.0	10.0 %	2	95.0 %	100.0 %	1.0
300.0	10.0 %	3	95.0 %	100.0 %	1.0
300.0	10.0 %	4	95.0 %	98.8 %	2.0
300.0	10.0 %	5	95.0 %	99.4 %	2.0
300.0	10.0 %	6	95.0 %	100.0 %	1.0
300.0	10.0 %	7	95.0 %	100.0 %	1.0
300.0	10.0 %	8	95.0 %	93.6 %	4.0
300.0	10.0 %	9	95.0 %	83.6 %	2.0
300.0	5.0 %	1	87.8 %	100.0 %	1.0
300.0	5.0 %	2	87.8 %	100.0 %	1.0
300.0	5.0 %	3	87.8 %	100.0 %	1.0
300.0	5.0 %	4	87.8 %	98.7 %	2.0
300.0	5.0 %	5	87.8 %	100.0 %	1.0
300.0	5.0 %	6	87.8 %	100.0 %	1.0
300.0	5.0 %	7	87.8 %	99.4 %	2.0
300.0	5.0 %	8	87.8 %	84.8 %	8.0
300.0	5.0 %	9	87.8 %	84.6 %	3.0
300.0	2.5 %	1	72.2 %	100.0 %	1.0
300.0	2.5 %	2	72.2 %	100.0 %	1.0
300.0	2.5 %	3	72.2 %	100.0 %	1.0
300.0	2.5 %	4	72.2 %	96.2 %	2.0
300.0	2.5 %	5	72.2 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
300.0	2.5 %	6	72.2 %	100.0 %	1.0
300.0	2.5 %	7	72.2 %	100.0 %	1.0
300.0	2.5 %	8	72.2 %	86.9 %	8.0
300.0	2.5 %	9	72.2 %	100.0 %	1.0
500.0	100.0 %	1	100.0 %	100.0 %	1.0
500.0	100.0 %	2	100.0 %	100.0 %	1.0
500.0	100.0 %	3	100.0 %	100.0 %	1.0
500.0	100.0 %	4	100.0 %	100.0 %	1.0
500.0	100.0 %	5	100.0 %	99.4 %	2.0
500.0	100.0 %	6	100.0 %	100.0 %	1.0
500.0	100.0 %	7	100.0 %	99.4 %	2.0
500.0	100.0 %	8	100.0 %	100.0 %	1.0
500.0	100.0 %	9	100.0 %	100.0 %	1.0
500.0	10.0 %	1	90.6 %	100.0 %	1.0
500.0	10.0 %	2	90.6 %	100.0 %	1.0
500.0	10.0 %	3	90.6 %	100.0 %	1.0
500.0	10.0 %	4	90.6 %	99.4 %	2.0
500.0	10.0 %	5	90.6 %	100.0 %	1.0
500.0	10.0 %	6	90.6 %	100.0 %	1.0
500.0	10.0 %	7	90.6 %	100.0 %	1.0
500.0	10.0 %	8	90.6 %	84.7 %	3.0
500.0	10.0 %	9	90.6 %	81.8 %	3.0
500.0	5.0 %	1	76.1 %	100.0 %	1.0
500.0	5.0 %	2	76.1 %	100.0 %	1.0
500.0	5.0 %	3	76.1 %	100.0 %	1.0
500.0	5.0 %	4	76.1 %	98.5 %	2.0
500.0	5.0 %	5	76.1 %	99.3 %	2.0
500.0	5.0 %	6	76.1 %	100.0 %	1.0
500.0	5.0 %	7	76.1 %	100.0 %	1.0
500.0	5.0 %	8	76.1 %	83.9 %	4.0
500.0	5.0 %	9	76.1 %	96.4 %	3.0
500.0	2.5 %	1	62.8 %	100.0 %	1.0
500.0	2.5 %	2	62.8 %	100.0 %	1.0
500.0	2.5 %	3	62.8 %	100.0 %	1.0
