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QUANTIZAÇÃO OPERATORIAL DE TEORIAS DE GAUGE NÃO-ABELIANAS EM UM GAUGE AXIAL COMPLETAMENTE FIXADO*

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RESUMO

Uma quantização consistente da cromodinâmica num gauge axial completamente fixado é realizada usando o procedimento de quantização por parênteses de Dirac. Os resultados centrais são: A translação de parênteses de Dirac a comutadores a tempos iguais é possível, sem ambigüidades, devido à ausência de problemas de ordenamento. Todos os comutadores a tempos iguais são compatíveis com os vínculos e condições de gauge valendo como identidades operatoriais fortes. Todos os comutadores a tempos iguais são compatíveis com os campos cromoelétricos, cromomagnéticos e fermiônicos anulando-se no infinito espacial. Os potenciais de gauge coloridos $A^{0,a}$, $A^{1,a}$ e $A^{2,a}$ apresentam um comportamento fisicamente significativo, embora do tipo gauge puro, em $x^3 = \pm\infty$, conforme exigido pela presença de um conteúdo topológico não-trivial. A invariança de Poincaré é satisfeita sem introduzir potenciais quanto-mecânicos "extras" no Hamiltoniano. O determinante da matriz de Faddeev-Popov não depende das variáveis de campo.

ABSTRACT

A consistent quantization of chromodynamics in a completely fixed axial gauge is carried out by using the Dirac bracket quantization procedure. The main results are: The translation of Dirac brackets into equal-time commutators is possible, without ambiguities, because of the absence of ordering problems. All equal-time commutators are compatible with constraints and gauge conditions holding as strong operator relations. All equal-time commutators are compatible with chromoelectric, chromomagnetic and fermionic fields vanishing at spatial infinity. The colored gauge potentials $A^{0,a}$, $A^{1,a}$ and $A^{2,a}$ are seen to develop a physically significant, although pure gauge, behavior at $x^3 = \pm\infty$, as required by the presence of a non-trivial topological content. Poincaré invariance is satisfied without introducing in the Hamiltonian "extra" quantum mechanical potentials. The determinant of the Faddeev-Popov matrix does not depend upon the field variables.

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I. INTRODUÇÃO

I.1 Motivação

Existem duas maneiras alternativas para quantizar uma certa teoria: o método operatorial ou o formalismo da integral funcional. Consideremos em primeiro lugar alguns dos problemas com que nos defrontamos ao quantizar uma teoria de campos de gauge não-Abelianos pelo método operatorial.

I.1.1 Problemas da quantização operatorial

a) Problema da escolha de um gauge confiável - Para ilustrar este problema consideremos algumas das condições de gauge mais utilizadas na literatura:

- O gauge de Coulomb [1] - Hoje em dia sabemos que a condição de Coulomb não fixa totalmente o gauge no caso não-Abeliano. Este fenômeno é conhecido como ambigüidade de Gribov [2,3] e indica a presença de campos intensos na teoria (p. ex., instantons [4]) os quais por sua vez refletem o conteúdo topológico não trivial da mesma [5].
- O gauge de Lorentz - A nosso conhecimento não existe uma quantização operatorial consistente neste gauge para campos não-Abelianos. A única quantização consistente no gauge covariante tem sido feita através da integral funcional. Algumas indicações de como proceder a nível operatorial foram dadas por Fradkin e Vilkovisky [6]. Entretanto, o gauge de Lorentz também está afetado por ambigüidades de Gribov [3].
- O gauge temporal - A condição de gauge temporal não fixa com

pletamente o gauge; a fixação completa do gauge temporal é incompatível com a álgebra dos comutadores básicos a tempos iguais. Além disso, no cálculo de certas quantidades invariantes de gauge tais como o loop de Wilson, o resultado que se obteve para esta quantidade a nível operatorial não coincidiu com o resultado obtido a nível da integral funcional no gauge covariante [7]. Esforços tem sido realizados para melhorar a situação dentro do formalismo operatorial e a nível perturbativo [8]. Por outro lado, o Hamiltoniano em termos de variáveis independentes obtido por Goldstone e Jackiw [9] para a teoria SU(2) de Yang-Mills é de duvidosa utilidade devido à sua grande complexidade. Além do mais, o tratamento de Goldstone e Jackiw só vale para o grupo SU(2).

- O gauge axial - A quantização neste gauge tem sido realizada com uma condição ($A^{3,a} = 0$) que não fixa totalmente o gauge. Isto levou a inconsistências as quais foram detectadas por Schwinger [10] em 1963. Este problema é discutido no item I.2, logo adiante.

b) Problema da escolha de uma prescrição de ordenamento - Em primeira quantização, o tradicional problema de ordenamento que em geral ocorre dá-se ao nível do operador Hamiltoniano que descreve o sistema. Por exemplo, o Hamiltoniano clássico que descreve uma partícula livre em coordenadas curvilíneas arbitrárias é $\frac{1}{2} p_r g^{rs}(q) p_s$. Além disso, o fato de trabalharmos em coordenadas curvilíneas exigem a presença de um potencial adicional quanto-mecânico de ordem $\hbar^2$ no Hamiltoniano. Este é o chamado problema de cartesianidade estudado com generalidade a nível operatorial por De Witt [11] em 1952.

Em teoria de campos, o problema de ordenamento pode ocor rer não somente a nível do operador Hamiltoniano mas também a nível dos comutadores básicos a tempos iguais (CTI's). Neste caso o problema é mais delicado porquanto para se construir os geradores das simetrias (externas e internas) que a teoria possui é necessário escolher, com algum critério, uma prescrição de ordenamento ao nível dos CTI's. Por exemplo, no caso das teorias não-Abelianas no gauge de Coulomb [1] os CTI's apresentam problemas de ordenamento, exigindo assim à adoção de uma prescrição definida. Claramente, a forma do Hamiltoniano quântico e do termo de potencial adicional (pois as coordenadas no gauge de Coulomb não são cartesianas) dependem dessa prescrição. É claro, espera-se que os resultados físicos independam da prescrição de ordenamento escolhida. Não é demais ressaltar, entretanto, que seria altamente desejável encontrar um gauge confiável e livre de problemas de ordenamento e cartesianidade.

c) Problema da obtenção de um formalismo perturbativo - Na situação não-Abeliana, a construção de um formalismo perturbativo para o cálculo das funções de Green da teoria não é simples em termos de variáveis vinculadas. Deveria ser simples em termos de variáveis independentes. Entretanto, a inexistência de um gauge natural característica das teorias não-Abelianas, complica em muito o problema de formular estas teorias na representação de interação.

I.1.2 Vantagens da quantização operatorial

a) Detectar diretamente os problemas de ordenamento e cartesianidade e, quando possível, resolvê-los.

b) As condições de contorno satisfeitas pelas coordenadas da teoria aparecem como um resultado da formulação.

I.1.3 Problemas da quantização funcional

a) A quantização funcional não detecta diretamente os problemas de ordenamento e cartesianidade. Em geral, para obter uma quantização funcional correta devemos começar utilizando coordenadas cartesianas. Caso se deseje utilizar coordenadas curvilíneas, deve-se realizar cuidadosamente, conforme mostrado por Gervais e Jevicki [12], a transformação canônica de ponto ao sistema de coordenadas desejado. Em conseqüência, na formulação quântica de campos de gauge não-Abelianos pela integral funcional devemos estabelecer inicialmente qual é o gauge no qual as coordenadas da teoria são cartesianas. O problema é que não existe critério *a priori* para saber se as coordenadas definidas num determinado gauge são cartesianas. A filosofia prática de análise da teoria não-Abeliana em diferentes gauges exposta por Christ e Lee [13,14] leva em conta este problema: Os autores partem de uma integral funcional escrita sem ambigüidades no gauge temporal no qual se supõe que as coordenadas são cartesianas; outros gauges são alcançados mediante uma transformação canônica de ponto. Note-se que o problema de ordenamento está embutido na integral funcional através da particular regra de ponto (ou discretização) que se utilize. Christ e Lee [13], bem como Gervais e Jevicki [12], usaram a regra de ponto médio a qual corresponde ao esquema de ordenamento de Weyl. Entretanto, conforme mostramos em dois trabalhos [15,16], podemos utilizar igualmente uma regra de ponto arbitrária.

b) As condições de contorno satisfeitas pelas coordenadas da teoria não aparecem como um resultado da formulação. Como se sabe, na integral funcional as condições de contorno especificam a coleção de funções sobre as quais é feita a integração. O problema é que não há forma de saber, a nível da integral funcional, quais são as condições de contorno adequadas à teoria num certo gauge. As condições de contorno são colocadas "à mão" na teoria, usando-se a intuição física.

I.1.4 Vantagens da quantização funcional

A principal vantagem é a simplicidade e automaticidade na obtenção de uma forma perturbativa para as funções de Green da teoria.

Com base no exposto, a razão de optarmos por uma quantização operatorial das teorias de gauge não-Abelianas prende-se ao nosso desejo de detectar e resolver os problemas de ordenamento e cartesianidade da QCD (cromodinâmica quântica) como parte integrante de um programa de obtenção de uma formulação quântica consistente. Este programa de obtenção de uma formulação consistente liga-se organicamente ao problema de encontrar um gauge confiável e implementável a nível quântico. Neste sentido, a escolha do gauge axial completamente fixado que denominamos superaxial decorre do fato que este gauge, o qual não é compactificável [3], permite uma eliminação completa e sem ambigüidades da liberdade de gauge e, além disso, é implementável a nível quântico. No próximo item, colocaremos o problema da quantização operatorial das teorias de gauge não-Abelianas no gauge axial.

I.2 Quantização Operatorial de Teorias de Gauge Não-Abelianas no Gauge Axial

A proposta de quantizar a teoria de campos de Yang-Mills acoplados a férmions massivos com grupo de gauge SU(N) (a cromodinâmica, para simplificar) descrita pelo Lagrangeano de 2ª ordem

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu,a} + \bar{\Psi}(i\gamma^{\mu}\tilde{D}_{\mu} - m)\Psi \qquad (1.1)$$

onde

$$F^{\mu\nu,a} \equiv \partial^{\mu}A^{\nu,a} - \partial^{\nu}A^{\mu,a} + g f^{abc} A^{\mu,b} A^{\nu,c} , \qquad (1.2)$$

$$\bar{\Psi} \equiv \Psi^{+}\gamma^{0} , \qquad (1.3)$$

$$\tilde{D}^{uv}_{\mu} \equiv \delta^{uv}\partial_{\mu} - ig\left(\frac{\lambda^{a}}{2}\right)^{uv} A^{a}_{\mu} \;{}^{*} , \qquad (1.4)$$

no gauge axial $A^{3,a} = 0$, foi levantada em 1962 por Arnowitt e Fickler [17]. Como é sabido, as equações de Euler-Lagrange que seguem de (1.1) podem ser agrupadas em duas categorias distin

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[*] Os $A^{a}_{\mu}(a = 1,\ldots,N^{2}-1)$ são os potenciais de gauge na representação adjunta de SU(N); os $\psi^{u}(u = 1,\ldots,N)$ são os campos fermiônicos (de massa m) na representação fundamental cujos geradores, os $\lambda^{a}/2$, satisfazem $[\lambda^{a},\lambda^{b}] = 2if^{abc}\lambda^{c}$. As f^{abc}'s são as constantes de estrutura do grupo e g é a constante de acoplamento. Nossa métrica espaço-tempo é $g_{00} = -g_{11} = -g_{22} = -g_{33} = +1$, $g_{\mu\nu} = 0$ se $\mu \neq \nu$. Índices de Lorentz gregos (latinos) vão de 0 a 3 (de 1 a 3). Soma sobre índices repetidos de Lorentz e/ou de cor é sempre implicada.

tas ($\pi^a_\mu \equiv \partial\mathcal{L}/\partial(\partial^0 A^{\mu,a}) = F^{0\mu,a}$):

(i) Equações de movimento

$$(i\gamma^\mu \tilde{D}_\mu - m)\psi = 0 \quad , \tag{1.5a}$$

$$D_0^{ab}\pi_k^b = D_j^{ab}F^{kj,b} - g J^{k,a} \quad , \tag{1.5b}$$

$$D_\mu^{ab} \equiv \delta^{ab}\partial_\mu + g f^{abc} A_\mu^c \quad ; \tag{1.6}$$

(ii) equações de vínculo

$$\pi_0^a = 0 \quad , \tag{1.7a}$$

$$D^{j,ab}\pi_j^b + g J^{0,a} = 0 \quad , \tag{1.7b}$$

onde ($\pi_\psi \equiv \partial\mathcal{L}/\partial(\partial^0\psi) = i\bar{\psi}\gamma^0$)

$$J^{\mu,a} \equiv \bar{\psi}\gamma^\mu \frac{\lambda^a}{2}\psi = -i\pi_\psi \gamma^0\gamma^\mu \frac{\lambda^a}{2}\psi \, . \tag{1.8}$$

A idéia básica de Arnowitt e Fickler [17] foi a de, lançando mão das equações (1.7) e da condição de gauge $A^{3,a} = 0$, eliminar as variáveis (dependentes) $A^{0,a}$ e π^a_3 e trabalhar apenas com as variáveis independentes restantes. A simplicidade do tratamento da ref. [17] transparece na estrutura canônica padrão dos comutadores a tempos iguais (CTI's) não nulos da teoria

$$[\hat{A}^{\alpha,a}_{(\underset{\sim}{x})}, \hat{\pi}^{b}_{\beta}(\underset{\sim}{y})] = i\,\delta^{\alpha\beta}\delta^{ab}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad (\alpha,\beta=1,2), \qquad (1.9)$$

$$\{\hat{\psi}^{u}(\underset{\sim}{x}), \hat{\pi}^{v}_{\psi}(\underset{\sim}{y})\} = i\,\delta^{uv}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad , \qquad (1.10)$$

onde o "chapéu" denota operadores de campo quânticos. Entretanto, conforme assinalado por Schwinger [10] e mais recentemente por Mandelstam [18], esta quantização apresenta o problema de que a eliminação de $\hat{\pi}^{a}_{3}$ a partir da Lei de Gauss (1.7b) é incompatível com a condição de energia finita

$$\lim_{x^3\to\pm\infty} \hat{\pi}^{a}_{3}(x^0,x^1,x^2,x^3) \longrightarrow 0 \qquad (1.11)$$

que deriva da expressão do correspondente Hamiltoniano quântico

$$\hat{H} = \int d^3x\, [\hat{\pi}^{a}_{3}(\underset{\sim}{x})]^2 + \text{termos positivo-definidos}. \qquad (1.12)$$

De fato, tomando (1.7b) operatorialmente para $\hat{A}^{3,a}=0$ e supondo que, com $x^3 \to \pm\infty$, $\hat{\pi}^{a}_{\alpha} \to 0$, $\hat{\psi} \to 0$ e $\hat{\pi}_{\psi} \to 0$ suficientemente rápido, encontra-se [10,18]

$$\hat{\pi}^{a}_{3}(x^0,x^1,x^2,+\infty) - \hat{\pi}^{a}_{3}(x^0,x^1,x^2,-\infty) = \int_{-\infty}^{+\infty} dx^3 \left[\hat{D}^{\alpha,ab}_{(\underset{\sim}{x})}\hat{\pi}^{b}_{\alpha}(\underset{\sim}{x}) + g\,\hat{J}^{0,a}_{(\underset{\sim}{x})}\right] \equiv \hat{Q}^{a}_{\perp}(x^0,x^1,x^2). \qquad (1.13)$$

A incompatibilidade de (1.13) com (1.11) decorre da impossibilidade de tomar $\hat{Q}^{a}_{\perp}(x^0,x^1,x^2)=0$ como identidade operatorial neste caso de fixação incompleta do gauge. Realmente, calculando a ação do operador $\hat{\Omega} = \frac{i}{g}\int dx^1 dx^2 \theta^a(x^0,x^1,x^2)\, \hat{Q}^a_\perp(x^0,x^1,x^2)$ sobre os campos básicos (ver (1.9) e (1.10))

$$\left[\hat{A}^{\alpha,a}_{(\underset{\sim}{x})}, \hat{\Omega}\right] = \frac{1}{g} \hat{D}^{\alpha,ab}_{(\underset{\sim}{x})}\, \theta^{b}(x^0,x^1,x^2) \quad , \qquad (1.14a)$$

$$\left[\hat{\pi}^{a}_{\alpha}(\underset{\sim}{x}), \hat{\Omega}\right] = -f^{acb}\, \theta^{c}(x^0,x^1,x^2)\, \hat{\pi}^{b}_{\alpha}(\underset{\sim}{x}) \, , \qquad (1.14b)$$

$$\left[\hat{\Psi}^{u}(\underset{\sim}{x}), \hat{\Omega}\right] = i\,\theta^{a}(x^0,x^1,x^2)\left(\frac{\lambda^a}{2}\right)^{uv} \hat{\Psi}^{v}(\underset{\sim}{x}) \quad , \qquad (1.14c)$$

$$\left[\hat{\pi}^{u}_{\Psi}(\underset{\sim}{x}), \hat{\Omega}\right] = -i\,\theta^{a}(x^0,x^1,x^2)\, \hat{\pi}^{v}_{\Psi}(\underset{\sim}{x})\left(\frac{\lambda^a}{2}\right)^{vu} \quad , \qquad (1.14d)$$

vemos que os operadores $\hat{Q}^a_\perp(x^0,x^1,x^2)$ são exatamente os geradores de transformações de gauge independentes de x^3 os quais representam a liberdade de gauge residual da teoria [10]. Existem duas maneiras alternativas de tratar este problema:

(i) Pode-se tentar quantizar a teoria em um gauge axial completamente fixado, i.e., completar primeiro a fixação do gauge e então quantizar. Obviamente, a eliminação da liberdade de gauge residual permite impor $\hat{Q}^a_\perp(x^0,x^1,x^2) = 0$ de modo consistente. Os trabalhos de Yao [19] e de Chodos [20] apontam nessa direção mas eles não resolveram o problema. Na fixação de gauge de Yao [19] a condição de gauge axial $A^{3,a} = 0$ é relaxada para pontos localizados em $x^3 = \pm\infty$ e lá substituída por uma condição de gauge do tipo Coulomb*. Entretanto, como já assinalamos, as ambiguidades de Gribov [2,3] que permeiam o gauge de Coulomb tornam insegura esta técnica para teorias de gauge não-Abelianas (no gauge de Coulomb, as condições de contorno não são suficientes para fixar completamente o gauge [2,3]). Por outro lado, a fixação de gauge de Chodos [20] leva a CTI's que não são compatíveis com intensidades de campo anulando-se no infinito espacial. Como conseqüência, a existência do conjunto inteiro de geradores de Poincaré não pode ser estabelecida**.

(ii) Pode-se optar por desistir da condição (1.11) como uma identidade operatorial e substituí-la por (ver (1.13)) [22,18]

$$\hat{\Pi}_3^a(x^0,x^1,x^2,x^3=-\infty) = 0 \quad , \tag{1.15a}$$

$$\hat{Q}_\perp^a(x^0,x^1,x^2)\,|\Psi\rangle = 0 \quad . \tag{1.15b}$$

Neste tipo de formulação, com a condição (1.15b) pretende-se separar o setor físico do espaço de Hilbert. Entretanto, esta condição coloca imediatamente em pauta o problema não-trivial [18] de encontrar o estado de vácuo físico $|0>$ da teoria, com energia e norma finitas (o vácuo "vestido"), que de fato exista satisfazendo

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*Um procedimento similar, porém com a condição de Coulomb num ponto próprio, foi seguido por Christ, Guth e Weinberg em sua formulação de gauge axial da teoria de Yang-Mills clássica acoplada a campos escalares de Higgs [21].

**Chodos, entretanto, sugere a possível necessidade de um termo tipo potencial adicional semelhante ao obtido por Schwinger no gauge de Coulomb [1].

$$\hat{Q}^{a}_{\perp}(x^0,x^1,x^2)|0\rangle = 0 \qquad (1.16)$$

e a partir do qual se possa calcular valores esperados de operadores [23,14]. Conforme discute Mandelstam [18], este problema, ao nível não-Abeliano no gauge axial*, ainda não foi resolvido. Desta forma, fica em aberto o problema crítico da invariança de Poincaré da cromodinâmica quântica (QCD) no gauge axial dentro de uma formulação quântica consistente. Além disso, na situação não-Abeliana em qualquer gauge, precisamos também de consistência da teoria em relação a seu conteúdo topológico não-trivial. De fato, como bem o ressaltam Marciano e Pagels [24], a carga topológica surge da dinâmica e topologia das configurações de campo não-Abelianas não sendo, portanto, colocada da "à mão" na teoria [24]. Especificamente, a existência de uma carga topológica não nula exige um comportamento assintótico não trivial dos potenciais de gauge axial em $x^3 = \pm\infty$**. Embora a necessidade deste comportamento assintótico tenha sido reconhecida pelos autores da ref. [22], tal comportamento não aparece como um resultado em sua formulação da QCD no gauge axial.

Pelas razões levantadas e motivados por trabalhos recentes relativos à quantização da eletrodinâmica em gauges

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*Na situação Abeliana, o problema tem sido resolvido em diferentes gauges [18,8]. Ao nível não-Abeliano em teoria de perturbações e no contexto da formulação de gauge temporal, Dahmen, Scholz e Steiner propuseram recentemente uma construção explícita para o vácuo vestido $|0\rangle$ [8].

**Assinalamos, como também o fizeram Bars e Green [22], que o problema das condições de contorno foi ignorado em investigações anteriores da QCD no gauge $A^{3,a} = 0$ [25] (ver item I.1.3b atrás).

axiais completamente fixados [26-28], o objetivo básico que buscamos com este trabalho foi a obtenção de uma formulação operatorial consistente para a QCD no gauge axial segundo a idéia apresentada em (i), ou seja, mediante a fixação completa do gauge. O particular gauge axial completamente fixado aqui introduzido foi denominado gauge superaxial, conforme adiantado.

Em resumo (ver cap. VII), a presente formulação da QCD no gauge superaxial tem como resultados explicitamente provados a consistência interna da teoria quanto aos requerimentos primordiais de:

a) Invariança relativística (com todos os geradores de Poincaré finitos, bem definidos matematicamente e transcritos diretamente dos análogos clássicos);

b) conteúdo topológico não-trivial, como exigido pela presença de fenômenos não-perturbativos do tipo instanton [4].

Além disso, a teoria nesta formulação apresenta os atributos de:

c) contar com um espaço de Hilbert, em princípio, bem definido (sem restrições sobre os vetores de estado, em particular, sobre o vácuo |0>);

d) simplicidade quanto a problemas de ordenamento (evidenciada pela estrutura dos CTI's);

e) ausência de partículas fictícias (fantasmas) de Faddeev-Popov [29,30,6] ao nível da integral funcional.

Ao tratarmos com um sistema físico vinculado, conforme vemos desde (1.7), é natural que usemos como método de quantização o procedimento de quantização por parênteses de Dirac (PQPD) [31,6] para os sistemas vinculados. O PQPD está baseado essencialmente na observação de que os vínculos de pri meira classe, característicos de teorias de gauge, junto com as condições subsidiárias (de gauge) formam um conjunto de vínculos de segunda classe [6]. Então, pode-se quantizar a teoria abstraindo os CTI's básicos dos correspondentes parênteses de Dirac (PD's) [31] com os vínculos e condições de gauge transladando-se, naturalmente, como relações operatoriais fortes. Com este método de quantização não é necessário distinguir entre graus de liberdade dependentes e independentes. O PQPD tem sido usado para quantizar sistemas tanto relativísticos quanto não-relativísticos [21,27,28,32-35].

O material foi organizado da seguinte maneira: começamos no capítulo II especificando as condições que definem o gauge superaxial. Após, o formalismo Hamiltoniano clássico de 1ª ordem para a cromodinâmica é construído. O formalismo de 1ª ordem resulta especialmente adequado às condições de gauge em questão. No capítulo III determinamos a inversa da matriz de Faddeev-Popov. Ficamos então aptos a calcular os PD's básicos e a executar a transição à teoria quântica. Apresentamos os CTI's no capítulo IV onde também discutimos o comportamento assintótico das variáveis canônicas. O tensor energia-momentum quântico simétrico é construído no capítulo V onde também determinamos a ação dos geradores de Poincaré sobre cada campo básico. Ainda neste capítulo, demonstramos a invariança de Poincaré da teoria, verificamos a validade da álgebra das cargas de cor e determinamos a álgebra de correntes no gauge superaxial. No capítulo VI, analisamos a dependência da matriz de Faddeev-Popov em relação aos campos com vistas a uma possível quantização da teoria através do formalismo funcional. Nossas conclusões e comentários finais encerram o trabalho, no capítulo VII.

II. FORMULAÇÃO HAMILTONIANA CLÁSSICA DE 1ª ORDEM DA CROMODINÂMICA NO GAUGE SUPERAXIAL

II.1 O Gauge Superaxial. Implementação do Gauge Superaxial na Forma Padrão de Dirac Através do Formalismo de 1ª Ordem

O particular gauge axial completamente fixado que implementamos nesta tese, e que denominamos gauge superaxial (ver Introdução), é definido pelas seguintes condições:

$$A^{3,a}(x^0, x^1, x^2, x^3) = 0 \quad , \qquad (2.1a)$$

$$A^{1,a}(x^0, x^1, x^2, x^3_{(0)}) = 0 \quad , \qquad (2.1b)$$

$$A^{2,a}(x^0, x^1_{(0)}, x^2, x^3_{(0)}) = 0 \quad , \qquad (2.1c)$$

$$A^{0,a}(x^0, \underset{\sim}{x}_{(0)}) = \int d^3z \; r_k(\underset{\sim}{z}; \underset{\sim}{x}_{(0)}) F^{0k,a}(x^0, \underset{\sim}{z}) \quad . \qquad (2.2)$$

Aqui, $\underset{\sim}{x}_{(0)} \equiv (x^1_{(0)}, x^2_{(0)}, x^3_{(0)})$ é um ponto arbitrário fixado e as r_k's são distribuições arbitrárias mas regulares de $\underset{\sim}{z}$ e $\underset{\sim}{x}_{(0)}$. Tais condições definem um gauge confiável no seguinte sentido:

a) O gauge está completamente fixado (é claro, a menos de uma transformação de gauge global) e, portanto, livre de ambigüidades de Gribov [2];

b) o gauge é implementável efetivamente a nível quântico. Com isto queremos dizer que as condições de gauge (2.1) e (2.2) são compatíveis, como identidades operatoriais, com a álgebra de comutação a tempos iguais.

Prova de a)

O gauge estará fixado a menos de uma transformação global de gauge se a imposição das condições (2.1) e (2.2) sobre configurações de campo $A'^{\mu,a}$ relacionadas às configurações $A^{\mu,a}$ por uma transformação de gauge não-Abeliana local

$$A^{\mu}(x^0,\underset{\sim}{x}) \rightarrow A'^{\mu}(x^0,\underset{\sim}{x}) = U(x^0,\underset{\sim}{x})\,A^{\mu}(x^0,\underset{\sim}{x})\,U^{\dagger}(x^0,\underset{\sim}{x}) + U(x^0,\underset{\sim}{x})\,\partial^{\mu}U^{\dagger}(x^0,\underset{\sim}{x}), \quad (2.3)$$

$$F^{\mu\nu}(x^0,\underset{\sim}{x}) \rightarrow F'^{\mu\nu}(x^0,\underset{\sim}{x}) = U(x^0,\underset{\sim}{x})\,F^{\mu\nu}(x^0,\underset{\sim}{x})\,U^{\dagger}(x^0,\underset{\sim}{x}) \;, \quad (2.4)$$

onde

$$A^{\mu} \equiv \frac{g}{i}\,\frac{\lambda^a}{2}\,A^{\mu,a} \;, \quad (2.5)$$

$$F^{\mu\nu} \equiv \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} + [\,A^{\mu}, A^{\nu}\,] \;, \quad (2.6)$$

$$U(x^0,\underset{\sim}{x}) \equiv \exp\left[\,i\,\frac{\lambda^a}{2}\,\theta^a(x^0,\underset{\sim}{x})\right] \;, \quad (2.7)$$

implicar que as funções de gauge θ^a's são constantes independentes de x^0 e $\underset{\sim}{x}$. Ora, a condição (2.1a) implica desde (2.3)

$$\partial^3 U^{\dagger}(x^0,x^1,x^2,x^3) = 0 \quad \therefore \quad U^{\dagger} = U^{\dagger}(x^0,x^1,x^2) \;. \quad (2.8)$$

O resultado (2.8) junto com (2.1b) conduz a

$$\partial^1 U^\dagger(x^0,x^1,x^2) = 0 \quad \therefore \quad U^\dagger = U^\dagger(x^0,x^2) \quad . \qquad (2.9)$$

Por sua vez, (2.9) junto com (2.1c) implica

$$\partial^2 U^\dagger(x^0,x^2) = 0 \quad \therefore \quad U^\dagger = U^\dagger(x^0) \quad . \qquad (2.10)$$

Finalmente, (2.10) e (2.2), através de (2.3) e (2.4) conduzem a

$$A'^0(x^0,\underset{\sim}{x}_{(0)}) = U(x^0)\, A^0(x^0,\underset{\sim}{x}_{(0)})\, U^\dagger(x^0) + U(x^0)\,\partial^0 U^\dagger(x^0)$$

$$= \int d^3\underset{\sim}{z}\; r_k(\underset{\sim}{z};\underset{\sim}{x}_{(0)}) \Big(U(x^0)\, F^{0k}(x^0,\underset{\sim}{z})\, U^\dagger(x^0) \Big) + U(x^0)\,\partial^0 U^\dagger(x^0)$$

$$= \int d^3\underset{\sim}{z}\; r_k(\underset{\sim}{z};\underset{\sim}{x}_{(0)})\, F'^{0k}(x^0,\underset{\sim}{z}) + U(x^0)\,\partial^0 U^\dagger(x^0)$$

$$\therefore \quad \partial^0 U^\dagger(x^0) = 0 \quad . \qquad (2.11)$$

Como se vê desde (2.11) e (2.7), θ^a é uma constante e o gauge está fixado (q.e.d.).

Observar que a operação de fixação de gauge poderia ser completada, em princípio, com (2.1) e com a imposição de [36]

$$A^{0,a}(x^0,\underset{\sim}{x}_{(0)}) = 0 \qquad (2.12)$$

conforme vemos a partir da primeira linha da expressão (2.11).

Entretanto, o gauge definido por (2.1) e (2.12) não é implementável no sentido que indicamos em b) atrás (ver também o capítulo IV). Isto explica nossa escolha da condição (2.2) ao invés de (2.12). À diferença do gauge especificado por (2.1) e (2.12), o gauge superaxial é implementável como demonstramos no capítulo IV.

Em nossa opinião o gauge superaxial é superior a tentativas anteriores realizadas no sentido de fixar completamente o gauge axial [20-22]. Em particular, é mais fácil desenvolver uma formulação matemática rigorosa da teoria quando a fixação é feita num ponto próprio $\underset{\sim}{x}_{(0)}$ do que quando o ponto de fixação de gauge é localizado no infinito [21,22]. Além disso, não está provado, a nosso conhecimento, que o gauge axial definido por (2.1a) e por uma condição da forma [21]

$$\partial^{\alpha} A^{\alpha,a} = 0 \quad , \quad \alpha = 1,2 \ ,$$

está livre de ambigüidades de Gribov [2].

Encerramos esta seção modificando a forma das condições (2.1) e (2.2) com vistas a obter uma formulação Hamiltoniana da cromodinâmica (o Lagrangeano de 2ª ordem da teoria foi introduzida em (1.1)). No tratamento de Dirac dos sistemas vinculados [31], a forma padrão em que as condições de gauge são introduzidas é $\chi^{\mu,a}(x^0,\underset{\sim}{x}) \approx 0$, valendo para todos os pontos espaço-tempo [37]. Isto não se verifica no caso do gauge superaxial. O que se deve notar é que, dentro de um formalismo <u>padrão</u> de Dirac [31], ainda não sabemos como tratar condições do tipo (2.1) e (2.2), que valem apenas para alguns pontos do espaço, usando apenas variáveis de 2ª ordem*. Entretanto, partindo da expressão do tensor intensidade de campo $F^{\mu\nu,a}$ em termos dos potenciais $A^{\mu,a}$ pode-se obter, de forma única [36], as fórmulas de inversão $A = A[F]$ (isto é feito no Apêndice A, ver expressões (A.7), (A.11), (A.12) e (A.18)) para o gauge superaxial. Tais fórmulas, por seu turno, permitem substituir (2.1) e (2.2) pelo seguinte conjunto de condições [27]

$$\chi^{1,a}(x^0,\underset{\sim}{x}) \equiv A^{1,a}(x^0,\underset{\sim}{x}) + \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,a}(x^0,x^1,x^2,x'^3) \approx 0 \,, \qquad (2.13a)$$

$$\chi^{2,a}(x^0,\underset{\sim}{x}) \equiv A^{2,a}(x^0,\underset{\sim}{x}) - \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{23,a}(x^0,x^1,x^2,x'^3) +$$

$$+ \int_{x^1_{(0)}}^{x^1} dx'^1\, F^{12,a}(x^0,x'^1,x^2,x^3_{(0)}) \approx 0 \,, \qquad (2.13b)$$

$$\chi^{3,a}(x^0,\underset{\sim}{x}) \equiv A^{3,a}(x^0,\underset{\sim}{x}) \approx 0 \,, \qquad (2.13c)$$

---

*No caso da eletrodinâmica, Kiefer e Rothe propuseram recentemente uma generalização da noção de parênteses de Dirac de modo a acomodar as condições não usuais que especificam um gauge do tipo do superaxial, em um formalismo de 2ª ordem [28].

$$\chi^{4,a}(x^0,\underset{\sim}{x}) \equiv A^{0,a}(x^0,\underset{\sim}{x}) - \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{03,a}(x^0,x^1,x^2,x'^3) -$$

$$- \int_{x^2_{(0)}}^{x^2} dx'^2\, F^{02,a}(x^0,x^1_{(0)},x^2,x^3_{(0)}) - \int_{x^1_{(0)}}^{x^1} dx'^1\, F^{01,a}(x^0,x'^1,x^2,x^3_{(0)}) -$$

$$- \int d^3z\, r_k(\underset{\sim}{z};\underset{\sim}{x}_{(0)})\, F^{0k,a}(x^0,\underset{\sim}{z}) \approx 0 \quad , \qquad (2.13d)$$

as quais valem para todos os pontos espaço-tempo, conforme desejado*. As relações (2.13) mostram o formalismo canônico de 1ª ordem, no qual os $A^{\mu,a}$ e as $F^{\mu\nu,a}$ são tratadas como variáveis independentes, como especialmente adequado para a implementação do gauge superaxial na forma padrão de Dirac.

II.2 Vínculos Primários e Secundários e o Hamiltoniano Total. Vínculos Irredutíveis de 1ª e 2ª Classe. Redução do Espaço de Fase

Com o propósito de estabelecer a formulação Hamiltoniana clássica de 1ª ordem da cromodinâmica segundo o método

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*Neste trabalho o sinal de igualdade fraca "≈", cujo significado é dado na p. 22, é definido como em [31].

sistemático* de Dirac [31, 6,37,38], introduzimos o Lagrangeano de 1ª ordem correspondente

$$\mathcal{L} = -\frac{1}{2} F^{a}_{\mu\nu}(\partial^{\mu}A^{\nu,a} - \partial^{\nu}A^{\mu,a} + g f^{abc} A^{\mu,b}A^{\nu,c}) + \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu,a} + i\bar{\psi}\gamma^{\mu}\tilde{D}_{\mu}\psi - m\bar{\psi}\psi \;, \qquad (2.14)$$

onde $F^{\mu\nu,a}$ é somente restringido a ser um tensor antissimétrico nos índices de Lorentz. Escolhemos $A^{\mu,a}$, $F^{0i,a}$, $F^{12,a}$, $F^{23,a}$, $F^{31,a}$ e ψ como as coordenadas canônicas e π^{a}_{μ}, π^{a}_{0i}, π^{a}_{12}, π^{a}_{23}, π^{a}_{31} e π_{ψ} como os correspondentes momenta canonicamente conjugados, respectivamente. Estas variáveis canônicas geram o espaço de fase Γ' do sistema. Reescrevendo (2.14) na forma

$$\mathcal{L} = \frac{1}{4} F^{ij,a}F^{ij,a} - \frac{1}{2} F^{0i,a}F^{0i,a} + F^{0i,a}(\partial^{0}A^{i,a} - \partial^{i}A^{0,a} + g f^{abc} A^{0,b}A^{i,c}) -$$
$$- \frac{1}{2} F^{ij,a}(\partial^{i}A^{j,a} - \partial^{j}A^{i,a} + g f^{abc} A^{i,b}A^{j,c}) + i\bar{\psi}\gamma^{0}\partial^{0}\psi - i\bar{\psi}\gamma^{k}\partial^{k}\psi +$$
$$+ g\bar{\psi}\gamma^{0}\frac{\lambda^{a}}{2}\psi A^{0,a} - g\bar{\psi}\gamma^{k}\frac{\lambda^{a}}{2}\psi A^{k,a} - m\bar{\psi}\psi \qquad (2.15)$$

---

*No restante deste capítulo seguimos os passos deste método procurando: a) discutir os pontos relevantes e b) facilitar a compreensão onde julgamos necessário. Desenvolvimentos detalhados podem ser encontrados nas referências [31, 6,37,38].

encontramos*

$$\pi_o^a = \frac{\partial \mathcal{L}}{\partial(\partial^o A^{o,a})} = 0 \quad , \tag{2.16}$$

$$\pi_j^a = \frac{\partial \mathcal{L}}{\partial(\partial^o A^{j,a})} = F^{oj,a} , \tag{2.17}$$

$$\pi_{oi}^a = \frac{\partial \mathcal{L}}{\partial(\partial^o F^{oi,a})} = 0 \quad , \tag{2.18}$$

$$\pi_{ij}^a = \frac{\partial \mathcal{L}}{\partial(\partial^o F^{ij,a})} = 0 \quad , \tag{2.19}$$

$$\pi_\psi^u = \frac{\partial \mathcal{L}}{\partial(\partial^o \psi^u)} = i\bar{\psi}^u \gamma^o \quad . \tag{2.20}$$

As expressões (2.16)-(2.19) são os vínculos primários da teoria (surgem apenas da forma funcional do Lagrangeano) que restringem as possíveis trajetórias no espaço de fase Γ' a uma hipersuperfície $\bar{\Sigma}$ ($\bar{\Sigma} \subset \Gamma'$) de dimensionalidade menor. Denotamos tais vínculos por

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[*] Para simplificar a notação omitiremos, sempre que possível, os argumentos espaço e/ou tempo dos campos.

$$\varphi_1^a = \pi_0^a \approx 0 \,, \quad \varphi_2^a = \pi_{01}^a \approx 0 \,, \quad \varphi_3^a = \pi_{02}^a \approx 0 \,, \quad \varphi_4^a = \pi_{03}^a \approx 0 \,, \qquad (2.21a,b,c,d)$$

$$\varphi_5^a = \pi_{12}^a \approx 0 \,, \quad \varphi_6^a = \pi_{23}^a \approx 0 \,, \quad \varphi_7^a = \pi_{31}^a \approx 0 \quad , \qquad (2.21e,f,g)$$

$$\varphi_8^a = \pi_1^a - F^{01,a} \approx 0 \,, \quad \varphi_9^a = \pi_2^a - F^{02,a} \approx 0 \,, \quad \varphi_{10}^a = \pi_3^a - F^{03,a} \approx 0 \,, \qquad (2.21h,i,j)$$

utilizando o conceito de <u>igualdade</u> <u>fraca</u> ($\approx$) de Dirac [31]*. As igualdades fortes (2.16)-(2.19) só valem sobre $\bar{\Sigma}$. As igualdades fracas (2.21) podem ser vistas como extensões (continuações analíticas) a todo o espaço de fase Γ' de funções definidas sobre $\bar{\Sigma}$.

O hamiltoniano definido sobre $\bar{\Sigma}$ é

$$\bar{H} = \int d^3x \left[\pi_j^a \partial^0 A^{j,a} + \pi_\psi \partial^0 \psi - \mathcal{L} \right] =$$

$$= \int d^3x \Big[\frac{1}{2} \pi_j^a \pi_j^a - \frac{1}{4} F^{ij,a} F^{ij,a} + \pi_j^a (\partial^j A^{0,a} - g f^{abc} A^{0,b} A^{j,c}) +$$

$$+ \frac{1}{2} F^{ij,a} (\partial^i A^{j,a} - \partial^j A^{i,a} + g f^{abc} A^{i,b} A^{j,c}) + i \bar{\psi} \gamma^k \partial^k \psi -$$

$$- g \bar{\psi} \gamma^0 \frac{\lambda^a}{2} A^{0,a} \psi + g \bar{\psi} \gamma^k \frac{\lambda^a}{2} A^{k,a} \psi + m \bar{\psi} \psi \Big] . \qquad (2.22)$$

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[*] Duas funções arbitrárias do espaço de fase Γ' são ditas fracamente iguais se forem iguais apenas sobre $\bar{\Sigma}$. Uma função arbitrária fracamente nula em Γ' pode ser representada por uma combinação linear arbitrária dos vínlos (2.21) e será fortemente nula (=0) quando tomada sobre $\bar{\Sigma}$.

Utilizando o fato

$$\int d^3x\, \pi_j^{a}\, \partial^j A^{0,a} = -\int d^3x\, A^{0,a}\, \partial^j \pi_j^{a} \qquad (2.23)$$

que surge de integrar por partes, assumindo que π_j^a tende a zero assintoticamente (o que será justificado a posteriori), podemos reescrever (2.22) na forma

$$\bar{H} = \int d^3x \Big[\frac{1}{2} \pi_j^a \pi_j^a - A^{0,a}\Big(\partial^j \pi_j^a + g f^{acb} A^{j,c} \pi_j^b - i g \pi_\psi \frac{\lambda^a}{2} \psi\Big) -$$

$$-\frac{1}{4} F^{ij,a} F^{ij,a} + \frac{1}{2} F^{ij,a}\big(\partial^i A^{j,a} - \partial^j A^{i,a} + g f^{abc} A^{i,b} A^{j,c}\big) +$$

$$+ \pi_\psi \gamma^0 \gamma^k \partial^k \psi - i g \pi_\psi \gamma^0 \gamma^k \frac{\lambda^a}{2} \psi A^{k,a} - i m \pi_\psi \gamma^0 \psi \Big] =$$

$$= \int d^3x \Big\{ \frac{1}{2} \pi_j^a \pi_j^a + \frac{1}{4} F^{ij,a} F^{ij,a} + \pi_\psi \gamma^0 (\gamma^k \tilde{D}^k - i m) \psi -$$

$$- A^{0,a}\Big(D^{j,ab} F^{0j,b} - i g \pi_\psi \frac{\lambda^a}{2} \psi\Big) - \frac{1}{2} F^{ij,a}\Big[F^{ij,a} - \big(\partial^i A^{j,a} - \partial^j A^{i,a} + g f^{abc} A^{i,b} A^{j,c}\big)\Big]\Big\}. \qquad (2.24)$$

Desde $\bar{H}$, define-se [31] um Hamiltoniano total, $H_{T'}$, sobre o espaço de fase total Γ'. Isto é feito adicionando os vínculos (2.21) (ϕ_A^a, $A = 1,\ldots,10$) a $\bar{H}$ através de multiplicadores de Lagrange arbitrários η_A^a:

$$H_{T'} = \bar{H} + \sum_{A=1}^{10} \int d^3x\, \eta_A^a(\underset{\sim}{x})\, \varphi_A^a(\underset{\sim}{x}) \qquad (2.25)$$

O parêntese de Poisson (PP) de dois funcionais regulares quaisquer, Ω_1 e Ω_2, das variáveis canônicas é definido por [6,38,39]

$$[\Omega_1,\Omega_2]_{PP} = \int d^3x \Big\{ \Big[\frac{\delta\Omega_1}{\delta A^{\mu,a}(\underset{\sim}{x})} \frac{\delta\Omega_2}{\delta\pi_\mu^a(\underset{\sim}{x})} - \exp(i\pi n_{\Omega_1} n_{\Omega_2}) \frac{\delta\Omega_2}{\delta A^{\mu,a}(\underset{\sim}{x})} \frac{\delta\Omega_1}{\delta\pi_\mu^a(\underset{\sim}{x})} \Big] +$$

$$+ \Big[\frac{\delta\Omega_1}{\delta F^{oj,a}(\underset{\sim}{x})} \frac{\delta\Omega_2}{\delta\pi_{oj}^a(\underset{\sim}{x})} - \exp(i\pi n_{\Omega_1} n_{\Omega_2}) \frac{\delta\Omega_2}{\delta F^{oj,a}(\underset{\sim}{x})} \frac{\delta\Omega_1}{\delta\pi_{oj}^a(\underset{\sim}{x})} \Big] +$$

$$+ \frac{1}{2}\Big[\frac{\delta\Omega_1}{\delta F^{jk,a}(\underset{\sim}{x})} \frac{\delta\Omega_2}{\delta\pi_{jk}^a(\underset{\sim}{x})} - \exp(i\pi n_{\Omega_1} n_{\Omega_2}) \frac{\delta\Omega_2}{\delta F^{jk,a}(\underset{\sim}{x})} \frac{\delta\Omega_1}{\delta\pi_{jk}^a(\underset{\sim}{x})} \Big] +$$

$$+ \Big[\frac{\Omega_1\overleftarrow{\delta}}{\delta\psi(\underset{\sim}{x})} \frac{\overrightarrow{\delta}\Omega_2}{\delta\pi_\psi(\underset{\sim}{x})} - \exp(i\pi n_{\Omega_1} n_{\Omega_2}) \frac{\Omega_2\overleftarrow{\delta}}{\delta\psi(\underset{\sim}{x})} \frac{\overrightarrow{\delta}\Omega_1}{\delta\pi_\psi(\underset{\sim}{x})} \Big]\Big\}, \quad (2.26)$$

onde $\overleftarrow{\delta}/\delta\psi$ e $\overrightarrow{\delta}/\delta\pi_\psi$ denotam derivadas funcionais pela direita e pela esquerda, respectivamente, enquanto n_Ω é igual a 0 ou 1 dependendo de Ω ser um bóson ou um férmion.

Os vínculos secundários da teoria surgem exigindo-se persistência no tempo dos vínculos primários. Assim,

$$\partial^0\varphi_1^a = [\pi_0^a(\underset{\sim}{x}), H_T]_{PP} \approx 0 \quad \Rightarrow$$

$$\Rightarrow \quad D^{j,ab}(\underset{\sim}{x})\, F^{oj,b}(\underset{\sim}{x}) - ig\,\pi_\psi(\underset{\sim}{x})\frac{\lambda^a}{2}\psi(\underset{\sim}{x}) \approx 0 \quad , \qquad (2.27)$$

$$\partial^0 \varphi^a_{5,6,7} = [\pi^a_{ij}(\underset{\sim}{x}), H_{T'}]_{PP} \approx 0 \quad \Longrightarrow$$

$$\Longrightarrow \quad F^{ij,a}_{(\underset{\sim}{x})} - (\partial^i A^{j,a}_{(\underset{\sim}{x})} - \partial^j A^{i,a}_{(\underset{\sim}{x})} + g f^{abc} A^{i,b}_{(\underset{\sim}{x})} A^{j,c}_{(\underset{\sim}{x})}) \approx 0 \,. \quad (2.28)$$

As condições de persistência restantes, i.e., $\partial^0 \phi^a_{2,3,4} \approx 0$ e $\partial^0 \phi^a_{8,9,10} \approx 0$, não implicam no aparecimento de novos vínculos.

Em continuação, deve-se exigir persistência no tempo dos vínculos secundários até esgotar o processo de geração de vínculos (i.e., até obter: (i) 0 ≈ 0 ou (ii) condições sobre os multiplicadores de Lagrange η^a_A). Ao fazer isto (ver Apêndice B), encontramos que não surgem outros vínculos além da lei de Gauss não-Abeliana (2.27) e dos vínculos (2.28). Na realidade, recaímos imediatamente na alternativa (ii).

Neste ponto, devemos distinguir quais dos vínculos da teoria são de 1ª classe e quais são de 2ª classe*. A importância desta classificação prende-se ao fato que os vínculos de 1ª classe são típicos de teorias de gauge (são os geradores de transformações de gauge locais). Por outro lado, um Lagrangeano que implique apenas vínculos de 2ª classe não apresenta liberdade de gauge, definindo um sistema não-degenerado. Numa análise superficial estaríamos em problemas pois, de nosso conjunto de vínculos (2.21), (2.27) e (2.28), apenas $\pi^a_0 \approx 0$ parece ser de 1ª classe. Isto na realidade reflete a necessi-

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*Um vínculo é dito <u>de 1ª classe</u> se tiver PP fracamente zero com <u>todos</u> os outros vínculos da teoria. Caso o vínculo não satisfizer esta condição, é dito <u>de 2ª classe</u>.

dade de descobrir os vínculos <u>irredutíveis</u> das duas categorias. Ou seja, devemos encontrar todas as combinações lineares diferentes de vínculos de 2ª classe que sejam vínculos de 1ª classe. Como fazê-lo? De nossa experiência com esta teoria e com modelos não-relativísticos, descobrimos que o próprio algoritmo de Dirac fornece, na etapa final de consistência, as combinações lineares apropriadas de 1ª classe (ver Apêndice B). De fato, ver (B.24) e (B.25), as (N^2-1) combinações lineares de vínculos de 2ª classe

$$\tau^a = D^{j,ab}\pi_j^{\,b} - ig\pi_\psi \frac{\lambda^a}{2}\psi + \frac{1}{2}gf^{abc}F^{ij,b}\pi_{ij}^{\,c} + gf^{abc}F^{0j,b}\pi_{0j}^{\,c} \approx 0 \qquad (2.29)$$

são vínculos de 1ª classe. Não existem outras combinações de vínculos de 2ª classe que sejam de 1ª classe. Após transferir os τ^a's ao conjunto de vínculos de 1ª classe ficamos com um número <u>par</u> de vínculos de 2ª classe na teoria, como deveria ser [31,32,37] (ver (B.27)).

As diferenças e semelhanças entre os formalismos de 1ª e de 2ª ordem devem ser notadas. Por um lado, quando a cromodinâmica é formulada usando o formalismo de 2ª ordem [35], encontramos somente vínculos de 1ª classe. Estes vínculos são $\pi_0^a \approx 0$ e a lei de Gauss (2.27) (ver (1.7)). Poder-se-ia argumentar que os vínculos de 2ª classe que surgem em conexão com o formalismo de 1ª ordem apenas indicam a presença de graus de liberdade sem importância física os quais, eventualmente, se poderia eliminar [31]. Entretanto, esta eliminação não é possível, no presente caso, devido à estrutura não usual das condições de gauge (ver (2.1) e (2.2)). Por outro lado, desde (2.24) seque que $A^{0,a}$ atua como um multiplicador de Lagrange para a lei de Gauss e, como tal, seus valores não são encontrados integrando-se as equações de movimento de Hamilton. Na realidade, $A^{0,a}$ será determinado em termos das restantes variáveis canônicas. Correspondentemente, π_0^a está diretamente fixado pela condição de vínculo $\pi_0^a \approx 0$ sem referência no que segue às equações de movimento, como deve ser, pois $\pi_0^a \approx 0$ é um vínculo primário. Como resultado, π_0^a desaparece do problema enquanto $A^{0,a}$ permanece como um multiplicador de Lagrange[31]. Designaremos, então, por Γ o espaço de fase obtido eliminando-se $A^{0,a}$ e π_0^a de Γ'. As mesmas razões possibilitam esta redução do espaço de fase, pela eliminação do setor $A^{0,a}$, π_0^a, no formalismo de 2ª ordem [21,31].

II.3 Introdução das Condições de Gauge e o Hamiltoniano Completo. Equações de Movimento. Matriz de Faddeev-Popov. Parênteses de Dirac.

Devemos agora introduzir as condições de gauge na teoria. Em princípio, isto poderia ser feito incorporando-se essas condições ao Hamiltoniano H_T via multiplicadores de Lagrange [6]. Entretanto, tal procedimento direto não funciona para o problema em questão devido à que os vínculos e condições de gauge não formam um conjunto de vínculos de 2ª classe irredutível em Γ [27]. De fato, as condições de gauge (2.13a,b,c) junto com a identidade de Bianchi temporal [36] (A.24)

$$\mathcal{B}^a = \frac{1}{2}\epsilon^{ijk}\partial^i F^{jk,a} + gf^{abc}\left[A^{1,b}[F]F^{23,c} + A^{2,b}[F]F^{31,c}\right] = 0, \quad (2.30)$$

por si só já implicam (2.28) (ver Apêndice A). Portanto, construiremos o Hamiltoniano completo H_C, descrevendo a cromodinâmica no gauge superaxial, da seguinte maneira: Adicionamos a $H_{T'}$ (dado em (2.25))* a identidade de Bianchi temporal (2.30) e as condições de gauge (2.13a,b,c) e retiramos de $H_{T'}$ os vínculos (2.28) e $\pi^a_0 \approx 0$. Portanto, H_C resulta igual a

$$H_c = H + \sum_{J=1}^{14} \int d^3x \, u^a_J(\underset{\sim}{x}) \, \Phi^a_J(\underset{\sim}{x}) \quad , \qquad (2.31)$$

onde

$$H = \int d^3x \left[\frac{1}{2} \pi^a_j \pi^a_j + \frac{1}{4} F^{ij,a} F^{ij,a} + \pi_\psi \gamma^0 (\gamma^k \tilde{D}^k - im) \psi \right] , \qquad (2.32)$$

$$\Phi^a_1 \equiv \pi^a_{01} \approx 0 \ , \ \Phi^a_2 \equiv \pi^a_{02} \approx 0 \ , \ \Phi^a_3 \equiv \pi^a_{03} \approx 0 \ , \qquad (2.33\,a,b,c)$$

$$\Phi^a_4 \equiv \pi^a_{12} \approx 0 \ , \ \Phi^a_5 \equiv \pi^a_{23} \approx 0 \ , \ \Phi^a_6 \equiv \pi^a_{31} \approx 0 \ , \qquad (2.33\,d,e,f)$$

$$\Phi^a_7 \equiv \pi^a_1 - F^{01,a} \approx 0 \ , \ \Phi^a_8 \equiv \pi^a_2 - F^{a2,a} \approx 0 \ , \ \Phi^a_9 \equiv \pi_3 - F^{03,a} \approx 0 \ , \quad (2.33\,g,h,i)$$

$$\Phi^a_{10} \equiv B^a \approx 0 \quad , \qquad (2.33\,j)$$

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*Aqui, seguimos o chamado formalismo do Hamiltoniano total [31]. É equivalente, neste caso, usar o formalismo do Hamiltoniano extendido [31]. Esta equivalência resulta trivial devido à particular forma funcional simples do vínculo primário $\pi^a_0 \approx 0$. Entretanto, recentemente provamos a equivalência de ambos formalismos, admitindo a forma funcional mais geral possível para os vínculos primários da teoria [40].

$$\Phi^{a}_{11} \equiv D_j^{ab} F^{oj,b} + ig\pi_\psi \frac{\lambda^a}{2}\psi \approx 0^{*}, \qquad (2.33k)$$

$$\Phi^{a}_{12} \equiv \chi^{1,a} \approx 0, \quad \Phi^{a}_{13} \equiv \chi^{2,a} \approx 0, \quad \Phi^{a}_{14} \equiv \chi^{3,a} \approx 0, \qquad (2.33\,l,m,n)$$

e

$$u^{a}_{1} \equiv \eta^{01,a}, \quad u^{a}_{2} \equiv \eta^{02,a}, \quad u^{a}_{3} \equiv \eta^{03,a}, \qquad (2.34\,a,b,c)$$

$$u^{a}_{4} \equiv \eta^{12,a}, \quad u^{a}_{5} \equiv \eta^{23,a}, \quad u^{a}_{6} = \eta^{31,a}, \qquad (2.34\,d,e,f)$$

$$u^{a}_{7} \equiv \eta^{1,a}, \quad u^{a}_{8} \equiv \eta^{2,a}, \quad u^{a}_{9} \equiv \eta^{3,a}, \qquad (2.34\,g,h,i)$$

$$u^{a}_{11} = +A^{0,a}. \qquad (2.34\,k)$$

É claro, u^a_{10}, u^a_{12}, u^a_{13} e u^a_{14} são os multiplicadores de Lagrange associados com B^a, $\chi^{1,a}$, $\chi^{2,a}$ e $\chi^{3,a}$, respectivamente, como se pode ver desde (2.31), (2.33j) e (2.33 l,m,n). Os vínculos (**2**.33) formam um conjunto irredutível e completo de vínculos independentes de 2ª classe [30,6]. Os vínculos (2.33) definem uma hipersuperfície em Γ que denotamos por Σ ($\Sigma\subset\Gamma$). Note-se que a condição de gauge (2.2), ou equivalentemente (2.13d), não entra na construção do formalismo Hamiltoniano. Conforme vere-

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*Φ^a_{11} é definido por (2.33k) no que segue, ao invés de $\Phi^a_{11} = D^{j,ab}F^{0j,b} -$ $- ig\pi_\psi(\lambda^a/2)\psi \approx 0$, para fins de definir u^a_{11} por (2.34k) ao invés de $u^a_{11} = -A^{0,a}$ (ver (2.24) e (B.1)).

mos, (2.2) proverá condições de contorno no processo de determinação dos multiplicadores de Lagrange $A^{0,a}$.

O Hamiltoniano clássico da teoria já foi construído. O desenvolvimento temporal de qualquer funcional regular Ω das variáveis canônicas pode agora ser encontrado pela integração das equações de movimento de Hamilton

$$\partial^0 \Omega = \left[\Omega, H_C\right]_{PP}\Big|_{\Sigma} \quad , \tag{2.35}$$

onde o parêntese de Poisson é o definido em (2.26) (mas com a troca do índice μ por j devido à eliminação do setor $A^{0,a}$, π^a_0). Em particular, para os campos básicos da teoria encontramos

$$\partial^0 A^{j,a}_{(\underset{\sim}{x})} = \left[A^{j,a}_{(\underset{\sim}{x})}, H_C \right]_{PP}\Big|_{\Sigma} = \frac{\delta H_c}{\delta \pi^{\;a}_{j}(\underset{\sim}{x})}\Bigg|_{\Sigma} = \pi^{\;a}_{j}(\underset{\sim}{x}) + \eta^{j,a}_{(\underset{\sim}{x})} \; , \tag{2.36a}$$

$$\partial^0 F^{0j,a}_{(\underset{\sim}{x})} = \left[F^{0j,a}_{(\underset{\sim}{x})}, H_C \right]_{PP}\Big|_{\Sigma} = \frac{\delta H_c}{\delta \pi^{\;a}_{0j}(\underset{\sim}{x})}\Bigg|_{\Sigma} = \eta^{0j,a}_{(\underset{\sim}{x})} \quad , \tag{2.36b}$$

$$\partial^0 F^{jk,a}_{(\underset{\sim}{x})} = \left[F^{jk,a}_{(\underset{\sim}{x})}, H_c \right]_{PP}\Big|_{\Sigma} = \frac{\delta H_c}{\delta \pi^{\;a}_{jk}(\underset{\sim}{x})}\Bigg|_{\Sigma} = \eta^{jk,a}_{(\underset{\sim}{x})} \quad , \tag{2.36c}$$

$$\partial^0\psi(\underset{\sim}{x}) = [\psi(\underset{\sim}{x}), H_c]_{PP}\Big|_\Sigma = \int d^3z \left(\frac{\psi(\underset{\sim}{x})\overleftarrow{\delta}}{\delta\psi(\underset{\sim}{z})} \frac{\overrightarrow{\delta} H_c}{\delta\pi_\psi(\underset{\sim}{z})}\right)\Bigg|_\Sigma = \frac{\overrightarrow{\delta} H_c}{\delta\pi_\psi(\underset{\sim}{x})}\Bigg|_\Sigma =$$

$$= \int d^3z\, \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{z})\left[\gamma^0(\gamma^k \tilde{D}^k(\underset{\sim}{z}) - im)\psi(\underset{\sim}{z}) + ig\, A^{0,a}(\underset{\sim}{z})\frac{\lambda^a}{2}\psi(\underset{\sim}{z})\right] =$$

$$= \gamma^0\gamma^k \partial^k_x \psi(\underset{\sim}{x}) + i\gamma^0\left[g\frac{\lambda^a}{2}\left(-\gamma^k A^{k,a}(\underset{\sim}{x}) + \gamma^0 A^{0,a}(\underset{\sim}{x})\right) - m\right]\psi(\underset{\sim}{x}) \quad , \qquad (2.36d)$$

$$\partial^0\pi_\psi(\underset{\sim}{x}) = [\pi_\psi(\underset{\sim}{x}), H_c]_{PP}\Big|_\Sigma = -\frac{H_c\overleftarrow{\delta}}{\delta\psi(\underset{\sim}{x})}\Bigg|_\Sigma =$$

$$= -\int d^3z \Big[\pi_\psi(\underset{\sim}{z})\gamma^0(\gamma^k\partial^k_z - im)\,\delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x}) - ig\,\pi_\psi(\underset{\sim}{z})\gamma^0\gamma^k\frac{\lambda^a}{2}A^{k,a}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x}) +$$

$$+ ig\,\pi_\psi(\underset{\sim}{z})\frac{\lambda^a}{2}A^{0,a}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x})\Big] =$$

$$= \partial^k_x \pi_\psi(\underset{\sim}{x})\gamma^0\gamma^k - i\,\pi_\psi(\underset{\sim}{x})\gamma^0\left[g\frac{\lambda^a}{2}\left(-\gamma^k A^{k,a}(\underset{\sim}{x}) + \gamma^0 A^{0,a}(\underset{\sim}{x})\right) - m\right] . \qquad (2.36e)$$

Os multiplicadores de Lagrange que aparecem em (2.36) e os restantes poderão ser, todos, determinados a partir da imposição de persistência no tempo dos vínculos (2.33) $\{\phi^a_J;\ J = 1,\dots,14\}$ os quais são de 2ª classe. De fato, por (2.35)

$$\dot{\Phi}^a_J(\underset{\sim}{x}) \approx 0 \Rightarrow \quad [\Phi^a_J(\underset{\sim}{x}), H_c]_{PP} \approx 0 \qquad (2.37)$$

a qual, levando em conta (2.31), conduz a

$$\left[\Phi^a_J(\underset{\sim}{x}), H\right]_{PP} + \sum_{K=1}^{14} \int d^3\underset{\sim}{y} \left[\Phi^a_J(\underset{\sim}{x}), \Phi^b_K(\underset{\sim}{y})\right]_{PP} u^b_K(\underset{\sim}{y}) \approx 0 \quad . \tag{2.38}$$

O sistema de equações (2.38) terá solução para os u^a_J's se o determinante da matriz de Faddeev-Popov [29,30,6], cujos elementos são definidos por

$$Q^{ab}_{JK}(\underset{\sim}{x};\underset{\sim}{y}) = \left[\Phi^a_J(\underset{\sim}{x}), \Phi^b_K(\underset{\sim}{y})\right]_{PP} \quad , \tag{2.39}$$

for diferente de zero, i.e., se

$$\det Q = \det\left[\Phi^a_J(\underset{\sim}{x}), \Phi^b_K(\underset{\sim}{y})\right] \neq 0 \quad . \tag{2.40}$$

A solução de (2.38) é então dada por

$$u^a_J(\underset{\sim}{x}) = - \sum_{K=1}^{14} \int d^3\underset{\sim}{y}\, R^{ab}_{JK}(\underset{\sim}{x};\underset{\sim}{y}) \left[\Phi^b_K(\underset{\sim}{y}), H\right]_{PP}\Big|_{\Sigma} \tag{2.41}$$

onde R denota a inversa da matriz Q, ou seja,

$$\sum_{K=1}^{14} \int d^3\underset{\sim}{z}\; R^{ac}_{JK}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{KL}(\underset{\sim}{z};\underset{\sim}{y}) = \delta^{ab}\delta_{JL}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad , \tag{2.42a}$$

$$\sum_{K=1}^{14} \int d^3\underset{\sim}{z}\; Q^{ac}_{JK}(\underset{\sim}{x};\underset{\sim}{z})\, R^{cb}_{KL}(\underset{\sim}{z};\underset{\sim}{y}) = \delta^{ab}\delta_{JL}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad . \tag{2.42b}$$

Voltando com (2.41) em (2.38), obtemos

$$\left[\underset{J}{\Phi}^{a}(\underset{\sim}{x}), H_c\right]_{PP} = \left[\underset{J}{\Phi}^{a}(\underset{\sim}{x}), H\right]_{PP} - \sum_{K,L=1}^{14} \int d^3z \int d^3z' \left[\underset{J}{\Phi}^{a}(\underset{\sim}{x}), \underset{K}{\Phi}^{b}(\underset{\sim}{z})\right]_{PP} R^{bc}_{KL}(\underset{\sim}{z};\underset{\sim}{z}') \left[\underset{L}{\Phi}^{c}(\underset{\sim}{z}'), H\right]_{PP} \approx 0.$$

(2.43)

Portanto, se definirmos o <u>parêntese de Dirac</u> (PD) de dois funcionais regulares das variáveis canônicas, Ω_1 e Ω_2, por [31]

$$[\Omega_1, \Omega_2]_{PD} \equiv [\Omega_1, \Omega_2]_{PP} -$$

$$- \sum_{J,K=1}^{14} \int d^3z \int d^3z' \left[\Omega_1, \underset{J}{\Phi}^{a}(\underset{\sim}{z})\right]_{PP} R^{ab}_{JK}(\underset{\sim}{z};\underset{\sim}{z}') \left[\underset{K}{\Phi}^{b}(\underset{\sim}{z}'), \Omega_2\right]_{PP} , \quad (2.44)$$

poderemos reescrever (2.35) na seguinte forma compacta

$$\partial^0 \Omega = [\Omega, H]_{PD}\Big|_{\Sigma} . \quad (2.45)$$

Mesmo para campos compostos de bósons e férmions, i.e., com a definição do PP dada em (2.26), mostra-se [35,39] que os PD's apresentam as propriedades gerais dos PP's:

(i) <u>Estatística</u>

$$[\Omega_1, \Omega_2]_{PD} = e^{i\pi(n_{\Omega_1} n_{\Omega_2} - 1)} [\Omega_2, \Omega_1]_{PD} \quad ; \quad (2.46a)$$

(ii) Linearidade (α e β são constantes)

$$[\Omega_1, \alpha\Omega_2 + \beta\Omega_3]_{PD} = \alpha[\Omega_1, \Omega_2]_{PD} + \beta[\Omega_1, \Omega_3]_{PD} \; ; \qquad (2.46b)$$

(iii) Lei do produto

$$[\Omega_1, \Omega_2\Omega_3]_{PD} = e^{i\pi n_{\Omega_1} n_{\Omega_2}} \Omega_2 [\Omega_1, \Omega_3]_{PD} + [\Omega_1, \Omega_2]_{PD} \Omega_3 \; ;$$

(2.46c)

(iv) Identidade de Jacobi

$$[\Omega_1, [\Omega_2, \Omega_3]_{PD}]_{PD} + e^{i\pi n_{\Omega_1}|n_{\Omega_2} - n_{\Omega_3}|}[\Omega_2, [\Omega_3, \Omega_1]_{PD}]_{PD} +$$

$$+ e^{i\pi n_{\Omega_3}|n_{\Omega_1} - n_{\Omega_2}|}[\Omega_3, [\Omega_1, \Omega_2]_{PD}]_{PD} = 0 .$$

(2.46d)

Estamos agora em boa posição para ilustrar, em termos gerais, a base do PQPD. A correspondência clássico-quântica é garantida pela abstração dos CTI's básicos dos correspondentes PD's, como se pode ver comparando (2.45) com a correspondente equação de movimento de Heisenberg. Além disso, dado que o PD envolvendo qualquer dos vínculos Φ_J^a é zero* [31], conclui-se que todos os CTI's serão compatíveis com $\{\Phi_J^a = 0\}$ como identidades operatoriais fortes. O cálculo explícito dos PD's

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*Por exemplo, tomar $\Omega_2 = \Phi_L^c(\underset{\sim}{x})$ em (2.44) e usar (2.42b).

básicos constitui, então, a pedra angular do PQPD. Por seu turno, para o cômputo dos PD's devemos conhecer os elementos de R como funções das variáveis canônicas (ver eq.(2.44)). Este último problema será resolvido no próximo capítulo e nos Apêndices C e D.

III. DETERMINAÇÃO DA INVERSA DA MATRIZ DE FADDEEV-POPOV

III.1 Determinação dos Elementos da Matriz Inversa que Controlam os Comutadores Básicos da Teoria Quântica

Desde (2.33) e (2.39) encontramos, para os elementos não-nulos de Q, as seguintes expressões

$$Q^{ab}_{j,k+6}(\underset{\sim}{x};y) = -Q^{ab}_{j+6,k+11}(\underset{\sim}{x};y) = \delta^{ab}\delta^{jk}\overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad , \qquad (3.1a)$$

$$Q^{ab}_{j,11}(\underset{\sim}{x};y) = -D^{j,ab}_{(\underset{\sim}{x})}\overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad , \qquad (3.1b)$$

$$Q^{ab}_{4,10}(\underset{\sim}{x};y) = \delta^{ab}\partial^{3}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad , \qquad (3.1c)$$

$$Q^{ab}_{5,10}(\underset{\sim}{x};y) = D^{1,ab}_{(\underset{\sim}{x})}\overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad , \qquad (3.1d)$$

$$Q^{ab}_{6,10}(\underset{\sim}{x};y) = D^{2,ab}_{(\underset{\sim}{x})}\overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad , \qquad (3.1e)$$

$$Q^{ab}_{4,13}(\underset{\sim}{x};y) = -\delta^{ab}\delta(x^2-y^2)\delta(x^3-x^3_{(0)})\Delta(y^1,x^1_{(0)};x^1) \; , \qquad (3.1f)$$

$$Q^{ab}_{5,13}(\underset{\sim}{x};y) = \delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) \; , \qquad (3.1g)$$

$$Q^{ab}_{6,12}(\underset{\sim}{x};y) = -\delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) \; , \qquad (3.1h)$$

$$Q^{ab}_{7,10}(\underset{\sim}{x};y) = g f^{abc}F^{23,c}_{(\underset{\sim}{x})}\overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad , \qquad (3.1i)$$

$$Q^{ab}_{8,10}(\underset{\sim}{x};\underset{\sim}{y}) = g f^{abc} F^{31,c}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \;, \qquad (3.1\, j)$$

$$Q^{ab}_{j+6,11}(\underset{\sim}{x};\underset{\sim}{y}) = -g f^{abc} F^{0j,c}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \;, \qquad (3.1\, k)$$

onde, sempre que possível, escrevemos os índices de vínculo J e K em termos dos índices de Lorentz espaciais j e k, respectivamente. Aqui, introduziu-se a notação

$$\Delta(x,y;z) = \int_y^x du\, \delta(u-z) = \theta(z-y) - \theta(z-x) \;, \qquad (3.2)$$

onde θ é a função degrau de Heaviside:

$$\theta(x) = \begin{cases} 0 \;, & \text{se} \quad x<0 \;, \\ 1 \;, & \text{se} \quad x>0 \;. \end{cases} \qquad (3.3)$$

Para ilustrar como surge a função definida em (3.2) apresentamos abaixo o cálculo de $Q^{ab}_{4,13}$. Os demais elementos da matriz (3.1) são também obtidos de forma direta.

Prova de (3.1f)

$$Q^{ab}_{4,13}(\underset{\sim}{x},\underset{\sim}{y}) = \left[\Phi^{a}_{4}(\underset{\sim}{x}), \Phi^{b}_{13}(\underset{\sim}{y}) \right]_{PP} = \left[\pi^{a}_{12}(\underset{\sim}{x}), \chi^{2,b}(\underset{\sim}{y}) \right]_{PP} =$$

$$= \left[\Pi_{12}^{a}(\underset{\sim}{x}), \int_{x^1_{(0)}}^{y^1} dz^1 \; F^{12,b}(z^1, y^2, x^3_{(0)}) \right]_{PP} = -\int_{x^1_{(0)}}^{y^1} dz^1 \, \delta^{ab} \delta(x^1 - z^1)\delta(x^2 - y^2)\delta(x^3 - x^3_{(0)}) =$$

$$= -\delta^{ab} \delta(x^2 - y^2)\delta(x^3 - x^3_{(0)}) \, \Delta(y^1, x^1_{(0)}; x^1) .$$

O passo seguinte consiste na substituição das eqs. (3.1) em (2.42a) para a obtenção de um sistema de equações diferenciais que governam os elementos da matriz inversa R. De forma direta mas um tanto longa (ver Apêndice C), obtemos o seguinte sistema de equações diferenciais acopladas:

$$\underset{J,k+6}{R^{ab}}(\underset{\sim}{x};y) - D^{k,bc}_{(y)} \underset{J,11}{R^{ac}}(\underset{\sim}{x};y) = -\delta^{ab}\delta^{J,k}\delta^{(3)}(\underset{\sim}{x}-y) \quad , \qquad (3.4a)$$

$$\partial^3_y \underset{J,10}{R^{ab}}(\underset{\sim}{x};y) - \delta(y^3 - x^3_{(0)}) \int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3 \, \underset{J,13}{R^{ab}}(\underset{\sim}{x}; z^1, y^2, z^3) \Delta(z^1, x^1_{(0)}; y^1) =$$

$$= -\delta^{ab}\delta^{J,4}\delta^{(3)}(\underset{\sim}{x}-y) \quad , \qquad (3.4b)$$

$$D^{1,bc}_{(y)} \underset{J,10}{R^{ac}}(\underset{\sim}{x};y) + \int_{-\infty}^{+\infty} dz^3 \, \underset{J,13}{R^{ab}}(\underset{\sim}{x}; y^1, y^2, z^3) \Delta(z^3, x^3_{(0)}; y^3) = -\delta^{ab}\delta^{J,5}\delta^{(3)}(\underset{\sim}{x}-y) ,$$

$$(3.4c)$$

$$D^{2,bc}_{(y)} \underset{J,10}{R^{ac}}(\underset{\sim}{x};y) - \int_{-\infty}^{+\infty} dz^3 \, \underset{J,12}{R^{ab}}(\underset{\sim}{x}; y^1, y^2, z^3) \Delta(z^3, x^3_{(0)}; y^3) = -\delta^{ab}\delta^{J,6}\delta^{(3)}(\underset{\sim}{x}-y) ,$$

$$(3.4d)$$

$$R^{ab}_{J,k}(\underset{\sim}{x};y) + R^{ab}_{J,k+11}(\underset{\sim}{x};y) + g f^{cbd}\left(\delta^{k1} F^{23,d}(y) + \delta^{k2} F^{31,d}(y)\right) R^{ac}_{J,10}(\underset{\sim}{x};y) -$$

$$- g f^{cbd} F^{0k,d}(y) R^{ac}_{J,11}(\underset{\sim}{x};y) = \delta^{ab}\delta^{J,k+6}\delta^{(3)}(\underset{\sim}{x}-y)\ , \tag{3.4e}$$

$$\partial^3_y R^{ab}_{J,4}(\underset{\sim}{x};y) + D^{1,bc}(y) R^{ac}_{J,5}(\underset{\sim}{x};y) + D^{2,bc}(y) R^{ac}_{J,6}(\underset{\sim}{x};y) -$$

$$- g f^{cbd}\left(F^{23,d}(y) R^{ac}_{J,7}(\underset{\sim}{x};y) + F^{31,d}(y) R^{ac}_{J,8}(\underset{\sim}{x};y)\right) = -\delta^{ab}\delta^{J,10}\delta^{(3)}(x-\underset{\sim}{y}), \tag{3.4f}$$

$$D^{k,bc}(y) R^{ac}_{J,k}(\underset{\sim}{x};y) + g f^{cdb} F^{0k,d}(y) R^{ac}_{J,k+6}(\underset{\sim}{x};y) = \delta^{ab}\delta^{J,11}\delta^{(3)}(\underset{\sim}{x}-y), \tag{3.4g}$$

$$R^{ab}_{J,7}(\underset{\sim}{x};y) + \int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,6}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(y^3,x^3_{(0)};z^3) = -\delta^{ab}\delta^{J,12}\delta^{(3)}(x-\underset{\sim}{y}), \tag{3.4h}$$

$$R^{ab}_{J,8}(\underset{\sim}{x};y) + \int_{-\infty}^{+\infty} dz^1\, R^{ab}_{J,4}(\underset{\sim}{x};z^1,y^2,x^3_{(0)})\,\Delta(y^1,x^1_{(0)};z^1) -$$

$$-\int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,5}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(y^3,x^3_{(0)};z^3) = -\delta^{ab}\delta^{J,13}\delta^{(3)}(\underset{\sim}{x}-y), \tag{3.4i}$$

$$R^{ab}_{J,9}(\underset{\sim}{x};y) = -\delta^{ab}\delta^{J,14}\delta^{(3)}(\underset{\sim}{x}-y) \quad . \tag{3.4j}$$

A integração do sistema acoplado de equações diferenciais (3.4) é extremamente enfadonha mas, a despeito disso, conseguimos realizá-la totalmente. Nesta seção, apresentamos o cálculo detalhado dos $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{y})$'s e, em particular (para, $J = k+11$), dos elementos de matriz inversa $R^{ab}_{k+11,11}(\underset{\sim}{x};\underset{\sim}{y})$ que controlam os CTI's básicos da teoria quântica. Os restantes elementos de R, além dos obtidos logo abaixo, são calculados no Apêndice C. Substituindo-se $R^{ab}_{J,k+6}$ desde (3.4 h,i,j) em (3.4a) chegamos, respectivamente, a

$$D^{1,bc}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) = +\delta^{ab}\left(\delta^{J,1}-\delta^{J,12}\right)\delta^{(3)}(\underset{\sim}{x}-y) - \int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,6}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(y^3,x^3_{(0)};z^3),$$

$$(3.5a)$$

$$D^{2,bc}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) = +\delta^{ab}\left(\delta^{J,2}-\delta^{J,13}\right)\delta^{(3)}(\underset{\sim}{x}-y) + \int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,5}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(y^3,x^3_{(0)};z^3) -$$

$$- \int_{-\infty}^{+\infty} dz^1\, R^{ab}_{J,4}(\underset{\sim}{x};z^1,y^2,x^3_{(0)})\,\Delta(y^1,x^1_{(0)};z^1) \quad , \qquad (3.5b)$$

$$\partial^3_y R^{ab}_{J,11}(\underset{\sim}{x};y) = +\delta^{ab}\left(\delta^{J,3}-\delta^{J,14}\right)\delta^{(3)}(\underset{\sim}{x}-y) \quad . \qquad (3.5c)$$

Desde (3.5a) e (3.5b), obtemos (ver (B.16))

$$\left(D^2_{(y)}D^1_{(y)} - D^1_{(y)}D^2_{(y)}\right)^{dc} R^{ac}_{J,11}(\underset{\sim}{x};y) = gf^{dec}F^{21,e}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) =$$

$$= \left(\delta^{J,1} - \delta^{J,12}\right) D^{2,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x} - y) - \left(\delta^{J,2} - \delta^{J,13}\right) D^{1,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x} - y) -$$

$$- \int_{-\infty}^{+\infty} dz^3 \, \Delta(y^3, x^3_{(0)}; z^3) \left(D^{2,db}_{(y)} \underset{J,6}{R}^{ab}(\underset{\sim}{x}; y^1, y^2, z^3) + D^{1,db}_{(y)} \underset{J,5}{R}^{ab}(\underset{\sim}{x}; y^1, y^2, z^3) \right) +$$

$$+ \int_{-\infty}^{+\infty} dz^1 \, \underset{J,4}{R}^{ab}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \, D^{1,db}_{(y)} \, \Delta(y^1, x^1_{(0)}; z^1) \quad . \qquad (3.6)$$

Por outro lado, usando (3.4f), podemos escrever

$$\int_{-\infty}^{+\infty} dz^3 \, \Delta(y^3, x^3_{(0)}; z^3) \left(D^{2,db}_{(y)} \underset{J,6}{R}^{ab}(\underset{\sim}{x}; y^1, y^2, z^3) + D^{1,db}_{(y)} \underset{J,5}{R}^{ab}(\underset{\sim}{x}; y^1, y^2, z^3) \right) =$$

$$= -\delta^{ad} \delta^{J,10} \delta(x^1 - y^1) \delta(x^2 - y^2) \Delta(y^3, x^3_{(0)}; x^3) + \underset{J,4}{R}^{ad}(\underset{\sim}{x}; y) - \underset{J,4}{R}^{ad}(\underset{\sim}{x}; y^1, y^2, x^3_{(0)}) +$$

$$+ g f^{cde} \int_{-\infty}^{+\infty} dz^3 \, \Delta(y^3, x^3_{(0)}; z^3) \Big[F^{23,e}(y^1, y^2, z^3) \underset{J,7}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) + F^{31,e}(y^1, y^2, z^3) \underset{J,8}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) +$$

$$+ \left(A^{1,e}_{(y)} - A^{1,e}(y^1, y^2, z^3)\right) \underset{J,5}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) + \left(A^{2,e}_{(y)} - A^{2,e}(y^1, y^2, z^3)\right) \underset{J,6}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) \Big],$$

$$(3.7)$$

onde levamos em conta que $\Lambda(y^3, x^3_{(0)}; +\infty) = 0$. Substituindo a expressão (3.7) em (3.6), encontramos

$$\boxed{\begin{aligned}
\underset{J,4}{R}^{ad}(\underset{\sim}{x};y) ={}& \left(\delta^{J,1}-\delta^{J,12}\right) D^{2,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x}-y) - \left(\delta^{J,2}-\delta^{J,13}\right) D^{1,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x}-y) + \\
&+ \delta^{ad}\delta^{J,10}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3, x^3_{(0)}; x^3) + g f^{dec} F^{12,e}_{(y)} \underset{J,11}{R}^{ac}(\underset{\sim}{x};y) + \\
&+ \Big\{ g f^{dec} A^{1,e}_{(y)} \int_{-\infty}^{+\infty} dz^1 \, \underset{J,4}{R}^{ac}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \Delta(y^1, x^1_{(0)}; z^1) - \\
&- g f^{dec} \int_{-\infty}^{+\infty} dz^3 \, \Delta(y^3, x^3_{(0)}; z^3) \Big[F^{23,e}_{(y^1,y^2,z^3)} \underset{J,7}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) + F^{31,e}_{(y^1,y^2,z^3)} \underset{J,8}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) + \\
&+ \left(A^{1,e}_{(y)} - A^{1,e}_{(y^1,y^2,z^3)}\right) \underset{J,5}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) + \left(A^{2,e}_{(y)} - A^{2,e}_{(y^1,y^2,z^3)}\right) \underset{J,6}{R}^{ac}(\underset{\sim}{x}; y^1, y^2, z^3) \Big] \Big\}.
\end{aligned}}$$

$$(3.8)$$

De forma similar, desde (3.5c) e (3.5b), obtemos

$$\begin{aligned}
\left[D^{2}_{(y)}, D^{3}_{(y)}\right]^{dc} \underset{J,11}{R}^{ac}(\underset{\sim}{x};y) &= g f^{dec} F^{23,e}_{(y)} \underset{J,11}{R}^{ac}(\underset{\sim}{x};y) = \\
&= (\delta^{J,3}-\delta^{J,14}) D^{2,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x}-y) - (\delta^{J,2}-\delta^{J,13}) D^{3,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x}-y) - \\
&\quad - \int_{-\infty}^{+\infty} dz^3 \, \underset{J,5}{R}^{ad}(\underset{\sim}{x}; y^1, y^2, z^3) \, \partial^3_y \Delta(y^3, x^3_{(0)}; z^3) \quad \Rightarrow
\end{aligned}$$

$$\Rightarrow \boxed{\begin{aligned}
\underset{J,5}{R}^{ad}(\underset{\sim}{x};y) ={}& \delta^{ad}(\delta^{J,2}-\delta^{J,13})\partial^3_y \overset{(3)}{\delta}(\underset{\sim}{x}-y) - (\delta^{J,3}-\delta^{J,14}) D^{2,da}_{(y)} \overset{(3)}{\delta}(\underset{\sim}{x}-y) + \\
&+ g f^{dec} F^{23,e}_{(y)} \underset{J,11}{R}^{ac}(\underset{\sim}{x};y) \quad ; \qquad (3.9)
\end{aligned}}$$

e, desde (3.5a) e (3.5c),

$$\left[\overset{3}{D}_{(y)},\overset{1}{D}_{(y)}\right]^{dc} R^{ac}_{J,11}(\underset{\sim}{x};y) = g f^{dec} F^{31,e}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) =$$

$$= (\delta^{J,1}-\delta^{J,12}) D^{3,da}_{(y)}\,\delta^{(3)}(\underset{\sim}{x}-y) - \int_{-\infty}^{+\infty} dz^3\, R^{ad}_{J,6}(\underset{\sim}{x};y^1,y^2,z^3)\,\overset{3}{\partial}_{y}\Delta(y^3,x^3_{(0)};z^3) -$$

$$-(\delta^{J,3}-\delta^{J,14}) D^{1,da}_{(y)}\,\delta^{(3)}(\underset{\sim}{x}-y) \qquad \Longrightarrow$$

$$\Longrightarrow$$

$$\boxed{\begin{aligned} R^{ad}_{J,6}(\underset{\sim}{x};y) = {} & -\delta^{da}(\delta^{J,1}-\delta^{J,12})\partial^3_y\,\delta^{(3)}(\underset{\sim}{x}-y) + (\delta^{J,3}-\delta^{J,14}) D^{1,da}_{(y)}\,\delta^{(3)}(\underset{\sim}{x}-y) + \\ & + g f^{dec} F^{31,e}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) \quad . \qquad (3.10) \end{aligned}}$$

A idéia agora é retornar com (3.8)-(3.10) em (3.5a,b) de forma a obter um sistema desacoplado de equações diferenciais parciais para os elementos $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{y})$. Calculamos primeiramente a integral que aparece em (3.5a):

$$-\int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,6}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(y^3,x^3_{(0)};z^3) =$$

$$= -\int_{-\infty}^{+\infty} dz^3\,\Delta(y^3,x^3_{(0)};z^3)\Big[(\delta^{J,3}-\delta^{J,14}) D^{1,ba}_{(y^1,y^2,z^3)}\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3) -$$

$$-\delta^{ab}(\delta^{J,1}-\delta^{J,12})\delta(x^1-y^1)\delta(x^2-y^2)\partial^3_z\delta(x^3-z^3) + g f^{bdc}\left(\partial^3_z A^{1,d}(y^1,y^2,z^3)\right) R^{ac}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big] =$$

$$= \delta^{ab}(\delta^{J,1}-\delta^{J,12})\delta(x^1-y^1)\delta(x^2-y^2)\left[\delta(x^3-x^3_{(0)})-\delta(x^3-y^3)\right] -$$

$$-(\delta^{J,3}-\delta^{J,14})\left[D^{1,ba}_{(y^1,y^2,x^3)}\delta(x^1-y^1)\right]\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3) -$$

$$-g f^{bdc}\int_{-\infty}^{+\infty} dz^3 A^{1,d}(y^1,y^2,z^3)\left\{\underset{J,11}{R^{ac}}(\underset{\sim}{x};y^1,y^2,z^3)\left[\delta(z^3-x^3_{(0)})-\delta(z^3-y^3)\right]+\Delta(y^3,x^3_{(0)};z^3)\,\overset{z}{\partial}_3 \underset{J,11}{R^{ac}}(\underset{\sim}{x};y^1,y^2,z^3)\right\}=$$

$$= \delta^{ab}(\delta^{J,1}-\delta^{J,12})\delta(x^1-y^1)\delta(x^2-y^2)\left[\delta(x^3-x^3_{(0)})-\delta(x^3-y^3)\right] -$$

$$-\delta^{ab}(\delta^{J,3}-\delta^{J,14})\left[\overset{1}{\underset{y}{\partial}}\delta(x^1-y^1)\right]\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3) +$$

$$+g f^{bdc} A^{1,d}(y)\,\underset{J,11}{R^{ac}}(\underset{\sim}{x};y) \quad . \qquad (3.11)$$

Aqui, fizemos uso de $\Delta(y^3,x^3_{(0)};\pm\infty) = 0$, de (2.1b) e da equação (3.5c). Levando (3.11) em (3.5a), obtemos a seguinte equação

$$\overset{1}{\underset{y}{\partial}}\underset{J,11}{R^{ab}}(\underset{\sim}{x};y) = \delta^{ab}(\delta^{J,1}-\delta^{J,12})\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-x^3_{(0)}) -$$

$$-\delta^{ab}(\delta^{J,3}-\delta^{J,14})\left[\overset{1}{\underset{y}{\partial}}\delta(x^1-y^1)\right]\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3).$$

$$(3.12)$$

Analogamente, a primeira integral que aparece no lado direito de (3.5b) é (ver (3.9))

$$\int_{-\infty}^{+\infty} dz^3\, \underset{J,5}{R}^{ab}(\underset{\sim}{x}; y^1, y^2, z^3)\, \Delta(y^3, x^3_{(0)}; z^3) =$$

$$= \int_{-\infty}^{+\infty} dz^3\, \Delta(y^3, x^3_{(0)}; z^3) \Big[\delta^{ab}(\delta^{J,2} - \delta^{J,13})\, \delta(x^1-y^1)\,\delta(x^2-y^2)\, \partial^3_z\, \delta(x^3-z^3) -$$

$$-(\delta^{J,3}-\delta^{J,14})\, D^{2,ba}(y^1,y^2,z^3)\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\delta(x^3-z^3) - gf^{bdc}\left(\partial^3_z A^{2,d}(y^1,y^2,z^3)\right) \underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,z^3)\Big] =$$

$$= \delta^{ab}(\delta^{J,2} - \delta^{J,13})\,\delta(x^1-y^1)\,\delta(x^2-y^2)\left[\delta(x^3-x^3_{(0)}) - \delta(x^3-y^3)\right] -$$

$$-(\delta^{J,3}-\delta^{J,14})\,\delta(x^1-y^1)\left[D^{2,ba}(y^1,y^2,x^3)\,\delta(x^2-y^2)\right]\Delta(y^3,x^3_{(0)};x^3) -$$

$$-gf^{bdc}\int_{-\infty}^{+\infty} dz^3\, A^{2,d}(y^1,y^2,z^3)\left\{\underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,z^3)\left[\delta(z^3-x^3_{(0)})-\delta(z^3-y^3)\right] + \Delta(y^3,x^3_{(0)};z^3)\,\partial^z_3\, \underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,z^3)\right\} =$$

$$= \delta^{ab}(\delta^{J,2} - \delta^{J,13})\,\delta(x^1-y^1)\,\delta(x^2-y^2)\left[\delta(x^3-x^3_{(0)}) - \delta(x^3-y^3)\right] -$$

$$-\delta^{ab}(\delta^{J,3}-\delta^{J,14})\,\delta(x^1-y^1)\left[\partial^2_y\,\delta(x^2-y^2)\right]\Delta(y^3,x^3_{(0)};x^3) -$$

$$-gf^{bdc}\left(A^{2,d}(y^1,y^2,x^3_{(0)})\,\underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,x^3_{(0)}) - A^{2,d}(y)\,\underset{J,11}{R}^{ac}(\underset{\sim}{x};y)\right), \qquad (3.13)$$

usando novamente $\Delta(y^3, x^3_{(0)}; \pm\infty) = 0$ e (3.5c). Desde (3.8), levando em conta $\Delta(x^3_{(0)}, x^3_{(0)}; x^3) = 0$ e (2.1b), calculamos a segunda integral no lado direito de (3.5b) como segue

$$-\int_{-\infty}^{+\infty} dz^1 \, R^{ab}_{J,4}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \, \Delta(y^1, x^1_{(0)}; z^1) \; =$$

$$= -\int_{-\infty}^{+\infty} dz^1 \, \Delta(y^1, x^1_{(0)}; z^1) \Big[(\delta^{J,1} - \delta^{J,12}) D^{2,ba}(z^1, y^2, x^3_{(0)}) \, \delta(x^1 - z^1) \delta(x^2 - y^2) \delta(x^3 - x^3_{(0)}) \; -$$

$$- \delta^{ab} (\delta^{J,2} - \delta^{J,13}) \big[\partial^1_z \, \delta(x^1 - z^1) \big] \delta(x^2 - y^2) \delta(x^3 - x^3_{(0)}) + g f^{bdc} \big(\partial^1_z A^{2,d}(z^1, y^2, x^3_{(0)}) \big) R^{ac}_{J,11}(\underset{\sim}{x}; z^1, y^2, x$$

$$= -(\delta^{J,1} - \delta^{J,12}) \, \Delta(y^1, x^1_{(0)}; x^1) \big[D^{2,ba}(x^1, y^2, x^3_{(0)}) \, \delta(x^2 - y^2) \big] \delta(x^3 - x^3_{(0)}) \; +$$

$$+ \delta^{ab} (\delta^{J,2} - \delta^{J,13}) \big[\delta(x^1 - x^1_{(0)}) - \delta(x^1 - y^1) \big] \delta(x^2 - y^2) \delta(x^3 - x^3_{(0)}) \; -$$

$$- g f^{bdc} \int_{-\infty}^{+\infty} dz^1 \, A^{2,d}(z^1, y^2, x^3_{(0)}) \Big\{ R^{ac}_{J,11}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \big[\delta(z^1 - x^1_{(0)}) - \delta(z^1 - y^1) \big] \; +$$

$$+ \Delta(y^1, x^1_{(0)}; z^1) \, \partial^z_1 R^{ac}_{J,11}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \Big\} \; =$$

$$= -\delta^{ab} (\delta^{J,1} - \delta^{J,12}) \, \Delta(y^1, x^1_{(0)}; x^1) \big[\partial^2_y \, \delta(x^2 - y^2) \big] \delta(x^3 - x^3_{(0)}) \; +$$

$$+ \delta^{ab} (\delta^{J,2} - \delta^{J,13}) \big[\delta(x^1 - x^1_{(0)}) - \delta(x^1 - y^1) \big] \delta(x^2 - y^2) \delta(x^3 - x^3_{(0)}) \; +$$

$$+ g f^{bdc} A^{2,d}(y^1, y^2, x^3_{(0)}) \, R^{ac}_{J,11}(\underset{\sim}{x}; y^1, y^2, x^3_{(0)}) \quad , \qquad (3.14)$$

onde, no último passo, usamos (2.1c) e a equação (3.12). A so ma das expressões (3.14) e (3.13) levada em (3.5b) nos deixa com a equação

$$\partial^2_y R^{ab}_{J,11}(\underset{\sim}{x};y) = -\delta^{ab}(\delta^{J,1}-\delta^{J,12})\Delta(y^1,x^1_{(0)};x^1)\left[\partial^2_y\delta(x^2-y^2)\right]\delta(x^3-x^3_{(0)}) +$$

$$+\delta^{ab}\left(\delta^{J,2}-\delta^{J,13}\right)\delta(x^1-x^1_{(0)})\delta(x^2-y^2)\delta(x^3-x^3_{(0)}) -$$

$$-\delta^{ab}(\delta^{J,3}-\delta^{J,14})\delta(x^1-y^1)\left[\partial^2_y\delta(x^2-y^2)\right]\Delta(y^3,x^3_{(0)};x^3).$$

(3.15)

As equações (3.5c), (3.12) e (3.15) constituem um sistema desacoplado para os elementos $R^{ab}_{J,11}$ cuja integração é direta. De fato, integrando (3.5c)

$$\int_{x^3_{(0)}}^{y^3} dy'^3\, \partial^{\prime y}_3 R^{ab}_{J,11}(\underset{\sim}{x};y^1,y^2,y'^3) = \delta^{ab}(\delta^{J,14}-\delta^{J,3})\delta(x^1-y^1)\delta(x^2-y^2)\int_{x^3_{(0)}}^{y^3} dy'^3\,\delta(x^3-y'^3)$$

obtemos

$$R^{ab}_{J,11}(\underset{\sim}{x};y) = R^{ab}_{J,11}(\underset{\sim}{x};y^1,y^2,x^3_{(0)}) + \delta^{ab}(\delta^{J,14}-\delta^{J,3})\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3).$$

(3.16 a)

Integrando (3.12)

$$\int_{x^1_{(0)}}^{y^1} dy'^1\partial^{\prime y}_1 R^{ab}_{J,11}(\underset{\sim}{x};y'^1,y^2,y^3) = \delta^{ab}(\delta^{J,12}-\delta^{J,1})\left(\int_{x^1_{(0)}}^{y^1} dy'^1\delta(x^1-y'^1)\right)\delta(x^2-y^2)\delta(x^3-x^3_{(0)})$$

obtemos

$$R^{ab}_{J,11}(\underset{\sim}{x};y) = R^{ab}_{J,11}(\underset{\sim}{x};x^1_{(0)},y^2,y^3) + \delta^{ab}(\delta^{J,12}-\delta^{J,1})\Delta(y^1,x^1_{(0)};x^1)\delta(x^2-y^2)\delta(x^3-x^3_{(0)}).$$

$$(3.16\,b)$$

Integrando (3.15)

$$\int_{x^2_{(0)}}^{y^2} dy'^2\, \partial^{\prime y}_{2} R^{ab}_{J,11}(\underset{\sim}{x};y^1,y'^2,y^3) = \delta^{ab}(\delta^{J,13}-\delta^{J,2})\delta(x^1-x^1_{(0)})\left(\int_{x^2_{(0)}}^{y^2} dy'^2\delta(x^2-y'^2)\right)\delta(x^3-x^3_{(0)})$$

obtemos

$$R^{ab}_{J,11}(\underset{\sim}{x};y) = R^{ab}_{J,11}(\underset{\sim}{x};y^1,x^2_{(0)},y^3) + \delta^{ab}(\delta^{J,13}-\delta^{J,2})\delta(x^1-x^1_{(0)})\Delta(y^2,x^2_{(0)};x^2)\,\delta(x^3-x^3_{(0)}).$$

$$(3.16c)$$

Usando (3.16b) para expressar $R^{ab}_{J,11}(\underset{\sim}{x};y^1,y^2,x^3_{(0)})$ em (3.16a), encontramos

$$R^{ab}_{J,11}(\underset{\sim}{x};y) = R^{ab}_{J,11}(\underset{\sim}{x};x^1_{(0)},y^2,x^3_{(0)}) + \delta^{ab}(\delta^{J,12}-\delta^{J,1})\Delta(y^1,x^1_{(0)};x^1)\delta(x^2-y^2)\delta(x^3-x^3_{(0)}) +$$

$$+\delta^{ab}(\delta^{J,14}-\delta^{J,3})\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3)\,. \quad (3.16d)$$

Através do uso de (3.16c) no lado direito de (3.16d) somos levados a

$$\boxed{\begin{aligned} R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{y}) = {} & R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \delta^{ab}\Big[(\delta^{J,12}-\delta^{J,1})\Delta(y^1,x^1_{(0)};x^1)\,\delta(x^2-y^2)\,\delta(x^3-x^3_{(0)}) + \\ & + (\delta^{J,13}-\delta^{J,2})\,\delta(x^1-x^1_{(0)})\,\Delta(y^2,x^2_{(0)};x^2)\,\delta(x^3-x^3_{(0)}) + \\ & + (\delta^{J,14}-\delta^{J,3})\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3)\Big] , \quad (3.17) \end{aligned}}$$

onde a presença das funções $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$ deve ser notada [27,32].

A determinação de $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{y})$ está terminada. Como subprodutos, determinamos também os elementos $R^{ab}_{J,5}(\underset{\sim}{x};\underset{\sim}{y})$ e $R^{ab}_{J,6}(\underset{\sim}{x};\underset{\sim}{y})$ (ver (3.9) e (3.10)). Para os demais elementos da matriz inversa R encontramos (ver Apêndices C e D) as seguintes expressões:

$$\boxed{\begin{aligned} R^{ab}_{J,k}(\underset{\sim}{x};\underset{\sim}{y}) = {} & \delta^{ab}\delta^{J,k+6}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + g f^{abc}\delta^{J,4}\Big[\delta^{k1}F^{23,c}(y) + \\ & + \delta^{k2}F^{31,c}(y)\Big]\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3,x^3_{(0)};y^3) + g f^{bdc}F^{0k,d}(y)\,R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{y}) - \\ & - R^{ab}_{J,k+11}(\underset{\sim}{x};\underset{\sim}{y}) , \quad (3.18) \end{aligned}}$$

$$\boxed{\begin{aligned} R^{ab}_{J,4}(\underset{\sim}{x};\underset{\sim}{y}) = {} & (\delta^{J,1}-\delta^{J,12})D^{2,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - (\delta^{J,2}-\delta^{J,13})D^{1,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \\ & + \delta^{ab}\delta^{J,10}\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3) + g f^{cbd}F^{12,d}(y)\,R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{y}) + \\ & + g f^{abc}(\delta^{J,12}F^{23,c}(\underset{\sim}{x}) + \delta^{J,13}F^{31,c}(\underset{\sim}{x}))\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3) , \quad (3.19) \end{aligned}}$$

$$\boxed{R^{ab}_{J,k+6}(\underset{\sim}{x};y) = -\delta^{ab}\delta^{J,k}\delta^{(3)}(\underset{\sim}{x}-y) + D^{k,bc}(y)\, R^{ac}_{J,11}(\underset{\sim}{x};y)}\,, \qquad (3.20)$$

$$\boxed{R^{ab}_{J,10}(\underset{\sim}{x};y) = -\delta^{ab}\delta^{J,4}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)}\,, \qquad (3.21)$$

$$\boxed{\begin{aligned}
R^{ab}_{J,12}(\underset{\sim}{x};y) = {} & \delta^{J,k} g f^{abc} F^{0k,c}(\underset{\sim}{x})\,\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\
& + \delta^{J,4}\left[D^{2,ab}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-y) + g f^{abc} F^{23,c}(y)\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\right] + \\
& + \delta^{J,6}\delta^{ab}\partial^3_y\delta^{(3)}(\underset{\sim}{x}-y) + \delta^{J,7}\left[\delta^{ab}\delta(x^1-y^1) + g f^{abc} A^{1,c}(\underset{\sim}{x})\Delta(x^1,x^1_{(0)};y^1)\right]\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\
& + \delta^{J,8}\left[\delta^{ab}\partial^2_y\delta(x^2-y^2) + g f^{abc} A^{2,c}(\underset{\sim}{x})\delta(x^2-y^2)\right]\Delta(x^1,x^1_{(0)};y^1)\delta(x^3_{(0)}-y^3) - \\
& - \delta^{J,11}\delta^{ab}\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + r^{ab}_{J(12)}(\underset{\sim}{x};y)\,, \qquad (3.22)
\end{aligned}}$$

$$\boxed{\begin{aligned}
R^{ab}_{J,13}(\underset{\sim}{x};y) = {} & \delta^{J,4}\left[D^{1,ba}(y)\delta^{(3)}(\underset{\sim}{x}-y) + g f^{abc} F^{31,c}(y)\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\right] + \\
& + \delta^{J,5}\delta^{ab}\partial^3_x\delta^{(3)}(\underset{\sim}{x}-y) + \delta^{J,8}\delta^{ab}\delta(x^1_{(0)}-y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\
& + \delta^{J,11}\delta^{ab}\delta(x^1_{(0)}-y^1)\Delta(y^2,x^2_{(0)};x^2)\delta(x^3_{(0)}-y^3) + r^{ab}_{J(13)}(\underset{\sim}{x};y)\,, \qquad (3.23)
\end{aligned}}$$

$$\boxed{\begin{aligned}R^{ab}_{J,14}(\underset{\sim}{x};y) ={}& \delta^{J,k} g f^{abc} F^{0k,c}_{(\underset{\sim}{x})}\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3,x^3_{(10)};y^3) + \\ &+\delta^{J,4} g f^{abc} F^{12,c}_{(y)}\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3,x^3_{(10)};y^3) + \delta^{J,5} D^{2,ba}_{(y)}\,\overset{(3)}{\delta}(\underset{\sim}{x}-y) - \\ &-\delta^{J,6} D^{1,ba}_{(y)}\,\overset{(3)}{\delta}(\underset{\sim}{x}-y) + \delta^{J,k+6} D^{ab}_{k}{}_{(\underset{\sim}{x})}\left(\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3,x^3_{(10)};y^3)\right) - \\ &-\delta^{J,11}\delta^{ab}\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3,x^2_{(10)};y^3) + r^{ab}_{J(14)}(\underset{\sim}{x};y)\,. \qquad (3.24)\end{aligned}}$$

Dentre os elementos da matriz inversa R, dados em (3.9), (3.10), (3.17)-(3.24), devemos analisar detalhadamente aqueles que controlam os PD's básicos da teoria (e conseqüentemente, os CTI's básicos), ou seja, os $R^{ab}_{k+11,11}(\underset{\sim}{x};\underset{\sim}{y})$ (ver(E.26), (E.29)-(E.33)). Particularizando, então, (3.17) para J = k+11, obtemos

$$\boxed{\begin{aligned}R^{ab}_{k+11,11}(\underset{\sim}{x};y) ={}& r^{ab}_{k}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \delta^{ab}\Big[\delta^{k1}\Delta(y^1,x^1_{(10)};x^1)\,\delta(x^2-y^2)\,\delta(x^3-x^3_{(0)}) + \\ &+\delta^{k2}\,\delta(x^1-x^1_{(10)})\,\Delta(y^2,x^2_{(10)};x^2)\,\delta(x^3-x^3_{(0)}) + \\ &+\delta^{k3}\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(y^3,x^3_{(10)};x^3)\quad, \qquad (3.25)\end{aligned}}$$

onde denotamos, para simplificar a escrita,

$$\boxed{r^{ab}_{k}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) \equiv R^{ab}_{k+11,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})}\ . \qquad (3.26)$$

Conforme mostramos no Apêndice D (ver (D.21)-(D.26)), a exigência de antissimetria $R^{ab}_{JK}(\underset{\sim}{x};\underset{\sim}{y}) = -R^{ba}_{KJ}(\underset{\sim}{y};\underset{\sim}{x})$ nos leva, na nova notação (3.26), a

$$\boxed{r^{ab}_{1}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^3-x^3_{(0)})\,\tilde{r}^{ab}_{1}(x^1,x^2;\underset{\sim}{x}_{(0)})} \qquad (3.27)$$

com

$$\boxed{\begin{aligned}\tilde{r}^{ab}_{1}(x^1,x^2;\underset{\sim}{x}_{(0)}) &= \tilde{r}^{ab}_{1}(\infty,x^2;\underset{\sim}{x}_{(0)})\,\Theta(x^1-x^1_{(0)}) + \tilde{r}^{ab}_{1}(-\infty,x^2;\underset{\sim}{x}_{(0)})\,\Theta(x^1_{(0)}-x^1) + \\ &+\int_{-\infty}^{+\infty} dz^1\,\Delta(z^1,x^1_{(0)};x^1)\Big[r^{ab}_{3}(z^1,x^2,\infty;\underset{\sim}{x}_{(0)}) - r^{ab}_{3}(z^1,x^2,-\infty;\underset{\sim}{x}_{(0)})\Big] \quad ;\end{aligned}} \qquad (3.28)$$

$$\boxed{r^{ab}_{2}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\,\tilde{r}^{ab}_{2}(x^2;\underset{\sim}{x}_{(0)})} \qquad (3.29)$$

com

$$\boxed{\begin{aligned}\tilde{r}^{ab}_{2}(x^2;\underset{\sim}{x}_{(0)}) &= \delta^{ab}\Big[\Theta(0)-\Theta(x^2-x^2_{(0)})\Big] + \tilde{r}^{ab}_{2}(x^2_{(0)};\underset{\sim}{x}_{(0)}) - \\ &-\int_{x^2_{(0)}}^{x^2} dz^2\Big[\tilde{r}^{ab}_{1}(\infty,z^2;\underset{\sim}{x}_{(0)}) - \tilde{r}^{ab}_{1}(-\infty,z^2;\underset{\sim}{x}_{(0)})\Big] - \\ &-\int_{-\infty}^{+\infty} dz^1\int_{x^2_{(0)}}^{x^2} dz^2\Big[r^{ab}_{3}(z^1,z^2,\infty;\underset{\sim}{x}_{(0)}) - r^{ab}_{3}(z^1,z^2,-\infty;\underset{\sim}{x}_{(0)})\Big] \quad ;\end{aligned}} \qquad (3.30)$$

$$\boxed{r^{ab}_{3}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = r^{ab}_{3}(x^1,x^2,\infty;\underset{\sim}{x}_{(0)})\,\Theta(x^3-x^3_{(0)}) + r^{ab}_{3}(x^1,x^2,-\infty;\underset{\sim}{x}_{(0)})\,\Theta(x^3_{(0)}-x^3).} \qquad (3.31)$$

Além disso, as funções $\tilde{r}^{ab}_{1}$, $\tilde{r}^{ab}_{2}$ e r^{ab}_{3} vinculam-se pela condição (ver (D.20))

$$\partial^{k}_{x} \underset{k}{r}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^{3}-x^{3}_{(0)})\,\partial^{1}_{x}\underset{1}{\tilde{r}}^{ab}(x^{1},x^{2};\underset{\sim}{x}_{(0)}) +$$

$$+\delta(x^{1}-x^{1}_{(0)})\delta(x^{3}-x^{3}_{(0)})\,\partial^{2}_{x}\underset{2}{\tilde{r}}^{ab}(x^{2};\underset{\sim}{x}_{(0)}) + \partial^{3}_{x}\underset{3}{r}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta^{ab}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}_{(0)}) \quad (3.32)$$

como se pode constatar facilmente a partir de (3.27) - (3.31). Na próxima seção, veremos que restrições adicionais sobre as funções $\tilde{r}^{ab}_{1}$, $\tilde{r}^{ab}_{2}$ e r^{ab}_{3} serão impostas pela condição de gauge (2.2) a qual ainda não foi utilizada.