500.0	2.5 %	4	62.8 %	99.1 %	2.0
500.0	2.5 %	5	62.8 %	99.1 %	2.0
500.0	2.5 %	6	62.8 %	100.0 %	1.0
500.0	2.5 %	7	62.8 %	100.0 %	1.0
500.0	2.5 %	8	62.8 %	100.0 %	1.0
500.0	2.5 %	9	62.8 %	100.0 %	1.0
1000.0	100.0 %	1	99.4 %	100.0 %	1.0
1000.0	100.0 %	2	99.4 %	100.0 %	1.0
1000.0	100.0 %	3	99.4 %	100.0 %	1.0
1000.0	100.0 %	4	99.4 %	98.9 %	2.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
1000.0	100.0 %	5	99.4 %	100.0 %	1.0
1000.0	100.0 %	6	99.4 %	100.0 %	1.0
1000.0	100.0 %	7	99.4 %	99.4 %	2.0
1000.0	100.0 %	8	99.4 %	100.0 %	1.0
1000.0	100.0 %	9	99.4 %	100.0 %	1.0
1000.0	10.0 %	1	76.1 %	100.0 %	1.0
1000.0	10.0 %	2	76.1 %	100.0 %	1.0
1000.0	10.0 %	3	76.1 %	100.0 %	1.0
1000.0	10.0 %	4	76.1 %	99.3 %	2.0
1000.0	10.0 %	5	76.1 %	100.0 %	1.0
1000.0	10.0 %	6	76.1 %	100.0 %	1.0
1000.0	10.0 %	7	76.1 %	100.0 %	1.0
1000.0	10.0 %	8	76.1 %	83.9 %	4.0
1000.0	10.0 %	9	76.1 %	96.4 %	3.0
1000.0	5.0 %	1	62.8 %	100.0 %	1.0
1000.0	5.0 %	2	62.8 %	100.0 %	1.0
1000.0	5.0 %	3	62.8 %	100.0 %	1.0
1000.0	5.0 %	4	62.8 %	100.0 %	1.0
1000.0	5.0 %	5	62.8 %	100.0 %	1.0
1000.0	5.0 %	6	62.8 %	100.0 %	1.0
1000.0	5.0 %	7	62.8 %	100.0 %	1.0
1000.0	5.0 %	8	62.8 %	100.0 %	1.0
1000.0	5.0 %	9	62.8 %	100.0 %	1.0
1000.0	2.5 %	1	61.1 %	100.0 %	1.0
1000.0	2.5 %	2	61.1 %	100.0 %	1.0
1000.0	2.5 %	3	61.1 %	100.0 %	1.0
1000.0	2.5 %	4	61.1 %	100.0 %	1.0
1000.0	2.5 %	5	61.1 %	100.0 %	1.0
1000.0	2.5 %	6	61.1 %	100.0 %	1.0
1000.0	2.5 %	7	61.1 %	100.0 %	1.0
1000.0	2.5 %	8	61.1 %	96.4 %	2.0
1000.0	2.5 %	9	61.1 %	97.9 %	2.0
5000.0	100.0 %	1	90.6 %	100.0 %	1.0
5000.0	100.0 %	2	90.6 %	100.0 %	1.0
5000.0	100.0 %	3	90.6 %	100.0 %	1.0
5000.0	100.0 %	4	90.6 %	100.0 %	1.0
5000.0	100.0 %	5	90.6 %	100.0 %	1.0
5000.0	100.0 %	6	90.6 %	100.0 %	1.0
5000.0	100.0 %	7	90.6 %	100.0 %	1.0
5000.0	100.0 %	8	90.6 %	84.7 %	3.0
5000.0	100.0 %	9	90.6 %	81.8 %	3.0
5000.0	10.0 %	1	60.6 %	100.0 %	1.0
5000.0	10.0 %	2	60.6 %	100.0 %	1.0
5000.0	10.0 %	3	60.6 %	100.0 %	1.0