III.2 Condição de Gauge sobre $A^{0,a}$. Conteúdo Topológico Não-Trivial da Teoria

Lançando mão dos elementos de matriz da inversa R, desde (2.41) calculamos facilmente o multiplicador de Lagrange $A^{0,a}$ encontrando (ver (E.5), Apêndice E)

$$A^{0,a}(\underset{\sim}{x}) = \int_{x^{1}_{(0)}}^{x^{1}} dz^{1}\, F^{01,a}(z^{1},x^{2},x^{3}_{(0)}) + \int_{x^{2}_{(0)}}^{x^{2}} dz^{2}\, F^{02,a}(x^{1}_{(0)}, z^{2}, x^{3}_{(0)}) +$$

$$+ \int_{x^{3}_{(0)}}^{x^{3}} dz^{3}\, F^{03,a}(x^{1},x^{2},z^{3}) + \int d^{3}z\, \underset{k}{r}^{ba}(\underset{\sim}{z};\underset{\sim}{x}_{(0)})\,F^{0k,b}(\underset{\sim}{z}) \quad . \qquad (3.33)$$

Claramente, (3.33) implica que $A^{0,a}$ deve obedecer a condição de contorno

$$A^{0,a}(\underset{\sim}{x}_{(0)}) = \int d^3 z \; r_k^{ba}(\underset{\sim}{z};\underset{\sim}{x}_{(0)}) \, F^{0k,b}(\underset{\sim}{z}) \; . \qquad (3.34)$$

De forma a compatibilizar (3.34) com (2.2) (ou, equivalentemente, (3.33) com (2.13d)) estamos forçados a escolher (ver (3.27), (3.29) e (3.31))

$$r_1^{ab} = \delta^{ab} r_1 \quad \Longrightarrow \quad \tilde{r}_1^{ab} = \delta^{ab} \tilde{r}_1 \; , \qquad (3.35a)$$

$$r_2^{ab} = \delta^{ab} r_2 \quad \Longrightarrow \quad \tilde{r}_2^{ab} = \delta^{ab} \tilde{r}_2 \; , \qquad (3.35b)$$

$$r_3^{ab} = \delta^{ab} r_3 \; , \qquad (3.35c)$$

onde as funções $\tilde{r}_1$, $\tilde{r}_2$ e r_3 devem satisfazer

$$\partial_x^k \, r_k(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^3 - x^3_{(0)}) \, \partial_x^1 \, \tilde{r}_1(x^1, x^2; \underset{\sim}{x}_{(0)}) +$$

$$+ \delta(x^1 - x^1_{(0)}) \delta(x^3 - x^3_{(0)}) \, \partial_x^2 \, \tilde{r}_2(x^2; \underset{\sim}{x}_{(0)}) +$$

$$+ \partial_x^3 \, r_3(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta^{(3)}(\underset{\sim}{x} - \underset{\sim}{x}_{(0)}) \qquad (3.36)$$

de modo a levar em conta (3.32). Portanto, (2.2) restringe todas as funções r_k^{ab} a serem diagonais nos índices de cor. Informação adicional pode ainda ser obtida desde (2.2). Referimo-nos ao comportamento assintótico das funções $\tilde{r}_1$, $\tilde{r}_2$ e r_3. Para ver como surge tal informação, listamos primeiramente os restantes multiplicadores de Lagrange da teoria calculados no Apêndice E:

$$\eta^{0k,a}_{(\underset{\sim}{x})} = D^{j,ab}_{(\underset{\sim}{x})} F^{jk,b}_{(\underset{\sim}{x})} - g f^{acb} A^{0,c}_{(\underset{\sim}{x})} F^{0k,b}_{(\underset{\sim}{x})} + i g \pi_{\psi}(\underset{\sim}{x}) \gamma^{0} \gamma^{k} \frac{\lambda^{a}}{2} \psi_{(\underset{\sim}{x})} , \quad (3.37a)$$

$$\eta^{jk,a}_{(\underset{\sim}{x})} = D^{j,ab}_{(\underset{\sim}{x})} F^{0k,b}_{(\underset{\sim}{x})} - D^{k,ab}_{(\underset{\sim}{x})} F^{0j,b}_{(\underset{\sim}{x})} + g f^{abc} F^{jk,b}_{(\underset{\sim}{x})} A^{0,c}_{(\underset{\sim}{x})} , \quad (3.37b)$$

$$\eta^{j,a}_{(\underset{\sim}{x})} = \partial^{j}_{x} A^{0,a}_{(\underset{\sim}{x})} - g f^{abc} A^{0,b}_{(\underset{\sim}{x})} A^{j,c}_{(\underset{\sim}{x})} , \quad (3.37c)$$

$$u^{a}_{10}(\underset{\sim}{x}) = \int_{-\infty}^{+\infty} dz^{3} \, \Delta(z^{3}, x^{3}_{(0)}; x^{3}) \, F^{12,a}_{(x^{1}, x^{2}, z^{3})} , \quad (3.37d)$$

$$\begin{aligned} u^{a}_{k+11}(\underset{\sim}{x}) = {} & D^{j,ab}_{(\underset{\sim}{x})} F^{kj,b}_{(\underset{\sim}{x})} + \\ & + g f^{abc} \left(\delta^{k1} F^{23,c}_{(\underset{\sim}{x})} + \delta^{k2} F^{31,c}_{(\underset{\sim}{x})} \right) \int_{-\infty}^{+\infty} dz^{3} \, \Delta(z^{3}, x^{3}_{(0)}; x^{3}) F^{12,b}_{(x^{1}, x^{2}, z^{3})} . \quad (3.37e) \end{aligned}$$

Agora, combinando (3.37c), (2.36a) e (2.33 g,h,i) obtemos, conforme esperávamos,

$$F^{0j,a}_{(\underset{\sim}{x})} = \partial^{0} A^{j,a}_{(\underset{\sim}{x})} - \partial^{j}_{x} A^{0,a}_{(\underset{\sim}{x})} + g f^{abc} A^{0,b}_{(\underset{\sim}{x})} A^{j,c}_{(\underset{\sim}{x})} . \quad (3.38)$$

A substituição de (3.38) em (3.34) (com (3.35), depois de integrar por partes, leva imediatamente a

$$A^{0,a}(\underset{\sim}{x}_{(0)}) = A^{0,a}(\underset{\sim}{x}_{(0)}) + \Big\{ \int_{-\infty}^{+\infty} dx^2 \Big[\tilde{r}_1(-\infty, x^2; \underset{\sim}{x}_{(0)}) A^{0,a}(-\infty, x^2, x^3_{(0)}) - \tilde{r}_1(\infty, x^2; \underset{\sim}{x}_{(0)}) A^{0,a}(\infty, x^2, x^3_{(0)}) \Big] +$$

$$+ \tilde{r}_2(-\infty; \underset{\sim}{x}_{(0)}) A^{0,a}(x^1_{(0)}, -\infty, x^3_{(0)}) - \tilde{r}_2(\infty; \underset{\sim}{x}_{(0)}) A^{0,a}(x^1_{(0)}, \infty, x^3_{(0)}) +$$

$$+ \int_{-\infty}^{+\infty} dx^1 \int_{-\infty}^{+\infty} dx^2 \Big[r_3(x^1, x^2, -\infty; \underset{\sim}{x}_{(0)}) A^{0,a}(x^1, x^2, -\infty) - r_3(x^1, x^2, \infty; \underset{\sim}{x}_{(0)}) A^{0,a}(x^1, x^2, \infty) \Big] \Big\} . \quad (3.39)$$

Para chegar a (3.39), levamos em conta (3.36) e

$$r_k(\underset{\sim}{x}; \underset{\sim}{x}_{(0)}) A^{k,a}(\underset{\sim}{x}) = \delta(x^3 - x^3_{(0)}) \tilde{r}_1(x^1, x^2; \underset{\sim}{x}_{(0)}) A^{1,a}(\underset{\sim}{x}) +$$

$$+ \delta(x^1 - x^1_{(0)}) \delta(x^3 - x^3_{(0)}) \tilde{r}_2(x^2; \underset{\sim}{x}_{(0)}) A^{2,a}(\underset{\sim}{x}) + r_3(\underset{\sim}{x}; \underset{\sim}{x}_{(0)}) A^{3,a}(\underset{\sim}{x}) = 0 \quad (3.40)$$

que segue diretamente das condições de gauge (2.1). Evidentemente, os termos de superfície na chave em (3.39) devem se anular. Dado que o gauge está totalmente fixado por (2.1) e (2.2), qualquer suposição sobre o comportamento assintótico de $A^{0,a}$ vai além da fixação do gauge. Por exemplo, se supusermos que

$$A^{0,a}(\pm\infty, x^2, x^3) = A^{0,a}(x^1, \pm\infty, x^3) = A^{0,a}(x^1, x^2, \pm\infty) = 0$$

estaremos eliminando efeitos não-perturbativos tipo-instanton, como se pode ver a partir da correspondente expressão da carba topológica no gauge axial [22] (ver Apêndice F)

$$q = \frac{ig^3}{192\pi^2} \int_{-\infty}^{+\infty} dx^0 \int_{-\infty}^{+\infty} dx^1 \int_{-\infty}^{+\infty} dx^2 \left| \overset{0,a}{A}(x^0, \underset{\sim}{x}) \overset{2,b}{A}(x^0, \underset{\sim}{x}) \overset{1,c}{A}(x^0, \underset{\sim}{x}) \right|_{x^3=-\infty}^{x^3=+\infty} Tr\left(\lambda^a [\lambda^b, \lambda^c] \right). \quad (3.41)$$

Então, a anulação da chave em (3.39) exige o comportamento assintótico

$$\boxed{\tilde{r}_1(\pm\infty, x^2; \underset{\sim}{x}_{(0)}) = 0} \;,\; \boxed{\boxed{\tilde{r}_2(\pm\infty; \underset{\sim}{x}_{(0)}) = 0}} \;,\; \boxed{r_3(x^1, x^2, \pm\infty; \underset{\sim}{x}_{(0)}) = 0}.$$

$$(3.42\ a,b,c)$$

Por seu turno, as condições (3.42a,c) levadas em (3.28) conduzem a (ver (3.35))

$$\tilde{r}_1(x^1, x^2; \underset{\sim}{x}_{(0)}) = 0 \quad . \qquad (3.43)$$

Além disso, (3.42c) substituída em (3.31) implica

$$r_3(\underset{\sim}{x}; \underset{\sim}{x}_{(0)}) = 0 \quad . \qquad (3.44)$$

Agora, os resultados (3.43) e (3.44), desde (3.30) e (3.35), implicam $\tilde{r}_2$ diferente de zero e dada por

$$\boxed{\tilde{r}_2(x^2; \underset{\sim}{x}_{(0)}) = \Theta(0) - \Theta(x^2 - x^2_{(0)}) + \tilde{r}_2(x^2_{(0)}; \underset{\sim}{x}_{(0)})} \quad , \qquad (3.45)$$

onde temos uma aparente indeterminação representada pela $\theta(0)$. Esta indeterminação é aparente pois, conforme veremos, é eliminada pelo termo $\tilde{r}_2(x_{(0)}^2;\underset{\sim}{x}_{(0)})$. Observe-se, por outro lado, que os resultados (3.43) e (3.44) combinados com (3.36) nos colocam o problema de procurar uma* solução à equação

$$\boxed{\partial_x^2 \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) = \delta(x^2-x_{(0)}^2)} \qquad (3.46)$$

tal que satisfaça (3.45) e (3.42b). Logo abaixo provamos que, respeitada certa prescrição, existe de fato uma solução para $\tilde{r}_2(x^2;\underset{\sim}{x}_{(0)})$ satisfazendo (3.42b), (3.45) e (3.46). Tal solução é

$$\tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) = \lim_{\epsilon\to 0^+}\left[-\theta(x^2-x_{(0)}^2)\,e^{-\epsilon(x^2-x_{(0)}^2)}\right] \qquad (3.47)$$

com a seguinte prescrição: Tome-se o limite $\varepsilon\to 0^+$ na última etapa dos cálculos exceto nas situações em que se precise tratar com integrais impróprias envolvendo a função $\tilde{r}_2$ as quais definiremos, sempre, pelo valor principal (V.P.). Nestas situações, o limite que define a integral deverá ser tomado por último.

Prova

Mostramos aqui que (3.47) satisfaz (3.45), (3.46) e (3.42b),

---

[*] Lembremo-nos que a função de Green do operador ∂_x não está univocamente definida.

respeitado o prescrito. Desde (3.47), obtemos a expressão

$$\tilde{r}_2(x^2_{(0)};\underset{\sim}{x}_{(0)}) = \lim_{\epsilon\to 0^+}\left[-\theta(0)\, e^{-\epsilon.0}\right] = -\theta(0)$$

a qual, levada em (3.45), nos deixa com

$$\tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) = -\theta(x^2-x^2_{(0)})$$

que é essencialmente (3.47). Conforme adiantado, a indeterminação $\theta(0)$ em (3.45) é cancelada pelo termo $\tilde{r}_2(x^2_{(0)};\underset{\sim}{x}_{(0)})$. A solução (3.47) também implica

$$\partial^x_2 \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) = \lim_{\epsilon\to 0^+}\left[-\delta(x^2-x^2_{(0)})\, e^{-\epsilon(x^2-x^2_{(0)})} + \epsilon\,\theta(x^2-x^2_{(0)})\, e^{-\epsilon(x^2-x^2_{(0)})}\right] =$$

$$= -\delta(x^2-x^2_{(0)})$$

o que reproduz (3.46). Além disso, é direto ver que (3.47) satisfaz (3.42b), i.e.,

$$\lim_{x^2\to\pm\infty} \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) \longrightarrow 0 \ .$$

O motivo da ressalva que fizemos em nossa prescrição pode ser explicitado ao analisarmos a compatibilidade de (3.46) com (3.42b). De fato, numa consideração superficial da função $\tilde{r}_2$, poder-se-ia concluir desde (3.46) que

$$\int_{-\infty}^{+\infty} dx^2\, \partial^x_2\, \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) = \tilde{r}_2(+\infty;\underset{\sim}{x}_{(0)}) - \tilde{r}_2(-\infty;\underset{\sim}{x}_{(0)}) = -\int_{-\infty}^{+\infty} dx^2\, \delta(x^2 - x^2_{(0)}) = -1 \; ,$$

em contradição com (3.42b). Agora, a prescrição introduzida significa definir $(-L<x^2_{(0)}<+L)$

$$\int_{-\infty}^{+\infty} dx^2\, \partial^x_2\, \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) = V.P.\int_{-\infty}^{+\infty} dx^2\, \partial^x_2\, \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) \equiv \lim_{L\to\infty} \int_{-L}^{+L} dx^2\, \partial^x_2\, \tilde{r}_2(x^2;\underset{\sim}{x}_{(0)}) =$$

$$= \lim_{L\to\infty} \left[\tilde{r}_2(+L;\underset{\sim}{x}_{(0)}) - \tilde{r}_2(-L;\underset{\sim}{x}_{(0)}) \right] =$$

$$= \lim_{L\to\infty} \left[\lim_{\epsilon\to 0^+} - e^{-\epsilon(L - x^2_{(0)})} \right] = \lim_{L\to\infty} (-e^0) = -1 \; .$$

Desta forma compatibilizamos (3.46) com (3.42b).

A análise das implicações de (2.2) está terminada. O caráter diagonal dos elementos de matriz $R^{ab}_{k+11,11}$ no espaço de cor

$$R^{ab}_{k+11,11}(\underset{\sim}{x};y) = \delta^{ab} R_k(\underset{\sim}{x};y) \qquad (3.48)$$

segue diretamente de (3.25) e (3.35). Uma condição de divergência da forma (3.36) para as funções R_k, i.e.,

$$\partial^k_x R_k(\underset{\sim}{x};y) = \delta^{(3)}(\underset{\sim}{x}-y) \qquad (3.49)$$

pode ser facilmente obtida combinando (3.25), (3.22) e (3.21). Além disso, (3.43), (3.44) e (3.47) implicam

$$\boxed{\begin{aligned} R_k(\underset{\sim}{x};y) = {} & \delta^{k1}\Delta(y^1,x^1_{(0)};x^1)\,\delta(x^2-y^2)\,\delta(x^3-x^3_{(0)}) + \\ & + \delta^{k2}\delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\left\{\Delta(y^2,x^2_{(0)};x^2) + \lim_{\epsilon\to 0^+}\left[-\Theta(x^2-x^2_{(0)})\,e^{-\epsilon(x^2-x^2_{(0)})}\right]\right\} + \\ & + \delta^{k3}\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3) \end{aligned}} \tag{3.50}$$

como se pode ver desde (3.25), (3.35) e (3.21). Note-se que as funções R_k dadas em (3.50) exibem o comportamento assintótico (para $\underset{\sim}{y}$ fixo)

$$\lim_{|\underset{\sim}{x}|\to\infty} R_k(\underset{\sim}{x};y) \longrightarrow 0 \,. \tag{3.51}$$

Tomaremos (3.48) e (3.50) como as expressões finais para os elementos de matriz $R^{ab}_{k+11,11}$. Completamos assim a determinação da matriz inversa R. É fácil mostrar que (2.36), (3.33) e (3.37) formam um conjunto de equações inteiramente equivalente às equações de Lagrange que surgem de (2.14).

IV. TRANSIÇÃO Ã TEORIA QUÂNTICA

IV.1 Os Comutadores Básicos a Tempos Iguais Não-Nulos da Cromodinâmica no Gauge Superaxial

No contexto do PQPD a transição à teoria quântica é feita através das substituições formais

$$\left.\begin{array}{lcl} i\left[B_1(\underset{\sim}{x}), B_2(\underset{\sim}{y})\right]_{PD} & \longrightarrow & \left[\hat{B}_1(\underset{\sim}{x}), \hat{B}_2(\underset{\sim}{y})\right] \\ i\left[B(\underset{\sim}{x}), F(\underset{\sim}{y})\right]_{PD} & \longrightarrow & \left[\hat{B}(\underset{\sim}{x}), \hat{F}(\underset{\sim}{y})\right] \\ i\left[F_1(\underset{\sim}{x}), F_2(\underset{\sim}{y})\right]_{PD} & \longrightarrow & \left\{\hat{F}_1(\underset{\sim}{x}), \hat{F}_2(\underset{\sim}{y})\right\} \end{array}\right\} , \qquad (4.1)$$

onde $B(\underset{\sim}{x})$ e $F(\underset{\sim}{x})$ referem-se a bósons e férmions, respectivamente, com o "chapéu" indicando operadores de campo quânticos.

Uma vez conhecida a matriz R, o cômputo dos PD's é direto (Apêndice E). A expressão (3.50) combinada com os resultados (E.26), (E.29)-(E.33), através de (4.1), nos diz que os CTI's básicos não-nulos da QCD no gauge superaxial são dados por

$$\left[\hat{A}^{j,a}(\underset{\sim}{x}), \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right] = i\left[\delta^{ab}\delta^{jk}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \hat{D}^{j,ab}(\underset{\sim}{x})\, R_k(\underset{\sim}{y};\underset{\sim}{x})\right], \qquad (4.2a)$$

$$\left[\hat{F}^{j\ell,a}(\underset{\sim}{x}), \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right] = i\left[\delta^{\ell k}\hat{D}^{j,ab}(\underset{\sim}{x}) - \delta^{jk}\hat{D}^{\ell,ab}(\underset{\sim}{x})\right]\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) +$$

$$+ ig f^{acb}\hat{F}^{j\ell,c}(\underset{\sim}{x})\, R_k(\underset{\sim}{y};\underset{\sim}{x}) \quad , \qquad (4.2b)$$

$$[\hat{\pi}^a_j(\underset{\sim}{x}), \hat{\pi}^b_k(\underset{\sim}{y})] = i g f^{acb} [\hat{\pi}^c_j(\underset{\sim}{x}) R_k(\underset{\sim}{y};\underset{\sim}{x}) + \hat{\pi}^c_k(\underset{\sim}{y}) R_j(\underset{\sim}{x};\underset{\sim}{y})], \quad (4.2c)$$

$$\{\hat{\psi}(\underset{\sim}{x}), \hat{\pi}_\psi(\underset{\sim}{y})\} = i \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) , \quad (4.2d)$$

$$[\hat{\psi}(\underset{\sim}{x}), \hat{\pi}^a_k(\underset{\sim}{y})] = -g \frac{\lambda^a}{2} \hat{\psi}(\underset{\sim}{x}) R_k(\underset{\sim}{y};\underset{\sim}{x}) , \quad (4.2e)$$

$$[\hat{\pi}_\psi(\underset{\sim}{x}), \hat{\pi}^a_k(\underset{\sim}{y})] = g \hat{\pi}_\psi(\underset{\sim}{x}) \frac{\lambda^a}{2} R_k(\underset{\sim}{y};\underset{\sim}{x}) \quad . \quad (4.2f)$$

Estes CTI's merecem vários comentários. Para o gauge superaxial a translação iPD → CTI é possível, sem ambigüidades, devido à ausência de problemas de ordenamento*. Observe-se que as funções $R_k(\underset{\sim}{x};\underset{\sim}{y})$ (3.52) não dependem das variáveis canônicas e são, portanto, funções "número-c". Esta não é a situação no caso da quantização canônica da QCD no gauge de Coulomb, onde a transição

$$i [\pi^a_j(\underset{\sim}{x}), \pi^b_k(\underset{\sim}{y})]_{PD} \longrightarrow [\hat{\pi}^a_j(\underset{\sim}{x}), \hat{\pi}^b_k(\underset{\sim}{y})]$$

só é possível depois de se adotar uma prescrição de ordenamento [1,35,42].

Por construção (ver (4.1)), os CTI's (4.2) são com

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*Um estudo relativamente completo do problema de ordenamento ao nível da Mecânica Quântica pode ser encontrado nas refs. [41,15,16].

patíveis com todos os vínculos (2.33) valendo como relações operatoriais fortes, i.e., $\{\hat{\Phi}^a_J = 0\}$. Recorde-se (p.32) que $[\Phi^a_J(\underset{\sim}{x}),\Omega]_{PD} = 0$, para Ω um funcional qualquer das variáveis de campo. Nenhuma restrição é imposta sobre o espaço de Hilbert dos estados físicos. A translação de (2.33) ao nível quântico é direta exceto para a lei de Gauss (2.33k), devido à não-comutatividade de $\hat{A}^{j,c}$ e $\hat{F}^{0k,b}$ $(\hat{F}^{0k,b} = \hat{\pi}^b_k)$. O modo mais simples de assegurar hermiticidade para o operador lei de Gauss, bem como para qualquer outro operador composto na teoria, é exigir simetrização ou antissimetrização nos produtos de campos de Bose e Fermi, respectivamente. Então, por exemplo, a lei de Gauss a nível quântico é

$$\hat{D}^{j,ab}\cdot\hat{F}^{oj,b} - ig\,\hat{\pi}_{\psi}\cdot\frac{\lambda^a}{2}\hat{\psi} = \partial^j\hat{F}^{oj,a} + g f^{acb}\hat{A}^{j,c}\cdot\hat{F}^{oj,b} - $$

$$- ig\,\hat{\pi}_{\psi}\cdot\frac{\lambda^a}{2}\hat{\psi} = 0 \qquad (4.3)$$

onde

$$\hat{A}^{j,c}\cdot\hat{F}^{oj,b} = \frac{1}{2}\left\{\hat{A}^{j,c},\hat{F}^{oj,b}\right\}, \quad \hat{\pi}_{\psi}\cdot\frac{\lambda^a}{2}\hat{\psi} = \frac{1}{2}\left[\hat{\pi}_{\psi},\frac{\lambda^a}{2}\hat{\psi}\right]. \qquad (4.4)$$

Desde (3.53) segue que os CTI's (4.2) são compatíveis com os campos cromoelétricos, cromomagnéticos e fermiônicos anulando-se no infinito espacial e, em particular, em $x^3 = \pm\infty$. É, portanto, consistente com as regras de quantização (4.2) supor que, com $|\underset{\sim}{x}| \to \infty$,

$$\hat{\pi}^a_j(x^o,\underset{\sim}{x}) \to 0\,, \quad \hat{F}^{jk,a}(x^o,\underset{\sim}{x}) \to 0\,, \quad \hat{\psi}(x^o,\underset{\sim}{x}) \to 0\,, \quad \hat{\pi}_{\psi}(x^o,\underset{\sim}{x}) \to 0\,. \qquad (4.5)$$

IV.2 Outros Comutadores Relevantes. Comportamento Assintótico dos Campos $\hat{A}^{0,a}$, $\hat{A}^{1,a}$ e $\hat{A}^{2,a}$.

Existem outros CTI's que merecem nossa atenção. Uma vez que $\hat{A}^{0,a}$ foi obtido em termos das variáveis canônicas elementares segundo (3.33), podemos pensá-lo como um operador de campo composto que pode aparecer como um dos termos de um CTI. Não é difícil verificar que (3.33), (3.35), (3.44), (3.45) e (4.2) nos levam a

$$[\hat{A}^{0,a}(\underset{\sim}{x}), \hat{A}^{j,b}(y)] = -i\delta^{ab}\left[R_j(y;\underset{\sim}{x}) + \partial^j_y\,\Omega(y,\underset{\sim}{x};\underset{\sim}{x}_{(0)})\right] +$$

$$+ i g f^{acb}\,\hat{A}^{j,c}(y)\,\Omega(y,\underset{\sim}{x};\underset{\sim}{x}_{(0)}) \quad , \qquad (4.6a)$$

$$[\hat{A}^{0,a}(\underset{\sim}{x}), \hat{\pi}^{b}_{j}(y)] = i g f^{acb}\Big[\hat{\pi}^{c}_{j}(y)\,\Omega(y,\underset{\sim}{x};\underset{\sim}{x}_{(0)}) +$$

$$+ \int_{x^1_{(0)}}^{x^1} dz^1\, R_j(y;z^1,x^2,x^3_{(0)})\,\hat{\pi}^{c}_{1}(z^1,x^2,x^3_{(0)}) +$$

$$+ \int_{x^2_{(0)}}^{x^2} dz^2\, R_j(y;x^1_{(0)},z^2,x^3_{(0)})\,\hat{\pi}^{c}_{2}(x^1_{(0)},z^2,x^3_{(0)}) +$$

$$+ \int_{x^3_{(0)}}^{x^3} dz^3\, R_j(y;x^1,x^2,z^3)\,\hat{\pi}^{c}_{3}(x^1,x^2,z^3) +$$

$$+ \int d^3z\, R_j(y;\underset{\sim}{z})\, r_2(\underset{\sim}{z};\underset{\sim}{x}_{(0)})\,\hat{\pi}^{c}_{2}(\underset{\sim}{z})\Big] \quad , \qquad (4.6b)$$

$$[\hat{A}^{0,a}(\underset{\sim}{x}), \hat{\psi}(y)] = g\,\frac{\lambda^a}{2}\,\hat{\psi}(y)\,\Omega(y, \underset{\sim}{x}; \underset{\sim}{x}_{(0)}) , \qquad (4.6c)$$

$$[\hat{A}^{0,a}(\underset{\sim}{x}), \hat{\pi}_{\psi}(y)] = -g\,\hat{\pi}_{\psi}(y)\,\frac{\lambda^a}{2}\,\Omega(y, \underset{\sim}{x}; \underset{\sim}{x}_{(0)}) , \qquad (4.6d)$$

onde

$$\Omega(y, \underset{\sim}{x}; \underset{\sim}{x}_{(0)}) = \int_{x^1_{(0)}}^{x^1} dz^1\, R_1(z^1, x^2, x^3_{(0)}; y) + \int_{x^2_{(0)}}^{x^2} dz^2\, R_2(x^1_{(0)}, z^2, x^3_{(0)}; y) +$$

$$+ \int_{x^3_{(0)}}^{x^3} dz^3\, R_3(x^1, x^2, z^3; y) + \int d^3z\; r_2(\underset{\sim}{z}; \underset{\sim}{x}_{(0)})\, R_2(\underset{\sim}{z}; y) . \qquad (4.7)$$

Podemos agora discutir em detalhe o comportamento das variáveis $\hat{A}^{1,a}(\underset{\sim}{x})$, $\hat{A}^{2,a}(\underset{\sim}{x})$ e $\hat{A}^{0,a}(\underset{\sim}{x})$ nos limites $x^3 \to \pm\infty$ (ver (3.41)). Desde (4.2a), (4.6a), (3.52) e para x^1, x^2 e $\underset{\sim}{y}$ fixos encontramos

$$[\hat{A}^{1,a}(x^1, x^2, x^3 = \pm\infty), \hat{\pi}^b_k(y)] = -i\,\delta^{ab}\Big\{\delta^{1k}\,\delta(x^1-y^1)\,\delta(x^2-y^2)\,\delta(x^3_{(0)}-y^3) -$$

$$- \delta^{3k}\left[\partial^1_x (x^1-y^1)\right] \delta(x^2-y^2)\,\Delta(x^3 = \pm\infty, x^3_{(0)}; y^3)\Big\} \quad +$$

$$+ i g f^{acb}\,\hat{A}^{1,c}(x^1, x^2, x^3 = \pm\infty)\, R_k(y; x^1, x^2, x^3 = \pm\infty) , \qquad (4.8a)$$

$$[\hat{A}^{2,a}(x^1, x^2, x^3 = \pm\infty), \hat{\Pi}^b_k(y)] =$$

$$= i\,\delta^{ab}\Big\{\delta^{1k}\,\Delta(x^1, x^1_{(0)}; y^1)\left[\partial^2_x\,\delta(x^2 - y^2)\right]\delta(x^3_{(0)} - y^3) -$$

$$-\delta^{2k}\,\delta(x^1_{(0)} - y^1)\,\delta(x^2 - y^2)\,\delta(x^3_{(0)} - y^3) +$$

$$+\delta^{3k}\,\delta(x^1 - y^1)\left[\partial^2_x\,\delta(x^2 - y^2)\right]\Delta(x^3 = \pm\infty, x^3_{(0)}; y^3)\Big\} +$$

$$+ i g f^{acb}\,\hat{A}^{2,c}(x^1, x^2, x^3 = \pm\infty)\,R_k(y; x^1, x^2, x^3 = \pm\infty) \quad , \qquad (4.8b)$$

$$[\hat{A}^{0,a}(x^1, x^2, x^3 = \pm\infty), \hat{A}^{j,b}(y)] = -i\,\delta^{ab}\Big[R_j(y; x^1, x^2, x^3 = \pm\infty) +$$

$$+\partial^j_y\,\Omega(y, x^1, x^2, x^3 = \pm\infty; \underset{\sim}{x}_{(0)})\Big] + i g f^{acb}\,\hat{A}^{j,c}(y)\,\Omega(y, x^1, x^2, x^3 = \pm\infty; \underset{\sim}{x}_{(0)}). \quad (4.8c)$$

Os termos número-c nos lados direitos de (4.8) claramente indicam que $\hat{A}^{1,a}$, $\hat{A}^{2,a}$ e $\hat{A}^{0,a}$ não se anulam em $x^3 = \pm\infty$. Assinalamos que este comportamento assintótico não-trivial, exigido pela presença de uma carga topológica não nula (ver (3.41)), surge como um resultado dentro da presente formulação da QCD. Este não é o caso na ref. [22] onde, entretanto, a necessidade de tal comportamento foi reconhecida.

Portanto, tivemos êxito em obter um conjunto consistente de regras de quantização para a QCD no gauge superaxial. Concluímos o capítulo mostrando que estas regras não permitem a implementação de (2.12) como uma identidade operatorial forte. De fato, por exemplo para $\underset{\sim}{x} = \underset{\sim}{x}_{(0)}$ e $j = 2$, a equação (4.6a) se reduz a

$$\left[\hat{A}^{0,a}(\underset{\sim}{x}_{(0)}), \hat{A}^{2,b}(y)\right] = -i\delta^{ab}\left[\underset{2}{r}(y;\underset{\sim}{x}_{(0)}) - \underset{2}{r}(x^1_{(0)},y^2,x^3_{(0)};\underset{\sim}{x}_{(0)})\right] + $$

$$+ igf^{acb}\hat{A}^{2,c}(y)\int d^3z\, \underset{2}{r}(\underset{\sim}{z};\underset{\sim}{x}_{(0)})\underset{2}{R}(\underset{\sim}{z};y) \qquad (4.9)$$

a qual não é compatível com $\hat{A}^{0,a}(\underset{\sim}{x}_{(0)}) = 0$. Conforme antecipado no capítulo II, p.17, $\hat{A}^{0,a}(\underset{\sim}{x}_{(0)}) = 0$ é descartada pela álgebra de comutação a tempos iguais.

V. INVARIANÇA DE POINCARÉ

V.1 Os Operadores Densidade de Energia e Densidade de Momentum da QCD no Gauge Superaxial e a Relação Fundamental de Schwinger da Teoria Quântica Relativística de Campos

Iniciamos este capítulo, propondo como candidatos para as componentes do tensor densidade de energia-momentum quântico simétrico $\hat{\theta}^{\mu\nu}$ os seguintes operadores hermiteanos

$$\hat{\Theta}^{00} = \hat{\Theta}^{00}_{B} + \hat{\Theta}^{00}_{F} \quad , \qquad (5.1)$$

onde

$$\hat{\Theta}^{00}_{B} = \frac{1}{2}\hat{\pi}^{a}_{j}\hat{\pi}^{a}_{j} + \frac{1}{4}\hat{F}^{jk,a}\hat{F}^{jk,a} \quad , \qquad (5.2)$$

$$\hat{\Theta}^{00}_{F} = \frac{1}{2}(\partial_k \hat{\pi}_{\psi}).\gamma^0\gamma^k\hat{\psi} - \frac{1}{2}\hat{\pi}_{\psi}.\gamma^0\gamma^k\partial_k\hat{\psi} -$$

$$-ig\,\hat{\pi}_{\psi}.\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}\hat{A}^{k,a} - im\,\hat{\pi}_{\psi}.\gamma^0\hat{\psi} \quad ; \qquad (5.3)$$

$$\hat{\Theta}^{0k} = \hat{\Theta}^{0k}_{B} + \hat{\Theta}^{0k}_{F} \quad , \qquad (5.4)$$

onde

$$\hat{\Theta}^{0k}_{B} = \hat{\pi}^{a}_{j}.\hat{F}^{kj,a} \quad , \qquad (5.5)$$

$$\hat{\Theta}^{ok}_{F} = \frac{1}{2}\hat{\pi}_{\psi}\cdot\partial^{k}\hat{\psi} - \frac{1}{2}(\partial^{k}\hat{\pi}_{\psi})\cdot\hat{\psi} - ig\,\hat{\pi}_{\psi}\cdot\frac{\lambda^{a}}{2}\hat{\psi}\hat{A}^{k,a} -$$

$$-\frac{i}{4}\partial_{j}(\hat{\pi}_{\psi}\cdot\sigma^{jk}\hat{\psi}) \qquad ; \qquad (5.6)$$

$$\hat{\Theta}^{jk} = \hat{\Theta}^{jk}_{B} + \hat{\Theta}^{jk}_{F} \qquad , \qquad (5.7)$$

onde

$$\hat{\Theta}^{jk}_{B} = \hat{F}^{jl,a}\hat{F}^{kl,a} - \hat{\pi}^{a}_{j}\hat{\pi}^{a}_{k} + \delta^{jk}\left[\frac{1}{2}\hat{\pi}^{a}_{l}\hat{\pi}^{a}_{l} - \frac{1}{4}\hat{F}^{lm,a}\hat{F}^{lm,a}\right],$$

$$(5.8)$$

$$\hat{\Theta}^{jk}_{F} = \frac{1}{4}\Big[\hat{\pi}_{\psi}\cdot\gamma^{0}\gamma^{k}\partial^{j}\hat{\psi} - (\partial^{j}\hat{\pi}_{\psi})\cdot\gamma^{0}\gamma^{k}\hat{\psi} + \hat{\pi}_{\psi}\cdot\gamma^{0}\gamma^{j}\partial^{k}\hat{\psi} -$$

$$-(\partial^{k}\hat{\pi}_{\psi})\cdot\gamma^{0}\gamma^{j}\hat{\psi}\Big] - \frac{ig}{2}\Big[\hat{\pi}_{\psi}\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}\hat{A}^{j,a} + \hat{\pi}_{\psi}\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}\hat{A}^{k,a}\Big], \quad (5.9)$$

com $\sigma^{jk} = \frac{1}{2i}[\gamma^{j},\gamma^{k}]$. Estas expressões para $\hat{\Theta}^{00}$, $\hat{\Theta}^{0k}$ e $\hat{\Theta}^{jk}$, por conveniência divididas em partes puramente bosônicas e partes fermiônicas com acoplamento, foram obtidas por translação direta desde suas análogas clássicas correspondentes.

A seguir, calcularemos o comutador $[\hat{\Theta}^{00}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]$. Desde (5.1), é evidente que

$$[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{\Theta}^{00}(\underset{\sim}{x}')] = [\hat{\Theta}^{00}_{B}(\underset{\sim}{x}), \hat{\Theta}^{00}_{B}(\underset{\sim}{x}')] + [\hat{\Theta}^{00}_{B}(\underset{\sim}{x}), \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')] +$$

$$+\left[\hat{\Theta}^{00}_F(\underset{\sim}{x}),\hat{\Theta}^{00}_B(\underset{\sim}{x}')\right]+\left[\hat{\Theta}^{00}_F(\underset{\sim}{x}),\hat{\Theta}^{00}_F(\underset{\sim}{x}')\right]=$$

$$=\left[\hat{\Theta}^{00}_B(\underset{\sim}{x}),\hat{\Theta}^{00}_B(\underset{\sim}{x}')\right]+\left[\hat{\Theta}^{00}_F(\underset{\sim}{x}),\hat{\Theta}^{00}_F(\underset{\sim}{x}')\right]+\left[\hat{\Theta}^{00}_B(\underset{\sim}{x}),\hat{\Theta}^{00}_F(\underset{\sim}{x}')\right]-$$

$$-(\underset{\sim}{x}\leftrightarrow\underset{\sim}{x}')\quad , \qquad (5.10)$$

onde a notação ($\underset{\sim}{x}$ ↔ $\underset{\sim}{x}'$), daqui por diante, significa o termo adjacente anterior (ou à esquerda) com a troca de $\underset{\sim}{x}$ por $\underset{\sim}{x}'$ e vice-versa. A partir de (5.2), usando os CTI's (4.2), computamos inicialmente

$$\left[\hat{\Theta}^{00}_B(\underset{\sim}{x}),\hat{\Theta}^{00}_B(\underset{\sim}{x}')\right]=\frac{1}{4}\left[\hat{\pi}^a_j(\underset{\sim}{x})\hat{\pi}^a_j(\underset{\sim}{x}),\hat{\pi}^b_\ell(\underset{\sim}{x}')\hat{\pi}^b_\ell(\underset{\sim}{x}')\right]+$$

$$+\frac{1}{8}\left[\hat{\pi}^a_j(\underset{\sim}{x})\hat{\pi}^a_j(\underset{\sim}{x}),\hat{F}^{\ell m,b}(\underset{\sim}{x}')\hat{F}^{\ell m,b}(\underset{\sim}{x}')\right]-(\underset{\sim}{x}\leftrightarrow\underset{\sim}{x}')\quad . \qquad (5.11)$$

Claramente (ver (4.4)),

$$\frac{1}{4}\left[\hat{\pi}^a_j(\underset{\sim}{x})\hat{\pi}^a_j(\underset{\sim}{x}),\hat{\pi}^b_\ell(\underset{\sim}{x}')\hat{\pi}^b_\ell(\underset{\sim}{x}')\right]=\frac{1}{2}\,\hat{\pi}^a_j(\underset{\sim}{x}).\left[\hat{\pi}^a_j(\underset{\sim}{x}),\hat{\pi}^b_\ell(\underset{\sim}{x}')\hat{\pi}^b_\ell(\underset{\sim}{x}')\right]=$$

$$=\frac{1}{2}\hat{\pi}^a_j(\underset{\sim}{x}).\left\{\left[\hat{\pi}^a_j(\underset{\sim}{x}),\hat{\pi}^b_\ell(\underset{\sim}{x}')\right]\hat{\pi}^b_\ell(\underset{\sim}{x}')+\hat{\pi}^b_\ell(\underset{\sim}{x}')\left[\hat{\pi}^a_j(\underset{\sim}{x}),\hat{\pi}^b_\ell(\underset{\sim}{x}')\right]\right\}=$$

$$=\hat{\pi}^a_j(\underset{\sim}{x}).\left\{\left[\hat{\pi}^a_j(\underset{\sim}{x}),\hat{\pi}^b_\ell(\underset{\sim}{x}')\right].\hat{\pi}^b_\ell(\underset{\sim}{x}')\right\}=$$

$$=\hat{\pi}^a_j(\underset{\sim}{x}).\left\{\left[igf^{acb}\left(\hat{\pi}^c_j(\underset{\sim}{x})\,R_\ell(\underset{\sim}{x}';\underset{\sim}{x})+\hat{\pi}^c_\ell(\underset{\sim}{x}')\,R_j(\underset{\sim}{x};\underset{\sim}{x}')\right)\right].\hat{\pi}^b_\ell(\underset{\sim}{x}')\right\}=$$

$$= \hat{\pi}^{a}_{j}(\underset{\sim}{x}) . \left\{ i g f^{acb} R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x})\, \hat{\pi}^{c}_{j}(\underset{\sim}{x}) . \hat{\pi}^{b}_{\ell}(\underset{\sim}{x}') + i g R_{j}(\underset{\sim}{x};\underset{\sim}{x}') f^{acb} \hat{\pi}^{c}_{\ell}(\underset{\sim}{x}') . \hat{\pi}^{b}_{\ell}(\underset{\sim}{x}') \right]$$

$$= i g f^{acb} R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x})\, \hat{\pi}^{a}_{j}(\underset{\sim}{x}) . \left(\hat{\pi}^{c}_{j}(\underset{\sim}{x}) . \hat{\pi}^{b}_{\ell}(\underset{\sim}{x}') \right) =$$

$$= i g f^{acb} R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) \left\{ \left(\hat{\pi}^{a}_{j}(\underset{\sim}{x}) . \hat{\pi}^{c}_{j}(\underset{\sim}{x}) \right) . \hat{\pi}^{b}_{\ell}(\underset{\sim}{x}') + \frac{1}{4} \left[\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}), \hat{\pi}^{b}_{\ell}(\underset{\sim}{x}') \right], \hat{\pi}^{c}_{j}(\underset{\sim}{x}) \right] \right\}$$

$$= \frac{i}{4} g f^{acb} R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) \left\{ i g f^{adb} \left[\left[\hat{\pi}^{d}_{j}(\underset{\sim}{x}), \hat{\pi}^{c}_{j}(\underset{\sim}{x}) \right] R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) + \right.\right.$$

$$\left.\left. + \left[\hat{\pi}^{d}_{\ell}(\underset{\sim}{x}'), \hat{\pi}^{c}_{j}(\underset{\sim}{x}) \right] R_{j}(\underset{\sim}{x};\underset{\sim}{x}') \right] \right\} =$$

$$= -\frac{g^{2}}{4} f^{acb} f^{adb} R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) R_{j}(\underset{\sim}{x};\underset{\sim}{x}') \left[\hat{\pi}^{d}_{\ell}(\underset{\sim}{x}'), \hat{\pi}^{c}_{j}(\underset{\sim}{x}) \right] =$$

$$= -\frac{i g^{3}}{4} R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) R_{j}(\underset{\sim}{x};\underset{\sim}{x}') f^{acb} f^{adb} f^{dec} \left(\hat{\pi}^{e}_{\ell}(\underset{\sim}{x}') R_{j}(\underset{\sim}{x};\underset{\sim}{x}') + \right.$$

$$\left. + \hat{\pi}^{e}_{j}(\underset{\sim}{x}) R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) \right) = 0 \quad , \qquad (5.12)$$

devido à antissimetria das constantes de estrutura. Na derivação de (5.12) usamos a identidade operatorial válida para bósons A, B e C

$$A.(B.C) = (A.B).C + \frac{1}{4}[[A,C], B] \quad . \qquad (5.13)$$

De modo similar,

$$\frac{1}{8} \left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}) \hat{\pi}^{a}_{j}(\underset{\sim}{x}), \hat{F}^{\ell m,b}(\underset{\sim}{x}') \hat{F}^{\ell m,b}(\underset{\sim}{x}') \right] =$$

$$= \frac{1}{4}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\,\hat{F}^{\ell m,b}(\underset{\sim}{x}')\,\hat{F}^{\ell m,b}(\underset{\sim}{x}')\right] =$$

$$= \frac{1}{2}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\left(\hat{F}^{\ell m,b}(\underset{\sim}{x}')\cdot\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\,\hat{F}^{\ell m,b}(\underset{\sim}{x}')\right]\right) =$$

$$= \frac{1}{2}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\Big\{\hat{F}^{\ell m,b}(\underset{\sim}{x}')\cdot\Big[-i\left(\delta^{mj}\hat{D}^{\ell,ba}(\underset{\sim}{x}')-\delta^{\ell j}\hat{D}^{m,ba}(\underset{\sim}{x}')\right)\delta^{(3)}(\underset{\sim}{x}'-\underset{\sim}{x}) -$$

$$- i g f^{bca}\,\hat{F}^{\ell m,c}(\underset{\sim}{x}')\,R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\Big]\Big]\Big\} =$$

$$= \frac{-i}{2}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\Big\{\hat{F}^{\ell j,a}(\underset{\sim}{x}')\,\partial^{\ell}_{x'}\delta^{(3)}(\underset{\sim}{x}'-\underset{\sim}{x}) + g f^{bca}\,\hat{F}^{\ell j,b}(\underset{\sim}{x}')\,\hat{A}^{\ell,c}(\underset{\sim}{x}')\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$- \hat{F}^{jm,a}(\underset{\sim}{x}')\,\partial^{m}_{x'}\delta^{(3)}(\underset{\sim}{x}'-\underset{\sim}{x}) - g f^{bca}\,\hat{F}^{jm,b}(\underset{\sim}{x}')\,\hat{A}^{m,c}(\underset{\sim}{x}')\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\} =$$

$$= i\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\,\hat{F}^{kj,a}(\underset{\sim}{x}')\,\partial^{x'}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ i g f^{acb}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\left(F^{kj,b}(\underset{\sim}{x}')\,\hat{A}^{k,c}(\underset{\sim}{x}')\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') \quad . \qquad (5.14)$$

Logo,

$$\frac{1}{8}\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\,\hat{F}^{\ell m,b}(\underset{\sim}{x}')\,\hat{F}^{\ell m,b}(\underset{\sim}{x}')\right] - (x \leftrightarrow x') =$$

$$= i\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\,\hat{F}^{kj,a}(\underset{\sim}{x}')\,\partial^{x'}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - i\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}').\,\hat{F}^{kj,a}(\underset{\sim}{x})\,\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') \qquad (5.15)$$

Levando os resultados (5.15) e (5.12) em (5.11), encontramos

$$[\hat{\Theta}^{00}_{B}(\underset{\sim}{x}), \hat{\Theta}^{00}_{B}(\underset{\sim}{x}')] = i(\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\hat{F}^{jk,a}(\underset{\sim}{x}') + \hat{\pi}^{a}_{j}(\underset{\sim}{x}').\hat{F}^{jk,a}(\underset{\sim}{x}))\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') . \tag{5.16}$$

Agora, usando a seguinte propriedade

$$\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\hat{\Omega}(\underset{\sim}{x}).\hat{\Lambda}(\underset{\sim}{x}') + \hat{\Omega}(\underset{\sim}{x}').\hat{\Lambda}(\underset{\sim}{x})\right] =$$

$$= \partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\left(\hat{\Omega}(\underset{\sim}{x})-\hat{\Omega}(\underset{\sim}{x}')\right).\left(\hat{\Lambda}(\underset{\sim}{x}')-\hat{\Lambda}(\underset{\sim}{x})\right) + \hat{\Omega}(\underset{\sim}{x}).\hat{\Lambda}(\underset{\sim}{x}) + \hat{\Omega}(\underset{\sim}{x}').\hat{\Lambda}(\underset{\sim}{x}')\right] =$$

$$= \partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\hat{\Omega}(\underset{\sim}{x}).\hat{\Lambda}(\underset{\sim}{x}) + \hat{\Omega}(\underset{\sim}{x}').\hat{\Lambda}(\underset{\sim}{x}')\right] , \tag{5.17}$$

válida para operadores arbitrários $\hat{\Omega}(\underset{\sim}{x})$ e $\hat{\Lambda}(\underset{\sim}{x})$ [43], podemos reescrever (5.16) na forma da equação fundamental [1,43] (ver (5.5))

$$[\hat{\Theta}^{00}_{B}(\underset{\sim}{x}), \hat{\Theta}^{00}_{B}(\underset{\sim}{x}')] = -i\left(\hat{\Theta}^{0k}_{B}(\underset{\sim}{x}) + \hat{\Theta}^{0k}_{B}(\underset{\sim}{x}')\right)\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') . \tag{5.18}$$

Em continuação, computamos

$$[\hat{\Theta}^{00}_{B}(\underset{\sim}{x}), \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')] = \left[\frac{1}{2}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x}) + \frac{1}{4}\hat{F}^{jk,a}(\underset{\sim}{x})\hat{F}^{jk,a}(\underset{\sim}{x}), \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\right] =$$

$$= \left[\frac{1}{2}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x}), \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\right] = \hat{\pi}^{a}_{j}(\underset{\sim}{x}).\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}), \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\right] =$$

$$= \hat{\pi}^{a}_{.j}(\underset{\sim}{x}).\Big\{\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ \tfrac{1}{2}(\partial^{x'}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\Big]-\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ \tfrac{1}{2}\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\partial^{x'}_{k}\hat{\psi}(\underset{\sim}{x}')\Big]-$$

$$-\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ ig\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,b}(\underset{\sim}{x}')\Big]-\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ im\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\Big]\Big\}, \qquad (5.19)$$

de acordo com (5.3). Utilizando (4.2), ē direto obter

$$\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ \tfrac{1}{2}(\partial^{x'}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\Big]=-\frac{g}{2}\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\partial^{x'}_{k}R_{.j}(\underset{\sim}{x};\underset{\sim}{x}')\ ; \qquad (5.20)$$

$$\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ \tfrac{1}{2}\,\pi_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\partial^{x'}_{k}\hat{\psi}(\underset{\sim}{x}')\Big]=\frac{g}{2}\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\partial^{x'}_{k}R_{.j}(\underset{\sim}{x};\underset{\sim}{x}')\ ; \qquad (5.21)$$

$$\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ ig\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,b}(\underset{\sim}{x}')\Big]=g\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-$$

$$-g\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\,\partial^{x'}_{k}R_{.j}(\underset{\sim}{x};\underset{\sim}{x}')\ ; \qquad (5.22)$$

$$\Big[\hat{\pi}^{a}_{.j}(\underset{\sim}{x}),\ im\,\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\Big] = 0 \ . \qquad (5.23)$$

Substituindo (5.20)-(5.23) em (5.19), ficamos com

$$\Big[\hat{\Theta}^{00}_{B}(\underset{\sim}{x}),\ \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\Big]= ig\,\hat{\pi}^{a}_{.j}(\underset{\sim}{x}).\,J^{j,a}(\underset{\sim}{x}')\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\ , \qquad (5.24)$$

onde introduzimos (para $\mu = j$) a versão quântica da corrente fermiônica (1.8), i.e.,

$$\hat{J}^{\mu,a}(\underset{\sim}{x}) = -i\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{\mu}\,\frac{\lambda^{a}}{2}\,\hat{\psi}(\underset{\sim}{x}) \quad . \qquad (5.25)$$

O lado direito de (5.24) é uma função simétrica de $\underset{\sim}{x}$ e $\underset{\sim}{x}'$, logo

$$\left[\hat{\Theta}^{00}_{B}(\underset{\sim}{x}),\ \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\right] - (\underset{\sim}{x} \leftrightarrow \underset{\sim}{x}') = 0 \quad . \qquad (5.26)$$

Por outro lado, desde (5.3) e (5.25), segue que

$$\begin{aligned}
\left[\hat{\Theta}^{00}_{F}(\underset{\sim}{x}),\ \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\right] &= \Big[\tfrac{1}{2}(\partial^{x}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x}) - \tfrac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\partial^{x}_{k}\hat{\psi}(\underset{\sim}{x}) + \\
&+ g\,\hat{J}^{k,a}(\underset{\sim}{x})\,\hat{A}^{k,a}(\underset{\sim}{x}) - im\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}),\ \tfrac{1}{2}(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x}') - \\
&- \tfrac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}') + g\,\hat{J}^{j,b}(\underset{\sim}{x}')\,\hat{A}^{j,b}(\underset{\sim}{x}') - im\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\Big] = \\
&= \left[\hat{L}(\underset{\sim}{x}),\ \hat{L}(\underset{\sim}{x}')\right] + g\left[\hat{L}(\underset{\sim}{x}),\ \hat{J}^{j,b}(\underset{\sim}{x}')\hat{A}^{j,b}(\underset{\sim}{x}')\right] - (\underset{\sim}{x} \leftrightarrow \underset{\sim}{x}') - \\
&- im\left[\hat{L}(\underset{\sim}{x}),\ \hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\right] - (\underset{\sim}{x} \leftrightarrow \underset{\sim}{x}') + g^{2}\left[\hat{J}^{k,a}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}),\ \hat{J}^{j,b}(\underset{\sim}{x}')\hat{A}^{j,b}(\underset{\sim}{x}')\right] - \\
&- img\left[\hat{J}^{k,a}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}),\ \hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\right] - (\underset{\sim}{x} \leftrightarrow \underset{\sim}{x}') - \\
&- m^{2}\left[\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}),\ \hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\right] \quad , \qquad (5.27)
\end{aligned}$$

onde definimos

$$\hat{L}(\underset{\sim}{x}) \equiv \frac{1}{2}(\partial^x_k \hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}) - \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\partial^x_k\hat{\psi}(\underset{\sim}{x}) \;. \qquad (5.28)$$

Com o uso de (4.2), calculamos, no que segue, os comutadores no lado direito de (5.27):

$$[\hat{L}(\underset{\sim}{x}), \hat{L}(\underset{\sim}{x}')] =$$

$$= \frac{1}{4}\left[(\partial^x_k\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}), (\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}')\right] -$$

$$-\frac{1}{4}\left[\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\partial^x_k\hat{\psi}(\underset{\sim}{x}), (\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}')\right] - (\underset{\sim}{x}\leftrightarrow\underset{\sim}{x}') +$$

$$+\frac{1}{4}\left[\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\partial^x_k\hat{\psi}(\underset{\sim}{x}), \hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}')\right] =$$

$$= -\frac{i}{4}(\partial^{x'}_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^x_k\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}') + \frac{i}{4}(\partial^x_k\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^{x'}_j\pi_\psi(\underset{\sim}{x}'))\cdot\gamma^j\gamma^k\hat{\psi}(\underset{\sim}{x})$$

$$+\frac{i}{4}(\partial^x_k\partial^{x'}_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}') - \frac{i}{4}(\partial^{x'}_k\partial^x_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x})$$

$$-\frac{i}{4}(\partial^x_k\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^k\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \frac{i}{4}(\partial^{x'}_k\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^k\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x})\,;$$

$$(5.29)$$

$$g\left[\hat{L}(\underset{\sim}{x}), \hat{J}^{j,b}(\underset{\sim}{x}')\hat{A}^{j,b}(\underset{\sim}{x}')\right] - (\underset{\sim}{x}\leftrightarrow\underset{\sim}{x}') =$$

$$= \frac{g}{2}\,\partial^x_k\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^j\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x}') + \hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^k\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x}')\right] -$$

$$-\frac{g}{2}\,\partial^{x'}_{k}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\left[\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{k}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\,\hat{A}^{j,a}(\underset{\sim}{x}) + \hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{j}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\,\hat{A}^{j,a}(\underset{\sim}{x})\right] =$$

$$=\frac{g}{2}\,\partial^{x}_{k}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\Bigg\{\left[\hat{\Pi}_{\psi}(\underset{\sim}{x}')\,\hat{A}^{j,a}(\underset{\sim}{x}')\cdot\gamma^{j}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}) + \hat{\Pi}_{\psi}(\underset{\sim}{x})\,\hat{A}^{j,a}(\underset{\sim}{x})\cdot\gamma^{j}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\right]+$$

$$+\left[\hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{k}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\,\hat{A}^{j,a}(\underset{\sim}{x}') + \hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{k}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\,\hat{A}^{j,a}(\underset{\sim}{x})\right]\Bigg\} =$$

$$=\frac{g}{2}\,\partial^{x}_{k}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\Bigg\{\left[\hat{\Pi}_{\psi}(\underset{\sim}{x})\,\hat{A}^{j,a}(\underset{\sim}{x})\cdot\gamma^{j}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}) + \hat{\Pi}_{\psi}(\underset{\sim}{x}')\,\hat{A}^{j,a}(\underset{\sim}{x}')\cdot\gamma^{j}\gamma^{k}\frac{\lambda^{a}}{2}\psi(\underset{\sim}{x}')\right]+$$

$$+\left[\hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{k}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\,\hat{A}^{j,a}(\underset{\sim}{x}) + \hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{k}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\,\hat{A}^{j,a}(\underset{\sim}{x}')\right]\Bigg\} =$$

$$=\partial^{x}_{k}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\left[-g\,\hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\,\hat{A}^{k,a}(\underset{\sim}{x}) - g\,\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\,\hat{A}^{k,a}(\underset{\sim}{x}')\right]\;; \tag{5.30}$$

$$\left[\hat{L}(\underset{\sim}{x}),\ \hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\right] - (\underset{\sim}{x}\leftrightarrow\underset{\sim}{x}') = 0 \qquad ; \tag{5.31}$$

$$\left[\hat{J}^{k,a}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}),\ \hat{J}^{j,b}(\underset{\sim}{x}')\hat{A}^{j,b}(\underset{\sim}{x}')\right] = 0 \qquad ; \tag{5.32}$$

$$\left[\hat{J}^{k,a}(\underset{\sim}{x})\,\hat{A}^{k,a}(\underset{\sim}{x}),\ \hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\right] - (\underset{\sim}{x}\leftrightarrow\underset{\sim}{x}') = 0 \qquad ; \tag{5.33}$$

$$\left[\hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}),\ \hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\right] = 0 \qquad ; \tag{5.34}$$

onde, na obtenção de (5.30), utilizamos (5.17) e o fato que

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}\ .$$

Antes de voltar com (5.29)-(5.34) em (5.27), reescreveremos o resultado (5.29). É direto encontrar a partir desse resultado, usando as propriedades da distribuição $\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')$,

$$[\hat{\mathcal{L}}(\underset{\sim}{x}), \hat{\mathcal{L}}(\underset{\sim}{x}')] = -\frac{i}{4}(\partial^x_k \delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})[\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^k\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_\psi(\underset{\sim}{x}').\gamma^k\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x})] -$$

$$-\frac{i}{4}(-\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).(\gamma^k\gamma^j + [\gamma^j,\gamma^k])\hat{\psi}(\underset{\sim}{x}') +$$

$$+\frac{i}{4}(\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).(\gamma^k\gamma^j + [\gamma^j,\gamma^k])\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{i}{4}(\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})[(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}') + (\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^j\gamma^k\hat{\psi}(\underset{\sim}{x})] =$$

$$= -\frac{i}{4}(\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})[\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^k\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_\psi(\underset{\sim}{x}').\gamma^k\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x}) -$$

$$-(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}') - (\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^k\gamma^j\psi(\underset{\sim}{x})] -$$

$$-i(\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})\Big\{-\frac{1}{2}(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^j\gamma^k\hat{\psi}(\underset{\sim}{x}) +$$

$$+\Big[\frac{1}{4}(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}) - \frac{1}{4}(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^j\gamma^k\hat{\psi}(\underset{\sim}{x}')\Big]\Big\} =$$

$$= -\frac{i}{4}(\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})[\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^k\gamma^j\partial^{x'}_j\psi(\underset{\sim}{x}') + \hat{\pi}_\psi(\underset{\sim}{x}').\gamma^k\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x}) -$$

$$-(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}') - (\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x})] +$$

$$+\frac{i}{2}(\partial^x_k\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')})\Big\{(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^j\gamma^k\hat{\psi}(\underset{\sim}{x}) + (\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^j\gamma^k\hat{\psi}(\underset{\sim}{x}')\Big\}\,; \qquad (5.35)$$

uma vez que

$$\frac{-i}{4}\left(\partial^{x}_{k}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left[\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{k}\gamma^{j}\hat{\psi}(\underset{\sim}{x})-\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\right]=$$

$$=\frac{i}{2}\left(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}') \quad . \qquad (5.36)$$

<u>Prova de (5.36)</u>

O lado esquerdo de (5.36) é igual a

$$\frac{-i}{4}\left\{\left(-\partial^{x'}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{k}\gamma^{j}\hat{\psi}(\underset{\sim}{x})-\left(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\right]=$$

$$=\frac{-i}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\left(\partial^{x'}_{k}\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{k}\gamma^{j}\hat{\psi}(\underset{\sim}{x})+\left(\partial^{x}_{k}\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\right]=$$

$$=\frac{-i}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\left(\partial^{x}_{j}\partial^{x}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')+\left(\partial^{x}_{k}\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\right]=$$

$$=-\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x}_{j}\partial^{x}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')=\frac{i}{2}\left(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right)\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}').$$

q.e.d.