Continued on next page

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
5000.0	10.0 %	4	60.6 %	100.0 %	1.0
5000.0	10.0 %	5	60.6 %	100.0 %	1.0
5000.0	10.0 %	6	60.6 %	100.0 %	1.0
5000.0	10.0 %	7	60.6 %	100.0 %	1.0
5000.0	10.0 %	8	60.6 %	93.6 %	3.0
5000.0	10.0 %	9	60.6 %	100.0 %	1.0
5000.0	5.0 %	1	56.1 %	100.0 %	1.0
5000.0	5.0 %	2	56.1 %	100.0 %	1.0
5000.0	5.0 %	3	56.1 %	100.0 %	1.0
5000.0	5.0 %	4	56.1 %	100.0 %	1.0
5000.0	5.0 %	5	56.1 %	100.0 %	1.0
5000.0	5.0 %	6	56.1 %	99.0 %	2.0
5000.0	5.0 %	7	56.1 %	100.0 %	1.0
5000.0	5.0 %	8	56.1 %	87.1 %	3.0
5000.0	5.0 %	9	56.1 %	95.7 %	2.0
5000.0	2.5 %	1	35.6 %	100.0 %	1.0
5000.0	2.5 %	2	35.6 %	100.0 %	1.0
5000.0	2.5 %	3	35.6 %	100.0 %	1.0
5000.0	2.5 %	4	35.6 %	95.3 %	4.0
5000.0	2.5 %	5	35.6 %	100.0 %	1.0
5000.0	2.5 %	6	35.6 %	95.3 %	2.0
5000.0	2.5 %	7	35.6 %	100.0 %	1.0
5000.0	2.5 %	8	35.6 %	64.1 %	2.0
5000.0	2.5 %	9	35.6 %	100.0 %	1.0

Table 50 – Summary of Unique Contracts Acquired for the 120-Month Hedging Strategy invertedbell Portfolio for nominal values from 100 MM BRL to 5000 MM BRL.(L_i of 100%, 10%, 5% and 2.5%)

Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
100.0	100.0 %	1	100.0 %	100.0 %	1.0
100.0	100.0 %	2	100.0 %	100.0 %	1.0
100.0	100.0 %	3	100.0 %	100.0 %	1.0
100.0	100.0 %	4	100.0 %	100.0 %	1.0
100.0	100.0 %	5	100.0 %	100.0 %	1.0
100.0	100.0 %	6	100.0 %	100.0 %	1.0
100.0	100.0 %	7	100.0 %	99.4 %	2.0
100.0	100.0 %	8	100.0 %	98.9 %	2.0
100.0	100.0 %	9	100.0 %	89.4 %	3.0
100.0	10.0 %	1	97.8 %	100.0 %	1.0
100.0	10.0 %	2	97.8 %	100.0 %	1.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
100.0	10.0 %	3	97.8 %	100.0 %	1.0
100.0	10.0 %	4	97.8 %	100.0 %	1.0
100.0	10.0 %	5	97.8 %	100.0 %	1.0
100.0	10.0 %	6	97.8 %	100.0 %	1.0
100.0	10.0 %	7	97.8 %	100.0 %	1.0
100.0	10.0 %	8	97.8 %	100.0 %	1.0
100.0	10.0 %	9	97.8 %	93.2 %	2.0
100.0	5.0 %	1	93.3 %	100.0 %	1.0
100.0	5.0 %	2	93.3 %	100.0 %	1.0
100.0	5.0 %	3	93.3 %	100.0 %	1.0
100.0	5.0 %	4	93.3 %	100.0 %	1.0
100.0	5.0 %	5	93.3 %	100.0 %	1.0
100.0	5.0 %	6	93.3 %	100.0 %	1.0
100.0	5.0 %	7	93.3 %	100.0 %	1.0
100.0	5.0 %	8	93.3 %	100.0 %	1.0
100.0	5.0 %	9	93.3 %	96.4 %	2.0
100.0	2.5 %	1	92.2 %	100.0 %	1.0
100.0	2.5 %	2	92.2 %	100.0 %	1.0
100.0	2.5 %	3	92.2 %	100.0 %	1.0
100.0	2.5 %	4	92.2 %	100.0 %	1.0
100.0	2.5 %	5	92.2 %	100.0 %	1.0
100.0	2.5 %	6	92.2 %	100.0 %	1.0
100.0	2.5 %	7	92.2 %	100.0 %	1.0
100.0	2.5 %	8	92.2 %	100.0 %	1.0
100.0	2.5 %	9	92.2 %	97.6 %	3.0
300.0	100.0 %	1	98.9 %	100.0 %	1.0
300.0	100.0 %	2	98.9 %	100.0 %	1.0
300.0	100.0 %	3	98.9 %	100.0 %	1.0
300.0	100.0 %	4	98.9 %	97.8 %	2.0
300.0	100.0 %	5	98.9 %	100.0 %	1.0
300.0	100.0 %	6	98.9 %	100.0 %	1.0
300.0	100.0 %	7	98.9 %	100.0 %	1.0
300.0	100.0 %	8	98.9 %	98.9 %	2.0
300.0	100.0 %	9	98.9 %	91.0 %	2.0
300.0	10.0 %	1	92.8 %	100.0 %	1.0
300.0	10.0 %	2	92.8 %	100.0 %	1.0
300.0	10.0 %	3	92.8 %	100.0 %	1.0
300.0	10.0 %	4	92.8 %	98.2 %	2.0
300.0	10.0 %	5	92.8 %	100.0 %	1.0
300.0	10.0 %	6	92.8 %	100.0 %	1.0
300.0	10.0 %	7	92.8 %	100.0 %	1.0
300.0	10.0 %	8	92.8 %	100.0 %	1.0
300.0	10.0 %	9	92.8 %	95.8 %	3.0
300.0	5.0 %	1	91.7 %	100.0 %	1.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
300.0	5.0 %	2	91.7 %	100.0 %	1.0
300.0	5.0 %	3	91.7 %	100.0 %	1.0
300.0	5.0 %	4	91.7 %	97.0 %	2.0
300.0	5.0 %	5	91.7 %	100.0 %	1.0
300.0	5.0 %	6	91.7 %	100.0 %	1.0
300.0	5.0 %	7	91.7 %	100.0 %	1.0
300.0	5.0 %	8	91.7 %	100.0 %	1.0
300.0	5.0 %	9	91.7 %	98.2 %	2.0
300.0	2.5 %	1	80.6 %	100.0 %	1.0
300.0	2.5 %	2	80.6 %	100.0 %	1.0
300.0	2.5 %	3	80.6 %	100.0 %	1.0
300.0	2.5 %	4	80.6 %	97.9 %	2.0
300.0	2.5 %	5	80.6 %	98.6 %	2.0
300.0	2.5 %	6	80.6 %	100.0 %	1.0
300.0	2.5 %	7	80.6 %	100.0 %	1.0
300.0	2.5 %	8	80.6 %	100.0 %	1.0
300.0	2.5 %	9	80.6 %	95.2 %	2.0
500.0	100.0 %	1	98.9 %	100.0 %	1.0
500.0	100.0 %	2	98.9 %	100.0 %	1.0
500.0	100.0 %	3	98.9 %	100.0 %	1.0
500.0	100.0 %	4	98.9 %	98.9 %	2.0
500.0	100.0 %	5	98.9 %	99.4 %	2.0
500.0	100.0 %	6	98.9 %	100.0 %	1.0
500.0	100.0 %	7	98.9 %	100.0 %	1.0
500.0	100.0 %	8	98.9 %	100.0 %	1.0
500.0	100.0 %	9	98.9 %	92.1 %	2.0
500.0	10.0 %	1	91.7 %	100.0 %	1.0
500.0	10.0 %	2	91.7 %	100.0 %	1.0
500.0	10.0 %	3	91.7 %	100.0 %	1.0
500.0	10.0 %	4	91.7 %	98.8 %	2.0
500.0	10.0 %	5	91.7 %	99.4 %	2.0
500.0	10.0 %	6	91.7 %	100.0 %	1.0
500.0	10.0 %	7	91.7 %	100.0 %	1.0
500.0	10.0 %	8	91.7 %	100.0 %	1.0
500.0	10.0 %	9	91.7 %	97.0 %	2.0
500.0	5.0 %	1	85.6 %	100.0 %	1.0
500.0	5.0 %	2	85.6 %	100.0 %	1.0
500.0	5.0 %	3	85.6 %	100.0 %	1.0
500.0	5.0 %	4	85.6 %	98.7 %	2.0
500.0	5.0 %	5	85.6 %	99.4 %	2.0
500.0	5.0 %	6	85.6 %	100.0 %	1.0
500.0	5.0 %	7	85.6 %	100.0 %	1.0
500.0	5.0 %	8	85.6 %	100.0 %	1.0
500.0	5.0 %	9	85.6 %	95.5 %	4.0

Continued on next page

Value	L_i	KRD	%Feasibles	% Single Contract	Max Contracts
500.0	2.5 %	1	77.2 %	100.0 %	1.0
500.0	2.5 %	2	77.2 %	100.0 %	1.0
500.0	2.5 %	3	77.2 %	100.0 %	1.0
500.0	2.5 %	4	77.2 %	99.3 %	2.0
500.0	2.5 %	5	77.2 %	99.3 %	2.0
500.0	2.5 %	6	77.2 %	100.0 %	1.0
500.0	2.5 %	7	77.2 %	100.0 %	1.0
500.0	2.5 %	8	77.2 %	97.8 %	2.0
500.0	2.5 %	9	77.2 %	92.8 %	2.0
1000.0	100.0 %	1	97.8 %	100.0 %	1.0
1000.0	100.0 %	2	97.8 %	100.0 %	1.0
1000.0	100.0 %	3	97.8 %	100.0 %	1.0
1000.0	100.0 %	4	97.8 %	98.3 %	2.0
1000.0	100.0 %	5	97.8 %	98.9 %	2.0
1000.0	100.0 %	6	97.8 %	100.0 %	1.0
1000.0	100.0 %	7	97.8 %	100.0 %	1.0
1000.0	100.0 %	8	97.8 %	100.0 %	1.0
1000.0	100.0 %	9	97.8 %	93.2 %	2.0
1000.0	10.0 %	1	85.6 %	100.0 %	1.0
1000.0	10.0 %	2	85.6 %	100.0 %	1.0
1000.0	10.0 %	3	85.6 %	100.0 %	1.0
1000.0	10.0 %	4	85.6 %	98.7 %	2.0
1000.0	10.0 %	5	85.6 %	98.7 %	2.0
1000.0	10.0 %	6	85.6 %	100.0 %	1.0
1000.0	10.0 %	7	85.6 %	100.0 %	1.0
1000.0	10.0 %	8	85.6 %	100.0 %	1.0
1000.0	10.0 %	9	85.6 %	93.5 %	4.0
1000.0	5.0 %	1	77.2 %	100.0 %	1.0
1000.0	5.0 %	2	77.2 %	100.0 %	1.0
1000.0	5.0 %	3	77.2 %	100.0 %	1.0
1000.0	5.0 %	4	77.2 %	98.6 %	2.0
1000.0	5.0 %	5	77.2 %	98.6 %	2.0
1000.0	5.0 %	6	77.2 %	100.0 %	1.0
1000.0	5.0 %	7	77.2 %	100.0 %	1.0
1000.0	5.0 %	8	77.2 %	97.8 %	2.0
1000.0	5.0 %	9	77.2 %	94.2 %	2.0
1000.0	2.5 %	1	60.0 %	100.0 %	1.0
1000.0	2.5 %	2	60.0 %	100.0 %	1.0
1000.0	2.5 %	3	60.0 %	100.0 %	1.0
1000.0	2.5 %	4	60.0 %	99.1 %	2.0
1000.0	2.5 %	5	60.0 %	98.1 %	2.0
1000.0	2.5 %	6	60.0 %	100.0 %	1.0
1000.0	2.5 %	7	60.0 %	100.0 %	1.0
1000.0	2.5 %	8	60.0 %	97.2 %	2.0