Por seu turno, a expressão (5.35) é idêntica a

$$\left[\hat{L}(\underset{\sim}{x}),\hat{L}(\underset{\sim}{x}')\right]=-i\,\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\hat{\Theta}^{ok}_{F,\,livre}(\underset{\sim}{x})+\hat{\Theta}^{ok}_{F,\,livre}(\underset{\sim}{x}')\right) \quad , \qquad (5.37)$$

onde a densidade de momentum fermiônica <u>livre</u> (ver (5.6)) é dada por

$$\hat{\Theta}^{ok}_{F,\,livre}=\frac{1}{2}\hat{\pi}_{\psi}.\partial^{k}\hat{\psi}-\frac{1}{2}(\partial^{k}\hat{\pi}_{\psi}).\hat{\psi}-\frac{i}{4}\partial_{j}(\hat{\pi}_{\psi}.\sigma^{jk}\hat{\psi}) \quad . \qquad (5.38)$$

Prova de (5.37)

Mostramos abaixo que o lado direito de (5.37), atravês do uso de (5.17) e das propriedades da $\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')$, ē idêntico ao lado esquerdo dado por (5.35). De fato, (5.38) implica

$$-i\,\partial^{x}_{k}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big(\hat{\Theta}^{0k}_{F,livre}(\underset{\sim}{x})+\hat{\Theta}^{0k}_{F,livre}(\underset{\sim}{x}')\Big)=$$

$$=-i\Big(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big)\Big\{\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}).\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})+\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}').\partial^{k}_{x'}\hat{\psi}(\underset{\sim}{x}')-\frac{1}{2}(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\hat{\psi}(\underset{\sim}{x})-$$

$$-\frac{1}{2}(\partial^{k}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\hat{\psi}(\underset{\sim}{x}')-\frac{i}{4}\Big[(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\hat{\psi}(\underset{\sim}{x})+(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\sigma^{jk}\hat{\psi}(\underset{\sim}{x}')+$$

$$+\hat{\pi}_{\psi}(\underset{\sim}{x}).\sigma^{jk}(\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}))+\hat{\pi}_{\psi}(\underset{\sim}{x}').\sigma^{jk}(\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}'))\Big]\Big\}=$$

$$=-i\Big(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big)\Big\{\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}).\partial^{k}_{x'}\hat{\psi}(\underset{\sim}{x}')+\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}').\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})-\frac{1}{2}(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\hat{\psi}(\underset{\sim}{x}')-$$

$$-\frac{1}{2}(\partial^{k}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\hat{\psi}(\underset{\sim}{x})-\frac{i}{4}\Big[(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\hat{\psi}(\underset{\sim}{x}')+(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\sigma^{jk}\hat{\psi}(\underset{\sim}{x})+$$

$$+\hat{\pi}_{\psi}(\underset{\sim}{x}).\sigma^{jk}(\partial^{x'}_{j}\psi(\underset{\sim}{x}'))+\hat{\pi}_{\psi}(\underset{\sim}{x}').\sigma^{jk}(\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}))\Big]\Big\}=$$

$$=-i\Big(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big)\Big\{\frac{1}{4}\Big[\hat{\pi}_{\psi}(\underset{\sim}{x}).2g^{kj}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}')+\hat{\pi}_{\psi}(\underset{\sim}{x}').2g^{kj}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})-$$

$$-(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).2g^{jk}\hat{\psi}(\underset{\sim}{x}')-(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')).2g^{jk}\hat{\psi}(\underset{\sim}{x})\Big]+$$

$$+\frac{1}{8}\Big[(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).[\gamma^{k},\gamma^{j}]\hat{\psi}(\underset{\sim}{x}')+(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')).[\gamma^{k},\gamma^{j}]\hat{\psi}(\underset{\sim}{x})+$$

$$+\ \hat{\pi}_{\psi}(\underset{\sim}{x}).[\gamma^k,\gamma^j]\,\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_{\psi}(\underset{\sim}{x}').[\gamma^k,\gamma^j]\,\partial^{x}_j\hat{\psi}(\underset{\sim}{x})\Big]\Big\} =$$

$$= \frac{-i}{8}\left(\partial^x_k\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\Big\{\hat{\pi}_{\psi}(\underset{\sim}{x}).\left(2\{\gamma^k,\gamma^j\}+[\gamma^k,\gamma^j]\right)\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') +$$

$$+\ \hat{\pi}_{\psi}(\underset{\sim}{x}').\left(2\{\gamma^k,\gamma^j\}+[\gamma^k,\gamma^j]\right)\partial^{x}_j\hat{\psi}(\underset{\sim}{x}) - \left(\partial^x_j\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\left(2\{\gamma^k,\gamma^j\}-[\gamma^k,\gamma^j]\right)\hat{\psi}(\underset{\sim}{x}') -$$

$$-\left(\partial^{x'}_j\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\left(2\{\gamma^k,\gamma^j\}-[\gamma^k,\gamma^j]\right)\hat{\psi}(\underset{\sim}{x})\Big\} . \qquad (5.39)$$

Usando as seguintes identidades

$$2\{\gamma^k,\gamma^j\}+[\gamma^k,\gamma^j] = 2\gamma^k\gamma^j+\{\gamma^k,\gamma^j\} ,$$

$$2\{\gamma^k,\gamma^j\}-[\gamma^k,\gamma^j] = 2\gamma^k\gamma^j+2[\gamma^j,\gamma^k]+\{\gamma^k,\gamma^j\} ,$$

em (5.39), obtemos

$$-i\left(\partial^x_k\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\underset{F,livre}{\hat{\Theta}^{0k}(\underset{\sim}{x})}+\underset{F,livre}{\hat{\Theta}^{0k}(\underset{\sim}{x}')}\right) =$$

$$= \frac{-i}{4}\left(\partial^x_k\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\Big[\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^k\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^k\gamma^j\partial^{x}_j\hat{\psi}(\underset{\sim}{x}) -$$

$$-\left(\partial^x_j\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x}') - \left(\partial^{x'}_j\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^k\gamma^j\hat{\psi}(\underset{\sim}{x})\Big] -$$

$$-\frac{i}{8}\left(\partial^x_k\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\Big\{\hat{\pi}_{\psi}(\underset{\sim}{x}).\{\gamma^k,\gamma^j\}\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_{\psi}(\underset{\sim}{x}').\{\gamma^k,\gamma^j\}\partial^{x}_j\hat{\psi}(\underset{\sim}{x}) -$$

$$-\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\left(3\gamma^{j}\gamma^{k}-\gamma^{k}\gamma^{j}\right)\hat{\psi}(\underset{\sim}{x}')-\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\left(3\gamma^{j}\gamma^{k}-\gamma^{k}\gamma^{j}\right)\hat{\psi}(\underset{\sim}{x})\Big\}.$$

$$(5.40)$$

A propriedade de antissimetria da $\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')$ permite reescrever o termo da chave em (5.40) como segue

$$\frac{i}{8}\Big\{\left(-\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}).\{\gamma^{k},\gamma^{j}\}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}')+\left(\partial^{x'}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}').\{\gamma^{k},\gamma^{j}\}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})-$$

$$-\left(\partial^{x'}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\left(3\gamma^{j}\gamma^{k}-\gamma^{k}\gamma^{j}\right)\hat{\psi}(\underset{\sim}{x}')\;+$$

$$+\left(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\left(3\gamma^{j}\gamma^{k}-\gamma^{k}\gamma^{j}\right)\hat{\psi}(\underset{\sim}{x})\Big\}=$$

$$=\frac{i}{8}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\left(\partial^{x}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\{\gamma^{k},\gamma^{j}\}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}')-\left(\partial^{x'}_{k}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\{\gamma^{k},\gamma^{j}\}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})\;+$$

$$+\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\left(3\gamma^{j}\gamma^{k}-\gamma^{k}\gamma^{j}\right)\partial^{x'}_{k}\hat{\psi}(\underset{\sim}{x}')-\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\left(3\gamma^{j}\gamma^{k}-\gamma^{k}\gamma^{j}\right)\partial^{x}_{k}\hat{\psi}(\underset{\sim}{x})\Big\}=$$

$$=\frac{i}{8}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\,4\gamma^{j}\gamma^{k}\left(\partial^{x'}_{k}\hat{\psi}(\underset{\sim}{x}')\right)-\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\,4\gamma^{j}\gamma^{k}\,\partial^{x}_{k}\hat{\psi}(\underset{\sim}{x})\Big]=$$

$$=\frac{i}{2}\Big[\left(-\partial^{x'}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')\;+$$

$$+\left(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x})\Big]=$$

$$=\frac{i}{2}\left(\partial^{x}_{k}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\Big[\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')+\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{j}\gamma^{k}\hat{\psi}(\underset{\sim}{x})\Big].\qquad(5.41)$$

Substituindo (5.41) em (5.40), obtemos (5.35) o que conclui a prova de (5.37) (q.e.d.).

Levando agora os resultados (5.37) e (5.30) - (5.34) em (5.27), encontramos (ver (5.38) e (5.6))

$$\left[\hat{\Theta}^{00}_{F}(\underset{\sim}{x}), \hat{\Theta}^{00}_{F}(\underset{\sim}{x}')\right] = -i\left(\hat{\Theta}^{0k}_{F}(\underset{\sim}{x}) + \hat{\Theta}^{0k}_{F}(\underset{\sim}{x}')\right)\partial^{x}_{k}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') \,. \quad (5.42)$$

Por sua vez, as expressões (5.42), (5.26) e (5.18) substituídas em (5.10) conduzem à relação fundamental de Schwinger da Teoria Quântica Relativística de Campos [1,43]

$$\boxed{\left[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{\Theta}^{00}(\underset{\sim}{x}')\right] = -i\left(\hat{\Theta}^{0k}(\underset{\sim}{x}) + \hat{\Theta}^{0k}(\underset{\sim}{x}')\right)\partial^{x}_{k}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')} \,. \quad (5.43)$$

Portanto, os candidatos (5.1) e (5.4) são aceitáveis como densidades de Poincaré da formulação de gauge superaxial da QCD.

À diferença da situação no gauge de Coulomb [1,35,44, 13,45], a relação (5.43) não exige "potenciais" quanto-mecanicos adicionais [11,16] quando a QCD é formulada no gauge superaxial. Isto se deve, em instância básica, a ausência de problemas de ordenamento em (4.2). O potencial quanto-mecânico [11,16] a que nos referimos pode também ser calculado, em princípio, usando-se os procedimentos de Christ e Lee [13] e de Falck e Hirschfeld [45] os quais não se baseiam na exigência de invariança de Poincaré. Entretanto, não é claro para nós se tais procedimentos podem englobar condições de gauge como as que especificam o gauge superaxial.

V.2 A Ação dos Geradores de Poincaré sobre cada Campo Básico. Equações de Movimento Quânticas.

Passemos ao problema de definir os geradores de Poincaré e de determinar a ação desses geradores sobre cada campo básico da teoria. Os geradores de Poincaré são definidos como segue:

(i) momenta:

$$\hat{P}^0 = \int d^3x \; \hat{\Theta}^{00}(\underset{\sim}{x}) \equiv \hat{H} \quad , \tag{5.44}$$

$$\hat{P}^k = \int d^3x \; \hat{\Theta}^{0k}(\underset{\sim}{x}) \quad ; \tag{5.45}$$

(ii) rotações espaciais:

$$\hat{J}^{k\ell} = \int d^3x \left(x^k \hat{\Theta}^{0\ell}(\underset{\sim}{x}) - x^\ell \hat{\Theta}^{0k}(\underset{\sim}{x}) \right) \quad ; \tag{5.46}$$

(iii) rotações espaço-temporais ("boosts" ou transformações de Lorentz puras)

$$\hat{J}^{0k} = x^0 \hat{P}^k - \hat{K}^k \quad , \tag{5.47}$$

$$\hat{K}^k = \int d^3x \; x^k \hat{\Theta}^{00}(\underset{\sim}{x}) \quad . \tag{5.48}$$

Note-se que todos os geradores de Poincaré são quantidades bem definidas matematicamente, dado que o comportamento assintótico (4.5) implica (ver (5.1)-(5.9))

$$\lim_{|\underset{\sim}{x}| \to \infty} \hat{\Theta}^{\mu\nu}(\underset{\sim}{x}) \longrightarrow 0 \ . \qquad (5.49)$$

A ação dos geradores de Poincaré sobre os campos básicos pode ser encontrada pelo uso de (4.2). Assim, o operador Hamiltoniano (5.44) leva às seguintes equações de movimento (Apêndice G, expressões (G.2), (G.5), (G.7), (G.9) e (G.11))

$$\partial^0 \hat{A}^{j,a} = i[\hat{H}, \hat{A}^{j,a}] = \hat{\pi}_j^{\ a} + \hat{D}^{j,ab} . \hat{A}^{0,b} \ , \qquad (5.50a)$$

$$\partial^0 \hat{F}^{jk,a} = i[\hat{H}, \hat{F}^{jk,a}] = \hat{D}^{j,ab} . \hat{\pi}_k^{\ b} - \hat{D}^{k,ab} . \hat{\pi}_j^{\ b} + g f^{acb} \hat{F}^{jk,c} . \hat{A}^{0,b} \ , \qquad (5.50b)$$

$$\partial^0 \hat{\pi}_j^{\ a} = i[\hat{H}, \hat{\pi}_j^{\ a}] = -g f^{acb} \hat{A}^{0,c} . \hat{\pi}_j^{\ b} + \hat{D}^{k,ab} \hat{F}^{kj,b} + i g \hat{\pi}_\psi . \gamma^0 \gamma^j \frac{\lambda^a}{2} \hat{\psi} \ ,$$

$$(5.50c)$$

$$\partial^0 \hat{\psi} = i[\hat{H}, \hat{\psi}] =$$

$$= -\gamma^0 \gamma^k \partial_k \hat{\psi} + i \gamma^0 \left[g \frac{\lambda^a}{2} (-\gamma^k \hat{A}^{k,a} + \gamma^0 \hat{A}^{0,a}) - m \right] . \hat{\psi} \ , \qquad (5.50d)$$

$$\partial^0 \hat{\pi}_\psi = i[\hat{H}, \hat{\pi}_\psi] =$$

$$= -(\partial_k \hat{\pi}_\psi) \gamma^0 \gamma^k - i \hat{\pi}_\psi . \gamma^0 \left[g \frac{\lambda^a}{2} (-\gamma^k \hat{A}^{k,a} + \gamma^0 \hat{A}^{0,a}) - m \right] . \qquad (5.50e)$$

A correspondência entre as formulações quântica e clássica da cromodinâmica no gauge superaxial é evidente (ver eqs. (2.36), (3.33), (3.35) e (3.37)). Desde (4.2), (4.6) e (5.50) também segue que os vínculos (2.33) são preservados no tempo. Os resultados da aplicação do gerador de translações espaciais infinitesimais sobre os campos básicos são (ver (G.16), (G.20), (G.25), (G.27) e (G.29))

$$[\hat{P}^k, \hat{A}^{j,a}(\underset{\sim}{x})] = -i\,\partial^k_x \hat{A}^{j,a}(\underset{\sim}{x}) - i\,\hat{D}^{j,ab}(\underset{\sim}{x})\,\hat{B}^{k,b}(\underset{\sim}{x}) \;, \qquad (5.51a)$$

$$[\hat{P}^k, \hat{F}^{j\ell,a}(\underset{\sim}{x})] = -i\,\partial^k_x \hat{F}^{j\ell,a}(\underset{\sim}{x}) - i g f^{acb}\,\hat{F}^{j\ell,c}(\underset{\sim}{x})\,\hat{B}^{k,b}(\underset{\sim}{x}) \,, \qquad (5.51b)$$

$$[\hat{P}^k, \hat{\pi}^{a}_{j\cdot}(\underset{\sim}{x})] = -i\,\partial^k_x \hat{\pi}^{a}_{j\cdot}(\underset{\sim}{x}) - i g f^{acb}\,\hat{\pi}^{c}_{j\cdot}(\underset{\sim}{x}).\hat{B}^{k,b}(\underset{\sim}{x}) \;, \qquad (5.51c)$$

$$[\hat{P}^k, \hat{\psi}(\underset{\sim}{x})] = -i\,\partial^k_x \hat{\psi}(\underset{\sim}{x}) + g\,\frac{\lambda^a}{2}\,\hat{\psi}(\underset{\sim}{x})\,\hat{B}^{k,a}(\underset{\sim}{x}) \;, \qquad (5.51d)$$

$$[\hat{P}^k, \hat{\pi}_{\psi}(\underset{\sim}{x})] = -i\,\partial^k_x \hat{\pi}_{\psi}(\underset{\sim}{x}) - g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\,\frac{\lambda^a}{2}\,\hat{B}^{k,a}(\underset{\sim}{x}) \;, \qquad (5.51e)$$

onde

$$\hat{B}^{k,a}(\underset{\sim}{x}) = \int d^3y\; R_\ell(\underset{\sim}{y};\underset{\sim}{x})\,\partial^k_y \hat{A}^{\ell,a}(\underset{\sim}{y}) \quad . \qquad (5.52)$$

Por outro lado, os resultados da aplicação do gerador de rotações espaciais infinitesimais sobre os campos básicos são (ver (G.33), (G.37), (G.39), (G.42) e (G.45))

$$\begin{aligned}[\hat{J}^{kl}, \hat{A}^{j,a}_{(\underset{\sim}{x})}] = {} & i(x^{l}\partial^{k} - x^{k}\partial^{l})\hat{A}^{j,a}_{(\underset{\sim}{x})} + i\delta^{jk}\hat{A}^{l,a}_{(\underset{\sim}{x})} - \\ & -i\delta^{jl}\hat{A}^{k,a}_{(\underset{\sim}{x})} + i\,\hat{D}^{j,ab}_{(\underset{\sim}{x})}\,\hat{C}^{kl,b}_{(\underset{\sim}{x})} \quad ,\end{aligned} \tag{5.53a}$$

$$\begin{aligned}[\hat{J}^{kl}, \hat{F}^{jm,a}_{(\underset{\sim}{x})}] = {} & i(x^{l}\partial^{k} - x^{k}\partial^{l})\hat{F}^{jm,a}_{(\underset{\sim}{x})} + \\ & + i(\delta^{mk}\hat{F}^{jl,a}_{(\underset{\sim}{x})} - \delta^{ml}\hat{F}^{jk,a}_{(\underset{\sim}{x})} - \delta^{jk}\hat{F}^{ml,a}_{(\underset{\sim}{x})} + \delta^{jl}\hat{F}^{mk,a}_{(\underset{\sim}{x})}) + \\ & + igf^{acb}\hat{F}^{jm,c}_{(\underset{\sim}{x})}\,\hat{C}^{kl,b}_{(\underset{\sim}{x})} \quad ,\end{aligned} \tag{5.53b}$$

$$\begin{aligned}[\hat{J}^{kl}, \hat{\pi}^{a}_{j}{}_{(\underset{\sim}{x})}] = {} & i(x^{l}\partial^{k} - x^{k}\partial^{l})\hat{\pi}^{a}_{j}{}_{(\underset{\sim}{x})} + i\delta^{jk}\hat{\pi}^{a}_{l}{}_{(\underset{\sim}{x})} - i\delta^{jl}\hat{\pi}^{a}_{k}{}_{(\underset{\sim}{x})} + \\ & + igf^{acb}\hat{\pi}^{c}_{j}{}_{(\underset{\sim}{x})}.\hat{C}^{kl,b}_{(\underset{\sim}{x})} \quad ,\end{aligned} \tag{5.53c}$$

$$\begin{aligned}[\hat{J}^{kl}, \hat{\psi}_{(\underset{\sim}{x})}] = {} & i(x^{l}\partial^{k} - x^{k}\partial^{l})\hat{\psi}_{(\underset{\sim}{x})} + \frac{1}{2}\sigma^{kl}\hat{\psi}_{(\underset{\sim}{x})} - \\ & -g\frac{\lambda^{a}}{2}\hat{\psi}_{(\underset{\sim}{x})}\,\hat{C}^{kl,a}_{(\underset{\sim}{x})} \quad ,\end{aligned} \tag{5.53d}$$

$$\begin{aligned}[\hat{J}^{kl}, \hat{\pi}_{\psi}{}_{(\underset{\sim}{x})}] = {} & i(x^{l}\partial^{k} - x^{k}\partial^{l})\hat{\pi}_{\psi}{}_{(\underset{\sim}{x})} - \frac{1}{2}\hat{\pi}_{\psi}{}_{(\underset{\sim}{x})}\sigma^{kl} + \\ & + g\,\hat{\pi}_{\psi}{}_{(\underset{\sim}{x})}\frac{\lambda^{a}}{2}\hat{C}^{kl,a}_{(\underset{\sim}{x})} \quad ,\end{aligned} \tag{5.53e}$$

onde

$$\hat{C}^{kl,a}_{(\underset{\sim}{x})} = \int d^3y \; R_j(y;\underset{\sim}{x}) \Big[\delta^{jm}\big(y^l \partial^k_y - y^k \partial^l_y\big) + \delta^{jk}\delta^{lm} -$$

$$- \delta^{jl}\delta^{km} \Big] \hat{A}^{m,a}_{(y)} \quad . \qquad (5.54)$$

Finalmente, a aplicação do gerador de transformações de Lorentz sobre os campos básicos conduz aos resultados (ver (G.46) e (G.48)-(G.51))

$$\left[\hat{J}^{ok}, \hat{A}^{j,a}_{(\underset{\sim}{x})}\right] = -ix^0\left(\partial^k_x \hat{A}^{j,a}_{(\underset{\sim}{x})} + \hat{D}^{j,ab}_{(\underset{\sim}{x})}\hat{B}^{k,b}_{(\underset{\sim}{x})}\right) +$$

$$+ i x^k \hat{\pi}^a_{j}\cdot(\underset{\sim}{x}) + i\,\hat{D}^{j,ab}_{(\underset{\sim}{x})} . \hat{E}^{k,b}_{(\underset{\sim}{x})} \quad , \qquad (5.55a)$$

$$\left[\hat{J}^{ok}, \hat{F}^{jl,a}_{(\underset{\sim}{x})}\right] = -ix^0\left(\partial^k_x \hat{F}^{jl,a}_{(\underset{\sim}{x})} + g f^{acb}\hat{F}^{jl,c}_{(\underset{\sim}{x})}\hat{B}^{k,b}_{(\underset{\sim}{x})}\right) +$$

$$+ i x^k\left(\hat{D}^{j,ab}_{(\underset{\sim}{x})} . \hat{\pi}^b_{l}\cdot(\underset{\sim}{x}) - \hat{D}^{l,ab}_{(\underset{\sim}{x})} . \hat{\pi}^b_{j}\cdot(\underset{\sim}{x})\right) + i g f^{acb}\hat{F}^{jl,c}_{(\underset{\sim}{x})} . \hat{E}^{k,b}_{(\underset{\sim}{x})} \, ,$$

$$(5.55b)$$

$$\left[\hat{J}^{ok}, \hat{\pi}^a_{j}(\underset{\sim}{x})\right] = -ix^0\left(\partial^k_x \hat{\pi}^a_{j}\cdot(\underset{\sim}{x}) + g f^{acb}\hat{\pi}^c_{j}\cdot(\underset{\sim}{x}) . \hat{B}^{k,b}_{(\underset{\sim}{x})}\right) -$$

$$- i x^k \hat{D}^{l,ab}_{(\underset{\sim}{x})}\hat{F}^{jl,b}_{(\underset{\sim}{x})} - g x^k \hat{\pi}_{\psi}(\underset{\sim}{x}) . \gamma^0\gamma^j \frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}) + i g f^{acb}\hat{\pi}^c_{j}\cdot(\underset{\sim}{x}) . \hat{E}^{k,b}_{(\underset{\sim}{x})} \, ,$$

$$(5.55c)$$

$$\left[\hat{J}^{ok}, \hat{\psi}(\underset{\sim}{x})\right] = -ix^0\left(\partial^k_x \hat{\psi}(\underset{\sim}{x}) + i g \frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{B}^{k,a}_{(\underset{\sim}{x})}\right) -$$

$$-ix^k\gamma^0\gamma^\ell\partial_\ell^x\hat{\psi}(\underset{\sim}{x})-\frac{i}{2}\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x})+gx^k\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{\ell,a}(\underset{\sim}{x})+$$

$$+mx^k\gamma^0\hat{\psi}(\underset{\sim}{x})-g\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}).\hat{E}^{k,a}(\underset{\sim}{x}) \quad , \qquad (5.55d)$$

$$[\hat{J}^{0k},\hat{\pi}_\psi(\underset{\sim}{x})]=-ix^0(\partial_x^k\hat{\pi}_\psi(\underset{\sim}{x})-ig\hat{\pi}_\psi(\underset{\sim}{x})\frac{\lambda^a}{2}\hat{B}^{k,a}(\underset{\sim}{x}))-$$

$$-ix^k(\partial_\ell^x\hat{\pi}_\psi(\underset{\sim}{x}))\gamma^0\gamma^\ell-\frac{i}{2}\hat{\pi}_\psi(\underset{\sim}{x})\gamma^0\gamma^k-gx^k\hat{\pi}_\psi(\underset{\sim}{x})\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{A}^{\ell,a}(\underset{\sim}{x})-$$

$$-mx^k\hat{\pi}_\psi(\underset{\sim}{x})\gamma^0+g\hat{\pi}_\psi(\underset{\sim}{x}).\frac{\lambda^a}{2}\hat{E}^{k,a}(\underset{\sim}{x}) \quad , \qquad (5.55e)$$

onde

$$\hat{E}^{k,a}(\underset{\sim}{x})=\int d^3y\, y^k R_j(\underset{\sim}{y};\underset{\sim}{x})\hat{\pi}_j^a(\underset{\sim}{y}) \quad . \qquad (5.56)$$

A presença dos operadores $\hat{B}^{k,a}$, $\hat{C}^{kl,a}$ e $\hat{E}^{k,a}$ em (5.51), (5.53) e (5.55), respectivamente, assinala a falta de invariança sob translações, rotações e transformações de Lorentz das condições de gauge que especificam o gauge superaxial. Entretanto, provaremos na próxima seção que a QCD no gauge superaxial é uma teoria totalmente invariante de Poincaré.

V.3 Álgebra de Poincaré. Álgebra de Correntes no Gauge Superaxial. Álgebra de Cargas.

Para completar a álgebra das densidades de momentum

$\hat{\Theta}^{0\mu}$ precisamos calcular, em adição a (5.43), os comutadores $[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]$ e $[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0\ell}(\underset{\sim}{x}')]$. Calculemos primeiramente o comutador $[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]$ fazendo uso dos resultados (G.12), (G.17), (G.24), (G.26) e (G.28). Desde (5.1)-(5.3), obtemos

$$[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')] = \Big[\hat{\Theta}^{0k}(\underset{\sim}{x}), \frac{1}{2}\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\hat{\pi}^{a}_{j}(\underset{\sim}{x}') + \frac{1}{4}\hat{F}^{j\ell,a}(\underset{\sim}{x}')\hat{F}^{j\ell,a}(\underset{\sim}{x}') + \frac{1}{2}(\partial^{x'}_{\ell}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') - ig\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}') - im\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\Big] =$$

$$= \hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{a}_{j}(\underset{\sim}{x}')] + \frac{1}{2}\hat{F}^{j\ell,a}(\underset{\sim}{x}')\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{F}^{j\ell,a}(\underset{\sim}{x}')] +$$

$$+\frac{1}{2}(\gamma^{0}\gamma^{\ell})_{rs}\Big\{\Big(\partial^{x'}_{\ell}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')]\Big)\cdot\hat{\psi}^{u}_{s}(\underset{\sim}{x}') + (\partial^{x'}_{\ell}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}'))\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{u}_{s}(\underset{\sim}{x}')]\Big\} -$$

$$-\frac{1}{2}(\gamma^{0}\gamma^{\ell})_{rs}\Big\{[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')]\cdot\partial^{x'}_{\ell}\hat{\psi}^{u}_{s}(\underset{\sim}{x}') + \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\partial^{x'}_{\ell}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{u}_{s}(\underset{\sim}{x}')]\Big\} -$$

$$-ig(\gamma^{0}\gamma^{\ell})_{rs}\left(\frac{\lambda^{a}}{2}\right)^{uv}\Big\{[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')]\cdot\hat{\psi}^{v}_{s}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}') + \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{v}_{s}(\underset{\sim}{x}')]\hat{A}^{\ell,a}(\underset{\sim}{x}') +$$

$$+\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\hat{\psi}^{v}_{s}(\underset{\sim}{x}')[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{A}^{\ell,a}(\underset{\sim}{x}')]\Big\} -$$

$$-im(\gamma^{0})_{rs}\Big\{[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')]\cdot\hat{\psi}^{u}_{s}(\underset{\sim}{x}') + \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{u}_{s}(\underset{\sim}{x}')]\Big\} =$$

$$= \hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\Big[\,i\Big(\hat{\pi}^{b}_{j}(\underset{\sim}{x})\cdot\hat{D}^{k,ba}(\underset{\sim}{x}) - \delta^{kj}\,\hat{\pi}^{b}_{\ell}(\underset{\sim}{x})\cdot\hat{D}^{\ell,ba}(\underset{\sim}{x})\Big)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ igf^{bca}\,\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{k\ell,b}(\underset{\sim}{x})\,R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') + \delta^{kj}g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big] +$$

$$+\frac{1}{2}\hat{F}^{jl,a}_{(\underset{\sim}{x}')}\cdot\Big[i\Big(\hat{F}^{kj,b}_{(\underset{\sim}{x})}\hat{D}^{l,ab}_{(\underset{\sim}{x}')}-\hat{F}^{kl,b}_{(\underset{\sim}{x})}\hat{D}^{j,ab}_{(\underset{\sim}{x}')}\Big)\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+igf^{abc}\hat{F}^{jl,c}_{(\underset{\sim}{x}')}\hat{F}^{km,b}_{(\underset{\sim}{x})}R_m(\underset{\sim}{x};\underset{\sim}{x}')\Big]+$$

$$+\frac{1}{2}(\gamma^0\gamma^l)_{rs}\Big\{\Big[-g\big(\partial^{x'}_l\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x}')\big)\Big(\frac{\lambda^a}{2}\Big)^{vu}R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}_{(\underset{\sim}{x})}-g\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x}')\Big(\frac{\lambda^a}{2}\Big)^{vu}\partial^{x'}_lR_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}_{(\underset{\sim}{x})}+$$

$$+\frac{i}{2}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\partial^{x'}_l\partial^k_x\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}-\frac{i}{2}\big(\partial^k_x\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\big)\partial^{x'}_l\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+g\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x})\Big(\frac{\lambda^a}{2}\Big)^{vu}\hat{A}^{k,a}_{(\underset{\sim}{x})}\partial^{x'}_l\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+$$

$$+\frac{1}{4}\big(\partial^x_j\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})\big)(\sigma^{jk})_{s'r}\partial^{x'}_l\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+\frac{1}{4}\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})(\sigma^{jk})_{s'r}\partial^{x'}_l\partial^x_j\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\Big]\cdot\hat{\psi}^{u}_{s}(\underset{\sim}{x}')+$$

$$+\big(\partial^{x'}_l\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\big)\cdot\Big[g\Big(\frac{\lambda^a}{2}\Big)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x}')R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}_{(\underset{\sim}{x})}-\frac{i}{2}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\partial^k_x\hat{\psi}^{u}_{s}(\underset{\sim}{x})+$$

$$+\frac{i}{2}\big(\partial^k_x\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\big)\hat{\psi}^{u}_{s}(\underset{\sim}{x})-g\Big(\frac{\lambda^a}{2}\Big)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\hat{A}^{k,a}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}-$$

$$-\frac{1}{4}\big(\partial^x_j\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\big)(\sigma^{jk})_{ss'}\hat{\psi}^{u}_{s'}(\underset{\sim}{x})-\frac{1}{4}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}(\sigma^{jk})_{ss'}\partial^x_j\hat{\psi}^{u}_{s'}(\underset{\sim}{x})\Big]\Big\}-$$

$$-\frac{1}{2}(\gamma^0\gamma^l)_{rs}\Big\{\Big[-g\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x}')\Big(\frac{\lambda^a}{2}\Big)^{vu}R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}_{(\underset{\sim}{x})}+\frac{i}{2}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\partial^k_x\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}-$$

$$-\frac{i}{2}\big(\partial^k_x\hat{\pi}^{u}_{\psi}(\underset{\sim}{x})\big)\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+g\hat{\pi}^{v}_{\psi}(\underset{\sim}{x})\Big(\frac{\lambda^a}{2}\Big)^{vu}\hat{A}^{k,a}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+$$

$$+\frac{1}{4}\big(\partial^x_j\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})\big)(\sigma^{jk})_{s'r}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}+\frac{1}{4}\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})(\sigma^{jk})_{s'r}\partial^x_j\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\Big]\cdot\partial^{x'}_l\hat{\psi}^{u}_{s}(\underset{\sim}{x}')+$$

$$+\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\Big[g\Big(\frac{\lambda^a}{2}\Big)^{uv}\big(\partial^{x'}_l\hat{\psi}^{v}_{s}(\underset{\sim}{x}')\big)R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}_{(\underset{\sim}{x})}+g\Big(\frac{\lambda^a}{2}\Big)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\partial^{x'}_lR_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}_{(\underset{\sim}{x})}-$$

$$-\frac{i}{2}\big(\partial^{x'}_l\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\big)\partial^k_x\hat{\psi}^{u}_{s}(\underset{\sim}{x})+\frac{i}{2}\big(\partial^{x'}_l\partial^k_x\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\big)\hat{\psi}^{u}_{s}(\underset{\sim}{x})-g\Big(\frac{\lambda^a}{2}\Big)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\hat{A}^{k,a}_{(\underset{\sim}{x})}\partial^{x'}_l\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\cdot$$

$$-\frac{1}{4}\big(\partial^{x'}_l\partial^x_j\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\big)(\sigma^{jk})_{ss'}\hat{\psi}^{u}_{s'}(\underset{\sim}{x})-\frac{1}{4}\big(\partial^{x'}_l\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\big)(\sigma^{jk})_{ss'}\partial^x_j\hat{\psi}^{u}_{s'}(\underset{\sim}{x})\Big]\Big\}-$$

$$-ig(\gamma^0\gamma^l)_{rs}\left(\frac{\lambda^a}{2}\right)^{uv}\Big\{\Big[-g\,\hat{\pi}^{w}_{\psi_r}(\underset{\sim}{x}')\left(\frac{\lambda^b}{2}\right)^{wu}R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x}) + \frac{i}{2}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\,\partial^k_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$-\frac{i}{2}\left(\partial^k_x\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + g\,\hat{\pi}^{w}_{\psi_r}(\underset{\sim}{x})\left(\frac{\lambda^b}{2}\right)^{wu}\hat{A}^{k,b}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+\frac{1}{4}\left(\partial^x_j\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})\right)(\sigma^{jk})_{s'r}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})(\sigma^{jk})_{s'r}\,\partial^x_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\cdot\hat{\psi}^{v}_{s}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')+$$

$$+\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')\cdot\Big[g\left(\frac{\lambda^b}{2}\right)^{vw}\hat{\psi}^{w}_{s}(\underset{\sim}{x}')R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x}) - \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^k_x\hat{\psi}^{v}_{s}(\underset{\sim}{x})+$$

$$+\frac{i}{2}\left(\partial^k_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\psi}^{v}_{s}(\underset{\sim}{x}) - g\left(\frac{\lambda^b}{2}\right)^{vw}\hat{\psi}^{w}_{s}(\underset{\sim}{x})\hat{A}^{k,b}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$-\frac{1}{4}\left(\partial^x_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)(\sigma^{jk})_{ss'}\hat{\psi}^{v}_{s'}(\underset{\sim}{x}) - \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\sigma^{jk})_{ss'}\,\partial^x_j\hat{\psi}^{v}_{s'}(\underset{\sim}{x})\Big] +$$

$$+\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\hat{\psi}^{v}_{s}(\underset{\sim}{x})\Big[i\hat{F}^{lk,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + i\hat{F}^{jk,b}(\underset{\sim}{x})\hat{D}^{l,ab}(\underset{\sim}{x}')R_j(\underset{\sim}{x};\underset{\sim}{x}')\Big]\Big\} -$$

$$-im(\gamma^0)_{rs}\Big\{\Big[-g\,\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x}')\left(\frac{\lambda^a}{2}\right)^{vu}R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x}) + \frac{i}{2}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\,\partial^k_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$-\frac{i}{2}\left(\partial^k_x\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + g\,\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x})\left(\frac{\lambda^a}{2}\right)^{vu}\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+\frac{1}{4}\left(\partial^x_j\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})\right)(\sigma^{jk})_{s'r}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\hat{\pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})(\sigma^{jk})_{s'r}\,\partial^x_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\cdot\hat{\psi}^{u}_{s}(\underset{\sim}{x}')+$$

$$+\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\Big[g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x}')R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x}) - \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^k_x\hat{\psi}^{u}_{s}(\underset{\sim}{x})+$$

$$+\frac{i}{2}\left(\partial^k_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\psi}^{u}_{s}(\underset{\sim}{x}) - g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$-\frac{1}{4}\left(\partial^x_j\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)(\sigma^{jk})_{ss'}\hat{\psi}^{u}_{s'}(\underset{\sim}{x}) - \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\sigma^{jk})_{ss'}\,\partial^x_j\hat{\psi}^{u}_{s'}(\underset{\sim}{x})\Big]\Big\}. \qquad (5.57)$$

A partir de (5.57), por conveniência, tomaremos separadamente os termos puramente fermiônicos (t.p.f.), os termos fermiônicos com acoplamento ao campo de gauge (t.f.c.a.) e os termos puramente bosônicos (t.p.b.) do comutador $[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]$. Assim, é direto ver que

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')\right]_{t.p.f.} = \frac{i}{4}\left(\partial^{x'}_{\ell}\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{i}{4}\left(\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}') + \frac{1}{8}\left(\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\sigma^{jk}\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}') +$$

$$+\frac{1}{8}\left(\partial^{x'}_{\ell}\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}).\sigma^{jk}\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}') - \frac{i}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x'}_{\ell}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{0}\gamma^{\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{i}{4}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x'}_{\ell}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}) - \frac{1}{8}\left(\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x'}_{\ell}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{0}\gamma^{\ell}\sigma^{jk}\hat{\psi}(\underset{\sim}{x}) -$$

$$-\frac{1}{8}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x'}_{\ell}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right).\gamma^{0}\gamma^{\ell}\sigma^{jk}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}) - \frac{i}{4}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') +$$

$$+\frac{i}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') - \frac{1}{8}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\sigma^{jk}\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{8}\left(\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}).\sigma^{jk}\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') + \frac{i}{4}\left(\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) -$$

$$-\frac{i}{4}\left(\partial^{x'}_{\ell}\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}) + \frac{1}{8}\left(\partial^{x'}_{\ell}\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{\ell}\sigma^{jk}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{1}{8}\left(\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}').\gamma^{0}\gamma^{\ell}\sigma^{jk}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}) - im\left[\frac{i}{2}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^{0}\hat{\psi}(\underset{\sim}{x}') -\right.$$

$$-\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\gamma^{0}\hat{\psi}(\underset{\sim}{x}') + \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\right).\sigma^{jk}\gamma^{0}\hat{\psi}(\underset{\sim}{x}') +$$

$$+\frac{1}{4}\left(\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\sigma^{jk}\gamma^{0}\hat{\psi}(\underset{\sim}{x}') - \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{i}{2}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}) - \frac{1}{4}\left(\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\sigma^{jk}\hat{\psi}(\underset{\sim}{x}) -$$

$$-\frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\sigma^{jk}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})\Big] \quad\Longrightarrow$$

$$\left[\hat{\Theta}^{ok}(\underset{\sim}{x}),\hat{\Theta}^{oo}(\underset{\sim}{x}')\right]_{t.p.f.} =$$

$$= -\frac{i}{2}\left(\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{k}_{x}\hat{\Pi}_{\psi}(\underset{\sim}{x})\right)\cdot\gamma^{0}\gamma^{\ell}\hat{\psi}(\underset{\sim}{x}') - \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x'}_{\ell}\hat{\Pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\gamma^{0}\gamma^{\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{k}_{x}\hat{\Pi}_{\psi}(\underset{\sim}{x})\right)\cdot\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') + \frac{i}{2}\left(\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) -$$

$$-im\Big[-i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{k}_{x}\hat{\Pi}_{\psi}(\underset{\sim}{x})\right)\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}') - i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})\Big] =$$

$$= -i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\left(\partial^{k}_{x}\hat{\Pi}_{\psi}(\underset{\sim}{x})\right)\cdot\gamma^{0}\gamma^{\ell}\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}') - \left(\partial^{x'}_{\ell}\hat{\Pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\gamma^{0}\gamma^{\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})\Big] -$$

$$-m\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\left(\hat{\Pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x})\right) \qquad (5.58)$$

Por outro lado, (5.57) implica

$$\left[\hat{\Theta}^{ok}(\underset{\sim}{x}),\hat{\Theta}^{oo}(\underset{\sim}{x}')\right]_{t.f.c.a.} =$$

$$= -\frac{g}{2}\left(\partial^{x'}_{\ell}\hat{\Pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\gamma^{0}\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x}) - \frac{g}{2}\hat{\Pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\partial^{x'}_{\ell}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x}) +$$

$$+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})\partial^{x'}_{l}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\frac{g}{2}(\partial^{x'}_{l}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})-$$

$$-\frac{g}{2}(\partial^{x'}_{l}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}(\partial^{x'}_{l}\hat{\psi}(\underset{\sim}{x}'))R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})-$$

$$-\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}(\partial^{x'}_{l}\hat{\psi}(\underset{\sim}{x}'))\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}(\partial^{x'}_{l}\hat{\psi}(\underset{\sim}{x}'))R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})-$$

$$-\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\partial^{x'}_{l}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\partial^{x'}_{l}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+ig^{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{b}}{2}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x})+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')$$

$$-\frac{g}{2}(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-ig^{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{b}}{2}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')\hat{A}^{k,b}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-$$

$$-\frac{ig}{4}(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\sigma^{jk}\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-\frac{ig}{4}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\sigma^{jk}\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-$$

$$-ig^{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x})-\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}(\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}))\hat{A}^{l,a}(\underset{\sim}{x}')\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}')\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+ig^{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}')\hat{A}^{k,b}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+\frac{ig}{4}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\sigma^{jk}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}')\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\frac{ig}{4}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\sigma^{jk}(\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}))\hat{A}^{l,a}(\underset{\sim}{x}')\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+g\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{F}^{lk,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+g\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{l}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{F}^{jk,b}(\underset{\sim}{x})\hat{D}^{l,ab}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')-$$

$$-im\Big[-g\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+g\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+g\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})-g\hat{\pi}_{\psi}(\underset{\sim}{x}')\gamma^{0}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\Longrightarrow$$

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')\right]_{t.f.c.a.} = -g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{F}^{kj;a}(\underset{\sim}{x})\,\partial^{x'}_{\ell}R_j(\underset{\sim}{x};\underset{\sim}{x}')\,+$$

$$+\frac{g}{2}\,\hat{\pi}_{+}(\underset{\sim}{x})\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})\,\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - \frac{g}{2}\,\hat{\pi}_{+}(\underset{\sim}{x})\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}(\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}'))\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,-$$

$$-\frac{g}{2}(\partial^{x'}_{\ell}\hat{\pi}_{+}(\underset{\sim}{x}'))\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{g}{2}\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\partial^{x'}_{\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,+$$

$$+\frac{g}{2}\,\hat{\pi}_{+}(\underset{\sim}{x})\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}')\,\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - \frac{g}{2}(\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x}))\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}')\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,-$$

$$-\frac{g}{2}\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}(\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}))\hat{A}^{\ell,a}(\underset{\sim}{x}')\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{g}{2}\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{\ell,a}(\underset{\sim}{x}')\,\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,+$$

$$+ig^2\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\left[\frac{\lambda^b}{2},\frac{\lambda^a}{2}\right]\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}')R_j(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x}) + ig^2\,\hat{\pi}_{+}(\underset{\sim}{x})\cdot\gamma^0\gamma^\ell\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]\hat{\psi}(\underset{\sim}{x})\hat{A}^{\ell,a}(\underset{\sim}{x})\hat{A}^{k,b}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,+$$

$$+g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\left(\partial^{\ell}_{x}\hat{A}^{k,a}(\underset{\sim}{x})-\partial^{k}_{x}\hat{A}^{\ell,a}(\underset{\sim}{x})+g f^{abc}\hat{A}^{\ell,b}(\underset{\sim}{x})\hat{A}^{k,c}(\underset{\sim}{x})\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,+$$

$$+g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{F}^{jk,a}(\underset{\sim}{x})\,\partial^{\ell}_{x'}R_j(\underset{\sim}{x};\underset{\sim}{x}') + g^2\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{F}^{jk,b}(\underset{\sim}{x})f^{acb}\hat{A}^{\ell,c}(\underset{\sim}{x}')R_j(\underset{\sim}{x};\underset{\sim}{x}') = {}^{*}$$

$$= -g(\partial^{x'}_{\ell}\hat{\pi}_{+}(\underset{\sim}{x}'))\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - g\,\hat{\pi}_{+}(\underset{\sim}{x})\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}(\partial^{x'}_{\ell}\hat{\psi}(\underset{\sim}{x}'))\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')$$

$$-g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})\,\partial^{\ell}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,-$$

$$-g(\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x}))\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}')\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}(\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}))\hat{A}^{\ell,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,-$$

$$-g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^0\gamma^\ell\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')(\partial^{k}_{x}\hat{A}^{\ell,a}(\underset{\sim}{x}))\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') =$$

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*Note-se o cancelamento de todos os termos dependentes das funções R_k.

$$= -g\,\partial^{x'}_{\ell}\Big(\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\Big)\hat{A}^{k,a}(\underset{\sim}{x})\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') - g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})\,\partial^{x'}_{\ell}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$-g\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\Big(\hat{\pi}_{+}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{\ell,a}(\underset{\sim}{x})\Big) \qquad \Longrightarrow$$

$$[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]_{t.f.c.a.} = -g\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\Big(\hat{\pi}_{+}(\underset{\sim}{x})\gamma^{0}\gamma^{\ell}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{\ell,a}(\underset{\sim}{x})\Big). \qquad (5.59)$$

Por último, desde (5.57)

$$[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]_{t.p.b.} =$$

$$= -i\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x}')\cdot\big(\hat{D}^{k,ab}(\underset{\sim}{x})\cdot\hat{\Pi}^{b}_{j}(\underset{\sim}{x})\big) + i\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\Pi}^{a}_{k}(\underset{\sim}{x}')\cdot\Big(\hat{D}^{\ell,ab}(\underset{\sim}{x})\cdot\hat{\Pi}^{b}_{\ell}(\underset{\sim}{x}) - ig\,\hat{\pi}_{+}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\Big) +$$

$$+ igf^{bca}R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x}')\cdot\big(\hat{\Pi}^{c}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{k\ell,b}(\underset{\sim}{x})\big) +$$

$$+ \frac{i}{2}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{j\ell,a}(\underset{\sim}{x}')\big[\hat{D}^{\ell,ab}(\underset{\sim}{x})\hat{F}^{kj,b}(\underset{\sim}{x}) - \hat{D}^{j,ab}(\underset{\sim}{x})\hat{F}^{k\ell,b}(\underset{\sim}{x})\big] \quad =$$

$$= -i\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x})\partial^{k}_{x}\hat{\Pi}^{a}_{j}(\underset{\sim}{x}) - igf^{acb}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x}')\cdot\big(\hat{A}^{k,c}(\underset{\sim}{x})\cdot\hat{\Pi}^{b}_{j}(\underset{\sim}{x})\big) +$$

$$+ igf^{abc}R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x}')\cdot\big(\hat{\Pi}^{c}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{k\ell,b}(\underset{\sim}{x})\big) - \frac{i}{2}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{j\ell,a}(\underset{\sim}{x}')\hat{D}^{k,ab}(\underset{\sim}{x})\hat{F}^{j\ell,b}(\underset{\sim}{x}), \qquad (5.60)$$

onde usamos a lei de Gauss (4.3) e a identidade de Bianchi (G.19). A expressão (5.60) implica ainda

$$[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')]_{t.p.b.} = -i\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x})\partial^{k}_{x}\hat{\Pi}^{a}_{j}(\underset{\sim}{x}) - \frac{i}{2}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{j\ell,a}(\underset{\sim}{x})\partial^{k}_{x}\hat{F}^{j\ell,a}(\underset{\sim}{x}) +$$

$$+ igf^{abc}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x}')\cdot\big(\hat{A}^{k,c}(\underset{\sim}{x})\cdot\hat{\Pi}^{b}_{j}(\underset{\sim}{x}')\big) + igf^{abc}R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}')\,\hat{\Pi}^{a}_{j}(\underset{\sim}{x}')\cdot\big(\hat{\Pi}^{c}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{k\ell,b}(\underset{\sim}{x})\big). \qquad (5.61$$

Analisemos os dois últimos termos em (5.61) a partir do uso dos CTI's (4.2). Lançando mão de (5.13), é claro que pela antissimetria das f^{abc}'s

$$ig f^{abc}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}').\left(\hat{A}^{k,c}(\underset{\sim}{x}).\hat{\pi}^{b}_{j}(\underset{\sim}{x}')\right) = ig\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\left(f^{abc}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}').\hat{\pi}^{b}_{j}(\underset{\sim}{x}')\right).\hat{A}^{k,c}(\underset{\sim}{x}) +$$

$$+\frac{ig}{4}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')f^{abc}\left[\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}'),\hat{A}^{k,c}(\underset{\sim}{x})\right],\hat{\pi}^{b}_{j}(\underset{\sim}{x}')\right] =$$

$$=\frac{ig}{4}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')f^{abc}\left[-igf^{cda}R_{j}(\underset{\sim}{x}';\underset{\sim}{x})\hat{A}^{k,d}(\underset{\sim}{x}),\hat{\pi}^{b}_{j}(\underset{\sim}{x}')\right] =$$

$$=\frac{-g^{2}}{4}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')f^{abc}f^{adc}R_{j}(\underset{\sim}{x}';\underset{\sim}{x})\left[i\delta^{db}\left(\delta^{jk}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')+\partial^{k}_{x}R_{j}(\underset{\sim}{x}';\underset{\sim}{x})\right)\right]$$

$$=\frac{-ig^{2}}{4}f^{abc}f^{abc}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\left[R_{k}(\underset{\sim}{x}';\underset{\sim}{x})\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')+R_{j}(\underset{\sim}{x}';\underset{\sim}{x})\partial^{k}_{x}R_{j}(\underset{\sim}{x}';\underset{\sim}{x})\right]; \quad (5.62)$$

$$ig f^{abc}R_{l}(\underset{\sim}{x};\underset{\sim}{x}')\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}').\left(\hat{\pi}^{c}_{j}(\underset{\sim}{x}').\hat{F}^{kl,b}(\underset{\sim}{x})\right) = igR_{l}(\underset{\sim}{x};\underset{\sim}{x}')\left(f^{abc}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}').\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\right).\hat{F}^{kl,b}(\underset{\sim}{x}) +$$

$$+\frac{ig}{4}R_{l}(\underset{\sim}{x};\underset{\sim}{x}')f^{abc}\left[\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}'),\hat{F}^{kl,b}(\underset{\sim}{x})\right],\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\right] =$$

$$=\frac{ig}{4}R_{l}(\underset{\sim}{x};\underset{\sim}{x}')f^{abc}\left[-i\left(\delta^{jl}gf^{bda}\hat{A}^{k,d}(\underset{\sim}{x})-\delta^{kj}gf^{bda}\hat{A}^{l,d}(\underset{\sim}{x})\right)\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')-\right.$$

$$\left.-igf^{bda}\hat{F}^{kl,d}(\underset{\sim}{x})R_{j}(\underset{\sim}{x}';\underset{\sim}{x}),\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\right] =$$

$$=\frac{g^{2}}{4}R_{l}(\underset{\sim}{x};\underset{\sim}{x}')f^{abc}f^{abd}\left\{\left(\left[\hat{A}^{k,d}(\underset{\sim}{x}),\hat{\pi}^{c}_{l}(\underset{\sim}{x}')\right]-\left[\hat{A}^{l,d}(\underset{\sim}{x}),\hat{\pi}^{c}_{k}(\underset{\sim}{x}')\right]\right)\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')+\right.$$

$$\left.+R_{j}(\underset{\sim}{x}';\underset{\sim}{x})\left[\hat{F}^{kl,d}(\underset{\sim}{x}),\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\right]\right\} =$$

$$= \frac{g^2}{4} R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') f^{abc} f^{abd} \Big\{ i\delta^{dc} \Big(\delta^{k\ell} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') + \partial^k_x R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) -$$

$$- \delta^{k\ell} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') - \partial^{\ell}_x R_k(\underset{\sim}{x}';\underset{\sim}{x}) \Big) \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ i\delta^{dc} R_j(\underset{\sim}{x}';\underset{\sim}{x}) \Big(\delta^{j\ell} \partial^k_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') - \delta^{jk} \partial^{\ell}_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') \Big) \Big\} =$$

$$= \frac{ig^2}{4} f^{abc} f^{abc} \Big[\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') \partial^k_x R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) - \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') \partial^{\ell}_x R_k(\underset{\sim}{x}';\underset{\sim}{x}) +$$

$$+ R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) \partial^k_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') - R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') R_k(\underset{\sim}{x}';\underset{\sim}{x}) \partial^{\ell}_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') \Big] =$$

$$= \frac{ig^2}{4} f^{abc} f^{abc} \Big[\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') \partial^k_x R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) - \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') \partial^{\ell}_x R_k(\underset{\sim}{x}';\underset{\sim}{x}) +$$

$$+ R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) \partial^k_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') + \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') \partial^{\ell}_x \Big(R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') R_k(\underset{\sim}{x}';\underset{\sim}{x}) \Big) \Big] =$$

$$= \frac{ig^2}{4} f^{abc} f^{abc} \Big[\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') \partial^k_x R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) + R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') R_{\ell}(\underset{\sim}{x}';\underset{\sim}{x}) \partial^k_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') R_k(\underset{\sim}{x}';\underset{\sim}{x}) \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') \Big] , \qquad (5.63)$$

onde, no último passo, utilizamos (3.49). Somando os resultados (5.62) e (5.63) ficamos com

$$i g f^{abc} \Big[\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') \hat{\pi}^a_j(\underset{\sim}{x}') . \big(\hat{A}^{k,c}(\underset{\sim}{x}) . \hat{\pi}^b_j(\underset{\sim}{x}') \big) + R_{\ell}(\underset{\sim}{x};\underset{\sim}{x}') \hat{\pi}^a_j(\underset{\sim}{x}') . \big(\hat{\pi}^c_j(\underset{\sim}{x}') . \hat{F}^{k\ell,b}(\underset{\sim}{x}) \big) \Big] =$$

$$= \frac{ig^2}{4} f^{abc} f^{abc} R_j(\underset{\sim}{x};\underset{\sim}{x}') R_j(\underset{\sim}{x}';\underset{\sim}{x}) \partial^k_x \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}') \quad . \qquad (5.64)$$

Para analisar o lado direito de (5.64) precisamos usar (3.50). Note-se que a definição (3.2) implica a propriedade

$$\Delta(x,y;z)\,\Delta(z,y;x) = 0 \quad . \qquad (5.65)$$

Prova

Desde (3.2), segue

$$\Delta(x,y;z)\Delta(z,y;x) = \left[\theta(z-y)-\theta(z-x)\right]\cdot\left[\theta(x-y)-\theta(x-z)\right] =$$

$$= \theta(z-y)\theta(x-y) - \theta(z-y)\theta(x-z) - \theta(z-x)\,\theta(x-y) \qquad \therefore$$

a) $x, z > y$

a1) $x < z \quad \Rightarrow \quad \Delta\cdot\Delta = 1-1 = 0$

a2) $x > z \quad \Rightarrow \quad \Delta\cdot\Delta = 1-1 = 0$;

b) $x, z < y$

b1) $x < z \quad \Rightarrow \quad \Delta\cdot\Delta = 0-0 = 0$

b2) $x > z \quad \Rightarrow \quad \Delta\cdot\Delta = 0-0 = 0$; etc. q.e.d.