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Value	L_i	**KRD**	**%Feasibles**	**% Single Contract**	**Max Contracts**
1000.0	2.5 %	9	60.0 %	73.1 %	3.0
5000.0	100.0 %	1	91.7 %	100.0 %	1.0
5000.0	100.0 %	2	91.7 %	100.0 %	1.0
5000.0	100.0 %	3	91.7 %	100.0 %	1.0
5000.0	100.0 %	4	91.7 %	100.0 %	1.0
5000.0	100.0 %	5	91.7 %	100.0 %	1.0
5000.0	100.0 %	6	91.7 %	100.0 %	1.0
5000.0	100.0 %	7	91.7 %	100.0 %	1.0
5000.0	100.0 %	8	91.7 %	100.0 %	1.0
5000.0	100.0 %	9	91.7 %	97.0 %	2.0
5000.0	10.0 %	1	55.0 %	100.0 %	1.0
5000.0	10.0 %	2	55.0 %	100.0 %	1.0
5000.0	10.0 %	3	55.0 %	100.0 %	1.0
5000.0	10.0 %	4	55.0 %	100.0 %	1.0
5000.0	10.0 %	5	55.0 %	100.0 %	1.0
5000.0	10.0 %	6	55.0 %	100.0 %	1.0
5000.0	10.0 %	7	55.0 %	100.0 %	1.0
5000.0	10.0 %	8	55.0 %	98.0 %	2.0
5000.0	10.0 %	9	55.0 %	72.7 %	2.0
5000.0	5.0 %	1	32.8 %	100.0 %	1.0
5000.0	5.0 %	2	32.8 %	100.0 %	1.0
5000.0	5.0 %	3	32.8 %	100.0 %	1.0
5000.0	5.0 %	4	32.8 %	100.0 %	1.0
5000.0	5.0 %	5	32.8 %	100.0 %	1.0
5000.0	5.0 %	6	32.8 %	94.9 %	2.0
5000.0	5.0 %	7	32.8 %	100.0 %	1.0
5000.0	5.0 %	8	32.8 %	100.0 %	1.0
5000.0	5.0 %	9	32.8 %	37.3 %	2.0
5000.0	2.5 %	1	5.0 %	100.0 %	1.0
5000.0	2.5 %	2	5.0 %	100.0 %	1.0
5000.0	2.5 %	3	5.0 %	100.0 %	1.0
5000.0	2.5 %	4	5.0 %	100.0 %	1.0
5000.0	2.5 %	5	5.0 %	100.0 %	1.0
5000.0	2.5 %	6	5.0 %	100.0 %	1.0
5000.0	2.5 %	7	5.0 %	100.0 %	1.0
5000.0	2.5 %	8	5.0 %	100.0 %	1.0
5000.0	2.5 %	9	5.0 %	0.0 %	3.0

APPENDIX E – Complete Results for R^2 Values of Hedge Effectiveness

E.1 Uniform Portfolios

Table 51 – Comparison of Hedge Effectiveness: R^2 Values for uniform Portfolios (L_i of 100%, 10%, 5% and 2.5%).

Months	**Value**	L_i	R^2
36	100	100.0 %	0.996
36	100	10.0 %	0.997
36	100	5.0 %	0.997
36	100	2.5 %	0.996
36	300	100.0 %	0.997
36	300	10.0 %	0.997
36	300	5.0 %	0.996
36	300	2.5 %	0.996
36	500	100.0 %	0.996
36	500	10.0 %	0.996
36	500	5.0 %	0.996
36	500	2.5 %	0.995
36	1000	100.0 %	0.997
36	1000	10.0 %	0.997
36	1000	5.0 %	0.996
36	1000	2.5 %	0.995
36	5000	100.0 %	0.995
36	5000	10.0 %	0.995
36	5000	5.0 %	0.994
36	5000	2.5 %	0.996
60	100	100.0 %	0.998
60	100	10.0 %	0.999
60	100	5.0 %	0.999
60	100	2.5 %	0.999
60	300	100.0 %	0.999
60	300	10.0 %	0.999
60	300	5.0 %	0.999
60	300	2.5 %	0.999
60	500	100.0 %	0.999
60	500	10.0 %	0.999
60	500	5.0 %	0.999
60	500	2.5 %	0.998
60	1000	100.0 %	0.999
60	1000	10.0 %	0.999
60	1000	5.0 %	0.999

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Months	**Value**	L_i	R^2
60	1000	2.5 %	0.998
60	5000	100.0 %	0.998
60	5000	10.0 %	0.998
60	5000	5.0 %	0.998
60	5000	2.5 %	0.998
84	100	100.0 %	0.999
84	100	10.0 %	0.999
84	100	5.0 %	0.999
84	100	2.5 %	0.999
84	300	100.0 %	0.999
84	300	10.0 %	0.999
84	300	5.0 %	0.999
84	300	2.5 %	0.999
84	500	100.0 %	0.999
84	500	10.0 %	0.998
84	500	5.0 %	0.999
84	500	2.5 %	0.998
84	1000	100.0 %	0.999
84	1000	10.0 %	0.998
84	1000	5.0 %	0.998
84	1000	2.5 %	0.998
84	5000	100.0 %	0.998
84	5000	10.0 %	0.998
84	5000	5.0 %	0.998
84	5000	2.5 %	0.998
120	100	100.0 %	0.999
120	100	10.0 %	0.999
120	100	5.0 %	0.999
120	100	2.5 %	0.999
120	300	100.0 %	0.999
120	300	10.0 %	0.999
120	300	5.0 %	0.999
120	300	2.5 %	0.999
120	500	100.0 %	0.999
120	500	10.0 %	0.999
120	500	5.0 %	0.999
120	500	2.5 %	0.999
120	1000	100.0 %	0.999
120	1000	10.0 %	0.999
120	1000	5.0 %	0.999
120	1000	2.5 %	0.999
120	5000	100.0 %	0.999
120	5000	10.0 %	0.999
120	5000	5.0 %	0.999

Continued on next page

Months	Value	L_i	R^2
120	5000	2.5 %	0.999

E.2 Bell Portfolios

Table 52 – Comparison of Hedge Effectiveness: R^2 Values for bell Portfolios (L_i of 100%, 10%, 5% and 2.5%).

Months	Value	L_i	R^2
36	100	100.0 %	0.997
36	100	10.0 %	0.997
36	100	5.0 %	0.997
36	100	2.5 %	0.996
36	300	100.0 %	0.998
36	300	10.0 %	0.997
36	300	5.0 %	0.995
36	300	2.5 %	0.995
36	500	100.0 %	0.995
36	500	10.0 %	0.994
36	500	5.0 %	0.994
36	500	2.5 %	0.995
36	1000	100.0 %	0.992
36	1000	10.0 %	0.993
36	1000	5.0 %	0.995
36	1000	2.5 %	0.995
36	5000	100.0 %	0.995
36	5000	10.0 %	0.994
36	5000	5.0 %	0.996
36	5000	2.5 %	0.996
60	100	100.0 %	0.997
60	100	10.0 %	0.997
60	100	5.0 %	0.997
60	100	2.5 %	0.997
60	300	100.0 %	0.998
60	300	10.0 %	0.997
60	300	5.0 %	0.997
60	300	2.5 %	0.997
60	500	100.0 %	0.997
60	500	10.0 %	0.997
60	500	5.0 %	0.997
60	500	2.5 %	0.997
60	1000	100.0 %	0.997

Continued on next page

Months	**Value**	L_i	R^2
60	1000	10.0 %	0.997
60	1000	5.0 %	0.997
60	1000	2.5 %	0.996
60	5000	100.0 %	0.997
60	5000	10.0 %	0.997
60	5000	5.0 %	0.997
60	5000	2.5 %	0.997
84	100	100.0 %	0.999
84	100	10.0 %	0.999
84	100	5.0 %	0.999
84	100	2.5 %	0.999
84	300	100.0 %	0.999
84	300	10.0 %	0.999
84	300	5.0 %	0.999
84	300	2.5 %	0.998
84	500	100.0 %	0.999
84	500	10.0 %	0.999
84	500	5.0 %	0.999
84	500	2.5 %	0.999
84	1000	100.0 %	0.999
84	1000	10.0 %	0.998
84	1000	5.0 %	0.999
84	1000	2.5 %	0.998
84	5000	100.0 %	0.998
84	5000	10.0 %	0.998
84	5000	5.0 %	0.998
84	5000	2.5 %	0.998
120	100	100.0 %	0.999
120	100	10.0 %	0.999
120	100	5.0 %	0.999
120	100	2.5 %	0.999
120	300	100.0 %	0.999
120	300	10.0 %	0.999
120	300	5.0 %	0.999
120	300	2.5 %	0.999
120	500	100.0 %	0.999
120	500	10.0 %	0.999
120	500	5.0 %	0.999
120	500	2.5 %	0.999
120	1000	100.0 %	0.999
120	1000	10.0 %	0.999
120	1000	5.0 %	0.999
120	1000	2.5 %	0.999
120	5000	100.0 %	0.999