Fazendo uso de (5.65), desde (3.50) encontramos

$$R_j(\underset{\sim}{x};\underset{\sim}{x}')\,R_j(\underset{\sim}{x}';\underset{\sim}{x}) = \Big\{\delta^{j1}\Delta(x'^1,x^1_{(0)};x^1)\,\delta(x^2-x'^2)\,\delta(x^3-x^3_{(0)}) \; +$$

$$+\,\delta^{j2}\delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\Big[\Delta(x'^2,x^2_{(0)};x^2) + \lim_{\epsilon\to 0^+}\Big(-\theta(x^2-x^2_{(0)})\,e^{-\epsilon(x^2-x^2_{(0)})}\Big)\Big] +$$

$$+\,\delta^{j3}\delta(x^1-x'^1)\,\delta(x^2-x'^2)\,\Delta(x'^3,x^3_{(0)};x^3)\Big\}\cdot\Big\{\delta^{j1}\Delta(x^1,x^1_{(0)};x'^1)\,\delta(x^2-x'^2)\,\delta(x'^2-x^3_{(0)}) \; +$$

$$+\,\delta^{j2}\delta(x'^1-x^1_{(0)})\,\delta(x'^3-x^3_{(0)})\Big[\Delta(x^2,x^2_{(0)};x'^2) + \lim_{\epsilon'\to 0^+}\Big(-\theta(x'^2-x^2_{(0)})\,e^{-\epsilon'(x'^2-x^2_{(0)})}\Big)\Big] +$$

$$+ \delta^{j3}\delta(x^1-x'^1)\delta(x^2-x'^2)\Delta(x^3,x^3_{(0)};x'^3)\Big\} =$$

$$= \delta(x^1-x^1_{(0)})\delta(x^3-x^3_{(0)})\delta(x'^1-x^1_{(0)})\delta(x'^3-x^3_{(0)})\Big\{\Delta(x'^2,x^2_{(0)};x^2)\lim_{\epsilon'\to 0^+}\Big(-\theta(x'^2-x^2_{(0)})e^{-\epsilon'(x'^2-x^2_{(0)})}\Big)+$$

$$+\Delta(x^2,x^2_{(0)};x'^2)\lim_{\epsilon\to 0^+}\Big(-\theta(x^2-x^2_{(0)})e^{-\epsilon(x^2-x^2_{(0)})}\Big) +$$

$$+\lim_{\epsilon\to 0^+}\lim_{\epsilon'\to 0^+}\Big(\theta(x^2-x^2_{(0)})e^{-\epsilon(x^2-x^2_{(0)})}\theta(x'^2-x^2_{(0)})e^{-\epsilon'(x'^2-x^2_{(0)})}\Big)\Big]\Big\} =$$

$$= \delta(x^1-x^1_{(0)})\delta(x^3-x^3_{(0)})\delta(x'^1-x^1_{(0)})\delta(x'^3-x^3_{(0)}).$$

$$\lim_{\epsilon\to 0^+}\lim_{\epsilon'\to 0^+}\Big\{\theta(x^2-x^2_{(0)})\theta(x'^2-x^2_{(0)})e^{-\epsilon(x^2-x^2_{(0)})}\Big[e^{-\epsilon'(x'^2-x^2_{(0)})}-1\Big]+$$

$$+\theta(x^2-x'^2)\theta(x'^2-x^2_{(0)})e^{-\epsilon'(x'^2-x^2_{(0)})}+\theta(x'^2-x^2)\theta(x^2-x^2_{(0)})e^{-\epsilon(x^2-x^2_{(0)})}-$$

$$-\theta(x^2-x^2_{(0)})\theta(x'^2-x^2_{(0)})e^{-\epsilon'(x'^2-x^2_{(0)})}\Big\}. \qquad (5.66)$$

Agora, realizando os limites ε,ε' → 0 de acordo com nossa prescrição estabelecida no capítulo III p.58, obtemos

$$R_j(\underset{\sim}{x};\underset{\sim}{x}')R_j(\underset{\sim}{x}';\underset{\sim}{x}) = \delta(x^1-x^1_{(0)})\delta(x^3-x^3_{(0)})\delta(x'^1-x^1_{(0)})\delta(x'^3-x^3_{(0)}) .$$

$$\cdot\Big[\theta(x^2-x'^2)\theta(x'^2-x^2_{(0)})+\theta(x'^2-x^2)\theta(x^2-x^2_{(0)})-\theta(x^2-x^2_{(0)})\theta(x'^2-x^2_{(0)})\Big]=0. \quad (5.67)$$

Prova de (5.67)

É direto provar que o colchete em (5.67) é identicamente nulo. De fato, se:

a) $x^2 > x^2_{(0)}$, temos que

$$\left[\underset{\cdots\cdots}{\text{em }(5.67)}\right] = \theta(x^2-x'^2)\,\theta(x'^2-x^2_{(0)}) + \theta(x'^2-x^2) - \theta(x'^2-x^2_{(0)}) =$$

$$= \theta(x'^2-x^2) - \theta(x'^2-x^2)\,\theta(x'^2-x^2_{(0)}) = \theta(x'^2-x^2)\,\theta(x^2_{(0)}-x'^2) \quad \therefore$$

a1) $x'^2 > x^2_{(0)} \Rightarrow [\ldots] = 0$

a2) $x'^2 < x^2_{(0)} \Rightarrow x'^2 < x^2 \Rightarrow [\ldots] = 0$;

b) $x^2 < x^2_{(0)}$, temos que

$$[\ldots] = \theta(x^2-x'^2)\,\theta(x'^2-x^2_{(0)}) \quad \therefore$$

b1) $x'^2 > x^2_{(0)} \Rightarrow x'^2 > x^2 \Rightarrow [\ldots] = 0$

b2) $x'^2 < x^2_{(0)} \Rightarrow [\ldots] = 0$.

q.e.d.

Desde (5.67), (5.64) e (5.61) encontramos o resultado

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\Theta}^{00}(\underset{\sim}{x}')\right]_{t.p.b.} = -i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\,\partial^{k}_{x}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}) -$$

$$-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{jl,a}(\underset{\sim}{x})\,\partial^{k}_{x}\,\hat{F}^{jl,a}(\underset{\sim}{x}) \,. \qquad (5.68)$$

As expressões (5.68), (5.59) e (5.58) somadas constituem nos so resultado final para o comutador $[\bar{\Theta}^{0k}(\underset{\sim}{x}),\bar{\Theta}^{00}(\underset{\sim}{x}')]$. Não é difícil mostrar que este resultado pode ser escrito em termos das densidades $\bar{\Theta}^{00}$ e $\bar{\Theta}^{jk}$ da seguinte maneira (ver Apêndice H)

$$\boxed{\begin{aligned}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')] &= -\frac{i}{2}\left(\hat{\Theta}^{kj}(\underset{\sim}{x})+\hat{\Theta}^{kj}(\underset{\sim}{x}')\right)\partial^{x}_{j}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\\ &+\frac{i}{2}\left(\hat{\Theta}^{00}(\underset{\sim}{x})+\hat{\Theta}^{00}(\underset{\sim}{x}')\right)\partial^{k}_{x}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-\\ &-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{k}_{x}\hat{\Theta}^{00}(\underset{\sim}{x})+\partial^{x}_{j}\hat{\Theta}^{jk}(\underset{\sim}{x})\right).\end{aligned}} \qquad (5.69)$$

Passemos, então, ao cômputo do comutador $[\bar{\Theta}^{0k}(\underset{\sim}{x}),\bar{\Theta}^{0\ell}(\underset{\sim}{x}')]$. Faremos uso novamente das expressões (G.12), (G.17), (G.24), (G.26) e (G.28). Desde (5.4)-(5.6), é claro que

$$\begin{aligned}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0\ell}(\underset{\sim}{x}')] &= \Big[\hat{\Theta}^{0k}(\underset{\sim}{x}),\ \hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{\ell j,a}(\underset{\sim}{x}')+\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\partial^{\ell}_{x'}\hat{\psi}(\underset{\sim}{x}')-\\ &-\frac{1}{2}\left(\partial^{\ell}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\hat{\psi}(\underset{\sim}{x}')-ig\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{d^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}')-\frac{i}{4}\partial^{x'}_{j}\left(\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\sigma^{j\ell}\hat{\psi}(\underset{\sim}{x}')\right)\Big]=\\ &=[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{a}_{j}(\underset{\sim}{x}')]\cdot\hat{F}^{\ell j,a}(\underset{\sim}{x}')+\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{F}^{\ell j,a}(\underset{\sim}{x}')]+\frac{1}{2}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')]\cdot\partial^{\ell}_{x'}\hat{\psi}_{r}\\ &+\frac{1}{2}\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')\cdot\partial^{\ell}_{x'}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(\underset{\sim}{x}')]-\frac{1}{2}\left(\partial^{\ell}_{x'}[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')]\right)\cdot\hat{\psi}^{u}_{r}(\underset{\sim}{x}')-\frac{1}{2}\left(\partial^{\ell}_{x'}\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')\right)\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(\underset{\sim}{x}')]-\\ &-ig\left(\frac{d^{a}}{2}\right)^{uv}\Big\{[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')]\cdot\hat{\psi}^{v}_{r}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}')+\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')\cdot[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\psi}^{v}_{r}(\underset{\sim}{x}')]\hat{A}^{\ell,a}(\underset{\sim}{x}')+\\ &\qquad+\hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}')\cdot\hat{\psi}^{v}_{r}(\underset{\sim}{x}')[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{A}^{\ell,a}(\underset{\sim}{x}')]\Big\}\quad -\end{aligned}$$

$$-\frac{i}{4}(\sigma^{jl})_{rs}\,\partial^{x'}_{j}\left\{\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Pi}^{u}_{\Psi_r}(\underset{\sim}{x}')\right]\cdot\hat{\Psi}^{u}_{s}(\underset{\sim}{x}')+\hat{\Pi}^{u}_{\Psi_r}(\underset{\sim}{x}')\cdot\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Psi}^{u}_{s}(\underset{\sim}{x}')\right]\right\}=$$

$$=\hat{F}^{lj,a}(\underset{\sim}{x}')\cdot\Big[\,i\Big(\hat{\pi}^{b}_{j}(\underset{\sim}{x})\cdot\hat{D}^{k,ba}(\underset{\sim}{x})-\delta^{kj}\hat{\pi}^{b}_{m}(\underset{\sim}{x})\cdot\hat{D}^{m,ba}(\underset{\sim}{x})\Big)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+igf^{bca}\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{km,b}(\underset{\sim}{x})R_{m}(\underset{\sim}{x};\underset{\sim}{x}')+\delta^{kj}\hat{\Pi}_{\Psi}(\underset{\sim}{x})\cdot\frac{\partial^{a}}{2}\hat{\Psi}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]+$$

$$+\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\Big[\,i\Big(\hat{F}^{kl,b}(\underset{\sim}{x})\hat{D}^{j,ab}(\underset{\sim}{x}')-\hat{F}^{kj,b}(\underset{\sim}{x})\hat{D}^{l,ab}(\underset{\sim}{x}')\Big)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$igf^{abc}\hat{F}^{lj,c}(\underset{\sim}{x}')\hat{F}^{km,b}(\underset{\sim}{x})R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\Big]+$$

$$+\frac{1}{2}\Big[-g\hat{\pi}^{v}_{\Psi_r}(\underset{\sim}{x}')\Big(\frac{\partial^{a}}{2}\Big)^{vu}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+\frac{i}{2}\hat{\Pi}^{u}_{\Psi_r}(\underset{\sim}{x})\partial^{k}_{x}\delta^{(1)}(\underset{\sim}{x}-\underset{\sim}{x}')-\frac{i}{2}\Big(\partial^{k}_{x}\hat{\pi}^{u}_{\Psi_r}(\underset{\sim}{x})\Big)\delta^{(1)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+g\hat{\pi}^{v}_{\Psi_r}(\underset{\sim}{x})\Big(\frac{\partial^{a}}{2}\Big)^{vu}\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\frac{1}{4}\Big(\partial^{x}_{j}\hat{\pi}^{u}_{\Psi_s}(\underset{\sim}{x})\Big)(\sigma^{jk})_{sr}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\frac{1}{4}\hat{\pi}^{u}_{\Psi_s}(\underset{\sim}{x})(\sigma^{jk})_{sr}\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\cdot$$

$$\cdot\,\partial^{l}_{x'}\hat{\Psi}^{u}_{r}(\underset{\sim}{x}')\quad+$$

$$+\frac{1}{2}\hat{\pi}^{u}_{\Psi_r}(\underset{\sim}{x}')\cdot\partial^{l}_{x'}\Big[g\Big(\frac{\partial^{a}}{2}\Big)^{uv}\hat{\Psi}^{v}_{r}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})-\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\partial^{k}_{x}\hat{\Psi}^{u}_{r}(\underset{\sim}{x})+$$

$$+\frac{i}{2}\Big(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big)\hat{\Psi}^{u}_{r}(\underset{\sim}{x})-g\Big(\frac{\partial^{a}}{2}\Big)^{uv}\hat{\Psi}^{v}_{r}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-$$

$$-\frac{1}{4}\Big(\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big)(\sigma^{jk})_{rs}\hat{\Psi}^{u}_{s}(\underset{\sim}{x})-\frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\sigma^{jk})_{rs}\partial^{x}_{j}\hat{\Psi}^{u}_{s}(\underset{\sim}{x})\Big]-$$

$$-\frac{1}{2}\partial^{l}_{x'}\Big[-g\hat{\pi}^{v}_{\Psi_r}(\underset{\sim}{x}')\Big(\frac{\partial^{a}}{2}\Big)^{vu}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+\frac{i}{2}\hat{\Pi}^{u}_{\Psi_r}(\underset{\sim}{x})\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')-\frac{i}{2}\Big(\partial^{k}_{x}\hat{\pi}^{u}_{\Psi_r}(\underset{\sim}{x})\Big)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+$$

$$+ g\,\hat{\Pi}^{v}_{\psi_r}(\underset{\sim}{x})\left(\frac{\lambda^a}{2}\right)^{vu}\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\left(\partial^{x}_{\cdot j}\hat{\Pi}^{u}_{\psi_s}(\underset{\sim}{x})\right)(\sigma^{jk})_{sr}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\hat{\Pi}^{u}_{\psi_s}(\underset{\sim}{x})(\sigma^{jk})_{sr}\,\partial^{x}_{\cdot j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\cdot\hat{\psi}^{u}_{r}(\underset{\sim}{x}')$$

$$-\frac{1}{2}\left(\partial^{\ell}_{x'}\hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x}')\right)\cdot\Big[g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}_{r}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x}) - \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\partial^{k}_{x}\hat{\psi}^{u}_{r}(\underset{\sim}{x}) + \frac{i}{2}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\psi}^{u}_{r}(\underset{\sim}{x}) -$$

$$- g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}_{r}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - \frac{1}{4}\left(\partial^{x}_{\cdot j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)(\sigma^{jk})_{rs}\hat{\psi}^{u}_{s}(\underset{\sim}{x}) - \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\sigma^{jk})_{rs}\,\partial^{x}_{\cdot j}\hat{\psi}^{u}_{s}(\underset{\sim}{x})\Big] -$$

$$- ig\left(\frac{\lambda^a}{2}\right)^{uv}\Big\{\Big[-g\,\hat{\Pi}^{w}_{\psi_r}(\underset{\sim}{x}')\left(\frac{\lambda^b}{2}\right)^{wu}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x}) + \frac{i}{2}\hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x})\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - \frac{i}{2}\left(\partial^{k}_{x}\hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x})\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ g\,\hat{\Pi}^{w}_{\psi_r}(\underset{\sim}{x})\left(\frac{\lambda^b}{2}\right)^{wu}\hat{A}^{k,b}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\left(\partial^{x}_{\cdot j}\hat{\Pi}^{u}_{\psi_s}(\underset{\sim}{x})\right)(\sigma^{jk})_{sr}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\hat{\Pi}^{u}_{\psi_s}(\underset{\sim}{x})(\sigma^{jk})_{sr}\,\partial^{x}_{\cdot j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\cdot\hat{\psi}^{v}_{r}(\underset{\sim}{x}')\hat{A}^{\ell,}(\underset{\sim}{x}$$

$$+ \hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\Big[g\left(\frac{\lambda^b}{2}\right)^{vw}\hat{\psi}^{w}_{r}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x}) - \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\partial^{k}_{x}\hat{\psi}^{v}_{r}(\underset{\sim}{x}) + \frac{i}{2}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\psi}^{v}_{r}(\underset{\sim}{x}) -$$

$$- g\left(\frac{\lambda^b}{2}\right)^{vw}\hat{\psi}^{w}_{r}(\underset{\sim}{x})\hat{A}^{k,b}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') - \frac{1}{4}\left(\partial^{x}_{\cdot j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)(\sigma^{jk})_{rs}\hat{\psi}^{v}_{s}(\underset{\sim}{x}) - \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\sigma^{jk})_{rs}\,\partial^{x}_{\cdot j}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\Big]\hat{A}^{\ell,a}(\underset{\sim}{x}') +$$

$$+ \hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\hat{\psi}^{v}_{r}(\underset{\sim}{x}')\Big[i\hat{F}^{\ell k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + i\hat{F}^{mk,b}(\underset{\sim}{x})\,\hat{D}^{\ell,ab}(\underset{\sim}{x}')\,R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\Big]\Big\} -$$

$$- \frac{i}{4}(\sigma^{j\ell})_{rs}\,\partial^{x'}_{j}\Big\{\Big[-g\,\hat{\Pi}^{v}_{\psi_r}(\underset{\sim}{x}')\left(\frac{\lambda^a}{2}\right)^{vu}R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{km,a}(\underset{\sim}{x}) + \frac{i}{2}\hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x})\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$- \frac{i}{2}\left(\partial^{k}_{x}\hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x})\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + g\,\hat{\Pi}^{v}_{\psi_r}(\underset{\sim}{x})\left(\frac{\lambda^a}{2}\right)^{vu}\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{1}{4}\left(\partial^{x}_{m}\hat{\Pi}_{\psi_{s'}}(\underset{\sim}{x})\right)(\sigma^{mk})_{s'r}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ \frac{1}{4}\hat{\Pi}^{u}_{\psi_{s'}}(\underset{\sim}{x})(\sigma^{mk})_{s'r}\,\partial^{x}_{m}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big]\cdot\hat{\psi}^{u}_{s}(\underset{\sim}{x}') + \hat{\Pi}^{u}_{\psi_r}(\underset{\sim}{x}')\cdot\Big[g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x}')R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{km,a}(\underset{\sim}{x}) -$$

$$- \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\partial^{k}_{x}\hat{\psi}^{u}_{s}(\underset{\sim}{x}) + \frac{i}{2}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\psi}^{u}_{s}(\underset{\sim}{x}) - g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$- \frac{1}{4}\left(\partial^{x}_{m}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)(\sigma^{mk})_{ss'}\hat{\psi}^{u}_{s'}(\underset{\sim}{x}) - \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\sigma^{mk})_{ss'}\,\partial^{x}_{m}\hat{\psi}^{u}_{s'}(\underset{\sim}{x})\Big]\Big\}. \quad (5.70)$$

Conforme procedemos em relação à expressão (5.57), tomaremos separadamente os termos puramente fermiônicos (t.p.f.), os termos fermiônicos com acoplamento (t.f.c.a.) e os termos puramente bosônicos (t.p.b.) da expressão (5.70), i.e., do comutador $[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0\ell}(\underset{\sim}{x}')]$. Assim,

$$[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0\ell}(\underset{\sim}{x}')]_{t.p.f.} = -\frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')(\partial^k_x\hat{\pi}_+(\underset{\sim}{x})).\partial^\ell_{x'}\hat{f}(\underset{\sim}{x}') - \frac{i}{4}(\partial^\ell_{x'}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}').\partial^k_x\hat{f}(\underset{\sim}{x}) +$$

$$+\frac{i}{4}(\partial^\ell_{x'}\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}').\hat{f}(\underset{\sim}{x}) - \frac{1}{8}(\partial^\ell_{x'}\partial^x_j\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}').\sigma^{jk}\hat{f}(\underset{\sim}{x}) - \frac{1}{8}(\partial^\ell_{x'}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}').\sigma^{jk}\partial^x_j\hat{f}(\underset{\sim}{x}) -$$

$$-\frac{i}{4}(\partial^\ell_{x'}\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}).\hat{f}(\underset{\sim}{x}') + \frac{i}{4}(\partial^\ell_{x'}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^k_x\hat{\pi}_+(\underset{\sim}{x})).\hat{f}(\underset{\sim}{x}') - \frac{1}{8}(\partial^\ell_{x'}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^x_j\hat{\pi}_+(\underset{\sim}{x})).\sigma^{jk}\hat{f}(\underset{\sim}{x}') -$$

$$-\frac{1}{8}(\partial^\ell_{x'}\partial^x_j\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}).\sigma^{jk}\hat{f}(\underset{\sim}{x}') + \frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')(\partial^\ell_{x'}\hat{\pi}_+(\underset{\sim}{x}')).\partial^k_x\hat{f}(\underset{\sim}{x}) +$$

$$+\frac{1}{8}(\partial^{x'}_j\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}).\sigma^{j\ell}\hat{f}(\underset{\sim}{x}') + \frac{1}{8}(\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}).\sigma^{j\ell}\partial^{x'}_j\hat{f}(\underset{\sim}{x}') -$$

$$-\frac{1}{8}(\partial^{x'}_j\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^k_x\hat{\pi}_+(\underset{\sim}{x})).\sigma^{j\ell}\hat{f}(\underset{\sim}{x}') - \frac{1}{8}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')(\partial^k_x\hat{\pi}_+(\underset{\sim}{x})).\sigma^{j\ell}\partial^{x'}_j\hat{f}(\underset{\sim}{x}') -$$

$$-\frac{i}{16}(\partial^{x'}_j\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^x_m\hat{\pi}_+(\underset{\sim}{x})).\sigma^{mk}\sigma^{j\ell}\hat{f}(\underset{\sim}{x}') - \frac{i}{16}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')(\partial^x_m\hat{\pi}_+(\underset{\sim}{x})).\sigma^{mk}\sigma^{j\ell}\partial^{x'}_j\hat{f}(\underset{\sim}{x}') -$$

$$-\frac{i}{16}(\partial^{x'}_j\partial^x_m\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}).\sigma^{mk}\sigma^{j\ell}\hat{f}(\underset{\sim}{x}') - \frac{i}{16}(\partial^x_m\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}).\sigma^{mk}\sigma^{j\ell}\partial^{x'}_j\hat{f}(\underset{\sim}{x}') -$$

$$-\frac{1}{8}(\partial^{x'}_j\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}').\sigma^{j\ell}\partial^k_x\hat{f}(\underset{\sim}{x}) - \frac{1}{8}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')(\partial^{x'}_j\hat{\pi}_+(\underset{\sim}{x}')).\sigma^{j\ell}\partial^k_x\hat{f}(\underset{\sim}{x}) +$$

$$+\frac{1}{8}(\partial^{x'}_j\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))\hat{\pi}_+(\underset{\sim}{x}').\sigma^{j\ell}\hat{f}(\underset{\sim}{x}) + \frac{1}{8}(\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}'))(\partial^{x'}_j\hat{\pi}_+(\underset{\sim}{x}')).\sigma^{j\ell}\hat{f}(\underset{\sim}{x}) +$$

$$+\frac{i}{16}\left(\partial^{x'}_{j}\partial^{x}_{m}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\sigma^{jl}\sigma^{mk}\hat{\psi}(\underset{\sim}{x})+\frac{i}{16}\left(\partial^{x}_{m}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\sigma^{jl}\sigma^{mk}\hat{\psi}(\underset{\sim}{x})+$$

$$+\frac{i}{16}\left(\partial^{x'}_{j}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\sigma^{jl}\sigma^{mk}\partial^{x}_{m}\hat{\psi}(\underset{\sim}{x})+\frac{i}{16}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\sigma^{jl}\sigma^{mk}\partial^{x}_{m}\hat{\psi}(\underset{\sim}{x})=$$

$$=\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\left(\partial^{l}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})-\left(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\right)\cdot\partial^{l}_{x'}\hat{\psi}(\underset{\sim}{x}')-$$

$$-\left(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\right)\cdot\partial^{l}_{x'}\hat{\psi}(\underset{\sim}{x}')+\left(\partial^{l}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})\Big]\qquad\Rightarrow$$

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0l}(\underset{\sim}{x}')\right]_{t.p.f.}=i\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\left(\partial^{l}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})-\left(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\right)\cdot\partial^{l}_{x'}\hat{\psi}(\underset{\sim}{x}')\Big].\tag{5.71}$$

Por outro lado, desde (5.70),

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0l}(\underset{\sim}{x}')\right]_{t.f.c.a.}=-\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\left(\partial^{l}_{x'}\hat{\psi}(\underset{\sim}{x}')\right)R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+$$

$$+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\left(\partial^{l}_{x'}\hat{\psi}(\underset{\sim}{x}')\right)\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\left(\partial^{l}_{x'}\hat{\psi}(\underset{\sim}{x}')\right)R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+$$

$$+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\left(\partial^{l}_{x'}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\right)\hat{F}^{kj,a}(\underset{\sim}{x})-\frac{g}{2}\left(\partial^{l}_{x'}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})+$$

$$+\frac{g}{2}\left(\partial^{l}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\left(\partial^{l}_{x'}R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\right)\hat{F}^{kj,a}(\underset{\sim}{x})-$$

$$-\frac{g}{2}\left(\partial^{l}_{x'}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\right)\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})-\frac{g}{2}\left(\partial^{l}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,a}(\underset{\sim}{x})+$$

$$+\frac{g}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left(\partial^{l}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')\right)\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})+ig^{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\frac{\lambda^{b}}{2}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x}')R_{j}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{kj,b}(\underset{\sim}{x})+$$

$$+\frac{g}{2}\left(\partial_x^k\delta^{(3)}(\mathbf{x}-\mathbf{x}')\right)\hat{\pi}_\psi(\mathbf{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{A}^{l,a}(\mathbf{x}')-\frac{g}{2}\delta^{(3)}(\mathbf{x}-\mathbf{x}')(\partial_x^k\hat{\pi}_\psi(\mathbf{x}))\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{A}^{l,a}(\mathbf{x}')\,-$$

$$-ig^2\hat{\pi}_\psi(\mathbf{x})\cdot\frac{\lambda^b}{2}\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{A}^{l,a}(\mathbf{x}')\hat{A}^{k,b}(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{x}')-ig^2\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\frac{\lambda^b}{2}\hat{\psi}(\mathbf{x}')\hat{A}^{l,a}(\mathbf{x}')R_j(\mathbf{x};\mathbf{x}')\hat{F}^{kj,b}(\mathbf{x})\,-$$

$$-\frac{g}{2}\delta^{(3)}(\mathbf{x}-\mathbf{x}')\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}(\partial_x^k\hat{\psi}(\mathbf{x}))\hat{A}^{l,a}(\mathbf{x}')+\frac{g}{2}\left(\partial_x^k\delta^{(3)}(\mathbf{x}-\mathbf{x}')\right)\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x})\hat{A}^{l,a}(\mathbf{x}')\,+$$

$$+ig^2\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\frac{\lambda^b}{2}\hat{\psi}(\mathbf{x})\hat{A}^{l,a}(\mathbf{x}')A^{k,b}(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{x}')+g\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{F}^{lk,a}(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{x}')\,+$$

$$+g\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{F}^{mk,b}(\mathbf{x})\hat{D}^{l,ab}(\mathbf{x}')R_m(\mathbf{x};\mathbf{x}')+\frac{ig}{4}\partial_j^{x'}\left(\hat{\pi}_\psi(\mathbf{x}')\cdot\sigma^{jl}\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')R_m(\mathbf{x};\mathbf{x}')\right)\hat{F}^{km,a}(\mathbf{x})\,-$$

$$-\frac{ig}{4}\left(\partial_j^{x'}\delta^{(3)}(\mathbf{x}-\mathbf{x}')\right)\hat{\pi}_\psi(\mathbf{x})\cdot\sigma^{jl}\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{A}^{k,a}(\mathbf{x})-\frac{ig}{4}\delta^{(3)}(\mathbf{x}-\mathbf{x}')\hat{\pi}_\psi(\mathbf{x})\cdot\sigma^{jl}\frac{\lambda^a}{2}\left(\partial_j^{x'}\hat{\psi}(\mathbf{x}')\right)\hat{A}^{k,a}(\mathbf{x})\,-$$

$$-\frac{ig}{4}\partial_j^{x'}\left(\hat{\pi}_\psi(\mathbf{x}')\cdot\sigma^{jl}\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')R_m(\mathbf{x};\mathbf{x}')\right)\hat{F}^{km,a}(\mathbf{x})+\frac{ig}{4}\left(\partial_j^{x'}\delta^{(3)}(\mathbf{x}-\mathbf{x}')\right)\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\sigma^{jl}\hat{\psi}(\mathbf{x})\hat{A}^{k,a}(\mathbf{x})\,+$$

$$+\frac{ig}{4}\delta^{(3)}(\mathbf{x}-\mathbf{x}')\left(\partial_j^{x'}\hat{\pi}_\psi(\mathbf{x})\right)\cdot\frac{\lambda^a}{2}\sigma^{jl}\hat{\psi}(\mathbf{x})\hat{A}^{k,a}(\mathbf{x}) \quad =$$

$$= g\,\delta^{(3)}(\mathbf{x}-\mathbf{x}')\hat{\pi}_\psi(\mathbf{x})\cdot\frac{\lambda^a}{2}(\partial_{x'}^l\hat{\psi}(\mathbf{x}'))\hat{A}^{k,a}(\mathbf{x})+g\,\delta^{(3)}(\mathbf{x}-\mathbf{x}')(\partial_{x'}^l\hat{\pi}_\psi(\mathbf{x}'))\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x})\hat{A}^{k,a}(\mathbf{x})\,+$$

$$+g\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')(\partial_{x'}^l R_j(\mathbf{x};\mathbf{x}'))\hat{F}^{kj,a}(\mathbf{x})+ig^2\hat{\pi}_\psi(\mathbf{x}')\cdot\left[\frac{\lambda^b}{2},\frac{\lambda^a}{2}\right]\hat{\psi}(\mathbf{x}')\hat{A}^{l,a}(\mathbf{x}')R_j(\mathbf{x};\mathbf{x}')\hat{F}^{kj,b}(\mathbf{x})\,-$$

$$-g\,\delta^{(3)}(\mathbf{x}-\mathbf{x}')(\partial_x^k\hat{\pi}_\psi(\mathbf{x}))\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{A}^{l,a}(\mathbf{x}')+ig^2\hat{\pi}_\psi(\mathbf{x})\cdot\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]\hat{\psi}(\mathbf{x})\hat{A}^{l,a}(\mathbf{x})A^{k,b}(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{x}')\,-$$

$$-g\,\delta^{(3)}(\mathbf{x}-\mathbf{x}')\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}(\partial_x^k\hat{\psi}(\mathbf{x}))\hat{A}^{l,a}(\mathbf{x}')+g\,\delta^{(3)}(\mathbf{x}-\mathbf{x}')\hat{\pi}_\psi(\mathbf{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x})\left(\partial_x^l\hat{A}^{k,a}(\mathbf{x})-\partial_x^k\hat{A}^{l,a}(\mathbf{x})+gf^{abc}\hat{A}^{l,b}(\mathbf{x})\hat{A}^{k,c}(\mathbf{x})\right)+$$

$$+g\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{F}^{mk,a}(\mathbf{x})\partial_{x'}^l R_m(\mathbf{x};\mathbf{x}')+g^2\hat{\pi}_\psi(\mathbf{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\mathbf{x}')\hat{F}^{mk,b}(\mathbf{x})f^{acb}\hat{A}^{l,c}(\mathbf{x}')R_m(\mathbf{x};\mathbf{x}')=$$

$$= g\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{l}_{x'}\Big(\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x}')\Big)\hat{A}^{k,a}(\underset{\sim}{x}) - g\,\hat{\pi}_{+}(\underset{\sim}{x}')\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x}')\,\hat{A}^{k,a}(\underset{\sim}{x})\,\partial^{l}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$- g\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\Big(\hat{\pi}_{+}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})\Big) =$$

$$= g\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{l}_{x'}\Big(\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x}')\Big)\hat{A}^{k,a}(\underset{\sim}{x}) + g\,\hat{\pi}_{+}(\underset{\sim}{x}')\cdot\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})\,\partial^{l}_{x'}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') -$$

$$- g\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\Big(\hat{\pi}_{+}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})\Big) \qquad \Longrightarrow$$

$$\big[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0l}(\underset{\sim}{x}')\big]_{t.f.c.a.} = -g\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\Big(\hat{\pi}_{+}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{f}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})\Big). \qquad (5.72)$$

Por último, da expressão (5.70) com o uso de (G.19) obtemos

$$\big[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0l}(\underset{\sim}{x}')\big]_{t.p.b.} = -i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{lj,a}(\underset{\sim}{x}')\cdot\Big(\hat{D}^{k,ab}(\underset{\sim}{x})\cdot\hat{\pi}^{b}_{j}(\underset{\sim}{x})\Big) +$$

$$+ igf^{abc}R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{lj,a}(\underset{\sim}{x}')\cdot\Big(\hat{F}^{km,b}(\underset{\sim}{x})\cdot\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\Big) + i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\Big(\hat{D}^{k,ab}(\underset{\sim}{x})\hat{F}^{jl,b}(\underset{\sim}{x})\Big) +$$

$$+ igf^{abc}R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\Big(\hat{F}^{km,b}(\underset{\sim}{x})\cdot\hat{F}^{lj,c}(\underset{\sim}{x}')\Big) =$$

$$= -i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{lj,a}(\underset{\sim}{x}')\cdot\Big(\partial^{k}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x}) + gf^{acb}\hat{A}^{k,c}(\underset{\sim}{x})\cdot\hat{\pi}^{b}_{j}(\underset{\sim}{x})\Big) +$$

$$+ igf^{abc}R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\hat{F}^{lj,a}(\underset{\sim}{x}')\cdot\Big(\hat{\pi}^{c}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{km,b}(\underset{\sim}{x})\Big) + i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\partial^{k}_{x}\hat{F}^{jl,a}(\underset{\sim}{x}) +$$

$$+ i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\Big(gf^{acb}\hat{A}^{k,c}(\underset{\sim}{x})\cdot\hat{F}^{jl,b}(\underset{\sim}{x})\Big) + igf^{abc}R_{m}(\underset{\sim}{x};\underset{\sim}{x}')\Big(\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\cdot\hat{F}^{km,b}(\underset{\sim}{x})\Big)\cdot\hat{F}^{lj,c}(\underset{\sim}{x}') =$$

$$= -i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\;\hat{F}^{lj,a}(\underset{\sim}{x})\cdot\partial^{k}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x}) - i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\;\hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\partial^{k}_{x}\hat{F}^{lj,a}(\underset{\sim}{x}) \;,$$

onde fizemos uso de (5.13) e (4.2). Assim, ficamos com

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\Theta}^{0\ell}(\underset{\sim}{x}')\right]_{t.p.b.} = -i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}\left(\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\hat{F}^{\ell j,a}(\underset{\sim}{x})\right) \qquad . \qquad (5.73)$$

Os resultados (5.71)-(5.73) somados constituem nosso resultado final para o comutador $[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0\ell}(\underset{\sim}{x}')]$. É direto mostrar (ver Apêndice H) que esse resultado pode ser convenientemente escrito na forma

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\Theta}^{0\ell}(\underset{\sim}{x}')\right] &= \frac{i}{2}\left[\hat{\Theta}^{0\ell}(\underset{\sim}{x})\partial^{k}_{x} - \hat{\Theta}^{0\ell}(\underset{\sim}{x}')\partial^{k}_{x'} + \hat{\Theta}^{0k}(\underset{\sim}{x})\partial^{\ell}_{x} - \hat{\Theta}^{0k}(\underset{\sim}{x}')\partial^{\ell}_{x'}\right]\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \\ &+ \frac{i}{2}\left[\partial^{\ell}_{x}\hat{\Theta}^{0k}(\underset{\sim}{x}) - \partial^{k}_{x}\hat{\Theta}^{0\ell}(\underset{\sim}{x})\right]\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') \quad . \qquad (5.74)\end{aligned}}$$

Tendo computado a álgebra das densidades de momentum $\hat{\Theta}^{0\mu}$ (ver (5.43), (5.69) e (5.74)) é direto obter, levando em conta (5.49), a correspondente álgebra dos geradores definidos em (5.44)-(5.48) (ver Apêndice I):

$$[\hat{P}^{\mu}, \hat{P}^{\nu}] = 0 \quad , \qquad (5.75a)$$

$$[\hat{P}^{\mu}, \hat{J}^{\rho\sigma}] = i(g^{\mu\rho}\hat{P}^{\sigma} - g^{\mu\sigma}\hat{P}^{\rho}) \quad , \qquad (5.75b)$$

$$[\hat{J}^{\nu\mu}, \hat{J}^{\rho\lambda}] = -i(g^{\rho\mu}\hat{J}^{\lambda\nu} + g^{\mu\lambda}\hat{J}^{\nu\rho} + g^{\lambda\nu}\hat{J}^{\rho\mu} + g^{\nu\rho}\hat{J}^{\mu\lambda}) \,. \quad (5.75c)$$

Esta é a álgebra de Poincaré. Portanto, por construção explícita, demonstramos que a QCD no gauge superaxial é uma teoria totalmente invariante de Poincaré.

Como teste final da consistência da formulação de gauge superaxial da QCD verificamos a seguir a álgebra das cargas de cor. Com esse propósito, introduzimos as definições das componentes temporal e espacial da densidade de corrente de cor total $\hat{j}^{\mu,a}$, respectivamente, dadas por

$$\hat{j}^{0,a} = -i\hat{\pi}_{\psi}\cdot\frac{\lambda^a}{2}\hat{\psi} + f^{abc}\hat{A}^{j,b}\cdot\hat{\pi}^{c}_{j} \quad , \qquad (5.76)$$

$$\hat{j}^{k,a} = -i\hat{\pi}_{\psi}\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi} + f^{abc}\hat{A}^{0,b}\cdot\hat{\pi}^{c}_{k} + f^{abc}\hat{A}^{j,b}\hat{F}^{kj,c} \quad . \qquad (5.77)$$

Conforme esperado, $\hat{j}^{\mu,a}$ obedece a lei de conservação local

$$\partial_\mu \hat{j}^{\mu,a} = 0 \quad . \qquad (5.78)$$

A prova de (5.78) envolve um cálculo extenso cujos detalhes são dados no Apêndice J. Por outro lado, podemos encontrar facilmente a relação que define a álgebra das densidades de corrente (5.76) no gauge superaxial

$$[\hat{j}^{0,a}(\underset{\sim}{x}), \hat{j}^{0,b}(\underset{\sim}{y})] = i f^{abc}[\hat{j}^{0,c}(\underset{\sim}{x}) + \hat{j}^{0,c}(\underset{\sim}{y})]\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) -$$

$$- \frac{i}{g} f^{abc}[\hat{\pi}^{c}_{j}(\underset{\sim}{x}) - \hat{\pi}^{c}_{j}(\underset{\sim}{y})]\partial^{j}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad . \qquad (5.79)$$

Prova de (5.79)

Desde (4.2c) é evidente que

$$[\partial_x^j \hat{\pi}_j^a(\underset{\sim}{x}), \hat{\pi}_k^b(\underset{\sim}{y})] = i g f^{acb}\Big[(\partial_x^j \hat{\pi}_j^c(\underset{\sim}{x})) R_k(\underset{\sim}{y};\underset{\sim}{x}) + \hat{\pi}_j^c(\underset{\sim}{x}) \partial_x^j R_k(\underset{\sim}{y};\underset{\sim}{x}) +$$

$$+ \hat{\pi}_k^c(\underset{\sim}{y}) \partial_x^j R_j(\underset{\sim}{x};\underset{\sim}{y})\Big] = i g f^{acb}\Big[(\partial_x^j \hat{\pi}_j^c(\underset{\sim}{x})) R_k(\underset{\sim}{y};\underset{\sim}{x}) +$$

$$+ \hat{\pi}_j^c(\underset{\sim}{x}) \partial_x^j R_k(\underset{\sim}{y};\underset{\sim}{x}) + \hat{\pi}_k^c(\underset{\sim}{y}) \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})\Big] ,$$

usando (3.49). A expressão acima implica, então,

$$[\partial_x^j \hat{\pi}_j^a(\underset{\sim}{x}), \partial_y^k \hat{\pi}_k^b(\underset{\sim}{y})] = i g f^{acb}\Big[(\partial_x^j \hat{\pi}_j^c(\underset{\sim}{x})) \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + (\partial_y^k \hat{\pi}_k^c(\underset{\sim}{y}))\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) +$$

$$+ \hat{\pi}_j^c(\underset{\sim}{x}) \partial_x^j \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + \hat{\pi}_k(\underset{\sim}{y}) \partial_y^k \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})\Big] . \qquad (5.80)$$

Notando que (5.76) permite escrever a lei de Gauss (4.3) como

$$\partial^j \hat{\pi}_j^a = -g(f^{abc} \hat{A}^{j,b} \hat{\pi}_j^c - i \hat{\pi}_\psi \cdot \frac{\lambda^a}{2} \hat{\psi}) = -g \hat{J}^{0,a} \quad , \qquad (5.81)$$

reescrevemos (5.80) como segue

$$g^2[\hat{J}^{0,a}(\underset{\sim}{x}), \hat{J}^{0,b}(\underset{\sim}{y})] = -i g^2 f^{acb}[\hat{J}^{0,c}(\underset{\sim}{x}) + \hat{J}^{0,c}(\underset{\sim}{y})]\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) +$$

$$+ i g f^{acb}[\hat{\pi}_j^c(\underset{\sim}{x}) - \hat{\pi}_j^c(\underset{\sim}{y})] \partial_x^j \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) \quad ,$$

de onde obtemos (5.79) (q.e.d.). A relação (5.79) mostra que $\hat{j}^{0,a}$ possui um caráter localizado em relação a comutadores a tempos iguais. É instrutivo comparar tal relação com o correspondente resultado no gauge de Coulomb, onde $[\hat{j}^{0,a}(\underset{\sim}{x}), \hat{j}^{0,b}(\underset{\sim}{y})]$ exibe uma dependência não-local em relação às variáveis espaciais [1]. Agora, integrando (5.79) sobre $\underset{\sim}{y}$ em todo o espaço, encontramos

$$[\hat{j}^{0,a}_{(\underset{\sim}{x})}, \int d^3y\, \hat{j}^{0,b}_{(\underset{\sim}{y})}] = i f^{abc} [\hat{j}^{0,c}_{(\underset{\sim}{x})} + \hat{j}^{0,c}_{(\underset{\sim}{x})}] +$$

$$+ \frac{i}{g} f^{abc} \partial^j_x \hat{\pi}^c_j (\underset{\sim}{x})$$

a qual, desde (5.81), conduz a

$$[\hat{j}^{0,a}_{(\underset{\sim}{x})}, \hat{T}^b] = i f^{abc} \hat{j}^{0,c}_{(\underset{\sim}{x})} \quad , \qquad (5.82)$$

onde

$$\hat{T}^a \equiv \int d^3x\, \hat{j}^{0,a}_{(\underset{\sim}{x})} \qquad (5.83)$$

são as cargas de cor. Uma integração adicional sobre $\underset{\sim}{x}$, em (5.82), confirma a álgebra das cargas de cor

$$[\hat{T}^a, \hat{T}^b] = i f^{abc} \hat{T}^c \quad . \qquad (5.84)$$

Por outro lado, desde (5.55c) e (1.8) é claro que

$$[\hat{J}^{0k}, \hat{\pi}^{a}_{j}(\underset{\sim}{x})] = -ix^{0}\left(\partial^{k}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x}) + gf^{acb}\hat{\pi}^{c}_{j}(\underset{\sim}{x}).\hat{B}^{k,b}(\underset{\sim}{x})\right) -$$

$$-ix^{k}\hat{D}^{\ell,ab}(\underset{\sim}{x})\hat{F}^{j\ell,b}(\underset{\sim}{x}) - igx^{k}\hat{J}^{j,a}(\underset{\sim}{x}) + igf^{acb}\hat{\pi}^{c}_{j}(\underset{\sim}{x}).\hat{E}^{k,b}(\underset{\sim}{x}), \quad (5.85)$$

onde

$$\hat{J}^{j,a} \equiv -i\hat{\pi}_{\psi}\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi} \quad . \qquad (5.86)$$

Desde (5.85) e (5.81), segue

$$[\hat{J}^{0k}, \partial^{x}_{j}\hat{\pi}^{a}_{j}(\underset{\sim}{x})] = [\hat{J}^{0k}, g\hat{\mathcal{J}}^{0,a}(\underset{\sim}{x})] =$$

$$= -ix^{0}\left[g\,\partial^{k}_{x}\hat{\mathcal{J}}^{0,a}(\underset{\sim}{x}) + gf^{acb}\partial^{x}_{j}\left(\hat{\pi}^{c}_{j}(\underset{\sim}{x}).\hat{B}^{k,b}(\underset{\sim}{x})\right)\right] -$$

$$-i\hat{D}^{\ell,ab}(\underset{\sim}{x})\hat{F}^{k\ell,b}(\underset{\sim}{x}) - igf^{acb}x^{k}\partial^{x}_{j}\left(\hat{A}^{\ell,c}(\underset{\sim}{x})\hat{F}^{j\ell,b}(\underset{\sim}{x})\right) - ig\hat{J}^{k,a}(\underset{\sim}{x}) -$$

$$-igx^{k}\partial^{x}_{j}\hat{J}^{j,a}(\underset{\sim}{x}) + igf^{acb}\partial^{x}_{j}\left(\hat{\pi}^{c}_{j}(\underset{\sim}{x}).\hat{E}^{k,b}(\underset{\sim}{x})\right) . \qquad (5.87)$$

Integrando ambos lados de (5.87) sobre $\underset{\sim}{x}$ em todo o espaço, levando em conta (4.5), obtemos

$$g[\hat{J}^{0k}, \hat{T}^{a}] = igx^{0}\int d^{3}x\,\partial^{x}_{k}\hat{\mathcal{J}}^{0,a}(\underset{\sim}{x}) - igf^{acb}x^{0}\int d^{3}x\,\partial^{x}_{j}\left(\hat{\pi}^{c}_{j}(\underset{\sim}{x}).\hat{B}^{k,b}(\underset{\sim}{x})\right) +$$

$$+i\int d^{3}x\,\partial^{x}_{\ell}\hat{F}^{k\ell,a}(\underset{\sim}{x}) - igf^{acb}\int d^{3}x\,\hat{A}^{\ell,c}(\underset{\sim}{x})\hat{F}^{k\ell,b}(\underset{\sim}{x}) -$$

$$- igf^{acb}\int d^3x\, x^k \partial^x_j \left(\hat{A}^{l,c}_{(\underset{\sim}{x})} \hat{F}^{jl,b}_{(\underset{\sim}{x})}\right) - ig\int d^3x\, \hat{J}^{k,a}_{(\underset{\sim}{x})} -$$

$$- ig\int d^3x\, x^k \partial^x_j \hat{J}^{j,a}_{(\underset{\sim}{x})} + igf^{acb}\int d^3x\, \partial^x_j\left(\hat{\Pi}^c_{j(\underset{\sim}{x})}.\hat{E}^{k,b}_{(\underset{\sim}{x})}\right) =$$

$$= - igf^{acb}\int d^3x\, \hat{A}^{l,c}_{(\underset{\sim}{x})} \hat{F}^{kl,b}_{(\underset{\sim}{x})} + igf^{acb}\int d^3x\, \hat{A}^{l,c}_{(\underset{\sim}{x})} \hat{F}^{kl,b}_{(\underset{\sim}{x})} -$$

$$- ig\int d^3x\, \hat{J}^{k,a}_{(\underset{\sim}{x})} + ig\int d^3x\, \hat{J}^{k,a}_{(\underset{\sim}{x})} = 0 \quad ,$$

ou seja,

$$[\hat{J}^{0k}, \hat{T}^a] = 0 \quad , \qquad (5.88)$$

o que expressa a invariança de Lorentz das cargas de cor.

VI. O DETERMINANTE DE FADDEEV-POPOV

Embora tratemos nesta tese da quantização operatorial da cromodinâmica no gauge superaxial, consideramos neste capítulo o determinante da matriz Q tendo em vista uma possível quantização da teoria através do formalismo da integral funcional.

Tem sido repetidamente enunciado na literatura que no (não-completamente fixado) gauge axial $\hat{A}^{3,a}(\underset{\sim}{x}) = 0$, o determinante de Faddeev-Popov não depende das variáveis de campo [46]. De fato, a ausência de fantasmas de Faddeev-Popov é considerada uma das vantagens da formulação de gauge axial das teorias de campos de gauge não-Abelianos [25]. No presente capítulo investigamos se o mesmo vale para a QCD no gauge superaxial. Este é um problema não trivial porque da simples inspeção de (3.1) vemos que alguns dos elementos da matriz Q dependem das variáveis de campo.