Continued on next page

Months	Value	L_i	R^2
120	5000	10.0 %	0.999
120	5000	5.0 %	0.999
120	5000	2.5 %	0.999

E.3 Inverted Bell Portfolios

Table 53 – Comparison of Hedge Effectiveness: R^2 Values for invertedbell Portfolios (L_i of 100%, 10%, 5% and 2.5%).

Months	Value	L_i	R^2
36	100	100.0 %	0.987
36	100	10.0 %	0.988
36	100	5.0 %	0.988
36	100	2.5 %	0.988
36	300	100.0 %	0.987
36	300	10.0 %	0.987
36	300	5.0 %	0.987
36	300	2.5 %	0.986
36	500	100.0 %	0.987
36	500	10.0 %	0.987
36	500	5.0 %	0.987
36	500	2.5 %	0.986
36	1000	100.0 %	0.987
36	1000	10.0 %	0.987
36	1000	5.0 %	0.986
36	1000	2.5 %	0.985
36	5000	100.0 %	0.987
36	5000	10.0 %	0.985
36	5000	5.0 %	0.984
36	5000	2.5 %	0.987
60	100	100.0 %	0.996
60	100	10.0 %	0.996
60	100	5.0 %	0.996
60	100	2.5 %	0.996
60	300	100.0 %	0.996
60	300	10.0 %	0.996
60	300	5.0 %	0.996
60	300	2.5 %	0.996
60	500	100.0 %	0.996
60	500	10.0 %	0.996
60	500	5.0 %	0.996

Continued on next page

Months	**Value**	L_i	R^2
60	500	2.5 %	0.995
60	1000	100.0 %	0.996
60	1000	10.0 %	0.996
60	1000	5.0 %	0.996
60	1000	2.5 %	0.996
60	5000	100.0 %	0.996
60	5000	10.0 %	0.996
60	5000	5.0 %	0.996
60	5000	2.5 %	0.996
84	100	100.0 %	0.998
84	100	10.0 %	0.998
84	100	5.0 %	0.998
84	100	2.5 %	0.997
84	300	100.0 %	0.998
84	300	10.0 %	0.997
84	300	5.0 %	0.997
84	300	2.5 %	0.997
84	500	100.0 %	0.998
84	500	10.0 %	0.997
84	500	5.0 %	0.997
84	500	2.5 %	0.997
84	1000	100.0 %	0.998
84	1000	10.0 %	0.997
84	1000	5.0 %	0.997
84	1000	2.5 %	0.997
84	5000	100.0 %	0.997
84	5000	10.0 %	0.997
84	5000	5.0 %	0.996
84	5000	2.5 %	0.997
120	100	100.0 %	0.999
120	100	10.0 %	0.999
120	100	5.0 %	0.999
120	100	2.5 %	0.999
120	300	100.0 %	0.999
120	300	10.0 %	0.999
120	300	5.0 %	0.999
120	300	2.5 %	0.999
120	500	100.0 %	0.999
120	500	10.0 %	0.999
120	500	5.0 %	0.999
120	500	2.5 %	0.998
120	1000	100.0 %	0.999
120	1000	10.0 %	0.999
120	1000	5.0 %	0.998

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Months	**Value**	L_i	R^2
120	1000	2.5 %	0.998
120	5000	100.0 %	0.999
120	5000	10.0 %	0.998
120	5000	5.0 %	0.998
120	5000	2.5 %	0.995