VI.1 Cálculo do Determinante de Faddeev-Popov pelo Método de Discretização

A partir de (2.39) e (3.1) segue que a estrutura total da matriz antissimétrica Q é a seguinte:

$$\underset{JK}{Q^{ab}}(\underset{\sim}{x};\underset{\sim}{y}) = \left[\begin{array}{cccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{1,7} & 0 & 0 & 0 & Q^{ab}_{1,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & 0 & Q^{ab}_{2,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{3,9} & 0 & Q^{ab}_{3,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & 0 & Q^{ab}_{4,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & 0 & Q^{ab}_{5,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & 0 & Q^{ab}_{6,12} & 0 & 0 \\
-Q^{ab}_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{7,10} & Q^{ab}_{7,11} & Q^{ab}_{7,12} & 0 & 0 \\
0 & -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & Q^{ab}_{8,11} & 0 & Q^{ab}_{8,13} & 0 \\
0 & 0 & -Q^{ab}_{3,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{9,11} & 0 & 0 & Q^{ab}_{9,14} \\
0 & 0 & 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{7,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 & 0 & 0 & 0 \\
-Q^{ab}_{1,11} & -Q^{ab}_{2,11} & -Q^{ab}_{3,11} & 0 & 0 & 0 & -Q^{ab}_{7,11} & -Q^{ab}_{8,11} & -Q^{ab}_{9,11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q^{ab}_{6,12} & -Q^{ab}_{7,12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q^{ab}_{9,14} & 0 & 0 & 0 & 0 & 0
\end{array}\right], \qquad (6.1)$$

onde lembramos que os índices de vínculo (J,K) vão de 1 a 14 enquanto os índices de cor vão de 1 a N^2-1. Como se vê em(6.1), denotamos por $Q_{J,K}$ a submatriz associada aos índices de vínculo J e K. Com a finalidade de definir precisamente o produto matricial de submatrizes e de aplicar o teorema de Laplace (ver adiante) no cálculo do determinante de Q, discretizaremos os índices espaciais contínuos $(\underset{\sim}{x},\underset{\sim}{y})$ com o auxílio da seguinte rede espacial:

$$\underset{\sim}{x} = (x^1, x^2, x^3) \longleftrightarrow \alpha_i \quad \text{com} \quad i = 1, \ldots, 3n \quad , \qquad (6.2a)$$

$$\underset{\sim}{y} = (y^1, y^2, y^3) \longleftrightarrow \beta_j \quad \text{com} \quad j = 1, \ldots, 3n \quad , \qquad (6.2b)$$

sendo que

$$\lim_{n\to\infty} \begin{Bmatrix} \alpha_i \\ \beta_j \end{Bmatrix} \to \begin{cases} \begin{Bmatrix} x^1 \\ y^1 \end{Bmatrix} ; \text{qdo. } i, j = 1, \ldots, n \ , \\ \begin{Bmatrix} x^2 \\ y^2 \end{Bmatrix} ; \text{qdo. } i, j = n+1, \ldots, 2n \ , \\ \begin{Bmatrix} x^3 \\ y^3 \end{Bmatrix} ; \text{qdo. } i, j = 2n+1, \ldots, 3n \ . \end{cases} \qquad (6.3)$$

Com isso, teremos por exemplo

$$\overset{(3)}{\delta}(\underset{\sim}{x} - \underset{\sim}{y}) \equiv \lim_{\substack{n\to\infty \\ \epsilon\to 0}} \prod_{i,j=1}^{3n} \frac{\delta_{\alpha_i \beta_j}}{\epsilon} \qquad (6.4)$$

subentendendo (6.4) idêntica a

$$\delta(x^1 - y^1)\delta(x^2 - y^2)\delta(x^3 - y^3) \equiv \lim_{\substack{n\to\infty \\ \epsilon\to 0}} \prod_{i,j=1}^{n} \frac{\delta_{\alpha_i \beta_j}}{\epsilon} \prod_{k,\ell=n+1}^{2n} \frac{\delta_{\alpha_k \beta_\ell}}{\epsilon} \prod_{m,n=2n+1}^{3n} \frac{\delta_{\alpha_m \beta_n}}{\epsilon} . \quad (6.5)$$

Para calcular o det Q lançaremos mão do seguinte teorema devido a Laplace [47].

Teorema: Todo determinante é igual à soma dos produtos obtidos multiplicando-se todos os menores de ordem h que se podem formar com h filas paralelas, pelos seus adjuntos respectivos. Aqui, por menor de ordem h queremos dizer o determinante da submatriz de ordem h obtida a partir da escolha de h linhas (colunas) paralelas das quais retiramos m-h colunas (linhas) paralelas quaisquer, onde m é a ordem da matriz total. O *adjunto* correspondente a um menor é seu menor complementar multiplicado por +1 (-1) se a soma dos índices de ordem das linhas e colunas deste menor complementar for par (ímpar). Isto é, o *adjunto* é o menor complementar com seu respectivo *sinal*.

Exemplo:

Calculemos o det da seguinte matriz 4 × 4 onde, evidentemente, a,b,c,d,e,f,g são números-c.

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & c & 0 \\ d & 0 & 0 & f \\ 0 & e & g & 0 \end{bmatrix} . \qquad (6.6)$$

a) Escolhemos, por exemplo, as duas primeiras linhas para expandir det A por menores de ordem 2.

Menores que se podem formar com essas linhas:

$$\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} \; ; \; \begin{vmatrix} a & 0 \\ 0 & c \end{vmatrix} .$$

Seus menores complementares, respectivos,

$$\begin{vmatrix} 0 & f \\ g & 0 \end{vmatrix} \; ; \; \begin{vmatrix} 0 & f \\ e & 0 \end{vmatrix} .$$

Os sinais, respectivos, destes últimos são

$$(3+4)+(3+4) = 14 \text{ (par)} \Rightarrow (+) \; ; \; (3+4)+(2+4) = 13 \text{(ímpar)} \Rightarrow (-) .$$

Logo,

$$\det A = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} \cdot \begin{vmatrix} 0 & f \\ g & 0 \end{vmatrix} - \begin{vmatrix} a & 0 \\ 0 & c \end{vmatrix} \cdot \begin{vmatrix} 0 & f \\ e & 0 \end{vmatrix} = -abgf + acef. \quad (6.7)$$

b) Uma escolha melhor é expandir det A pela 2ª e 3ª colunas de onde obtemos diretamente (6.7)

$$\det A = \begin{vmatrix} b & c \\ e & g \end{vmatrix} \cdot \left(- \begin{vmatrix} a & 0 \\ d & f \end{vmatrix} \right) = (bg - ec).(-af) = -bgaf + ecaf. \quad (6.8)$$

Por outro lado, para verificar o teorema, se expandíssemos det A desde (6.6) pela 1ª linha (expansão de Laplace), obteríamos, aplicando a regra de Sarrus,

$$\det A = (-1)^{1+1} a \cdot \begin{vmatrix} b & c & 0 \\ 0 & 0 & f \\ e & g & 0 \end{vmatrix} = a \left| \begin{array}{ccc|cc} b & c & 0 & b & c \\ 0 & 0 & f & 0 & 0 \\ e & g & 0 & e & g \end{array} \right| =$$

$$= a(cfe - gfb) = acfe - agfb \qquad (6.9)$$

Claramente, (6.9) reproduz (6.7) e (6.8). Do exposto, vemos que o teorema acima reduz o desenvolvimento de um determinante ao de outros de ordem inferior. Para fazer o desenvolvimento pelos menores de h linhas (colunas) é conveniente escolher aquelas em que apareça o maior número de colunas (linhas) formadas por elementos nulos pois todo menor em que figure uma destas colunas (linhas) é zero.

Calculamos então o determinante de Q desde (6.1) expandindo o mesmo pelos menores das últimas (3n) linhas. Conforme pode-se ver em (6.1) o único menor diferente de zero corresponde à submatriz $-Q^{ab}_{9,14}$, o qual denotamos por $-|Q^{ab}_{9,14}|$. Seu correspondente menor complementar segue diretamente de (3.1).

Para calcular o sinal respectivo teríamos de fixar um dado valor para n. Entretanto, como veremos a seguir, os sinais dos adjuntos não serão necessários em nossa análise da dependência de det Q em relação aos campos. Por outro lado, para tornar clara a aplicação do teorema enunciado, fixaremos n um número par. Assim, expandindo o det Q pelas últimas 3n linhas

$$\det Q = |Q^{ab}_{9,14}| \cdot \left|\begin{array}{ccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{1,7} & 0 & 0 & Q^{ab}_{1,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & Q^{ab}_{2,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{3,11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & 0 & Q^{ab}_{4,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & 0 & Q^{ab}_{5,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & 0 & Q^{ab}_{6,12} & 0 & 0 \\
-Q^{ab}_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{7,10} & Q^{ab}_{7,11} & Q^{ab}_{7,12} & 0 & 0 \\
0 & -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & Q^{ab}_{8,11} & 0 & Q^{ab}_{8,13} & 0 \\
0 & 0 & -Q^{ab}_{3,9} & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{9,11} & 0 & 0 & Q^{ab}_{9,14} \\
0 & 0 & 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{7,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 & 0 & 0 \\
-Q^{ab}_{1,11} & -Q^{ab}_{2,11} & -Q^{ab}_{3,11} & 0 & 0 & 0 & -Q^{ab}_{7,11} & -Q^{ab}_{8,11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q^{ab}_{6,12} & -Q^{ab}_{7,12} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 & 0 & 0
\end{array}\right| , \quad (6.10)$$

a menos do sinal. Apliquemos o mesmo teorema ao segundo determinante no lado direito de (6.10). Expandindo tal menor pelas últimas (3n) colunas vemos que o único menor diferente de zero na expansão corresponde à submatriz $Q^{ab}_{9,14}$. Logo,

$$\det Q = |Q^{ab}_{9,14}|^{2} \cdot \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{1,7} & 0 & 0 & Q^{ab}_{1,11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & Q^{ab}_{2,11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{3,11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & 0 & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{7,10} & Q^{ab}_{7,11} & Q^{ab}_{7,12} & 0 \\ 0 & -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & Q^{ab}_{8,11} & 0 & Q^{ab}_{8,13} \\ 0 & 0 & 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{7,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 & 0 \\ -Q^{ab}_{1,11} & -Q^{ab}_{2,11} & -Q^{ab}_{3,11} & 0 & 0 & 0 & -Q^{ab}_{7,11} & -Q^{ab}_{8,11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q^{ab}_{6,12} & -Q^{ab}_{7,12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 & 0 \end{vmatrix} . \qquad (6.11)$$

Expandindo agora o menor em (6.11) pelas 3(3n)-2(3n) linhas entre as linhas de índices 2(3n) e 3(3n) incluindo a última, encontramos

$$\det Q = |Q^{ab}_{9,14}|^{2} \cdot |Q^{ab}_{3,11}| \cdot \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{1,7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{7,10} & Q^{ab}_{7,12} & 0 \\ 0 & -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & 0 & Q^{ab}_{8,13} \\ 0 & 0 & 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{7,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 \\ -Q^{ab}_{1,11} & -Q^{ab}_{2,11} & -Q^{ab}_{3,11} & 0 & 0 & 0 & -Q^{ab}_{7,11} & -Q^{ab}_{8,11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q^{ab}_{6,12} & -Q^{ab}_{7,12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 \end{vmatrix} , \qquad (6.12)$$

Podemos continuar o processo, expandindo o 3º determinante em (6.12) pelas 3(3n)-2(3n) colunas entre as colunas de índices 2(3n) e 3(3n) incluída a última, e obter

$$\det Q = |Q^{ab}_{9,14}|^2 \cdot |Q^{ab}_{3,11}|^2 \cdot \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & Q^{ab}_{1,7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{7,10} & Q^{ab}_{7,12} & 0 \\ 0 & -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & 0 & Q^{ab}_{8,13} \\ 0 & 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{7,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q^{ab}_{6,12} & -Q^{ab}_{7,12} & 0 & 0 & 0 & 0 \\ 0 & 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 \end{vmatrix} . \quad (6.13)$$

Expandindo o determinante em (6.13) pelas primeiras (3n) linhas, ficamos com

$$\det Q = |Q^{ab}_{9,14}|^2 \cdot |Q^{ab}_{3,11}|^2 \cdot |Q^{ab}_{1,7}| \cdot \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{1,7} & 0 & 0 & 0 & 0 & 0 & Q^{ab}_{7,10} & Q^{ab}_{7,12} & 0 \\ 0 & -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & 0 & Q^{ab}_{8,13} \\ 0 & 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q^{ab}_{6,12} & 0 & 0 & 0 & 0 \\ 0 & 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 \end{vmatrix} , \quad (6.14)$$

Expandindo o último menor no lado direito de (6.14) pelas primeiras (3n) colunas, obtemos

$$\det Q = |Q^{ab}_{9,14}|^2 . |Q^{ab}_{3,11}|^2 . |Q^{ab}_{1,7}|^2 . \begin{vmatrix} 0 & 0 & 0 & 0 & Q^{ab}_{2,8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{2,8} & 0 & 0 & 0 & 0 & Q^{ab}_{8,10} & 0 & Q^{ab}_{8,13} \\ 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & -Q^{ab}_{8,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q^{ab}_{6,12} & 0 & 0 & 0 & 0 \\ 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & -Q^{ab}_{8,13} & 0 & 0 & 0 \end{vmatrix} \quad (6.15)$$

Desenvolvendo o menor em (6.15) pelas primeiras (3n) linhas, encontramos

$$\det Q = |Q^{ab}_{9,14}|^2 . |Q^{ab}_{3,11}| |Q^{ab}_{1,7}|^2 . |Q^{ab}_{2,8}| . \begin{vmatrix} 0 & 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{2,8} & 0 & 0 & 0 & Q^{ab}_{8,10} & 0 & Q^{ab}_{8,13} \\ 0 & -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q^{ab}_{6,12} & 0 & 0 & 0 \\ 0 & -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & 0 & 0 \end{vmatrix} . \quad (6.16)$$

Expandindo agora o último menor em (6.16) pelas primeiras (3n) colunas, ficamos com

$$\det Q = |Q^{ab}_{9,14}|^2 |Q^{ab}_{3,11}|^2 . |Q^{ab}_{1,7}|^2 . |Q^{ab}_{2,8}|^2 . \begin{vmatrix} 0 & 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ 0 & 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ 0 & 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & -Q^{ab}_{6,10} & 0 & 0 & 0 \\ 0 & 0 & -Q^{ab}_{6,12} & 0 & 0 & 0 \\ -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & 0 & 0 \end{vmatrix} , \quad (6.17)$$

Agora, expandindo o menor em (6.17) pelas 5(3n)-4(3n) linhas que se localizam entre as linhas de índices 4(3n) e 5(3n) incluindo esta última, obtemos

$$\det Q = |Q^{ab}_{9,14}|^2 |Q^{ab}_{3,11}|^2 |Q^{ab}_{1,7}|^2 |Q^{ab}_{2,8}|^2 |Q^{ab}_{6,12}| \left| \begin{array}{c:c:c:c:c} 0 & 0 & Q^{ab}_{4,10} & 0 & Q^{ab}_{4,13} \\ \hdashline 0 & 0 & Q^{ab}_{5,10} & 0 & Q^{ab}_{5,13} \\ \hdashline 0 & 0 & Q^{ab}_{6,10} & Q^{ab}_{6,12} & 0 \\ \hdashline -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & 0 & 0 & 0 \\ \hdashline -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & 0 & 0 & 0 \end{array} \right| , \qquad (6.18)$$

Desenvolvendo o último menor em (6.18) pelas 4(3n)-3(3n) colunas que se localizam entre as colunas de índices 3(3n) e 4(3n) incluída a última, chegamos a

$$\det Q = |Q^{ab}_{9,14}|^2 |Q^{ab}_{3,11}|^2 |Q^{ab}_{1,7}|^2 |Q^{ab}_{2,8}|^2 |Q^{ab}_{6,12}|^2 \cdot \left| \begin{array}{c:c:c:c} 0 & 0 & Q^{ab}_{4,10} & Q^{ab}_{4,13} \\ \hdashline 0 & 0 & Q^{ab}_{5,10} & Q^{ab}_{5,13} \\ \hdashline -Q^{ab}_{4,10} & -Q^{ab}_{5,10} & & \\ \hdashline -Q^{ab}_{4,13} & -Q^{ab}_{5,13} & & \end{array} \right| =$$

$$= |Q^{ab}_{9,14}|^2 |Q^{ab}_{3,11}|^2 |Q^{ab}_{1,7}|^2 |Q^{ab}_{2,8}|^2 |Q^{ab}_{6,12}|^2 \cdot \left(\left| \begin{array}{c:c} Q^{ab}_{4,10} & Q^{ab}_{4,13} \\ \hdashline Q^{ab}_{5,10} & Q^{ab}_{5,13} \end{array} \right| \right)^2 , \qquad (6.19)$$

a menos de um sinal, o qual conforme adiantamos não é relevante. Dado que por definição (ver eq.(2.39)) Q é uma matriz totalmente antissimétrica seu determinante pode sempre ser escrito [48]

$$\det Q = [pf\,Q]^2 , \qquad (6.20)$$

onde pf Q é o pfaffiano associado à matriz Q. Note-se que o lado direito de (6.19) é da forma (6.20).

Como se pode ver a partir de (3.1), somente o último fator no lado direito de (6.19) poderia eventualmente depender das variáveis de campo (através da submatriz $Q^{ab}_{5,10}$). Iremos, então, focalizar nossa atenção sobre esse fator. Não é difícil verificar que

$$\left[\begin{array}{c|c} Q^{ab}_{4,10} & Q^{ab}_{4,13} \\ \hline Q^{ab}_{5,10} & Q^{ab}_{5,13} \end{array}\right] = \sum_{c=1}^{N^2-1} \left[\begin{array}{c|c} I\delta^{ac} & (Q_{4,13}\cdot Q^{-1}_{5,13})^{ac} \\ \hline 0 & I\delta^{ac} \end{array}\right] \cdot \left[\begin{array}{c|c} Q^{cb}_{4,10} - (Q_{4,13}.Q^{-1}_{5,13}.Q_{5,10})^{cb} & 0 \\ \hline Q^{cb}_{5,10} & Q^{cb}_{5,13} \end{array}\right] ,$$

(6.21)

onde I é a matriz identidade (3n) × (3n) do espaço de Minkowski. Note-se que, pela discretização adotada, os produtos de submatrizes no lado direito de (6.21) estão bem definidos matematicamente.

Prova de (6.21)

Mostramos aqui que o lado direito de (6.21) é idêntico ao lado esquerdo. Para isso, definimos

$$(Q_{4,13}\cdot Q^{-1}_{5,13})^{ac}_{3n\times 3n} \equiv A^{ac}_{3n\times 3n} , \qquad (6.22)$$

$$\left[Q^{cb}_{4,10} - (Q_{4,13}\cdot Q^{-1}_{5,13}\cdot Q_{5,10})^{cb}\right]_{3n\times 3n} \equiv B^{cb}_{3n\times 3n} , \qquad (6.23)$$

de modo que o lado direito de (6.21) pode ser escrito como

$$\sum_{c=1}^{N^2-1}\left[\begin{array}{c|c}\delta^{ac}(I)_{3n\times3n} & A^{ac}_{3n\times3n}\\\hline 0 & \delta^{ac}(I)_{3n\times3n}\end{array}\right]\cdot\left[\begin{array}{c|c}B^{cb}_{3n\times3n} & 0\\\hline (Q_{5,10})^{cb}_{3n\times3n} & (Q_{5,13})^{cb}_{3n\times3n}\end{array}\right]=$$

$$=\sum_{c=1}^{N^2-1}\begin{matrix}3n\left\{\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right.\\3n\left\{\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right.\end{matrix}\left[\begin{array}{c|c}\overbrace{\begin{matrix}\delta^{ac} & 0 & \cdots & 0\\ 0 & \delta^{ac} & & \\ \vdots & & \ddots & \\ 0 & & & \delta^{ac}\end{matrix}}^{3n} & \begin{matrix}A^{ac}_{11} & A^{ac}_{12} & \cdots & A^{ac}_{1,3n}\\ A^{ac}_{21} & A^{ac}_{22} & \ddots & \\ \vdots & & & \\ A^{ac}_{3n,1} & & & A^{ac}_{3n,3n}\end{matrix}\\\hline \underbrace{\begin{matrix}0 & \cdots & 0\\ \vdots & & \vdots\\ 0 & \cdots & 0\end{matrix}}_{3n} & \underbrace{\begin{matrix}\delta^{ac} & 0 & \cdots & 0\\ 0 & \delta^{ac} & & \\ \vdots & & \ddots & \\ 0 & & & \delta^{ac}\end{matrix}}_{3n}\end{array}\right]\left[\begin{array}{c|c}\begin{matrix}B^{cb}_{11} & \cdots & B^{cb}_{1,3n}\\ \vdots & \ddots & \\ B^{cb}_{3n,1} & & B^{cb}_{3n,3n}\end{matrix} & \begin{matrix}0 & \cdots & 0\\ \vdots & & \vdots\\ 0 & \cdots & 0\end{matrix}\\\hline \begin{matrix}(Q_{5,10})^{cb}_{1,1} & \cdots & (Q_{5,10})^{cb}_{1,3n}\\ \vdots & \ddots & \\ (Q_{5,10})^{cb}_{3n,1} & & (Q_{5,10})^{cb}_{3n,3n}\end{matrix} & \begin{matrix}(Q_{5,13})^{cb}_{1,1} & \cdots & (Q_{5,13})^{cb}_{1,3n}\\ \vdots & & \\ (Q_{5,13})^{cb}_{3n,1} & \cdots & (Q_{5,13})^{cb}_{3n,3n}\end{matrix}\end{array}\right].$$

$$(6.24)$$

Tomemos, por exemplo, o elemento $(\;)_{11}$ da matriz produto desde (6.24) (índices de cor repetidos $\Longrightarrow \Sigma$):

$$(\;)_{11} = B^{ab}_{11} + \sum_{m=1}^{3n} A^{ac}_{1m}(Q_{5,10})^{cb}_{m1} \qquad (6.25)$$

Mas, de acordo com (6.22) e (6.23), o lado direito de (6.25) é igual a

$$(\;)_{11} = \left[Q^{ab}_{4,10} - (Q_{4,13}\cdot Q^{-1}_{5,13}\cdot Q_{5,10})^{ab}\right]_{11} + \sum_{n=1}^{3n}(Q_{4,13}\cdot Q^{-1}_{5,13})^{ac}_{1m}(Q_{5,10})^{cb}_{m1}$$

$$= (Q_{4,10})^{ab}_{11} - \cancel{(Q_{4,13}\cdot Q^{-1}_{5,13}\cdot Q_{5,10})^{ab}_{11}} + \cancel{(Q_{4,13}\cdot Q^{-1}_{5,13}\cdot Q_{5,10})^{ab}_{11}} =$$

$$= (Q_{4,10})^{ab}_{11} \qquad (6.26)$$

que coincide com o elemento $(\ )_{11}$ da matriz no lado esquerdo de (6.21). De modo similar, pode-se provar que <u>todos</u> os elementos da matriz produto no lado direito de (6.24) coincidem com os elementos da matriz no lado esquerdo de (6.24) (q.e.d.).

A importância da igualdade (6.21) é a de fornecer diretamente (det(A.B) = det A. det B)

$$\det\left[\begin{array}{c|c} Q^{ab}_{4,10} & Q^{ab}_{4,13} \\ \hline Q^{ab}_{5,10} & Q^{ab}_{5,13} \end{array}\right] = \det(Q^{ab}_{5,13})\det\left[Q^{ab}_{4,10}-(Q_{4,13}.Q^{-1}_{5,13}.Q_{5,10})^{ab}\right], \tag{6.27}$$

onde usamos duas vezes o teorema de Laplace anterior e levamos em conta o fato que $\det(\delta^{ab}I) = 1$. Fazemos notar que qualquer dependência nos campos em (6.27) pode somente proceder dos elementos da submatriz $Q_{5,10}$ (ver eqs. (3.1)). A obtenção de (6.27) encerra nossos desenvolvimentos no discreto. Na próxima seção tomamos o limite contínuo nas variáveis espaciais.

VI.2 <u>O Determinante de Faddeev-Popov no Limite Contínuo</u>

Passando ao limite contínuo nas variáveis espaciais, i.e., tomando os limites $n \to \infty$, $\varepsilon \to 0$ (ver (6.2)-(6.5)), é evidente desde (6.27), (3.1c), (3.1d), (3.1f) e (3.1g) que

$$\det\left(Q^{ab}_{4,10}-(Q_{4,13}.Q^{-1}_{5,13}.Q_{5,10})^{ab}\right) =$$

$$= det\left(Q^{ab}_{4,10} - Q^{ac}_{4,13} \cdot (Q^{-1}_{5,13})^{cd} \cdot Q^{db}_{5,10}\right) \longrightarrow det\left[M^{ab}(\underset{\sim}{x},\underset{\sim}{y})\right], \quad (6.28)$$

onde o operador diferencial $M^{ab}(\underset{\sim}{x},\underset{\sim}{y})$ é dado por

$$M^{ab}(\underset{\sim}{x},\underset{\sim}{y}) = \delta^{ab}\partial^3_x \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) +$$

$$+\int d^3u \int d^3z \; \delta(x^2-u^2)\,\Delta(u^1, x^1_{(0)}; x^1)\,\delta(x^3-x^3_{(0)})\,K^{-1}(\underset{\sim}{u},\underset{\sim}{z})\,D^{1,ab}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{y}) \,, \quad (6.29)$$

com

$$K(u,z) = \delta(u^1-z^1)\,\delta(u^2-z^2)\,\Delta(z^3, x^3_{(0)}; u^3) \,. \quad (6.30)$$

A presença da derivada covariante $D^{1,ab}$ no lado direito de (6.29) parece implicar que o operador $M^{ab}(\underset{\sim}{x},\underset{\sim}{y})$ depende da variável de campo $A^{1,a}$. Isto não acontece. Na realidade, depois de calcular a inversa de K*,

$$K^{-1}(\underset{\sim}{u},\underset{\sim}{z}) = \delta(u^1-z^1)\,\delta(u^2-z^2)\,\partial^u_3\,\delta(u^3-z^3) \quad , \quad (6.31)$$

obtemos

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*É imediato verificar que (6.31) $\Longrightarrow$

$$\int d^3z \, K^{-1}(\underset{\sim}{u},\underset{\sim}{z})K(\underset{\sim}{u}',\underset{\sim}{z}) = \delta^{(3)}(\underset{\sim}{u}-\underset{\sim}{u}').$$

$$\int d^3u \int d^3z\, \delta(x^2-u^2)\, \Delta(u^1, x^1_{(0)}; x^1)\, \delta(x^3-x^3_{(0)})\, \delta(u^1-z^1)\, \delta(u^2-z^2)\, \partial^u_3\, \delta(u^3-z^3) .$$

$$\cdot\, D^{1,ab}_{(\underset{\sim}{z})}\, \delta^{(3)}(\underset{\sim}{z}-y) = 0 \quad , \qquad (6.32)$$

devido à integração sobre $\underline{u}^3$. Portanto, $M^{ab}(\underset{\sim}{x},\underset{\sim}{y})$ se reduz a

$$M^{ab}(\underset{\sim}{x}, y) = \delta^{ab}\, \partial^3_x\, \delta^{(3)}(\underset{\sim}{x}-y) \quad . \qquad (6.33)$$

Desde (6.33), (6.28), (6.27) e (6.19) concluímos que o determinante de Faddeev-Popov correspondente à formulação de gauge superaxial da cromodinâmica não depende das variáveis de campo.

VII. CONCLUSÕES

Os resultados obtidos nesta tese mostram que nosso propósito de conseguir uma formulação consistente para a Cromodinâmica Quântica no gauge axial foi alcançado. Conforme vimos, nosso tratamento desenvolveu-se a partir da idéia de fixação completa do gauge. Assim, elaboramos a formulação de gauge superaxial da teoria lançando mão do método operatorial sistemático para quantizar teorias de campos de gauge criado por Dirac [31]. Aqui, com base em nossos resultados, sumarizamos as conclusões centrais do trabalho:

1) Para os campos básicos a translação iDB → CTI é possível, sem ambigüidades, devido à ausência de problemas de ordenamento [41]. Esta é uma das feições mais atrativas da formulação de gauge superaxial de teorias de gauge. Fazemos notar que para a QCD no gauge de Coulomb os correspondentes CTI's são afetados por problemas de ordenamento [1,35]; isto vale mesmo para o setor de carga topológica nula, onde a condição de Coulomb torna-se uma condição aceitável de fixação de gauge [2,3].

2) Todos os CTI's são compatíveis com os campos fermiônicos e intensidades de campo anulando-se no infinito espacial. É claro, o limite $x^3 \to \pm\infty$ está incluído aqui.

3) Da inspeção dos correspondentes CTI's segue que os potenciais de gauge desenvolvem um comportamento assintótico não-trivial fisicamente significativo em $x^3 \to \pm\infty$, como exigido pela presença de fenômenos do tipo instanton [4].

4) Todos os CTI's resultam compatíveis com os vínculos e con

dições de gauge valendo como relações operatoriais fortes. Com isso, o correspondente espaço de Hilbert dos estados físicos está, em princípio, bem definido (sem restrições sobre os vetores de estado).

5) A QCD no gauge superaxial é uma teoria invariante de Poincaré. Conforme demonstramos, nesta formulação não surgem "potenciais" quanto-mecânicos extras do tipo encontrado por Schwinger [1,11,16] para a QCD no gauge de Coulomb. A construção do conjunto inteiro de geradores de Poincaré é direta e a existência desses geradores, como entidades matemáticas bem definidas, é garantida pelo comportamento assintótico das variáveis canônicas no infinito espacial. A validade da correspondência clássica-quântica para os geradores de Poincaré deve-se essencialmente à ausência de problemas de ordenamento ao nível dos CTI's.

6) Para a cromodinâmica no gauge superaxial os elementos da matriz de Faddeev-Popov [29,30,6] resultam dependentes dos potenciais de gauge. Entretanto, mostramos que o determinante desta matriz é, como esperado, independente das variáveis de campo. Assim, o gauge superaxial está livre de fantasmas de Faddeev-Popov*.

Para finalizar gostaríamos de comentar brevemente sobre dois projetos de importância para pesquisa futura:

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*Note-se que Basseto et al.[49], notando as inconsistências da formulação de gauge axial $A^{3,a}(x) = 0$, propuseram recentemente a substituição desta condição em favor da condição de gauge planar. O preço a pagar, entretanto, é a inclusão de fantasmas.

1º) Obtenção das regras de Feynman da QCD no gauge superaxial:

a) Pelo método funcional [29,30,6], levando em conta nosso resultado 6 (ver capítulo VI desta tese);

b) pelo método operatorial canônico, comparando os resultados obtidos aqui com os encontrados pelo método a). Exatamente este problema de comparar regras de Feynman, obtidas por ambos métodos, em diferentes gauges, foi bem recentemente considerado por H.Cheng e E.C.Tsai para o campo de Yang-Mills puro [50].

2º) A nível do formalismo canônico, construir o espaço de Hilbert da teoria, em particular, o estado de vácuo físico da QCD, com norma e energia finitas, no gauge superaxial. Em conexão com este ponto, além do tipo de construção proposta por Dahmen, Scholz e Steiner [8], surgiu recentemente uma construção modificada proposta por Zeppenfeld [51], para a QCD no gauge temporal. O estudo de tais construções, no contexto da formulação de gauge superaxial, coloca-se, portanto, na ordem do dia.

APÊNDICE A

FÓRMULAS DE INVERSÃO A = A[F] PARA O GAUGE SUPERAXIAL. CONDIÇÃO NECESSÁRIA E SUFICIENTE PARA A INVERSÃO ÚNICA $A^j = A^j[F]$

Começamos escrevendo as intensidades de campo $F^{\mu\nu,a} = \partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a} + g\, f^{abc} A^{\mu,b} A^{\nu,c}$ em termos dos potenciais $A^{\mu,a}$ sujeitos às condições que definem o gauge superaxial. Usando (2.1a), obtemos

$$F^{01,a} = \partial^0 A^{1,a} - \partial^1 A^{0,a} + g f^{abc} A^{0,b} A^{1,c} , \qquad (A.1)$$

$$F^{02,a} = \partial^0 A^{2,a} - \partial^2 A^{0,a} + g f^{abc} A^{0,b} A^{2,c} , \qquad (A.2)$$

$$F^{03,a} = -\partial^3 A^{0,a} , \qquad (A.3)$$

$$F^{12,a} = \partial^1 A^{2,a} - \partial^2 A^{1,a} + g f^{abc} A^{1,b} A^{2,c} , \qquad (A.4)$$

$$F^{23,a} = -\partial^3 A^{2,a} , \qquad (A.5)$$

$$F^{31,a} = \partial^3 A^{1,a} . \qquad (A.6)$$

Integrando ambos lados de (A.6) sobre a 3ª variável espacial, segue que ($\partial_j \equiv \partial/\partial x^j$)

$$\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,a}(x^0,x^1,x^2,x'^3) = -\int_{x^3_{(0)}}^{x^3} dx'^3\, \partial'_3 A^{1,a}(x^0,x^1,x^2,x'^3) = A^{1,a}(x^0,x^1,x^2,x^3_{(0)}) -$$

$$- A^{1,a}(x^0,x^1,x^2,x^3) .$$

A fixação (2.1b) nos permite escrever, consistentemente,

$$A^{1,a}(x^0,x^1,x^2,x^3) = -\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,a}(x^0,x^1,x^2,x'^3) \quad . \qquad (A.7)$$

Agora, integrando (A.5) sobre a 3ª variável espacial, encontramos

$$\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{23,a}(x^0,x^1,x^2,x'^3) = \int_{x^3_{(0)}}^{x^3} dx'^3\, \partial'_3 A^{2,a}(x^0,x^1,x^2,x'^3) = A^{2,a}(x^0,x^1,x^2,x^3) - A^{2,a}(x^0,x^1,x^2,x^3_{(0)})$$

$$\therefore$$

$$A^{2,a}(x^0,x^1,x^2,x^3) = A^{2,a}(x^0,x^1,x^2,x^3_{(0)}) + \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{23,a}(x^0,x^1,x^2,x'^3) . \qquad (A.8)$$

Substituindo as expressões (A.7) e (A.8) para $A^{1,a}$ e $A^{2,a}$, respectivamente, no lado direito de (A.4), ficamos com

$$F^{12,a}(x^0,x^1,x^2,x^3) = \partial^1 A^{2,a}(x^0,x^1,x^2,x^3_{(0)}) +$$

$$+ \int_{x^3_{(0)}}^{x^3} dx'^3\, \partial^1 F^{23,a}(x^0,x^1,x^2,x'^3) + \int_{x^3_{(0)}}^{x^3} dx'^3\, \partial^2 F^{31,a}(x^0,x^1,x^2,x'^3) -$$

$$-gf^{abc}\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,b}_{(x^0,x^1,x^2,x'^3)}\left[A^{2,c}_{(x^0,x^1,x^2,x^3_{(0)})} + \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{23,a}_{(x^0,x^1,x^2,x'^3)}\right]$$

de onde obtemos

$$F^{12,a}_{(x^0,x^1,x^2,x^3_{(0)})} = \partial^1 A^{2,a}_{(x^0,x^1,x^2,x^3_{(0)})} \quad . \qquad (A.9)$$

Integrando (A.9) sobre a 1ª variável espacial e introduzindo a condição (2.1c):

$$\int_{x^1_{(0)}}^{x^1} dx'^1\, F^{12,a}_{(x^0,x'^1,x^2,x^3_{(0)})} = -\int_{x^1_{(0)}}^{x^1} dx'^1\, \partial'_1 A^{2,a}_{(x^0,x'^1,x^2,x^3_{(0)})} =$$

$$= A^{2,a}_{(x^0,x^1_{(0)},x^2,x^3_{(0)})} - A^{2,a}_{(x^0,x^1,x^2,x^3_{(0)})} = -A^{2,a}_{(x^0,x^1,x^2,x^3_{(0)})} . \qquad (A.10)$$

Voltando com (A.10) em (A.8), temos que

$$A^{2,a}_{(x^0,x^1,x^2,x^3)} = \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{23,a}_{(x^0,x^1,x^2,x'^3)} - \int_{x^1_{(0)}}^{x^1} dx'^1\, F^{12,a}_{(x^0,x'^1,x^2,x^3_{(0)})} \quad . \qquad (A.11)$$

Obviamente, a equação (**2**.1a), i.e.,

$$A^{3,a}_{(x^0,x^1,x^2,x^3)} = 0 \qquad (A.12)$$

é sua própria inversão. Por outro lado, desde (A.3) podemos es crever

$$\int_{x^3_{(0)}}^{x^3} dx'^3 \, F^{03,a}(x^0,x^1,x^2,x'^3) = \int_{x^3_{(0)}}^{x^3} dx'^3 \, \partial'_3 A^{0,a}(x^0,x^1,x^2,x'^3) =$$

$$= A^{0,a}(x^0,x^1,x^2,x^3) - A^{0,a}(x^0,x^1,x^2,x^3_{(0)})$$

$$\therefore$$

$$A^{0,a}(x^0,x^1,x^2,x^3) = A^{0,a}(x^0,x^1,x^2,x^3_{(0)}) + \int_{x^3_{(0)}}^{x^3} dx'^3 \, F^{03,a}(x^0,x^1,x^2,x'^3) \quad . \qquad (A.13)$$

Levando (A.13) e (A.7) no lado direito de (A.1) e colocando $x^3 = x^3_{(0)}$, ficamos com

$$F^{01,a}(x^0,x^1,x^2,x^3_{(0)}) = -\partial^1 A^{0,a}(x^0,x^1,x^2,x^3_{(0)}) \qquad (A.14)$$

depois de usar (2.1b). Desde (A.14), é direto encontrar

$$\int_{x^1_{(0)}}^{x^1} dx'^1 \, F^{01,a}(x^0,x'^1,x^2,x^3_{(0)}) = \int_{x^1_{(0)}}^{x^1} dx'^1 \, \partial'_1 A^{0,a}(x^0,x'^1,x^2,x^3_{(0)}) =$$

$$= A^{0,a}(x^0,x^1,x^2,x^3_{(0)}) - A^{0,a}(x^0,x^1_{(0)},x^2,x^3_{(0)})$$

que, em (A.13), fornece

$$A^{0,a}(x^0,x^1,x^2,x^3) = A^{0,a}(x^0,x^1_{(0)},x^2,x^3_{(0)}) + \int_{x^1_{(0)}}^{x^1} dx'^1 \, F^{01,a}(x^0,x'^1,x^2,x^3_{(0)}) +$$

$$+\int_{x^3_{(0)}}^{x^3} dx'^3 \, F^{03,a}_{(x^0,x^1,x^2,x'^3)} \qquad . \qquad (A.15)$$

As expressões (A.15) e (A.11) substituídas no lado direito de (A.2), conduzem a

$$F^{02,a}_{(x^0,x^1_{(0)},x^2,x^3_{(0)})} = -\partial^2 A^{0,a}_{(x^0,x^1_{(0)},x^2,x^3_{(0)})} \qquad (A.16)$$

depois de fixar $x^1 = x^1_{(0)}$ e $x^3 = x^3_{(0)}$. Esta última expressão implica

$$\int_{x^2_{(0)}}^{x^2} dx'^2 \, F^{02,a}_{(x^0,x^1_{(0)},x'^2,x^3_{(0)})} = \int_{x^2_{(0)}}^{x^2} dx'^2 \, \partial'_2 A^{0,a}_{(x^0,x^1_{(0)},x'^2,x^3_{(0)})} =$$

$$= A^{0,a}_{(x^0,x^1_{(0)},x^2,x^3_{(0)})} - A^{0,a}_{(x^0,x^1_{(0)},x^2_{(0)},x^3_{(0)})}$$

a qual, junto com (2.2), leva ao resultado

$$A^{0,a}_{(x^0,x^1_{(0)},x^2,x^3_{(0)})} = \int_{x^2_{(0)}}^{x^2} dx'^2 \, F^{02,a}_{(x^0,x^1_{(0)},x'^2,x^3_{(0)})} + \int d^3z \, r_k(\underset{\sim}{z};\underset{\sim}{x}_{(0)}) \, F^{0k,a}_{(x^0,\underset{\sim}{z})} \qquad . \qquad (A.17)$$

Substituindo (A.17) em (A.15), encontramos a fórmula de inversão que faltava

$$A^{0,a}(x^0,x^1,x^2,x^3) = \int_{x^3_{(0)}}^{x^3} dx'^3 \, F^{03,a}(x^0,x^1,x^2,x'^3) + \int_{x^2_{(0)}}^{x^2} dx'^2 \, F^{02,a}(x^0, x^1_{(0)}, x'^2, x^2_{(0)}) +$$

$$+ \int_{x^1_{(0)}}^{x^1} dx'^1 \, F^{01,a}(x^0, x'^1, x^2, x^3_{(0)}) + \int d^3 z \, r_k(\underset{\sim}{z}; \underset{\sim}{x}_{(0)}) \, F^{0k,a}(x^0, \underset{\sim}{z}) \; . \qquad (A.18)$$

Dado que a condição de gauge (2.11d) (ou, equivalentemente, a inversão (A.18)) não entra na construção do formalismo Hamiltoniano clássico, conforme discutimos nas págs. 27 e 29, precisamos considerar a condição necessária e suficiente que nos leva, de forma única, às inversões (A.7), (A.11) e (A.12), i.e., $A^j = A^j[F]$. Como mostrou Halpern [36], tal condição é a "identidade de Bianchi temporal" que surge da imposição de consistência $F\{A[F]\} = F$. Explicitamente (ver (A.7), (A.11)e (A.12)), devemos impor

$$F^{23,a}\{A^j[F^{kl}]\} = F^{23,a} \, , \; F^{31,a}\{A^j[F^{kl}]\} = F^{31,a} \, , \; F^{12,a}\{A^j[F^{kl}]\} = F^{12,a} .$$

$$(A.19a,b,c)$$

A validade das relações (A.19a,b) não exige condições adicionais.

Prova

Desde (A.11) e (A.12), segue

$$F^{23,a}\{A^j[F^{kl}]\} = -\partial^3 \left[\int_{x^3_{(0)}}^{x^3} dx'^3 \, F^{23,a}(x^0,x^1,x^2,x'^3) \right] = F^{23,a}(x^0,x^1,x^2,x^3) ; \; (A.20)$$

desde (A.7) e (A.12)

$$F^{31,a}\{A^j[F^{k\ell}]\} = \partial^3\left[-\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,a}(x^0,x^1,x^2,x'^3)\right] = F^{31,a}(x^0,x^1,x^2,x^3) . \quad (A.21)$$

Por outro lado, usando (A.7) e (A.11), o lado esquerdo de (A.19c) é igual a

$$F^{12,a}\{A^j[F^{k\ell}]\} = \partial^1 A^{2,a}[F] - \partial^2 A^{1,a}[F] + g f^{abc} A^{1,b}[F]\, A^{2,c}[F] =$$

$$= \int_{x^3_{(0)}}^{x^3} dx'^3\, \partial^1 F^{23,a}(x^0,x^1,x^2,x'^3) + F^{12,a}(x^0,x^1,x^2,x^3_{(0)}) +$$

$$+ \int_{x^3_{(0)}}^{x^3} dx'^3\, \partial^2 F^{31,a}(x^0,x^1,x^2,x'^3) - g f^{abc} \int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,b}(x^0,x^1,x^2,x'^3) .$$

$$\cdot\left[\int_{x^3_{(0)}}^{x^3} dx''^3\, F^{23,c}(x^0,x^1,x^2,x''^3) - \int_{x^1_{(0)}}^{x^1} dx'^1\, F^{12,c}(x^0,x'^1,x^2,x^3_{(0)})\right] . \quad (A.22)$$

Notar que o lado esquerdo de (A.19c) (a expressão (A.22)) já é manifestamente igual ao lado direito de (A.19c), i.e., a $F^{12,a}(x^0,\underset{\sim}{x})$, tomado no ponto $x^3 = x^3_{(0)}$. Mas, por consistência, (A.19c) deve valer para todo $\underset{\sim}{x}$ o que nos força a impor

$$\int_{x^3_{(0)}}^{x^3} dx'^3\, \partial^1 F^{23,a}(x^0,x^1,x^2,x'^3) + F^{12,a}(x^0,x^1,x^2,x^3_{(0)}) + \int_{x^3_{(0)}}^{x^3} dx'^3\, \partial^2 F^{31,a}(x^0,x^1,x^2,x'^3) -$$

$$-gf^{abc}\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,b}_{(x^0,x^1,x^2,x'^3)}\left[\int_{x^3_{(0)}}^{x^3} dx''^3\, F^{23,c}_{(x^0,x^1,x^2,x''^3)} - \int_{x^1_{(0)}}^{x^1} dx'^1\, F^{12,c}_{(x^0,x'^1,x^2,x^3_{(0)})}\right] =$$

$$= F^{12,a}_{(x^0,x^1,x^2,x^3)} \quad . \qquad (A.23)$$

A condição (A.23) é essencialmente a "identidade de Bianchi temporal" [36]. De fato, sem perda de informação, podemos diferenciar ambos lados de (A.23) em relação a x^3 e obter

$$\partial^1 F^{23,a}_{(x^0,\underset{\sim}{x})} + \partial^2 F^{31,a}_{(x^0,\underset{\sim}{x})} - gf^{abc} F^{31,b}_{(x^0,\underset{\sim}{x})}\left[\int_{x^3_{(0)}}^{x^3} dx''^3\, F^{23,c}_{(x^0,x^1,x^2,x''^3)} -\right.$$

$$\left. - \int_{x^1_{(0)}}^{x^1} dx'^1\, F^{12,c}_{(x^0,x'^1,x^2,x^3_{(0)})}\right] + gf^{abc}\left[-\int_{x^3_{(0)}}^{x^3} dx'^3\, F^{31,b}_{(x^0,x^1,x^2,x'^3)}\right] F^{23,c}_{(x^0,\underset{\sim}{x})} =$$

$$= \partial_3 F^{12,a}_{(x^0,\underset{\sim}{x})}$$

a qual, depois de usar (A.7) e (A.11), pode ser escrita*

$$\frac{1}{2}\epsilon^{ijk}\partial^i F^{jk,a} + gf^{abc}\left[A^{1,b}[F]\,F^{23,c} + A^{2,b}[F]\,F^{31,c}\right] = 0 \; .$$

$$(A.24)$$

A condição (A.24) é a identidade de Bianchi temporal que assegura a unicidade da inversão $A^j = A^j[F]$.

---

*ε^{ijk} é o tensor totalmente antissimétrico de Levi-Civita.

APÊNDICE B

PERSISTÊNCIA NO TEMPO DE TODOS OS VÍNCULOS DA TEORIA. DETERMINAÇÃO DOS VÍNCULOS IRREDUTÍVEIS DE 1ª E 2ª CLASSE.

Desde (2.25) e (2.24), tendo em vista o aparecimento dos vínculos secundários (2.27) e (2.28), definimos um novo Hamiltoniano total, H_T, sobre Γ', na forma

$$H_T = H + \int d^3x \Big\{ \eta^{0,a}\pi_0^a + \eta^{0j,a}\pi_{0j}^a + \frac{1}{2}\eta^{ij,a}\pi_{ij}^a + \eta^{j,a}(\pi_j^a - F^{0j,a}) + \eta^a(D^{j,ab}F^{0j,b} - ig\pi_\psi \frac{\lambda^a}{2}\psi) + \frac{1}{2}\bar{\eta}_{ij}^a\left[F^{ij,a} - (\partial^i A^{j,a} - \partial^j A^{i,a} + g f^{abc}A^{i,b}A^{j,c})\right]\Big\} , \quad (B.1)$$

onde

$$H = \int d^3x \left[\frac{1}{2}\pi_j^a\pi_j^a + \frac{1}{4}F^{ij,a}F^{ij,a} + \pi_\psi\gamma^0(\gamma^k\tilde{D}^k - im)\psi\right]. \quad (B.2)$$

Os multiplicadores de Lagrange $\eta^{0,a}$, $\eta^{0j,a}$, $\eta^{ij,a}$ $(= -\eta^{ji,a})$ e $\eta^{j,a}$ associam-se aos vínculos primários $\pi_0^a \approx 0$, $\pi_{0j}^a \approx 0$, $\pi_{ij}^a \approx 0$ e $\pi_j^a - F^{0j,a} \approx 0$ enquanto os multiplicadores η^a e $\bar{\eta}_{ij}^a (= -\bar{\eta}_{ji}^a)$ associam-se aos vínculos secundários (2.27) e (2.28), respectivamente.

A seguir, mostramos que a persistência no tempo de todos os vínculos obtidos até aqui (ver (B.1)) não implica em novos vínculos mas estabelece relações entre alguns dos multi plicadores de Lagrange:

$$a)\ \dot{\pi}_{o}^{a}(\underset{\sim}{x}) \approx 0 \quad \Longrightarrow \quad [\pi_{o}^{a}(\underset{\sim}{x}), H_T]_{PP} \approx 0 \quad \therefore \quad 0 \approx 0 \ ; \qquad (B.3)$$

$$b)\ \dot{\pi}_{oj}^{a}(\underset{\sim}{x}) \approx 0 \quad \Longrightarrow \quad [\pi_{oj}^{a}(\underset{\sim}{x}), H_T]_{PP} = \frac{-\delta H_T}{\delta F^{oj,a}(\underset{\sim}{x})} \approx 0 \quad \therefore$$

$\therefore$

$$\eta^{j,a}(\underset{\sim}{x}) - \int d^3z\, \eta^{b}(\underset{\sim}{z})\, D^{j,bc}(\underset{\sim}{z}) \left(\delta^{ca}\, \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x})\right) \approx 0 \quad \Longrightarrow$$

$$\Longrightarrow \eta^{j,a}(\underset{\sim}{x}) \approx -D^{j,ab}(\underset{\sim}{x})\, \eta^{b}(\underset{\sim}{x}) \quad ; \qquad (B.4)$$

$$c)\ \dot{\pi}_{ij}^{a}(\underset{\sim}{x}) \approx 0 \quad \Longrightarrow \quad [\pi_{ij}^{a}(\underset{\sim}{x}), H_T]_{PP} = \frac{-\delta H_T}{\delta F^{ij,a}(\underset{\sim}{x})} \approx 0 \quad \therefore$$

$\therefore$

$$-\frac{1}{2} F^{ij,a}(\underset{\sim}{x}) - \bar{\eta}_{ij}^{a}(\underset{\sim}{x}) \approx 0 \quad \Longrightarrow$$

$$\Longrightarrow \quad \bar{\eta}_{ij}^{a}(\underset{\sim}{x}) \approx -\frac{1}{2} F^{ij,a}(\underset{\sim}{x}) \qquad (B.5)$$

$$d)\ \dot{\pi}_{j}^{a}(\underset{\sim}{x}) - \dot{F}^{oj,a}(\underset{\sim}{x}) \approx 0 \quad \Longrightarrow \quad [\pi_{j}^{a}(\underset{\sim}{x}), H_T]_{PP} - [F^{oj,a}(\underset{\sim}{x}), H_T]_{PP} \approx 0 \Rightarrow$$

$$\Longrightarrow \quad \frac{-\delta H_T}{\delta A^{j,a}(\underset{\sim}{x})} - \frac{\delta H_T}{\delta \pi_{oj}^{a}(\underset{\sim}{x})} \approx 0 \qquad \therefore$$

$\therefore$

$$\eta^{oj,a}_{(\underset{\sim}{x})} \approx \frac{-\delta H_T}{\delta A^{j,a}_{(\underset{\sim}{x})}} = \frac{-\delta}{\delta A^{j,a}_{(\underset{\sim}{x})}} \Bigg\{ \int d^3z \Big[\pi_{\psi}(\underset{\sim}{z}) \gamma^0 \gamma^k (-ig) \frac{\lambda^b}{2} A^{k,b}_{(\underset{\sim}{z})} \psi(\underset{\sim}{z}) +$$

$$+ \eta^b(\underset{\sim}{z}) g f^{bdc} A^{k,d}_{(\underset{\sim}{z})} F^{ok,c}_{(\underset{\sim}{z})} - \frac{1}{2} \bar{\eta}^b_{kl}(\underset{\sim}{z}) \Big(\partial^k_z A^{l,b}(\underset{\sim}{z}) - \partial^l_z A^{k,b}(\underset{\sim}{z}) + g f^{bcd} A^{k,c}(\underset{\sim}{z}) A^{l,d}(\underset{\sim}{z}) \Big) \Big] \Bigg\} =$$

$$= ig \pi_{\psi}(\underset{\sim}{x}) \gamma^0 \gamma^j \frac{\lambda^a}{2} \psi(\underset{\sim}{x}) - g f^{bac} \eta^b(\underset{\sim}{x}) F^{oj,c}_{(\underset{\sim}{x})} +$$

$$+ \Bigg\{ \frac{1}{2} \int d^3z\, \bar{\eta}^b_{kl}(\underset{\sim}{z}) \Big[\delta^{lj} \delta^{ab} \partial^k_z \delta^{(3)}(\underset{\sim}{z} - \underset{\sim}{x}) - \delta^{kj} \delta^{ab} \partial^l_z \delta^{(3)}(\underset{\sim}{z} - \underset{\sim}{x}) +$$

$$+ g f^{bcd} \big(\delta^{kj} \delta^{ac} A^{l,d}(\underset{\sim}{z}) + \delta^{lj} \delta^{ad} A^{k,c}(\underset{\sim}{z}) \big) \delta^{(3)}(\underset{\sim}{z} - \underset{\sim}{x}) \Big] \Bigg\}.$$

(B.6)

A chave em (B.6) ē igual a

$$\frac{1}{2} \Big[\partial^l_x \bar{\eta}^a_{jl}(\underset{\sim}{x}) - \partial^l_x \bar{\eta}^a_{lj}(\underset{\sim}{x}) + g f^{bac} \bar{\eta}^b_{jl}(\underset{\sim}{x}) A^{l,c}(\underset{\sim}{x}) + g f^{bca} \bar{\eta}^b_{kj}(\underset{\sim}{x}) A^{k,c}(\underset{\sim}{x}) \Big] =$$

$$= \partial^k_x \bar{\eta}^a_{jk}(\underset{\sim}{x}) + g f^{acb} \bar{\eta}^b_{jk}(\underset{\sim}{x}) A^{k,c}(\underset{\sim}{x}) = D^{k,ab}(\underset{\sim}{x}) \bar{\eta}^b_{jk}(\underset{\sim}{x})$$

$$= \frac{1}{2} D^{k,ab}_{(\underset{\sim}{x})} F^{kj,b}_{(\underset{\sim}{x})} \quad , \qquad (B.7)$$

onde usamos (B.5). Levando (B.7) em (B.6), obtemos

$$\eta^{oj,a}_{(\underset{\sim}{x})} \approx \frac{1}{2} D^{k,ab}_{(\underset{\sim}{x})} F^{kj,b}_{(\underset{\sim}{x})} + ig \pi_{\psi}(\underset{\sim}{x}) \gamma^0 \gamma^j \frac{\lambda^a}{2} \psi(\underset{\sim}{x}) - g f^{acb} \eta^b(\underset{\sim}{x}) F^{oj,c}_{(\underset{\sim}{x})} .$$

(B.8)

$$\text{e)}\left[\partial^j_x F^{0j,a}_{(\underset{\sim}{x})} + g f^{acb} A^{j,c}_{(\underset{\sim}{x})} F^{0j,b}_{(\underset{\sim}{x})} - i g \pi_\psi(\underset{\sim}{x}) \frac{\lambda^a}{2}\psi(\underset{\sim}{x}) , H_T\right]_{PP} \approx 0 \Longrightarrow$$

$$\Longrightarrow \partial^j_x\left(\frac{\delta H_T}{\delta \pi^{\;a}_{0j}(\underset{\sim}{x})}\right) + g f^{acb}\left[A^{j,c}_{(\underset{\sim}{x})}\left(\frac{\delta H_T}{\delta \pi^{\;b}_{0j}(\underset{\sim}{x})}\right) + F^{0j,b}_{(\underset{\sim}{x})}\left(\frac{\delta H_T}{\delta \pi^{\;c}_{j}(\underset{\sim}{x})}\right)\right] -$$

$$- i g \left(\frac{\lambda^a}{2}\right)^{uv}\left[\left(- \frac{H_T \overleftarrow{\delta}}{\delta \psi^u_t(\underset{\sim}{x})}\right)\psi^v_t(\underset{\sim}{x}) + \pi^{\;u}_{\psi_t}(\underset{\sim}{x})\left(\frac{\overrightarrow{\delta} H_T}{\delta \pi^{\;v}_{\psi_t}(\underset{\sim}{x})}\right)\right] \approx 0 . \tag{B.9}$$

Mas

$$\frac{\delta H_T}{\delta \pi^{\;a}_{0j}(\underset{\sim}{x})} = \eta^{0j,a}(\underset{\sim}{x}) \quad , \tag{B.10}$$

$$\frac{\delta H_T}{\delta \pi^{\;c}_{j}(\underset{\sim}{x})} = \eta^{j,c}(\underset{\sim}{x}) + \pi^{\;c}_{j}(\underset{\sim}{x}) \quad , \tag{B.11}$$

$$\frac{H_T \overleftarrow{\delta}}{\delta \psi^u_t(\underset{\sim}{x})} = \left\{ \int d^3 z \left[\pi^{\;w}_{\psi_r}(\underset{\sim}{z}) (\gamma^0\gamma^k)_{rs} \partial^k_z \psi^w_s(\underset{\sim}{z}) - i g \left(\frac{\lambda^b}{2}\right)^{wu'} \pi^{\;w}_{\psi_r}(\underset{\sim}{z}) (\gamma^0\gamma^k)_{rs} A^{k,b}(\underset{\sim}{z}) \psi^{u'}_s(\underset{\sim}{z}) - \right.\right.$$

$$\left.\left. - i m \pi^{\;w}_{\psi_r}(\underset{\sim}{z}) (\gamma^0)_{rs} \psi^w_s(\underset{\sim}{z}) - i g \eta^b(\underset{\sim}{z}) \left(\frac{\lambda^b}{2}\right)^{wu'} \pi^{\;w}_{\psi_r}(\underset{\sim}{z}) \psi^{u'}_r(\underset{\sim}{z}) \right]\right\} \frac{\overleftarrow{\delta}}{\delta \psi^u_t(\underset{\sim}{x})} =$$

$$= -\left(\partial^k_x \pi^{\;u}_{\psi_r}(\underset{\sim}{x})\right)(\gamma^0\gamma^k)_{rt} - i g \pi^{\;w}_{\psi_r}(\underset{\sim}{x}) (\gamma^0\gamma^k)_{rt} \left(\frac{\lambda^b}{2}\right)^{wu} A^{k,b}(\underset{\sim}{x}) -$$

$$- i m \pi^{\;u}_{\psi_r}(\underset{\sim}{x}) (\gamma^0)_{rt} - i g \eta^b(\underset{\sim}{x}) \left(\frac{\lambda^b}{2}\right)^{wu} \pi^{\;v}_{\psi_t}(\underset{\sim}{x}) \quad , \tag{B.12}$$

$$\frac{\overrightarrow{\delta} H_T}{\delta \pi^{\;v}_{\psi_t}(\underset{\sim}{x})} = \frac{\overrightarrow{\delta}}{\delta \pi^{\;v}_{\psi_t}(\underset{\sim}{x})} \left(\text{chave em} (B.12) \right) =$$

$$= (\gamma^0\gamma^k)_{ts}\, \partial^k_x \psi^v_s(\underset{\sim}{x}) - ig\,(\gamma^0\gamma^k)_{ts} \left(\frac{\lambda^b}{2}\right)^{vu'} \psi^{u'}_s(\underset{\sim}{x})\, A^{k,b}(\underset{\sim}{x}) -$$

$$- im\,(\gamma^0)_{ts}\, \psi^v_s(\underset{\sim}{x}) - ig\, \eta^b(\underset{\sim}{x}) \left(\frac{\lambda}{2}^b\right)^{vu'} \psi^{u'}_t(\underset{\sim}{x}) \quad . \qquad (B.13)$$

Levando (B.10)-(B.13) em (B.9), encontramos

$$D^{j,ab}(\underset{\sim}{x})\, \eta^{0j,b}(\underset{\sim}{x}) \approx -g f^{acb} F^{0j,b}(\underset{\sim}{x}) \left(- D^{j,cd}(\underset{\sim}{x})\, \eta^d(\underset{\sim}{x})\right) +$$

$$+ ig\, D^{k,ab}(\underset{\sim}{x}) \left(\pi_\psi(\underset{\sim}{x})\, \gamma^0\gamma^k \frac{\lambda^b}{2} \psi(\underset{\sim}{x}) \right) - ig^2 f^{acb}\, \pi_\psi(\underset{\sim}{x}) \frac{\lambda^c}{2} \psi(\underset{\sim}{x})\, \eta^b(\underset{\sim}{x}) \qquad (B.14)$$

usando (B.4) e o fato que $\left[\frac{\lambda^b}{2}, \frac{\lambda^a}{2}\right] = i f^{bac} \frac{\lambda^c}{2}$.

f) $\left[F^{ij,a}(\underset{\sim}{x}) - (\partial^i_x A^{j,a}(\underset{\sim}{x}) - \partial^j_x A^{i,a}(\underset{\sim}{x}) + g f^{abc} A^{i,b}(\underset{\sim}{x}) A^{j,c}(\underset{\sim}{x})), H_T \right]_{PP} \approx 0 \Longrightarrow$

$$\Longrightarrow \frac{\delta H_T}{\delta \pi^a_{ij}(\underset{\sim}{x})} - \left\{ \partial^i_x \left(\frac{\delta H_T}{\delta \pi^a_{\cdot j}(\underset{\sim}{x})} \right) - \partial^j_x \left(\frac{\delta H_T}{\delta \pi^a_{\cdot i}(\underset{\sim}{x})} \right) + g f^{abc} \left[\left(\frac{\delta H_T}{\delta \pi^b_{\cdot i}(\underset{\sim}{x})} \right) A^{j,c}(\underset{\sim}{x}) + \right.\right.$$

$$\left.\left. + A^{i,b}(\underset{\sim}{x}) \left(\frac{\delta H_T}{\delta \pi^c_{\cdot j}(\underset{\sim}{x})} \right) \right] \right\} \approx 0 \qquad \therefore$$

$$\eta^{ij,a}(\underset{\sim}{x}) \approx \partial^i_x (\eta^{j,a}(\underset{\sim}{x}) + \pi^a_{\cdot j}(\underset{\sim}{x})) - \partial^j_x (\eta^{i,a}(\underset{\sim}{x}) + \pi^a_{\cdot i}(\underset{\sim}{x})) +$$

$$+ g f^{abc} \left[(\eta^{i,b}(\underset{\sim}{x}) + \pi^b_{\cdot i}(\underset{\sim}{x}))\, A^{j,c}(\underset{\sim}{x}) + A^{i,b}(\underset{\sim}{x}) (\eta^{j,c}(\underset{\sim}{x}) + \pi^c_{\cdot j}(\underset{\sim}{x})) \right] =$$

$$= D^{i,ab}(\underset{\sim}{x})\, \pi^b_{\cdot j}(\underset{\sim}{x}) - D^{j,ab}(\underset{\sim}{x})\, \pi^b_{\cdot i}(\underset{\sim}{x}) + D^{i,ab}(\underset{\sim}{x})\, \eta^{j,b}(\underset{\sim}{x}) - D^{j,ab}(\underset{\sim}{x})\, \eta^{i,b}(\underset{\sim}{x}) \quad . \qquad (B.15)$$

Fazendo uso da propriedade

$$[D_\mu, D_\nu]^{ab} = g f^{acb} F^c_{\mu\nu} \tag{B.16}$$

e do resultado (B.4), podemos escrever

$$D^{i,ab}_{(\underset{\sim}{x})}\eta^{j,b}_{(\underset{\sim}{x})} - D^{j,ab}_{(\underset{\sim}{x})}\eta^{i,b}_{(\underset{\sim}{x})} = -g f^{abc} F^{ij,b}_{(\underset{\sim}{x})}\eta^{c}_{(\underset{\sim}{x})} \tag{B.17}$$

o qual, levado em (B.15), fornece

$$\eta^{ij,a}_{(\underset{\sim}{x})} \approx D^{i,ab}_{(\underset{\sim}{x})}\Pi^{b}_{j}{}_{(\underset{\sim}{x})} - D^{j,ab}_{(\underset{\sim}{x})}\Pi^{b}_{i}{}_{(\underset{\sim}{x})} - g f^{abc} F^{ij,b}_{(\underset{\sim}{x})}\eta^{c}_{(\underset{\sim}{x})} . \tag{B.18}$$

Observar que a relação estabelecida por (B.14) entre os multiplicadores $\eta^{0j,a}$ e η^a é idêntica à relação entre os mesmos multiplicadores estabelecida por (B.8). De fato, é simples provar que aplicando-se a derivada $D^{j,ab}(\underset{\sim}{x})$ sobre $\eta^{0j,b}(\underset{\sim}{x})$, definida por (B.8), encontra-se (B.14).

Prova

Desde (B.8), segue imediatamente que

$$\begin{aligned} D^{j,ab}_{(\underset{\sim}{x})}\eta^{0j,b}_{(\underset{\sim}{x})} \approx{}& ig D^{j,ab}_{(\underset{\sim}{x})}\left(\pi_{\psi}{}_{(\underset{\sim}{x})}\gamma^0\gamma^j\frac{\lambda^b}{2}\psi_{(\underset{\sim}{x})}\right) - \\ & -g f^{bcd} D^{j,ab}_{(\underset{\sim}{x})}\left(\eta^{d}_{(\underset{\sim}{x})} F^{0j,c}_{(\underset{\sim}{x})}\right) . \end{aligned} \tag{B.19}$$

Comparando (B.19) com (B.14), resta provar que

$$g f^{acb}\Big[F^{0j,b}_{(\underset{\sim}{x})} D^{j,cd}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} - i g\, \pi_{\psi}{}_{(\underset{\sim}{x})} \frac{\lambda^{c}}{2} \psi_{(\underset{\sim}{x})} \eta^{b}_{(\underset{\sim}{x})} \Big] =$$

$$= -g f^{bcd} D^{j,ab}_{(\underset{\sim}{x})} \Big(\eta^{d}_{(\underset{\sim}{x})} F^{0j,c}_{(\underset{\sim}{x})} \Big) \quad . \qquad (B.20)$$

O lado esquerdo de (B.20), com o uso de (2.27), é igual a

$$g f^{acb}\Big[F^{0j,b}_{(\underset{\sim}{x})} D^{j,cd}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} - \Big(D^{j,cd}_{(\underset{\sim}{x})} F^{0j,d}_{(\underset{\sim}{x})} \Big) \eta^{b}_{(\underset{\sim}{x})} \Big] =$$

$$= g f^{acb}\Big[F^{0j,b}_{(\underset{\sim}{x})} \Big(\partial^{j}_{x} \eta^{c}_{(\underset{\sim}{x})} + g f^{ced} A^{j,e}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} \Big) -$$

$$- \Big(\partial^{j}_{x} F^{0j,c}_{(\underset{\sim}{x})} \Big) \eta^{b}_{(\underset{\sim}{x})} - g f^{ced} A^{j,e}_{(\underset{\sim}{x})} F^{0j,d}_{(\underset{\sim}{x})} \eta^{b}_{(\underset{\sim}{x})} \Big] =$$

$$= g f^{acb} \partial^{j}_{x} \Big(F^{0j,b}_{(\underset{\sim}{x})} \eta^{c}_{(\underset{\sim}{x})} \Big) + g^{2} \Big(f^{acb} f^{ced} - f^{acd} f^{ceb} \Big) .$$

$$. \, F^{0j,b}_{(\underset{\sim}{x})} A^{j,e}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} =$$

$$= g f^{bdc} \delta^{ab} \partial^{j}_{x} \Big(F^{0j,c}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} \Big) + g^{2} f^{cdb} f^{cae} A^{j,e}_{(\underset{\sim}{x})} F^{0j,b}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} =$$

$$= -g f^{bcd} \delta^{ab} \partial^{j}_{x} \Big(F^{0j,c}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} \Big) - g^{2} f^{bcd} f^{aeb} A^{j,e}_{(\underset{\sim}{x})} F^{0j,c}_{(\underset{\sim}{x})} \eta^{d}_{(\underset{\sim}{x})} =$$

$$= -g f^{bcd} D^{j,ab}_{(\underset{\sim}{x})} \Big(\eta^{d}_{(\underset{\sim}{x})} F^{0j,c}_{(\underset{\sim}{x})} \Big) \quad ,$$

que coincide com o lado direito de (B.20) (q.e.d.). Nesta prova, fizemos uso da identidade de Jacobi

$$f^{abc} f^{cde} + f^{adc} f^{ceb} + f^{bdc} f^{cae} = 0 \qquad \text{(B.21)}$$

para as constantes de estrutura de SU(N).

Substituindo (B.18), (B.8), (B.5) e (B.4) em (B.1), ficamos com

$$\begin{aligned} H_T = H + \int d^3_x \Big\{ & \eta^{0,a} \pi_0^a + \Big(\frac{1}{2} D^{k,ab} F^{kj,b} + i g \pi_\psi \gamma^0 \gamma^j \frac{\lambda^a}{2} \psi - \\ & - g f^{acb} \eta^b F^{0j,c} \Big) \pi_{0j}^a + \frac{1}{2} \Big(D^{i,ab} \pi_j^b - D^{j,ab} \pi_i^b - g f^{abc} F^{ij,b} \eta^c \Big) \pi_{ij}^a - \\ & - (D^{j,ab} \eta^b)(\pi_j^a - F^{0j,a}) + \eta^a (D^{j,ab} F^{0j,b} - i g \pi_\psi \frac{\lambda^a}{2} \psi) - \\ & - \frac{1}{4} F^{ij,a} \Big[F^{ij,a} - (\partial^i A^{j,a} - \partial^j A^{i,a} + g f^{abc} A^{i,b} A^{j,c}) \Big] \Big\} \; . \qquad \text{(B.22)} \end{aligned}$$

Integrando por partes como em (2.23), podemos escrever

$$\int d^3_x \left(- D^{j,ab} \eta^b \right) \left(\pi_j^a - F^{0j,a} \right) = \int d^3_x \, \eta^a D^{j,ab} \left(\pi_j^b - F^{0j,b} \right) . \qquad \text{(B.23)}$$

Logo (com (B.23) em (B.22)),

$$\begin{aligned} H_T = H + \int d^3_x \Big\{ & \eta^{0,a} \pi_0^a + \left(\frac{1}{2} D^{k,ab} F^{kj,b} + i g \pi_\psi \gamma^0 \gamma^j \frac{\lambda^a}{2} \psi \right) \pi_{0j}^a + \\ & + \frac{1}{2} (D^{i,ab} \pi_j^b - D^{j,ab} \pi_i^b) \pi_{ij}^a - \end{aligned}$$

$$-\frac{1}{4} F^{ij,a}\left[F^{ij,a} - (\partial^i A^{j,a} - \partial^j A^{i,a} + g f^{abc} A^{i,b} A^{j,c})\right] +$$

$$+ \eta^a \left(D^{j,ab} \pi^b_j - i g \pi_\psi \frac{\lambda^a}{2} \psi + g f^{acb} F^{0j,c} \pi^b_{0j} + \frac{g}{2} f^{acb} F^{ij,c} \pi^b_{ij} \right)\Big\} .$$

(B.24)

Note-se que em H_T ainda temos dois (na realidade, $2(N^2-1)$) multiplicadores de Lagrange indeterminados, $\eta^{0,a}$ e η^a, os quais representam a liberdade de gauge da cromodinâmica. A combinação linear de vínculos que multiplica a η^a em (B.24), ou seja, a "lei de Gauss generalizada"

$$\tau^a = D^{j,ab} \pi^b_j - i g \pi_\psi \frac{\lambda^a}{2} \psi + g f^{acb} F^{0j,c} \pi^b_{0j} + \frac{g}{2} f^{acb} F^{ij,c} \pi^b_{ij} \qquad (B.25)$$

é, de fato, um vínculo de 1ª classe.

Prova

A expressão (B.25) implica:

a) $$[\tau^a(\underset{\sim}{x}), \pi^a_0(\underset{\sim}{y})]_{PP} \approx 0 \quad ; \qquad (B.26a)$$

b) $$[\tau^a(\underset{\sim}{x}), \pi^b_{0j}(y)]_{PP} = \frac{\delta \tau^a(x)}{\delta F^{0j,b}_{(y)}} = g f^{abc} \pi^c_{0j}(\underset{\sim}{x}) \delta^{(3)}(\underset{\sim}{x}-y) \approx 0 \; ; \qquad (B.26b)$$

c) $$[\tau^a(\underset{\sim}{x}), \pi^b_{ij}(y)]_{PP} = \frac{\delta \tau^a(\underset{\sim}{x})}{\delta F^{ij,b}_{(y)}} = g f^{abc} \pi^c_{ij}(\underset{\sim}{x}) \delta^{(3)}(\underset{\sim}{x}-y) \approx 0 \; ; \qquad (B.26c)$$

d) $$\left[\tau^a_{(\underset{\sim}{x})}, \pi^b_{j(y)} - F^{0j,b}_{(y)}\right]_{PP} = -\frac{\delta \tau^a_{(\underset{\sim}{x})}}{\delta A^{j,b}_{(y)}} - \frac{\delta \tau^a_{(\underset{\sim}{x})}}{\delta \pi^b_{0j}(y)} =$$

$$= g f^{acb}\left(\pi^c_j(\underset{\sim}{x}) - F^{0j,c}_{(\underset{\sim}{x})}\right)\overset{(3)}{\delta}(\underset{\sim}{x}-y) \approx 0 \quad ;$$

(B.26 d)

e) $$\left[\tau^a_{(\underset{\sim}{x})}, D^{j,bc}_{(y)} F^{0j,c}_{(y)} - i g \pi_{\psi(y)} \frac{\lambda^b}{2} \psi_{(y)}\right]_{PP} =$$

$$= \partial^j_y\left(-\frac{\delta \tau^a_{(\underset{\sim}{x})}}{\delta \pi^b_{0j}(y)}\right) + g f^{bdc}\left[\left(-\frac{\delta \tau^a_{(\underset{\sim}{x})}}{\delta \pi^d_{j}(y)}\right) F^{0j,c}_{(y)} + A^{j,d}_{(y)}\left(-\frac{\delta \tau^a_{(\underset{\sim}{x})}}{\delta \pi^c_{0j}(y)}\right)\right] -$$

$$- i g \left(\frac{\lambda^b}{2}\right)^{uv}\left[\left(\frac{\tau^a_{(\underset{\sim}{x})}\overleftarrow{\delta}}{\delta \psi^u_{(y)}}\right)\psi^v_{(y)} - \pi^u_{\psi(y)}\left(\frac{\overrightarrow{\delta}\, \tau^a_{(\underset{\sim}{x})}}{\delta \pi^v_{\psi}(y)}\right)\right] =$$

$$= -g f^{acb} F^{0j,c}_{(\underset{\sim}{x})} \partial^j_y \overset{(3)}{\delta}(\underset{\sim}{x}-y) - g f^{bdc}\left[\left(D^{j,ad}_{(\underset{\sim}{x})} \overset{(3)}{\delta}(\underset{\sim}{x}-y)\right) F^{0j,c}_{(y)} + \right.$$

$$\left. + A^{j,d}_{(y)}\, g f^{aec} F^{0j,e}_{(\underset{\sim}{x})} \overset{(3)}{\delta}(\underset{\sim}{x}-y)\right] -$$

$$- i g \left(\frac{\lambda^b}{2}\right)^{uv}\left[-i g \pi^w_{\psi}(\underset{\sim}{x})\left(\frac{\lambda^a}{2}\right)^{wu}\psi^v_{(y)} + i g \pi^u_{\psi}(y)\left(\frac{\lambda^a}{2}\right)^{vw}\psi^w_{(\underset{\sim}{x})}\right]\overset{(3)}{\delta}(\underset{\sim}{x}-y) =$$

$$= -g f^{acb}\left[F^{0j,c}_{(\underset{\sim}{x})} \partial^j_y \overset{(3)}{\delta}(\underset{\sim}{x}-y) + F^{0j,c}_{(y)} \partial^j_x \overset{(3)}{\delta}(\underset{\sim}{x}-y)\right] -$$

$$- g^2\left(f^{bdc} f^{aed} + f^{bed} f^{acd}\right) A^{j,e}_{(\underset{\sim}{x})} F^{0j,c}_{(\underset{\sim}{x})} \overset{(3)}{\delta}(\underset{\sim}{x}-y) \quad -$$

$$-ig^2 f^{abc}\,\Pi_{\psi}(\underset{\sim}{x})\frac{\lambda^c}{2}\psi(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) =$$

$$= -gf^{acb}\left[D^{j,cd}_{(\underset{\sim}{x})}F^{oj,d}_{(\underset{\sim}{x})} - ig\,\Pi_{\psi}(\underset{\sim}{x})\frac{\lambda^c}{2}\psi(\underset{\sim}{x})\right]\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \approx 0\ , \tag{B.26e}$$

onde usamos novamente (B.21);

$$f)\ \left[\tau^a(\underset{\sim}{x}),\ F^{ij,b}(\underset{\sim}{y}) - (\partial^i A^{j,b}(\underset{\sim}{y}) - \partial^j A^{i,b}(\underset{\sim}{y}) + gf^{bcd}A^{i,c}(\underset{\sim}{y})A^{j,d}(\underset{\sim}{y})\right]_{PP} =$$

$$= -\frac{\delta\tau^a(\underset{\sim}{x})}{\delta\Pi^b_{ij}(\underset{\sim}{y})} + \partial^i_y\left(\frac{\delta\tau^a(\underset{\sim}{x})}{\delta\Pi^b_j(\underset{\sim}{y})}\right) - \partial^j_y\left(\frac{\delta\tau^a(\underset{\sim}{x})}{\delta\Pi^b_i(\underset{\sim}{y})}\right) +$$

$$+ gf^{bcd}\left[\left(\frac{\delta\tau^a(\underset{\sim}{x})}{\delta\Pi^c_i(\underset{\sim}{y})}\right)A^{j,d}(\underset{\sim}{y}) + A^{i,c}(\underset{\sim}{y})\left(\frac{\delta\tau^a(\underset{\sim}{x})}{\delta\Pi^d_j(\underset{\sim}{y})}\right)\right] =$$

$$= -gf^{acb}F^{ij,c}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \partial^i_y\left(D^{j,ab}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\right) -$$

$$-\partial^j_y\left(D^{i,ab}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\right) + gf^{bcd}\left[\left(D^{i,ac}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\right)A^{j,d}(\underset{\sim}{y}) + A^{i,c}(\underset{\sim}{y})\left(D^{j,ad}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\right)\right] =$$

$$= -gf^{acb}F^{ij,c}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + gf^{acb}A^{j,c}(\underset{\sim}{x})\,\partial^i_y\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - gf^{acb}A^{i,c}(\underset{\sim}{x})\,\partial^j_y\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) +$$

$$+ gf^{bac}(\partial^i_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}))A^{j,c}(\underset{\sim}{y}) - gf^{bac}(\partial^j_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}))A^{i,c}(\underset{\sim}{y}) \quad -$$

$$-g^2\left(f^{bcd}f^{aed}+f^{bde}f^{acd}\right)A^{i,e}_{(\underset{\sim}{x})}A^{j,c}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}=$$

$$=gf^{abc}\left[F^{ij,c}_{(\underset{\sim}{x})}-\left(\partial^i_x A^{j,c}_{(\underset{\sim}{x})}-\partial^j A^{i,c}_{(\underset{\sim}{x})}+gf^{ced}A^{i,e}_{(\underset{\sim}{x})}A^{j,d}_{(\underset{\sim}{x})}\right)\right]\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}\approx 0\,;$$

$$(B.26f)$$

g) $\left[\tau^a_{(\underset{\sim}{x})},\ \tau^b_{(\underset{\sim}{y})}\right]_{PP}=$

$$=\left[\tau^a_{(\underset{\sim}{x})},\ D^{j,bc}_{(\underset{\sim}{y})}\pi^c_{j}{}_{(\underset{\sim}{y})}-ig\pi_\psi{}_{(\underset{\sim}{y})}\frac{\lambda^b}{2}\psi_{(\underset{\sim}{y})}\right]_{PP}+$$

$$+gf^{bcd}\left[\tau^a_{(\underset{\sim}{x})},\ F^{0j,c}_{(\underset{\sim}{y})}\pi^d_{0j}{}_{(\underset{\sim}{y})}+\frac{1}{2}F^{ij,c}_{(\underset{\sim}{y})}\pi^d_{ij}{}_{(\underset{\sim}{y})}\right]_{PP}=$$

$$=gf^{abc}\left(D^{j,cd}_{(\underset{\sim}{x})}\pi^d_{j}{}_{(\underset{\sim}{x})}-ig\pi_\psi{}_{(\underset{\sim}{x})}\frac{\lambda^c}{2}\psi_{(\underset{\sim}{x})}\right)\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}+$$

$$+\left\{gf^{bcd}\left[\left(-\frac{\delta\tau^a_{(\underset{\sim}{x})}}{\delta\pi^c_{0j}{}_{(\underset{\sim}{y})}}\right)\pi^d_{0j}{}_{(\underset{\sim}{y})}+F^{0j,c}_{(\underset{\sim}{y})}\left(\frac{\delta\tau^a_{(\underset{\sim}{x})}}{\delta F^{0j,d}_{(\underset{\sim}{y})}}\right)+\frac{1}{2}\left(-\frac{\delta\tau^a_{(\underset{\sim}{x})}}{\delta\pi^c_{ij}{}_{(\underset{\sim}{y})}}\right)\pi^d_{ij}{}_{(\underset{\sim}{y})}+\right.\right.$$

$$\left.\left.+\frac{1}{2}F^{ij,c}_{(\underset{\sim}{y})}\left(\frac{\delta\tau^a_{(\underset{\sim}{x})}}{\delta F^{ij,d}_{(\underset{\sim}{y})}}\right)\right]\right\}\ ,$$

onde usamos o resultado (B.26e). Por outro lado, a chave acima é igual a

$$gf^{bcd}\left(gf^{ace}F^{0j,e}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}\pi^d_{0j}{}_{(\underset{\sim}{y})}+gf^{ade}F^{0j,c}_{(\underset{\sim}{y})}\pi^e_{0j}{}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}+\right.$$

$$+\frac{1}{2}gf^{ace}F^{ij,e}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}\Pi^{d}_{ij}{}_{(\underset{\sim}{y})}+\frac{1}{2}gf^{ade}F^{ij,c}_{(\underset{\sim}{y})}\Pi^{\cdot e}_{ij}{}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}\Big)=$$

$$=gf^{abc}\Big[\Big(f^{bcd}f^{ace}+f^{bec}f^{acd}\Big)F^{oj,e}_{(\underset{\sim}{x})}\Pi^{\cdot d}_{oj}{}_{(\underset{\sim}{x})}+$$

$$+\frac{1}{2}\Big(f^{bcd}f^{ace}+f^{bec}f^{acd}\Big)F^{ij,e}_{(\underset{\sim}{x})}\Pi^{d}_{ij}{}_{(\underset{\sim}{x})}\Big]\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}=$$

$$=gf^{abc}\Big(gf^{ced}F^{oj,e}_{(\underset{\sim}{x})}\Pi^{d}_{oj}{}_{(\underset{\sim}{x})}+\frac{1}{2}gf^{ced}F^{ij,e}_{(\underset{\sim}{x})}\Pi^{d}_{ij}{}_{(\underset{\sim}{x})}\Big)\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})},$$

usando (B.21). Logo, desde (B.25), temos que

$$\Big[\tau^{a}_{(\underset{\sim}{x})},\tau^{b}_{(\underset{\sim}{y})}\Big]_{PP}=gf^{abc}\tau^{c}_{(\underset{\sim}{x})}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}\approx 0\ . \qquad (B.26g)$$

As expressões (B.26) mostram que os τ^a's dados por (B.25) são vínculos de 1ª classe (q.e.d.). Com isto, os vínculos irredutíveis de 1ª classe são $\pi^a \approx 0$ e $\tau^a \approx 0$, os restantes são os vínculos irredutíveis de 2ª classe:

$$\Pi^{\cdot a}_{oj}\approx 0\ ,\ \Pi^{\cdot a}_{ij}\approx 0\ ,\ \Pi^{\cdot a}_{j}-F^{oj,a}\approx 0\ , \qquad (B.27\ a,b,c)$$

$$F^{ij,a}-\big(\partial^{i}A^{j,a}-\partial^{j}A^{i,a}+gf^{abc}A^{i,b}A^{j,c}\big)\approx 0\ . \qquad (B.27d)$$

Note-se que o número de vínculos irredutíveis de 2ª classe é <u>par</u> ($12(N^2-1)$ = número par), como deveria ser [31,32,37].

APÊNDICE C

OBTENÇÃO E RESOLUÇÃO DO SISTEMA DE EQUAÇÕES DIFERENCIAIS ACOPLADAS PARA OS ELEMENTOS DA INVERSA DA MATRIZ DE FADDEEV-POPOV

Começamos este Apêndice pela obtenção das equações (3.4) através da substituição de (3.1) em (2.42a), i.e., em

$$\sum_{K=1}^{14} \int d^3z\, R^{ac}_{JK}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{KL}(\underset{\sim}{z};\underset{\sim}{y}) = \delta^{ab}\, \delta^{JL}\, \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) . \qquad (C.1)$$

a) Fixando L = k em (C.1), as expressões (3.1 a,b) implicam

$$\int d^3z \left[R^{ac}_{J,j+6}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{j+6,k}(\underset{\sim}{z};\underset{\sim}{y}) + R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{11,k}(\underset{\sim}{z};\underset{\sim}{y}) \right] =$$

$$= -\int d^3z\, R^{ac}_{J,j+6}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{cb}\, \delta^{jk}\, \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{y}) + D^{k,bc}(\underset{\sim}{y}) \int d^3z\, R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{y}) =$$

$$= -R^{ab}_{J,k+6}(\underset{\sim}{x};\underset{\sim}{y}) + D^{k,bc}(\underset{\sim}{y})\, R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{y}) = \delta^{ab}\, \delta^{J,k}\, \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) , \qquad (C.2a)$$

de onde segue (3.4a).

b) Fixando L = 4 em (C.1) e levando em conta (3.1 c,f)

$$\int d^3z \left[R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{10,4}(\underset{\sim}{z};\underset{\sim}{y}) + R^{ac}_{J,13}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{13,4}(\underset{\sim}{z};\underset{\sim}{y}) \right] =$$

$$= -\partial_y^3 \int d^3z \, \underset{J,10}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{bc}\, \delta^{(3)}(\underset{\sim}{z}-y) + \int d^3z\, \underset{J,13}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{bc}\,\delta(z^2-y^2)\,\delta(x^3_{(0)}-y^3)\,\Delta(z^1,x^1_{(0)};y^1) =$$

$$= -\partial_y^3 \underset{J,10}{R}^{ab}(\underset{\sim}{x};y) + \delta(y^3-x^3_{(0)}) \int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \underset{J,13}{R}^{ab}(\underset{\sim}{x};z^1,y^2,z^3)\,\Delta(z^1,x^1_{(0)};y^1) = \delta^{ab}\,\delta^{J,4}\,\delta^{(3)}(\underset{\sim}{x}-y),$$

$$(C.2b)$$

de onde segue (3.4b).

c) L = 5 em (C.1) e (3.1 d,g) ⟹

$$\int d^3z \left[\underset{J,10}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\, \underset{10,5}{Q}^{cb}(\underset{\sim}{z};y) + \underset{J,13}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\, \underset{13,5}{Q}^{cb}(\underset{\sim}{z};y) \right] =$$

$$= -D^{1,bc}(y) \int d^3z\, \underset{J,10}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-y) - \int d^3z\, \underset{J,13}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{bc}\,\delta(y^1-z^1)\,\delta(y^2-z^2)\,\Delta(z^3,x^3_{(0)};y^3) =$$

$$= -D^{1,bc}(y)\, \underset{J,10}{R}^{ac}(\underset{\sim}{x};y) - \int_{-\infty}^{+\infty} dz^3\, \underset{J,13}{R}^{ab}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(z^3,x^3_{(0)};y^3) = \delta^{ab}\,\delta^{J,5}\,\delta^{(3)}(x-y),$$

$$(C.2c)$$

que implica (3.4c).

d) L = 6 em (C.1) e (3.1 e,h) ⟹

$$\int d^3z \left[\underset{J,10}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\, \underset{10,6}{Q}^{cb}(\underset{\sim}{z};y) + \underset{J,12}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\, \underset{12,6}{Q}^{cb}(\underset{\sim}{z};y) \right] =$$

$$= -D^{2,bc}_{(y)}\int d^3z\, R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-y) + \int d^3z\, R^{ac}_{J,12}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{bc}\,\delta(z^1-y^1)\,\delta(z^2-y^2)\,\Delta(z^3,x^3_{(0)};y^3) =$$

$$= -D^{2,bc}_{(y)}\, R^{ac}_{J,10}(\underset{\sim}{x};y) + \int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,12}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(z^3,x^3_{(0)};y^3) = \delta^{ab}\,\delta^{J,6}\,\delta^{(3)}(\underset{\sim}{x}-y),$$

$$(C.2d)$$

que implica (3.4d).

e) L = k+6 em (C.1) e (3.1a,i,j,k) $\Longrightarrow$

$$\int d^3z\Big[R^{ac}_{J,j}(\underset{\sim}{x};\underset{\sim}{z})\,Q^{cb}_{j,k+6}(\underset{\sim}{z};y) + R^{ac}_{J,j+11}(\underset{\sim}{x};\underset{\sim}{z})\,Q^{cb}_{j+11,k+6}(\underset{\sim}{z};y) + R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{z})\,Q^{cb}_{10,k+6}(\underset{\sim}{z};y) +$$

$$+ R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{z})\,Q^{cb}_{11,k+6}(\underset{\sim}{z};y)\Big] =$$

$$= \int d^3z\Big[R^{ac}_{J,j}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{cb}\delta^{jk}\delta^{(3)}(\underset{\sim}{z}-y) + R^{ac}_{J,j+11}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{bc}\delta^{jk}\delta^{(3)}(\underset{\sim}{z}-y) +$$

$$+ R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{z})\,g f^{bdc}\big(\delta^{k1}F^{23,d}_{(y)} + \delta^{k2}F^{31,d}_{(y)}\big)\delta^{(3)}(\underset{\sim}{z}-y) -$$

$$- R^{ac}_{J,11}(\underset{\sim}{x};\underset{\sim}{z})\,g f^{bdc}F^{ok,d}_{(y)}\,\delta^{(3)}(\underset{\sim}{z}-y)\Big] =$$

$$= R^{ab}_{J,k}(\underset{\sim}{x};y) + R^{ab}_{J,k+11}(\underset{\sim}{x};y) + g f^{cbd}\big(\delta^{k1}F^{23,d}_{(y)} + \delta^{k2}F^{31,d}_{(y)}\big)R^{ac}_{J,10}(\underset{\sim}{x};y) -$$

$$- g f^{cbd}F^{ok,d}_{(y)}\,R^{ac}_{J,11}(\underset{\sim}{x};y) = \delta^{ab}\,\delta^{J,k+6}\,\delta^{(3)}(\underset{\sim}{x}-y), \qquad (C.2e)$$

que coincide com (3.4e).

f) L = 10 em (C.1) e (3.1c,d,e,i,j) $\Longrightarrow$

$$\int d^3z \Big[\underset{J,4}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{4,10}{Q}^{cb}(\underset{\sim}{z};y) + \underset{J,5}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{5,10}{Q}^{cb}(\underset{\sim}{z};y) + \underset{J,6}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{6,10}{Q}^{cb}(\underset{\sim}{z};y) +$$

$$+ \underset{J,7}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{7,10}{Q}^{cb}(\underset{\sim}{z};y) + \underset{J,8}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{8,10}{Q}^{cb}(\underset{\sim}{z};y) \Big] =$$

$$= \int d^3z \Big[- \underset{J,4}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\delta^{bc}\,\partial^3_y\,\delta^{(3)}(\underset{\sim}{z}-y) - \underset{J,5}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,D^{1,bc}(y)\,\delta^{(3)}(\underset{\sim}{z}-y) -$$

$$- \underset{J,6}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,D^{2,bc}(y)\,\delta^{(3)}(\underset{\sim}{z}-y) + g f^{cbd}\,\underset{J,7}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,F^{23,d}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-y) + g f^{cbd}\,\underset{J,8}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,F^{31,d}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-y) \Big] =$$

$$= -\partial^3_y\,\underset{J,4}{R}^{ab}(\underset{\sim}{x};y) - D^{1,bc}(y)\,\underset{J,5}{R}^{ac}(\underset{\sim}{x};y) - D^{2,bc}(y)\,\underset{J,6}{R}^{ac}(\underset{\sim}{x};y) +$$

$$+ g f^{cbd}\Big(F^{23,d}(y)\,\underset{J,7}{R}^{ac}(\underset{\sim}{x};y) + F^{31,d}(y)\,\underset{J,8}{R}^{ac}(\underset{\sim}{x};y) \Big) = \delta^{ab}\,\delta^{J,10}\,\delta^{(3)}(\underset{\sim}{x}-y), \tag{C.2f}$$

de onde segue (3.4f).

g) L = 11 em (C.1) e (3.1b,k) $\Longrightarrow$

$$\int d^3z \Big[\underset{J,k}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{k,11}{Q}^{cb}(\underset{\sim}{z};y) + \underset{J,k+6}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,\underset{k+6,11}{Q}^{cb}(\underset{\sim}{z};y) \Big] =$$

$$= \int d^3z \Big[-\underset{J,k}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,D^{k,cb}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-y) - g f^{cbd}\,\underset{J,k+6}{R}^{ac}(\underset{\sim}{x};\underset{\sim}{z})\,F^{0k,d}(\underset{\sim}{z})\,\delta^{(3)}(\underset{\sim}{z}-y) \Big] =$$

$$= D^{k,bc}(y)\, R^{ac}_{J,k}(\underset{\sim}{x};y) + g f^{cdb} F^{0k,d}(y)\, R^{ac}_{J,k+6}(\underset{\sim}{x};y) = \delta^{ab}\delta^{J,11}\delta^{(3)}(\underset{\sim}{x}-y), \qquad (C.2g)$$

que coincide com (3.4g).

h) L = 12 em (C.1) e (3.1a,h) $\Longrightarrow$

$$\int d^3z \left[R^{ac}_{J,7}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{7,12}(\underset{\sim}{z};y) + R^{ac}_{J,6}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{6,12}(\underset{\sim}{z};y) \right] =$$

$$= \int d^3z \left[-R^{ac}_{J,7}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{cb}\delta^{(3)}(\underset{\sim}{z}-y) - R^{ac}_{J,6}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{cb}\delta(z^1-y^1)\delta(z^2-y^2)\Delta(y^3,x^3_{(0)};z^3) \right] =$$

$$= -R^{ab}_{J,7}(\underset{\sim}{x};y) - \int_{-\infty}^{+\infty} dz^3\, R^{ab}_{J,6}(\underset{\sim}{x};y^1,y^2,z^3)\,\Delta(y^3,x^3_{(0)};z^3) = \delta^{ab}\delta^{J,12}\delta^{(3)}(\underset{\sim}{x}-y), \qquad (C.2h)$$

que nos leva a (3.4h).

i) L = 13 em (C.1) e (3.1a,f,g) $\Longrightarrow$

$$\int d^3z \left[R^{ac}_{J,8}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{8,13}(\underset{\sim}{z};y) + R^{ac}_{J,4}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{4,13}(\underset{\sim}{z};y) + R^{ac}_{J,5}(\underset{\sim}{x};\underset{\sim}{z})\, Q^{cb}_{5,13}(\underset{\sim}{z};y) \right] =$$

$$= \int d^3z \left[-R^{ac}_{J,8}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{bc}\delta^{(3)}(\underset{\sim}{z}-y) - R^{ac}_{J,4}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{bc}\delta(z^2-y^2)\delta(z^3-x^3_{(0)})\,\Delta(y^1,x^1_{(0)};z^1) + \right.$$

$$\left. + R^{ac}_{J,5}(\underset{\sim}{x};\underset{\sim}{z})\, \delta^{bc}\delta(z^1-y^1)\,\delta(z^2-y^2)\,\Delta(y^3,x^3_{(0)};z^3) \right] =$$

$$= -\underset{J,8}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{y}) - \int_{-\infty}^{+\infty} dz^1 \underset{J,4}{R^{ab}}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \Delta(y^1, x^1_{(0)}; z^1) +$$

$$+ \int_{-\infty}^{+\infty} dz^3 \underset{J,5}{R^{ab}}(\underset{\sim}{x}; y^1, y^2, z^3) \Delta(y^3, x^3_{(0)}; z^3) = \delta^{ab} \delta^{J,13} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) , \qquad (C.2\,i)$$

que implica (3.4 i).

j) L = 14 em (C.1) e (3.1a)

$$\int d^3z \underset{J,9}{R^{ac}}(\underset{\sim}{x};\underset{\sim}{z}) \underset{9,14}{Q^{cb}}(\underset{\sim}{z};\underset{\sim}{y}) = -\int d^3z \underset{J,9}{R^{ac}}(\underset{\sim}{x};\underset{\sim}{z}) \delta^{bc} \overset{(3)}{\delta}(\underset{\sim}{z}-\underset{\sim}{y}) =$$

$$= -\underset{J,9}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{y}) = \delta^{ab} \delta^{J,14} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) , \qquad (C.2j)$$

que fornece (3.4j).

Continuamos agora a integração do sistema de equações (3.4) (ou (C.2)) iniciada na seção III.1, p.35, com o cálculo dos elementos $R^{ab}_{J,11}$, $R^{ab}_{J,5}$ e $R^{ab}_{J,6}$ (ver (3.17), (3.9) e (3.10)). Notar que, desde (3.17) e (3.4a), já podemos encontrar expressões explícitas para os elementos $R^{ab}_{J,k+6}$

$$\boxed{\underset{J,k+6}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{y}) = -\delta^{ab} \delta^{J,k} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + D^{k,bc}(\underset{\sim}{y}) \underset{J,11}{R^{ac}}(\underset{\sim}{x};\underset{\sim}{y})} \qquad . \qquad (C.3)$$

Com os elementos conhecidos até aqui, calculamos $R^{ab}_{J,4}$. De fato, usando (C.3), começamos o cálculo da chave em (3.8) computando a seguinte integral:

$$gf^{bcd}\int_{-\infty}^{+\infty} dz^3\,\Delta(y^3,x^3_{(0)};z^3)\Big[F^{23,d}(y^1,y^2,z^3)\,R^{ac}_{J,7}(\underset{\sim}{x};y^1,y^2,z^3)+F^{31,d}(y^1,y^2,z^3)\,R^{ac}_{J,8}(\underset{\sim}{x};y^1,y^2,z^3)\Big]=$$

$$=gf^{bcd}\int_{-\infty}^{+\infty} dz^3\,\Delta(y^3,x^3_{(0)};z^3)\Big\{F^{23,d}(y^1,y^2,z^3)\Big[-\delta^{ac}\delta^{J,1}\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3)+D^{1,ce}(y^1,y^2,z^3)R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big]+$$

$$+F^{31,d}(y^1,y^2,z^3)\Big[-\delta^{ac}\delta^{J,2}\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3)+D^{2,ce}(y^1,y^2,z^3)\,R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big]\Big\}=$$

$$=gf^{abc}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3)\left(\delta^{J,1}F^{23,c}(\underset{\sim}{x})+\delta^{J,2}F^{31,c}(\underset{\sim}{x})\right)+$$

$$+gf^{bed}\int_{-\infty}^{+\infty} dz^3\,\Delta(y^3,x^3_{(0)};z^3)\Big[\left(\partial^z_3 A^{2,d}(y^1,y^2,z^3)\right)D^{1,ce}(y^1,y^2,z^3)\,R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)+$$

$$+\left(-\partial^z_3 A^{1,d}(y^1,y^2,z^3)\right)D^{2,ce}(y^1,y^2,z^3)\,R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big]\,. \quad (C.4)$$

Levando em conta que $\Delta(y^3,x^3_{(0)};\pm\infty)=0$, calculamos por partes a integral no lado direito de (C.4) como segue

$$gf^{bcd}\int_{-\infty}^{+\infty} dz^3\Big\{A^{1,d}(y^1,y^2,z^3)\Big[\left(D^{2,ce}(y^1,y^2,z^3)\,R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\right)\left(\delta(z^3-x^3_{(0)})-\delta(z^3-y^3)\right)+$$

$$+\Delta(y^3,x^3_{(0)};z^3)\;\partial^z_3\left(\partial^2_y R^{ac}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)+gf^{cfe}A^{2,f}(y^1,y^2,z^3)R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\right)\Big]-$$

$$-A^{2,d}_{(y^1,y^2,z^3)}\Big[\Big(D^{1,ce}_{(y^1,y^2,z^3)}\underset{J,11}{R}^{ae}(\underset{\sim}{x};y^1,y^2,z^3)\Big)\Big(\delta(z^3-x^3_{(0)})-\delta(z^3-y^3)\Big)+$$

$$+\Delta(y^3,x^3_{(0)};z^3)\,\partial^z_3\Big(\partial^1_y\underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,z^3)+gf^{cfe}A^{1,f}(y^1,y^2,z^3)\underset{J,11}{R}^{ae}(\underset{\sim}{x};y^1,y^2,z^3)\Big)\Big]\Big\}=$$

$$=-gf^{bcd}A^{1,d}_{(y)}D^{2,ce}_{(y)}\underset{J,11}{R}^{ae}(\underset{\sim}{x};y)-$$

$$-gf^{bcd}\Big[A^{2,d}_{(y^1,y^2,x^3_{(0)})}D^{1,ce}_{(y^1,y^2,x^3_{(0)})}R^{ae}(\underset{\sim}{x};y^1,y^2,x^3_{(0)})-A^{2,d}_{(y)}D^{1,ce}_{(y)}\underset{J,11}{R}^{ae}(\underset{\sim}{x};y)\Big]+$$

$$+gf^{bcd}\int_{-\infty}^{+\infty}dz^3\,\Delta(y^3,x^3_{(0)};z^3)\Big\{A^{1,d}_{(y^1,y^2,z^3)}\Big[\partial^z_3\Big(\partial^2_y\underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,z^3)\Big)+$$

$$+gf^{cfe}\Big(\underset{J,11}{R}^{ae}(\underset{\sim}{x};y^1,y^2,z^3)\,\partial^z_3A^{2,f}_{(y^1,y^2,z^3)}+A^{2,f}_{(y^1,y^2,z^3)}\,\partial^z_3\underset{J,11}{R}^{ae}(\underset{\sim}{x};y^1,y^2,z^3)\Big)\Big]-$$

$$-A^{2,d}_{(y^1,y^2,z^3)}\Big[\partial^z_3\Big(\partial^1_y\underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,z^3)\Big)+$$

$$+gf^{cfe}\Big(\underset{J,11}{R}^{ae}(\underset{\sim}{x};y^1,y^2,z^3)\,\partial^z_3A^{1,f}_{(y^1,y^2,z^3)}+A^{1,f}_{(y^1,y^2,z^2)}\,\partial^z_3\underset{J,11}{R}^{ae}(\underset{\sim}{x};y^1,y^2,z^3)\Big)\Big]\Big\}=$$

$$=-gf^{bcd}A^{1,d}_{(y)}\,\partial^2_y\underset{J,11}{R}^{ac}(\underset{\sim}{x};y)-$$

$$-gf^{bcd}\Big[A^{2,d}_{(y^1,y^2,x^3_{(0)})}\,\partial^1_y\underset{J,11}{R}^{ac}(\underset{\sim}{x};y^1,y^2,x^3_{(0)})-A^{2,d}_{(y)}\,\partial^1_y\underset{J,11}{R}^{ac}(\underset{\sim}{x};y)\Big]+$$

$$+g^{2} f^{bcd} f^{cfe} \left(A^{1,f}_{(y)} A^{2,d}_{(y)} - A^{1,d}_{(y)} A^{2,f}_{(y)} \right) R^{ae}_{J,11}(\underset{\sim}{x}; y) \quad +$$

$$+g^{2} f^{bcd} f^{cfe} \int_{-\infty}^{+\infty} dz^{3}\, \Delta(y^{3}, x^{3}_{(10)}; z^{3})\, R^{ae}_{J,11}(\underset{\sim}{x}; y^{1}, y^{2}, z^{3}) \left[A^{1,d}(y^{1}, y^{2}, z^{3})\, \partial^{z}_{3} A^{2,f}(y^{1}, y^{2}, z^{3}) - A^{2,d}(y^{1}, y^{2}, z^{3})\, \partial^{z}_{3} A^{1,f}(y^{1}, y^{2}, z^{3}) \right] +$$

$$+g^{2} f^{bcd} f^{cfe} \int_{-\infty}^{+\infty} dz^{3}\, \Delta(y^{3}, x^{3}_{(10)}; z^{3}) \Big\{ \left[A^{1,d}(y^{1}, y^{2} z^{3}) A^{2,f}(y^{1}, y^{2} z^{3}) - A^{2,d}(y^{1}, y^{2}, z^{3}) A^{1,f}(y^{1}, y^{2}, z^{3}) \right] \cdot$$

$$\cdot \left[-\delta^{ae} (\delta^{J,3} - \delta^{J,14}) \delta(x^{1} - y^{1}) \delta(x^{2} - y^{2}) \delta(x^{3} - z^{3}) \right] \Big\} +$$

$$+g f^{bcd} \int_{-\infty}^{+\infty} dz^{3}\, \Delta(y^{3}, x^{3}_{(10)}; z^{3}) \Big\{ A^{1,d}(y^{1}, y^{2}, z^{3}) \left[-\delta^{ac} (\delta^{J,3} - \delta^{J,14}) \delta(x^{1} - y^{1}) (\partial^{2}_{y} \delta(x^{2} - y^{2})) \delta(x^{3} - z^{3}) \right] -$$

$$- A^{2,d}(y^{1}, y^{2}, z^{3}) \left[-\delta^{ac} (\delta^{J,3} - \delta^{J,14}) (\partial^{1}_{y} \delta(x^{1} - y^{1})) \delta(x^{2} - y^{2}) \delta(x^{3} - z^{3}) \right] \Big\} =$$

$$= g f^{abc} A^{1,c}_{(y)} \Big[-(\delta^{J,1} - \delta^{J,12}) \Delta(y^{1}, x^{1}_{(10)}; x^{1})\, \partial^{2}_{y} \delta(x^{2} - y^{2}) \delta(x^{3} - x^{3}_{(10)}) \; +$$

$$+ (\delta^{J,2} - \delta^{J,13}) \delta(x^{1} - x^{1}_{(10)}) \delta(x^{2} - y^{2}) \delta(x^{3} - x^{3}_{(10)}) -$$

$$- (\delta^{J,3} - \delta^{J,14}) \delta(x^{1} - y^{1})\, \partial^{2}_{y} \delta(x^{2} - y^{2})\, \Delta(y^{3}, x^{3}_{(10)}; x^{3}) \Big] \; +$$

$$+g f^{abc} \left(A^{2,c}(y^{1}, y^{2}, x^{3}_{(10)}) - A^{2,c}_{(y)} \right) (\delta^{J,1} - \delta^{J,12}) \delta(x^{1} - y^{1}) \delta(x^{2} - y^{2}) \delta(x^{3} - x^{3}_{(10)}) \; +$$

$$+g f^{abc} A^{2,c}_{(y)} (\delta^{J,3} - \delta^{J,14})\, \partial^{1}_{y} \delta(x^{1} - y^{1}) \delta(x^{2} - y^{2}) \Delta(y^{3}, x^{3}_{(10)}; x^{3}) \; +$$

$$+g^2 f^{ebc} f^{cfd} A^{1,f}(y)\, A^{2,d}(y)\, R^{ae}_{J,11}(\underset{\sim}{x};y) \quad +$$

$$+g^2 f^{dbc} f^{cfe} \int_{-\infty}^{+\infty} dz^3\, \Delta(y^3, x^3_{(0)}; z^3)\, R^{ae}_{J,11}(\underset{\sim}{x}; y^1, y^2, z^3) \left[A^{1,d}(y^1,y^2,z^3)\, \partial^z_3 A^{2,f}(y^1,y^2,z^3) - A^{2,d}(y^1,y^2,z^3)\, \partial^z_3 A^{1,f}(y^1,y^2,z^3) \right] +$$

$$+g^2 f^{abc} f^{cfd} (\delta^{J,3} - \delta^{J,14})\, \delta(x^1-y^1)\, \delta(x^2-y^2)\, \Delta(y^3, x^3_{(0)}; x^3)\, A^{1,f}(\underset{\sim}{x})\, A^{2,d}(\underset{\sim}{x}) \quad +$$

$$+g f^{abc} A^{1,c}(y^1,y^2,x^3)\, (\delta^{J,3}-\delta^{J,14})\, \delta(x^1-y^1)\, \partial^2_y \delta(x^2-y^2)\, \Delta(y^3,x^3_{(0)};x^3) \quad -$$

$$-g f^{abc} A^{2,c}(y^1,y^2,x^3)\, (\delta^{J,3}-\delta^{J,14})\, \partial^1_y \delta(x^1-y^1)\, \delta(x^2-y^2)\, \Delta(y^3,x^3_{(0)};x^3) \, . \quad (C.5)$$

Para completar o cãlculo de $R^{ab}_{J,4}$ devemos ainda calcular a integral (ver (3.8), (3.11), (3.13) e (3.14))

$$g f^{bdc} \int_{-\infty}^{+\infty} dz^3\, \Delta(y^3, x^3_{(0)}; z^3) \left[A^{1,d}(y^1,y^2,z^3)\, R^{ac}_{J,5}(\underset{\sim}{x}; y^1,y^2,z^3) + A^{2,d}(y^1,y^2,z^3)\, R^{ac}_{J,6}(\underset{\sim}{x}; y^1,y^2,z^3) \right] =$$

$$= g f^{bdc} \int_{-\infty}^{+\infty} dz^3\, \Delta(y^3, x^3_{(0)}; z^3) \Big\{ A^{1,d}(y^1,y^2,z^3) \Big[\delta^{ac} (\delta^{J,2} - \delta^{J,13})\, \delta(x^1-y^1)\, \delta(x^2-y^2) \left(-\partial^z_3 \delta(x^3-z^3)\right) -$$

$$-(\delta^{J,3}-\delta^{J,14})\, \delta(x^1-y^1) \left(D^{2,ca}(y^1,y^2,z^3)\, \delta(x^2-y^2)\right) \delta(x^3-z^3) + g f^{cfe} \left(\partial^z_3 A^{2,f}(y^1,y^2,z^3)\right) R^{ae}_{J,11}(\underset{\sim}{x}; y^1,y^2,z^3) \Big] +$$

$$+ A^{2,d}(y^1,y^2,z^3) \Big[\delta^{ac} (\delta^{J,1} - \delta^{J,12})\, \delta(x^1-y^1)\, \delta(x^2-y^2) \left(\partial^z_3 \delta(x^3-z^3)\right) +$$

$$+(\delta^{J,3}-\delta^{J,14})\Big(D^{1,ca}(y^1,y^2,z^3)\delta(x^1-y^1)\Big)\delta(x^2-y^2)\delta(x^3-z^3)+gf^{cfe}\Big(-\partial^z_3 A^{1,f}(y^1,y^2,z^3)\Big)R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big]\Big\}=$$

$$=-g^2 f^{dbc} f^{cfe}\int_{-\infty}^{+\infty} dz^3\,\Delta(y^3,x^3_{(0)};z^3)\,R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big[A^{1,d}(y^1,y^2,z^3)\partial^z_3 A^{2,f}(y^1,y^2,z^3)-A^{2,d}(y^1,y^2,z^3)\partial^z_3 A^{1,f}(y^1,y^2,z^3)\Big]-$$

$$-gf^{bdc}(\delta^{J,3}-\delta^{J,14})\delta(x^1-y^1)\Big(D^{2,ca}(y^1,y^2,x^3)\delta(x^2-y^2)\Big)\Delta(y^3,x^3_{(0)};x^3)\,A^{1,d}(y^1,y^2,x^3)+$$

$$+gf^{bdc}(\delta^{J,3}-\delta^{J,14})\Big(D^{1,ca}(y^1,y^2,x^3)\delta(x^1-y^1)\Big)\delta(x^2-y^2)\,\Delta(y^3,x^2_{(0)};x^3)\,A^{2,d}(y^1,y^2,x^3)\ +$$

$$+gf^{abc}A^{1,c}(y^1,y^2,x^3)\Big(\delta(x^3-x^3_{(0)})-\delta(x^3-y^3)\Big)(\delta^{J,2}-\delta^{J,13})\delta(x^1-y^1)\delta(x^2-y^2)\ -$$

$$-gf^{abc}A^{2,c}(y^1,y^2,x^3)\Big(\delta(x^3-x^3_{(0)})-\delta(x^3-y^3)\Big)(\delta^{J,1}-\delta^{J,12})\delta(x^1-y^1)\delta(x^2-y^2)\ +$$

$$+gf^{abc}\Big(\partial^x_3 A^{1,c}(\underset{\sim}{x})\Big)(\delta^{J,2}-\delta^{J,13})\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3)\ -$$

$$-gf^{abc}\Big(\partial^x_3 A^{2,c}(\underset{\sim}{x})\Big)(\delta^{J,1}-\delta^{J,12})\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3)\quad=$$

$$=-g^2 f^{dbc} f^{cfe}\int_{-\infty}^{+\infty} dz^3\,\Delta(y^3,x^3_{(0)};z^3)\,R^{ae}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3)\Big[A^{1,d}(y^1,y^2,z^3)\partial^z_3 A^{2,f}(y^1,y^2,z^3)-A^{2,d}(y^1,y^2,z^3)\partial^z_3 A^{1,f}(y^1,y^2,z^3)\Big]-$$

$$-gf^{abc}A^{1,c}(y^1,y^2,x^3)(\delta^{J,3}-\delta^{J,14})\delta(x^1-y^1)\partial^2_y\delta(x^2-y^2)\ \Delta(y^3,x^3_{(0)};x^3)\ +$$

$$+gf^{abc}A^{2,c}(y^1,y^2,x^3)(\delta^{J,3}-\delta^{J,14})\,\partial^1_y\delta(x^1-y^1)\,\delta(x^2-y^2)\ \Delta(y^3,x^3_{(0)};x^3)\ +$$

$$+g^2 f^{abc} f^{cde} A^{1,e}_{(\underset{\sim}{x})} A^{2,d}_{(\underset{\sim}{x})} (\delta^{J,3} - \delta^{J,14}) \delta(x^1 - y^1) \delta(x^2 - y^2) \Delta(y^3, x^3_{(0)}; x^3) -$$

$$-g f^{abc} A^{1,c}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x} - \underset{\sim}{y}) (\delta^{J,2} - \delta^{J,13}) -$$

$$-g f^{abc} A^{2,c}_{(\underset{\sim}{x})} \delta(x^1 - y^1) \delta(x^2 - y^2) \left(\delta(x^3 - x^3_{(0)}) - \delta(x^3 - y^3) \right) (\delta^{J,1} - \delta^{J,12}) -$$

$$-g f^{abc} F^{31,c}_{(\underset{\sim}{x})} (\delta^{J,2} - \delta^{J,13}) \delta(x^1 - y^1) \delta(x^2 - y^2) \Delta(y^3, x^3_{(0)}; x^3) -$$

$$-g f^{abc} F^{23,c}_{(\underset{\sim}{x})} (\delta^{J,1} - \delta^{J,12}) \delta(x^1 - y^1) \delta(x^2 - y^2) \Delta(y^3, x^3_{(0)}; x^3) \,. \qquad (C.6)$$

As demais integrais em (3.8) já foram calculadas. Realmente, o resultado (3.14) implica

$$g f^{bdc} A^{1,d}_{(y)} \int_{-\infty}^{+\infty} dz^1 \, R^{ac}_{J,4}(\underset{\sim}{x}; z^1, y^2, x^3_{(0)}) \Delta(y^1, x^1_{(0)}; z^1) =$$

$$= g f^{abc} A^{1,c}_{(y)} (\delta^{J,1} - \delta^{J,12}) \Delta(y^1, x^1_{(0)}; x^1) \partial^2_y \delta(x^2 - y^2) \delta(x^3 - x^3_{(0)}) -$$

$$-g f^{abc} A^{1,c}_{(y)} (\delta^{J,2} - \delta^{J,13}) \left[\delta(x^1 - x^1_{(0)}) - \delta(x^1 - y^1) \right] \delta(x^2 - y^2) \delta(x^3 - x^3_{(0)}) -$$

$$-g^2 f^{bdc} f^{cfe} A^{1,d}_{(y)} A^{2,f}_{(y^1, y^2, x^3_{(0)})} R^{ae}_{J,11}(\underset{\sim}{x}; y^1, y^2, x^3_{(0)}) \;; \qquad (C.7)$$

o resultado (3.13) implica

$$-g f^{bdc} A^{1,d}_{(y)} \int_{-\infty}^{+\infty} dz^3 \, \Delta(y^3, x^3_{(0)}; z^3) \, R^{ac}_{J,5}(\underset{\sim}{x}; y^1, y^2, z^3) =$$

$$
\begin{aligned}
&= -g f^{abc} A^{1,c}(y)\left(\delta^{J,2} - \delta^{J,13}\right) \delta(x^1 - y^1)\, \delta(x^2 - y^2) \left[\delta(x^3 - x^3_{(0)}) - \delta(x^3 - y^3)\right] + \\
&+ g f^{abc} A^{1,c}(y)\left(\delta^{J,3} - \delta^{J,14}\right) \delta(x^1 - y^1)\, \partial^2_y \delta(x^2 - y^2)\, \Delta(y^3, x^3_{(0)}; x^3) + \\
&+ g^2 f^{bdc} f^{cfe} A^{1,d}(y)\left(A^{2,f}(y^1, y^2, x^3_{(0)})\, R^{ae}_{J,11}(\underset{\sim}{x}; y^1, y^2, x^3_{(0)}) - A^{2,f}(y)\, R^{ae}_{J,11}(\underset{\sim}{x}; y)\right) \qquad (C.8)
\end{aligned}
$$

e o resultado (3.11) fornece

$$
\begin{aligned}
&-g f^{bdc} A^{2,d}(y) \int_{-\infty}^{+\infty} dz^3\, \Delta(y^3, x^3_{(0)}; z^3)\, R^{ac}_{J,6}(\underset{\sim}{x}; y^1, y^2, z^3) = \\
&= g f^{abc} A^{2,c}(y)\left(\delta^{J,1} - \delta^{J,12}\right) \delta(x^1 - y^1)\, \delta(x^2 - y^2) \left[\delta(x^3 - x^3_{(0)}) - \delta(x^3 - y^3)\right] - \\
&- g f^{abc} A^{2,c}(y)\left(\delta^{J,3} - \delta^{J,14}\right) \partial^1_y \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \Delta(y^3, x^3_{(0)}; x^3) + \\
&+ g^2 f^{bdc} f^{cfe} A^{2,d}(y)\, A^{1,f}(y)\, R^{ae}_{J,11}(\underset{\sim}{x}; y) \quad . \qquad (C.9)
\end{aligned}
$$

Levando (C.5) em (C.4) e realizando a soma dos resultados (C.4)+ +(C.6)+(C.7)+(C.8)+(C.9) (ou seja, calculando toda a <u>chave</u> em (3.8)), produz-se um cancelamento notável de quase todos os termos que nos deixa com

$$
\left\{ \begin{matrix} \text{chave em} \\ (3.8) \text{ para } R^{ab}_{J,4} \end{matrix} \right\} = g f^{abc} \left(\delta^{J,12} F^{23,c}(\underset{\sim}{x}) + \delta^{J,13} F^{31,c}(\underset{\sim}{x})\right) \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \Delta(y^3, x^3_{(0)}; x^3)
$$

o que, por sua vez, nos leva a

$$\boxed{\begin{aligned}
\underset{J,4}{R}^{ab}(\underset{\sim}{x};y) &= (\delta^{J,1}-\delta^{J,12})D^{2,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) - (\delta^{J,2}-\delta^{J,13})D^{1,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) + \\
&+\delta^{ab}\delta^{J,10}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) + gf^{cbd}F^{12,d}(y)\underset{J,11}{R}^{ac}(\underset{\sim}{x};y) + \\
&+gf^{abc}(\delta^{J,12}F^{23,c}(\underset{\sim}{x})+\delta^{J,13}F^{31,c}(\underset{\sim}{x}))\,\delta(x^1-y^1)\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3). \quad (C.10)
\end{aligned}}$$

Em continuação, passamos ao cálculo dos elementos $R^{ab}_{J,k+11}$. Desde (C.2b,c), é direto obter

$$\begin{aligned}
&[D^1(y),D^3(y)]^{dc}\underset{J,10}{R}^{ac}(\underset{\sim}{x};y) = gf^{dec}F^{13,e}(y)\underset{J,10}{R}^{ac}(\underset{\sim}{x};y) = \\
&= -\delta^{J,4}D^{1,da}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) + \delta^{J,5}D^{3,da}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) + \\
&+\delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\,\underset{J,13}{R}^{ab}(\underset{\sim}{x};z^1,y^2,z^3)\,D^{1,db}(y)\,\Delta(z^1,x^1_{(0)};y^1) + \\
&+\int_{-\infty}^{+\infty}dz^3\,\underset{J,13}{R}^{ab}(\underset{\sim}{x};y^1,y^2,z^3)\,D^{3,db}(y)\,\Delta(z^3,x^3_{(0)};y^3) = \\
&= -\delta^{J,4}D^{1,da}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) + \delta^{da}\delta^{J,5}\partial^3_y\,\delta^{(3)}(\underset{\sim}{x}-y) + \\
&+gf^{deb}A^{1,e}(y)\,\delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\,\underset{J,13}{R}^{ab}(\underset{\sim}{x};z^1,y^2,z^3)\,\Delta(z^1,x^1_{(0)};y^1) + \\
&+\delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^3\int_{-\infty}^{+\infty}dz^1\,\underset{J,13}{R}^{ad}(\underset{\sim}{x};z^1,y^2,z^3)\left[\delta(z^1-y^1)-\delta(y^1-x^1_{(0)})\right] + \\
&+\int_{-\infty}^{+\infty}dz^3\,\underset{J,13}{R}^{ad}(\underset{\sim}{x};y^1,y^2,z^3)\left[\delta(z^3-y^3)-\delta(y^3-x^3_{(0)})\right] \qquad \therefore
\end{aligned}$$

$$\therefore$$

$$\underset{J,13}{R}^{ad}(\underset{\sim}{x};y) = -g f^{dec} F^{31,e}(y)\, \underset{J,10}{R}^{ac}(\underset{\sim}{x};y) + \delta^{J,4} D^{1,da}(y)\, \delta^{(3)}(\underset{\sim}{x-y}) - \delta^{da}\delta^{J,5}\partial^{3}_{y}\delta^{(3)}(\underset{\sim}{x-y}) +$$

$$+ \delta(y^1-x^1_{(0)})\,\delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\; \underset{J,13}{R}^{ad}(\underset{\sim}{x};z^1,y^2,z^3) \,. \tag{C.11}$$

De forma similar, (C.2b,d) implicam

$$[D^2(y), D^3(y)]^{dc}\, \underset{J,10}{R}^{ac}(\underset{\sim}{x};y) = g f^{dec} F^{23,e}(y)\, \underset{J,10}{R}^{ac}(\underset{\sim}{x};y) =$$

$$= -\delta^{J,4} D^{2,da}(y)\, \delta^{(3)}(\underset{\sim}{x-y}) + \delta^{J,6} D^{3,da}(y)\, \delta^{(3)}(\underset{\sim}{x-y}) +$$

$$+ \delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\; \Delta(z^1,x^1_{(0)};y^1)\, D^{2,db}(y)\, \underset{J,13}{R}^{ab}(\underset{\sim}{x};z^1,y^2,z^3) -$$

$$- \int_{-\infty}^{+\infty}dz^3\; \underset{J,12}{R}^{ad}(\underset{\sim}{x};y^1,y^2,z^3)\left[\delta(z^3-y^3) - \delta(y^3-x^3_{(0)})\right] \qquad \therefore$$

$$\therefore$$

$$\underset{J,12}{R}^{ad}(\underset{\sim}{x};y) = -g f^{dec} F^{23,e}(y)\, \underset{J,10}{R}^{ac}(\underset{\sim}{x};y) - \delta^{J,4} D^{2,da}(y)\, \delta^{(3)}(\underset{\sim}{x-y}) + \delta^{da}\delta^{J,6}\partial^{3}_{y}\delta^{(3)}(\underset{\sim}{x-y}) +$$

$$+ \delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\; \Delta(z^1,x^1_{(0)};y^1)\, D^{2,db}(y)\, \underset{J,13}{R}^{ab}(\underset{\sim}{x};z^1,y^2,z^3) +$$

$$+ \delta(y^3-x^3_{(0)})\int_{-\infty}^{+\infty}dz^3\; \underset{J,12}{R}^{ad}(\underset{\sim}{x};y^1,y^2,z^3) \,. \tag{C.12}$$

Além disso, desde (C.2c,d), obtemos

$$[D^{2}_{(\underset{\sim}{y})}, D^{1}_{(\underset{\sim}{y})}]^{dc} \underset{J,10}{R}^{ac}_{(\underset{\sim}{x};\underset{\sim}{y})} = g f^{dec} F^{21,e}_{(\underset{\sim}{y})} \underset{J,10}{R}^{ac}_{(\underset{\sim}{x};\underset{\sim}{y})} =$$

$$= -\delta^{J,5} D^{2,da}_{(\underset{\sim}{y})} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + \delta^{J,6} D^{1,da}_{(\underset{\sim}{y})} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) -$$

$$-\int_{-\infty}^{+\infty} dz^3 \, \Delta(z^3, x^3_{(0)}; y^3) D^{2,db}_{(\underset{\sim}{y})} \underset{J,13}{R}^{ab}_{(\underset{\sim}{x}; y^1, y^2, z^3)} -$$

$$-\int_{-\infty}^{+\infty} dz^3 \, \Delta(z^3, x^3_{(0)}; y^3) D^{1,db}_{(\underset{\sim}{y})} \underset{J,12}{R}^{ab}_{(\underset{\sim}{x}; y^1, y^2, z^3)} \qquad \therefore$$

$$\int_{-\infty}^{+\infty} dz^3 \, \Delta(z^3, x^3_{(0)}; y^3) \left[D^{1,db}_{(\underset{\sim}{y})} \underset{J,12}{R}^{ab}_{(\underset{\sim}{x}; y^1, y^2, z^3)} + D^{2,db}_{(\underset{\sim}{y})} \underset{J,13}{R}^{ab}_{(\underset{\sim}{x}; y^1, y^2, z^3)} \right] =$$

$$= -\delta^{J,5} D^{2,da}_{(\underset{\sim}{y})} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + \delta^{J,6} D^{1,da}_{(\underset{\sim}{y})} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) +$$

$$+ g f^{dec} F^{12,e}_{(\underset{\sim}{y})} \underset{J,10}{R}^{ac}_{(\underset{\sim}{x};\underset{\sim}{y})} \quad . \qquad (C.13)$$

Por outro lado, aplicando a derivada $D^{k,eb}(\underset{\sim}{y})$ à equação (C.2e) (ou (3.4e)) e somando sobre os índices k e b, obtemos

$$D^{k,eb}_{(\underset{\sim}{y})} \underset{J,k}{R}^{ab}_{(\underset{\sim}{x};\underset{\sim}{y})} = \delta^{J,k+6} D^{k,ea}_{(\underset{\sim}{y})} \overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + g f^{cbd} D^{k,eb}_{(\underset{\sim}{y})} \left(F^{0k,d}_{(\underset{\sim}{y})} \underset{J,11}{R}^{ac}_{(\underset{\sim}{x};\underset{\sim}{y})} \right) -$$

$$- D^{k,eb}_{(\underset{\sim}{y})} \underset{J,k+11}{R}^{ab}_{(\underset{\sim}{x};\underset{\sim}{y})} - g f^{cbd} D^{1,eb}_{(\underset{\sim}{y})} \left(F^{23d}_{(\underset{\sim}{y})} \underset{J,10}{R}^{ac}_{(\underset{\sim}{x};\underset{\sim}{y})} \right) -$$

$$-gf^{cbd}\, D^{2,eb}_{(\underset{\sim}{y})}\Big(F^{31,d}_{(\underset{\sim}{y})}\, R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{y})\Big) \quad . \qquad (C.14)$$

A equação (C.2g), por sua vez, é igual a (ver (C.3))

$$D^{k,eb}_{(\underset{\sim}{y})}\, R^{ab}_{J,k}(\underset{\sim}{x};\underset{\sim}{y}) = \delta^{ae}\delta^{J,11}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} - gf^{ecd}\, F^{ok,d}_{(\underset{\sim}{y})}\, R^{ac}_{J,k+6}(\underset{\sim}{x};\underset{\sim}{y}) =$$

$$=\delta^{ae}\delta^{J,11}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} + gf^{ead}\, F^{ok,d}_{(\underset{\sim}{y})}\,\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})}\delta^{J,k} -$$

$$-gf^{ecd}\, F^{ok,d}_{(\underset{\sim}{y})}\, D^{k,cb}_{(\underset{\sim}{y})}\, R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{y}) \quad . \qquad (C.15)$$

Tomando a diferença (C.14)-(C.15), encontramos

$$D^{k,eb}_{(\underset{\sim}{y})}\, R^{ab}_{J,k+11}(\underset{\sim}{x};\underset{\sim}{y}) = \delta^{J,k+6}\, D^{k,ea}_{(\underset{\sim}{y})}\,\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} - \delta^{ae}\delta^{J,11}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} -$$

$$-gf^{cbd}\Big[D^{1,eb}_{(\underset{\sim}{y})}\Big(F^{23,d}_{(\underset{\sim}{y})}\, R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{y})\Big) + D^{2,eb}_{(\underset{\sim}{y})}\Big(F^{31,d}_{(\underset{\sim}{y})}\, R^{ac}_{J,10}(\underset{\sim}{x};\underset{\sim}{y})\Big)\Big] -$$

$$-g\delta^{J,k} f^{ead}\, F^{ok,d}_{(\underset{\sim}{y})}\,\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} + gf^{bec}\Big(D^{k,cd}_{(\underset{\sim}{y})}F^{ok,d}_{(\underset{\sim}{y})}\Big)\, R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{y}) \,,$$

$$(C.16)$$

usando (B.21). Utilizando (C.16), reescrevemos o lado esquerdo de (C.13) como segue

$$\int_{-\infty}^{+\infty} dz^3\, \Delta(z^3, x^3_{(0)}; y^3) \Big[D^{1,db}(y)\, R^{ab}_{J,12}(\underset{\sim}{x}; y^1, y^2, z^3) + D^{2,db}(y)\, R^{ab}_{J,13}(\underset{\sim}{x}; y^1, y^2, z^3) \Big] =$$

$$= \int_{-\infty}^{+\infty} dz^3\, \Delta(z^3, x^3_{(0)}; y^3) \Big\{ \partial^z_3 R^{ad}_{J,14}(\underset{\sim}{x}; y^1, y^2, z^3) + \delta^{J,k+6} D^{k,da}(y^1, y^2, z^3)\, \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \delta(x^3 - z^3) -$$

$$- \delta^{ad} \delta^{J,11} \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \delta(x^3 - z^3) - g f^{dae} \delta^{J,k} F^{0k,e}(y^1, y^2, z^3)\, \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \delta(x^3 - z^3) -$$

$$- g f^{cbe} \Big[D^{1,db}(y^1, y^2, z^3) \Big(F^{23,e}(y^1, y^2, z^3)\, R^{ac}_{J,10}(\underset{\sim}{x}; y^1, y^2, z^3) \Big) + D^{2,db}(y^1, y^2, z^3) \Big(F^{31,e}(y^1, y^2, z^3)\, R^{ac}_{J,16}(\underset{\sim}{x}; y^1, y^2, z^3) \Big) \Big] +$$

$$+ g f^{bdc} \Big(D^{k,ce}(y^1, y^2, z^3)\, F^{0k,e}(y^1, y^2, z^3) \Big) R^{ab}_{J,11}(\underset{\sim}{x}; y^1, y^2, z^3) +$$

$$+ g f^{dcb} \Big[\Big(A^{1,c}(y) - A^{1,c}(y^1, y^2, z^3) \Big) R^{ab}_{J,12}(\underset{\sim}{x}; y^1, y^2, z^3) + \Big(A^{2,c}(y) - A^{2,c}(y^1, y^2, z^3) \Big) R^{ab}_{J,13}(\underset{\sim}{x}; y^1, y^2, z^3) \Big] \Big\} =$$

$$= - R^{ad}_{J,14}(\underset{\sim}{x}; y) + r'^{ad}_{J(14)}(\underset{\sim}{x}; y) + \delta^{J,k+6} D^{ad}_{k}(\underset{\sim}{x}) \Big(\delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \Delta(x^3, x^3_{(0)}; y^3) \Big) -$$

$$- \delta^{ad} \delta^{J,11} \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \Delta(x^3, x^3_{(0)}; y^3) - g f^{dae} \delta^{J,k} F^{0k,e}(\underset{\sim}{x})\, \delta(x^1 - y^1)\, \delta(x^2 - y^2)\, \Delta(x^3 x^3_{(0)}; y^3) +$$

$$+ r''^{ad}_{J(14)}(\underset{\sim}{x}; y) \quad , \qquad (C.17)$$

onde

$$\boxed{\begin{aligned} r'^{ad}_{J(14)}(\underset{\sim}{x};y) &= \Big[\big(\Theta(y^3-x^3_{(0)})-\Theta(y^3-z^3)\big) R^{ad}_{J,14}(\underset{\sim}{x};y^1,y^2,z^3)\Big]^{z^3=+\infty}_{z^3=-\infty} = \\ &= R^{ad}_{J,14}(\underset{\sim}{x};y^1,y^2,\infty)\,\Theta(y^3-x^3_{(0)}) + R^{ad}_{J,14}(\underset{\sim}{x};y^1,y^2,-\infty)\,\Theta(x^3_{(0)}-y^3) \qquad (C.18) \end{aligned}}$$

e

$$\boxed{\begin{aligned} r''^{ad}_{J(14)}(\underset{\sim}{x};y) = \int_{-\infty}^{+\infty} dz^3\, \Delta(z^3,x^3_{(0)};y^3)\Big\{ & g f^{bdc}\Big(D^{k,ce}(y^1,y^2,z^3)\,F^{0k,e}(y^1,y^2,z^3)\Big) R^{ab}_{J,11}(\underset{\sim}{x};y^1,y^2,z^3) - \\ & -g f^{cbe}\Big[D^{1,db}(y^1,y^2,z^3)\Big(F^{23,e}(y^1,y^2,z^2)\,R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,z^3)\Big) + D^{2,db}(y^1,y^2,z^3)\Big(F^{31,e}(y^1,y^2,z^3)\,R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,z^3)\Big)\Big] + \\ & + g f^{dcb}\Big[\Big(A^{1,c}(y)-A^{1,c}(y^1,y^2,z^3)\Big) R^{ab}_{J,12}(\underset{\sim}{x};y^1,y^2,z^3) + \Big(A^{2,c}(y)-A^{2,c}(y^1,y^2,z^3)\Big) R^{ab}_{J,13}(\underset{\sim}{x};y^1,y^2,z^3)\Big]\Big\}\,. \end{aligned}}$$

$$(C.19)$$

Substituindo (C.17) em (C.13), encontramos

$$\boxed{\begin{aligned} R^{ad}_{J,14}(\underset{\sim}{x};y) = {} & \delta^{J,5} D^{2,da}_{(y)}\,\delta^{(3)}(\underset{\sim}{x}-y) - \delta^{J,6} D^{1,da}_{(y)}\,\delta^{(3)}(\underset{\sim}{x}-y) - \\ & - g f^{dbc} F^{12,b}_{(y)}\, R^{ac}_{J,10}(\underset{\sim}{x};y) + \delta^{J,k+6} D^{ad}_{k\,(\underset{\sim}{x})}\left(\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\right) - \\ & - \delta^{ad}\delta^{J,11}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) + g f^{adc}\delta^{J,k} F^{0k,c}_{(\underset{\sim}{x})}\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) + \\ & + r^{ad}_{J(14)}(\underset{\sim}{x};y) \, , \qquad\qquad (C.20) \end{aligned}}$$

onde (ver (C.17))

$$\boxed{r^{ad}_{J(14)}(\underset{\sim}{x};y) \equiv r'^{ad}_{J(14)}(\underset{\sim}{x};y) + r''^{ad}_{J(14)}(\underset{\sim}{x};y)} \; . \tag{C.21}$$

A expressão (C.20) exibe o motivo da notação $r^{ab}_{J(14)}$ que usamos em (C.18) e (C.19): Tais funções surgem no processo de determinação dos elementos $R^{ab}_{J,14}$. Note-se que o resultado (C.20) é ainda <u>parcial</u> pois não conhecemos os $R^{ab}_{J,10}$'s e as funções $r^{ab}_{J(14)}$ dependem destes elementos e dos $R^{ab}_{J,k+11}$'s. A seguir, usando (C.16) nas equações (C.12) e (C.11), obtemos resultados também parciais para os elementos $R^{ab}_{J,12}$ e $R^{ab}_{J,13}$, respectivamente. De fato, através de (C.16), calculamos

$$\int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \Delta(z^1,x^1_{(0)};y^1)\, D^{2,db}_{(y)}\, R^{ab}_{J,13}(\underset{\sim}{x};z^1,y^2,z^3) =$$

$$= \int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \Delta(z^1,x^1_{(0)};y^1) \Big\{ \partial^z_3 R^{ad}_{J,14}(\underset{\sim}{x};z^1,y^2,z^3) - D^{1,db}_{(z^1,y^2,z^3)}\, R^{ab}_{J,12}(\underset{\sim}{x};z^1,y^2,z^3) +$$

$$+\delta^{J,k+6}D^{k,da}(z^1,y^2,z^3)\delta(x^1-z^1)\delta(x^2-y^2)\delta(x^3-z^3)-\delta^{da}\delta^{J,11}\delta(x^1-z^1)\delta(x^2-y^2)\delta(x^3-z^3)-$$

$$-gf^{cbe}\Big[D^{1,db}(z^1,y^2,z^3)\Big(F^{23,e}(z^1,y^2,z^3)\underset{J,10}{R^{ac}}(\underset{\sim}{x};z^1,y^2,z^3)\Big)+D^{2,db}(z^1,y^2,z^3)\Big(F^{31,e}(z^1,y^2,z^3)\underset{J,10}{R^{ac}}(\underset{\sim}{x};z^1,y^2,z^3)\Big)\Big]-$$

$$-g\delta^{J,k}f^{dac}F^{0k,c}(z^1,y^2,z^3)\delta(x^1-z^1)\delta(x^2-y^2)\delta(x^3-z^3)+gf^{bdc}\Big(D^{k,ce}(z^1,y^2,z^3)F^{0k,e}(z^1,y^2,z^3)\Big)\underset{J,11}{R^{ab}}(\underset{\sim}{x};z^1,y^2,z^3)+$$

$$+gf^{deb}\Big(A^{2,e}(y)-A^{2,e}(z^1,y^2,z^3)\Big)\underset{J,13}{R^{ab}}(\underset{\sim}{x};z^1,y^2,z^3)\Big\}=$$

$$=-\int_{-\infty}^{+\infty}dz^3\,\underset{J,12}{R^{ad}}(\underset{\sim}{x};y^1,y^2,z^3)+\underset{J(12)}{r'^{ad}}(\underset{\sim}{x};y^1,y^2)+\delta^{J,7}\delta^{ad}\delta(x^1-y^1)\delta(x^2-y^2)+$$

$$+\delta^{J,7}gf^{dca}A^{1,c}(\underset{\sim}{x})\delta(x^2-y^2)\Delta(x^1,x^1_{(0)};y^1)+\delta^{J,8}\delta^{ad}\Delta(x^1,x^1_{(0)};y^1)\partial^2_y\delta(x^2-y^2)+$$

$$+\delta^{J,8}gf^{dca}A^{2,c}(\underset{\sim}{x})\delta(x^2-y^2)\Delta(x^1,x^1_{(0)};y^1)+g\delta^{J,k}f^{dca}F^{0k,c}(\underset{\sim}{x})\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)-$$

$$-\delta^{ad}\delta^{J,11}\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)+\underset{J(12)}{r''^{ad}}(\underset{\sim}{x};y)\quad,\qquad(C.22)$$

onde

$$\boxed{\begin{aligned}\underset{J(12)}{r'^{ad}}(\underset{\sim}{x};y^1,y^2)&=\int_{-\infty}^{+\infty}dz^1\,\Delta(z^1,x^1_{(0)};y^1)\Big[\underset{J,14}{R^{ad}}(\underset{\sim}{x};z^1,y^2,\infty)-\underset{J,14}{R^{ad}}(\underset{\sim}{x};z^1,y^2,-\infty)\Big]+\\&+\int_{-\infty}^{+\infty}dz^3\Big[\underset{J,12}{R^{ad}}(\underset{\sim}{x};\infty,y^2,z^3)\,\theta(y^1-x^1_{(0)})+\underset{J,12}{R^{ad}}(\underset{\sim}{x};-\infty,y^2,z^3)\,\theta(x^1_{(0)}-y^1)\Big]\end{aligned}}$$

$$(C.23)$$

e

$$\boxed{\begin{aligned}
\overset{ad}{\underset{J(12)}{r''}}(\underset{\sim}{x};y) = & \int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \Delta(z^1,x^1_{(0)};y^1)\Big\{ g f^{bdc} \big(D^{k,ce}(z^1,y^2,z^3)\, F^{ok,e}(z^1,y^2,z^3)\big) \underset{J,11}{R^{ab}}(\underset{\sim}{x};z^1,y^2,z^3) - \\
& - g f^{cbe}\Big[D^{1,db}(z^1,y^2,z^3)\big(F^{23,e}(z^1,y^2,z^3)\underset{J,10}{R^{ac}}(\underset{\sim}{x};z^1,y^2,z^3)\big) + D^{2,db}(z^1,y^2,z^3)\big(F^{31,e}(z^1,y^2,z^3)\underset{J,10}{R^{ac}}(\underset{\sim}{x};z^1,y^2,z^3)\big)\Big] + \\
& + g f^{dcb}\big(A^{2,c}(y) - A^{2,c}(z^1,y^2,z^3)\big) \underset{J,13}{R^{ab}}(\underset{\sim}{x};z^1,y^2,z^3)\Big\} \quad . \qquad (C.24)
\end{aligned}}$$

Substituindo (C.24) em (C.12), obtemos o resultado parcial

$$\boxed{\begin{aligned}
\underset{J,12}{R^{ad}}(\underset{\sim}{x};y) = & -g f^{dec} F^{23,e}(y)\, \underset{J,10}{R^{ac}}(\underset{\sim}{x};y) - \delta^{J,4} D^{2,da}(y)\,\delta^{(3)}(\underset{\sim}{x-y}) + \delta^{da}\delta^{J,6}\delta^{(3)}(\underset{\sim}{x-y}) + \\
& + \delta^{J,7}\delta^{ad}\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \delta^{J,8}\delta^{ad}\Delta(x^1,x^1_{(0)};y^1)\partial^2_y\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) - \\
& - \delta^{J,11}\delta^{ad}\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \delta^{J,7} g f^{dca} A^{1,c}(\underset{\sim}{x})\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\
& + \delta^{J,8} g f^{dca} A^{2,c}(\underset{\sim}{x})\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\
& + \delta^{J,k} g f^{dca} F^{ok,c}(\underset{\sim}{x})\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \underset{J(12)}{r^{ad}}(\underset{\sim}{x};y) \quad , \qquad (C.25)
\end{aligned}}$$

onde (ver (C.22))

$$\boxed{\begin{aligned} \underset{J(12)}{r}^{ad}(\underset{\sim}{x};y) &\equiv \delta(y^3-x^3_{(0)})\Big(\underset{J(12)}{r'}^{ad}(\underset{\sim}{x};y^1,y^2)+\underset{J(12)}{r''}^{ad}(\underset{\sim}{x};y)\Big)\equiv \\ &\equiv \delta(y^3-x^3_{(0)})\;\underset{J(12)}{\tilde{r}}^{ad}(\underset{\sim}{x};y^1,y^2) \;. \end{aligned}} \qquad (C.26)$$

Consideremos agora a expressão (C.11). Desde (C.16), é evidente que

$$\int_{x^2_{(0)}}^{y^2} dy'^2\,\partial^2_{y'}\,\underset{J,13}{R}^{ad}(\underset{\sim}{x};z^1,y'^2,z^3) = \int_{x^2_{(0)}}^{y^2} dy'^2\Big\{-\partial^1_z\,\underset{J,12}{R}^{ad}(\underset{\sim}{x};z^1,y'^2,z^3)-\partial^3_z\,\underset{J,14}{R}^{ad}(\underset{\sim}{x};z^1,y'^2,z^3)+$$

$$+\delta^{J,7}\delta^{da}\partial^1_z\delta(x^1-z^1)\,\delta(y'^2-x^2)\delta(x^3-z^3)-\delta^{da}\delta^{J,11}\delta(x^1-z^1)\delta(x^2-y'^2)\delta(x^3-z^3)+$$

$$+\delta^{J,8}\delta^{da}\partial^2_{y'}\delta(y'^2-x^2)\,\delta(x^1-z^1)\delta(x^3-z^3)+\delta^{J,9}\delta^{da}\delta(x^1-z^1)\delta(x^2-y'^2)\partial^3_z\delta(x^3-z^3)+$$

$$+\;g f^{dca}\Big[\overset{1,c}{A}(z^1,y'^2,z^3)\delta^{J,7}+\overset{2,c}{A}(z^1,y'^2,z^3)\delta^{J,8}\Big]\delta(x^1-z^1)\delta(y'^2-x^2)\delta(x^3-z^3)\;-$$

$$-\,g f^{dcb}\Big[\overset{1,c}{A}(z^1,y'^2,z^3)\,\underset{J,12}{R}^{ab}(\underset{\sim}{x};z^1,y'^2,z^3)+\overset{2,c}{A}(z^1,y'^2,z^3)\,\underset{J,13}{R}^{ab}(\underset{\sim}{x};z^1,y'^2,z^3)\Big]\;-$$

$$-g f^{ecb}\Big[\overset{1,db}{D}(z^1,y'^2,z^3)\Big(\overset{23,e}{F}(z^1,y'^2,z^3)\,\underset{J,10}{R}^{ac}(\underset{\sim}{x};z^1,y'^2,z^3)\Big)+\overset{2,db}{D}(z^1,y'^2,z^3)\Big(\overset{31,e}{F}(z^1,y'^2,z^3)\,\underset{J,10}{R}^{ac}(\underset{\sim}{x};z^1,y'^2,z^3)\Big)\Big]\cdot$$

$$-g\,\delta^{J,k} f^{dac}\,\overset{0k,c}{F}(z^1,y'^2,z^3)\delta(x^1-z^1)\delta(y'^2-x^2)\delta(x^3-z^3)\quad+$$

$$+ g f^{bdc}\Big(D^{k,ce}_{(z^1,y'^2,z^3)}\, F^{0k,e}(z^1,y'^2,z^3)\Big)\, \underset{J,11}{R^{ab}}(\underset{\sim}{x};z^1,y'^2,z^3)\Big\} \therefore \qquad (C.27)$$

$\therefore$

$$\underset{J,13}{R^{ad}}(\underset{\sim}{x};z^1,y^2,z^3) = \underset{J,13}{R^{ad}}(\underset{\sim}{x};z^1,x^2_{(0)},z^3) - \int_{x^2_{(0)}}^{y^2} dy'^2 \left\{ \begin{matrix} \text{chave} \\ \text{em} \\ (C.27) \end{matrix} \right\} . \qquad (C.28)$$

Notando que

$$\int_{x^2_{(0)}}^{y^2} dy'^2\, \delta(x^2-y'^2) = \Delta(y^2,x^2_{(0)};x^2) \quad , \qquad (C.29)$$

$$\int_{x^2_{(0)}}^{y^2} dy'^2\, \partial^2_{y'}\, \delta(x^2-y'^2) = \delta(x^2-x^2_{(0)}) - \delta(x^2-y^2) \quad , \qquad (C.30)$$

integramos ambos lados de (C.28) sobre $\underline{z^1}$ e $\underline{z^3}$ em todo o espaço:

$$\int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \underset{J,13}{R^{ad}}(\underset{\sim}{x};z^1,y^2,z^3) =$$

$$= \delta^{da}\left(\delta^{J,11}\Delta(y^2,x^2_{(0)};x^2) + \delta^{J,8}\delta(x^2-y^2)\right) + \underset{J(13)}{r'^{ad}}(\underset{\sim}{x};y^2) \quad , \qquad (C.31)$$

com

$$\boxed{\begin{aligned}
&r'^{ad}_{J(13)}(\underset{\sim}{x};y^2) \equiv -\delta^{da}\delta^{J,8}\delta(x^2-x^2_{(0)}) + gf^{acd}\left[A^{1,c}(\underset{\sim}{x})\delta^{J,7} + A^{2,c}(\underset{\sim}{x})\delta^{J,8} - F^{0k,c}(\underset{\sim}{x})\delta^{J,k}\right]+\\
&+\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\, R^{ad}_{J,13}(\underset{\sim}{x};z^1,x^2_{(0)},z^3) + \int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\int_{x^2_{(0)}}^{y^2}dy'^2\Bigg\{\partial^1_z R^{ad}_{J,12}(\underset{\sim}{x};z^1,y'^2,z^3) +\\
&+\partial^3_z R^{ad}_{J,14}(\underset{\sim}{x};z^1,y'^2,z^3) + gf^{dcb}\Big(A^{1,c}(z^1,y'^2,z^3)R^{ab}_{J,12}(\underset{\sim}{x};z^1,y^2,z^3) + A^{2,c}(z^1,y'^2,z^3)R^{ab}_{J,13}(\underset{\sim}{x};z^1,y'^2,z^3)\Big)+\\
&+gf^{ecb}\Big[D^{1,db}(z^1,y'^2,z^3)\Big(F^{23,e}(z^1,y'^2,z^3)R^{ac}_{J,10}(\underset{\sim}{x};z^1,y'^2,z^3)\Big)+D^{2,db}(z^1,y'^2,z^3)\Big(F^{31,e}(z^1,y'^2,z^3)R^{ac}_{J,10}(\underset{\sim}{x};z^1,y'^2,z^3)\Big)\Big]+\\
&+gf^{dbc}\Big(D^{k,ce}(z^1,y'^2,z^3)F^{0k,e}(z^1,y'^2,z^3)\Big)R^{ab}_{J,11}(\underset{\sim}{x};z^1,y'^2,z^3)\Bigg\} \quad . \qquad (C.32)
\end{aligned}}$$

Substituindo (C.31) em (C.11), encontramos

$$\begin{aligned}
R^{ad}_{J,13}(\underset{\sim}{x};y) &= -gf^{dec}F^{31,e}(y)\,R^{ac}_{J,10}(\underset{\sim}{x};y) + \delta^{J,4}D^{1,da}(y)\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) -\\
&-\delta^{da}\delta^{J,5}\partial^3_y\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \delta^{da}\delta(y^1-x^1_{(0)})\delta(y^3-x^3_{(0)})\Big(\delta^{J,11}\Delta(y^2,x^2_{(0)};x^2) +\\
&+\delta^{J,8}\delta(x^2-y^2)\Big) + \delta(y^1-x^1_{(0)})\delta(y^3-x^3_{(0)})\, r'^{ad}_{J(13)}(\underset{\sim}{x};y^2) \quad , \qquad (C.33)
\end{aligned}$$

ou seja, o outro resultado parcial

$$\boxed{\begin{aligned} R^{ad}_{J,13}(\underset{\sim}{x};y) &= -g f^{dec} F^{31,e}(y)\, R^{ac}_{J,10}(\underset{\sim}{x};y) + \delta^{J,4} D^{1,da}(y)\, \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - \\ & -\delta^{da}\delta^{J,5}\, \partial^3_y \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \delta^{da}\delta^{J,8}\delta(x^1_{(0)}-y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\ & + \delta^{da}\delta^{J,11}\delta(x^1_{(0)}-y^1)\Delta(y^2,x^2_{(0)};x^2)\delta(x^3_{(0)}-y^3) + r^{ad}_{J(13)}(\underset{\sim}{x};y), \quad (C.34) \end{aligned}}$$

onde definimos (ver (C.32))

$$\boxed{r^{ad}_{J(13)}(\underset{\sim}{x};y) \equiv \delta(x^1_{(0)}-y^1)\delta(x^3_{(0)}-y^3)\, r'^{ad}_{J(13)}(\underset{\sim}{x};y^2)} \quad . \qquad (C.35)$$

Da mesma forma em que procedemos no capítulo III, a idéia agora é retornar com as expressões (C.34) e (C.25) nas equações (C.2 b,c,d) com o propósito de obter um sistema desacoplado de equações para os elementos $R^{ab}_{J,10}$. Por exemplo, em (C.2d), podemos calcular

$$\int_{-\infty}^{+\infty} dz^3\, \Delta(z^3,x^3_{(0)};y^3)\, R^{ab}_{J,12}(\underset{\sim}{x};y^1,y^2,z^3) =$$

$$= \int_{-\infty}^{+\infty} dz^3\, \Delta(z^3,x^3_{(0)};y^3)\Big\{ g f^{bdc}\left(\partial^3_z A^{2,d}(y^1,y^2,z^3)\right) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,z^3) -$$

$$- \delta^{J,4}\delta^{ab}\delta(x^1-y^1)\,\partial^2_y\delta(x^2-y^2)\delta(x^3-z^3) - \delta^{J,4} g f^{bca} A^{2,c}(y^1,y^2,z^3)\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3) +$$

$$+\delta^{ab}\delta^{J,6}\left(\partial^3_z\,\delta(x^3-z^3)\right)\delta(x^1-y^1)\delta(x^2-y^2)\Big\} \quad =$$

$$= -\delta^{J,4}\delta^{ab}\delta(x^1-y^1)\partial^2_y\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) - \delta^{J,4} g f^{bca} A^{2,c}(\underset{\sim}{x})\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)$$

$$+\delta^{ab}\delta^{J,6}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + g f^{bdc} A^{2,d}(y) R^{ac}_{J,10}(\underset{\sim}{x};y) \quad -$$

$$-\Big[\left(\theta(y^3-x^3_{(0)})-\theta(y^3-z^3)\right) g f^{bdc} A^{2,d}(y^1,y^2,z^3) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,z^3)\Big]_{z^3=-\infty}^{z^3=+\infty} \quad +$$

$$+ g f^{bdc}\int_{-\infty}^{+\infty} dz^3\, A^{2,d}(y^1,y^2,z^3)\Delta(z^3,x^3_{(0)};y^3)\,\delta^{ac}\delta^{J,4}\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3),$$

$$(C.36)$$

onde integramos por partes e usamos (C.26). O penúltimo termo no lado direito de (C.36) é a seguinte função

$$\boxed{\begin{aligned} & g f^{cdb}\Big[A^{2,d}(y^1,y^2,\infty) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,\infty)\,\theta(y^3-x^3_{(0)}) + A^{2,d}(y^1,y^2,-\infty) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,-\infty)\,\theta(x^3_{(0)}-y^3)\Big] \equiv \\ & \equiv r'^{ab}_{J(10)}(\underset{\sim}{x};y) \quad . \qquad\qquad (C.37) \end{aligned}}$$

Levando (C.37) em (C.36) e o resultado em (C.2d), obtemos

$$\boxed{\begin{aligned} \partial^2_y \underset{J,10}{R}{}^{ab}(\underset{\sim}{x};y) = {}& -\delta^{J,4}\delta^{ab}\delta(x^1-y^1)\,\partial^2_y\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) \; + \\ & + \underset{J(10)}{r'}{}^{ab}(\underset{\sim}{x};y) \qquad (C.38) \end{aligned}}$$

Agora, usando (C.34), segue que

$$\int_{-\infty}^{+\infty} dz^3\,\Delta(z^3,x^3_{(0)};y^3)\,\underset{J,13}{R}{}^{ab}(\underset{\sim}{x};y^1,y^2,z^3) =$$

$$= \int_{-\infty}^{+\infty} dz^3\,\Delta(z^3,x^3_{(0)};y^3)\Big\{ g f^{bdc}\Big(\partial^z_3 A^{1,d}(y^1,y^2,z^3)\Big)\underset{J,10}{R}{}^{ac}(\underset{\sim}{x};y^1,y^2,z^3) \; +$$

$$+ \delta^{J,4}\delta^{ab}\,\partial^1_y\delta(x^1-y^1)\,\delta(x^2-y^2)\delta(x^3-z^3) + \delta^{J,4} g f^{bca} A^{1,c}(y^1,y^2,z^3)\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3) +$$

$$+ \delta^{ab}\delta^{J,5}\,\partial^z_3\delta(x^3-z^3)\;\delta(x^1-y^1)\delta(x^2-y^2)\Big\} =$$

$$= \delta^{J,4}\delta^{ab}\,\partial^1_y\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) + \delta^{J,4} g f^{bca} A^{1,c}(\underset{\sim}{x})\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) -$$

$$- \delta^{ab}\delta^{J,5}\,\delta^{(3)}(\underset{\sim}{x}-y) \quad - g f^{bdc} A^{1,d}(y)\,\underset{J,10}{R}{}^{ac}(\underset{\sim}{x};y) \quad -$$

$$- g f^{bdc}\int_{-\infty}^{+\infty} dz^3\, A^{1,d}(y^1,y^2,z^3)\,\Delta(z^3,x^3_{(0)};y^3)\,\delta^{ac}\delta^{J,4}\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3-z^3) +$$

$$+\left[\left(\theta(y^3-x^3_{(0)})-\theta(y^3-z^3)\right)g f^{bdc} A^{1,d}(y^1,y^2,z^3) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,z^3)\right]_{z^3=-\infty}^{z^3=+\infty} . \qquad (C.39)$$

Desta vez, denotamos o termo integrado por

$$\boxed{\begin{aligned} r''^{ab}_{J(10)}(\underset{\sim}{x};y) \equiv g f^{bdc}\Big[& A^{1,d}(y^1,y^2,\infty) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,\infty)\theta(y^3-x^3_{(0)}) + \\ & + A^{1,d}(y^1,y^2,-\infty) R^{ac}_{J,10}(\underset{\sim}{x};y^1,y^2,-\infty)\theta(x^3_{(0)}-y^3)\Big] . \quad (C.40)\end{aligned}}$$

Levando (C.40) em (C.39) e o resultado em (C.2c), achamos

$$\boxed{\begin{aligned} \partial^1_y R^{ab}_{J,10}(\underset{\sim}{x};y) = & -\delta^{J,4}\delta^{ab}\partial^1_y \delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) - \\ & - r''^{ab}_{J(10)}(\underset{\sim}{x};y) \qquad . \qquad (C.41)\end{aligned}}$$

A partir de (C.34) e com o uso de (C.2b), escrevemos

$$\int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \Delta(z^1,x^1_{(0)};y^1)\, R^{ab}_{J,13}(\underset{\sim}{x};z^1,y^2,z^3) =$$

$$= \int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \Delta(z^1,x^1_{(0)};y^1) \Big\{ \delta^{J,4}\delta^{ab}\partial^1_z \delta(x^1-z^1)\delta(x^2-y^2)\delta(x^3-z^3) +$$

$$+ \delta^{J,5}\delta^{ab}\partial^{z}_{3}\delta(x^3-z^3)\delta(x^1-z^1)\delta(x^2-y^2) +$$

$$+gf^{bdc}\left[\left(\partial_3^{z} A^{1,d}(z^1,y^2,z^3)\right)\underset{J,10}{R}^{ac}(\underset{\sim}{x};z^1,y^2,z^3)+\delta^{J,4}\delta^{ca}A^{1,d}(z^1,y^2,z^3)\delta(z^1-x^1)\delta(x^2-y^2)\delta(z^3-x^3)\right]\Bigg\}=$$

$$=\delta^{ab}\delta^{J,4}\delta(x^1-y^1)\delta(x^2-y^2)+$$

$$+gf^{bdc}\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\,\Delta(z^1,x^1_{(0)};y^1)\,\partial_3^{z}\left[A^{1,d}(z^1,y^2,z^3)\,\underset{J,10}{R}^{ac}(\underset{\sim}{x};z^1,y^2,z^3)\right]\ =$$

$$=\delta^{ab}\delta^{J,4}\delta(x^1-y^1)\delta(x^2-y^2)\ +\ \underset{J(10)}{r'''}^{ab}(\underset{\sim}{x};y^1,y^2)\quad,\qquad(C.42)$$

onde

$$\boxed{\begin{aligned}&\underset{J(10)}{r'''}^{ab}(\underset{\sim}{x};y^1,y^2)\ \equiv\\&\equiv gf^{bdc}\int_{-\infty}^{+\infty}dz^1\,\Delta(z^1,x^1_{(0)};y^1)\left[A^{1,d}(z^1,y^2,\infty)\underset{J,10}{R}^{ac}(\underset{\sim}{x};z^1,y^2,\infty)-A^{1,d}(z^1,y^2,-\infty)\underset{J,10}{R}^{ac}(\underset{\sim}{x};z^1,y^2,-\infty)\right].\end{aligned}}$$

$$(C.43)$$

Substituindo (C.42) em (C.2b), encontramos

$$\boxed{\begin{aligned}\partial_y^3\underset{J,10}{R}^{ab}(\underset{\sim}{x};y)=-\delta^{J,4}\delta^{ab}\left(\delta^{(3)}(\underset{\sim}{x}-y)-\delta(x^1-y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3)\right)+&\\+\ \underset{J(10)}{r'''}^{ab}(\underset{\sim}{x};y)\quad,\qquad\qquad(C.44)&\end{aligned}}$$

com

$$\boxed{\underset{J(10)}{r'''^{ab}}(\underset{\sim}{x};y) = \delta(x^3_{(0)}-y^3)\, \underset{J(10)}{r'''^{ab}}(\underset{\sim}{x};y^1,y^2)} \; . \qquad (C.45)$$

As equações (C.38), (C.41) e (C.44) constituem o sistema desacoplado de equações, para os elementos $R^{ab}_{J,10}$, que buscāva-mos. Passemos ā integração dessas equações. Desde (C.41), segue que

$$\int_{x^1_{(0)}}^{y^1} dy'^1\, \partial^{y'}_1 \underset{J,10}{R^{ab}}(\underset{\sim}{x};y'^1,y^2,y^3) = -\delta^{J,4}\delta^{ab}\left(\int_{x^1_{(0)}}^{y^1} dy'^1\, \partial^{y'}_1 \delta(x^1-y'^1)\right)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) +$$

$$+\int_{x^1_{(0)}}^{y^1} dy'^1\, \underset{J(10)}{r''^{ab}}(\underset{\sim}{x};y'^1,y^2,y^3)$$

$$\therefore$$

$$\underset{J,10}{R^{ab}}(\underset{\sim}{x};y) = \underset{J,10}{R^{ab}}(\underset{\sim}{x};x^1_{(0)},y^2,y^3) - \delta^{J,4}\delta^{ab}\left[\delta(x^1-y^1)-\delta(x^1-x^1_{(0)})\right]\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) +$$

$$+\int_{x^1_{(0)}}^{y^1} dy'^1\, \underset{J(10)}{r''^{ab}}(\underset{\sim}{x};y'^1,y^2,y^3) \; . \qquad (C.46a)$$

Desde (C.38),

$$\int_{x^2_{(0)}}^{y^2} dy'^2\, \partial^{y'}_2 \underset{J,10}{R^{ab}}(\underset{\sim}{x};y^1,y'^2,y^3) = -\delta^{J,4}\delta^{ab}\delta(x^1-y^1)\left(\int_{x^2_{(0)}}^{y^2} dy'^2\, \partial^{y'}_2 \delta(x^2-y'^2)\right)\Delta(x^3,x^3_{(0)};y^3) +$$

$$+\int_{x^2_{(0)}}^{y^2} dy'^2\, \underset{J(10)}{r'^{ab}}(\underset{\sim}{x};y^1,y'^2,y^3) \qquad \therefore$$

$$R^{ab}_{J,10}(\underset{\sim}{x};y) = R^{ab}_{J,10}(\underset{\sim}{x};y^1,x^2_{(0)},y^3) - \delta^{J,4}\delta^{ab}\delta(x^1-y^1)\left[\delta(x^2-y^2)-\delta(x^2-x^2_{(0)})\right]\Delta(x^3,x^3_{(0)};y^3) + \int_{x^2_{(0)}}^{y^2} dy'^2\, r'^{ab}_{J(10)}(\underset{\sim}{x};y^1,y'^2,y^3) \quad . \tag{C.46b}$$

Desde (C.44),

$$\int_{x^3_{(0)}}^{y^3} dy'^3\, \partial_3^{y'} R^{ab}_{J,10}(\underset{\sim}{x};y^1,y^2,y'^3) = \delta^{J,4}\delta^{ab}\left[\delta(x^1-y^1)\delta(x^2-y^2)\left(\Delta(y^3,x^3_{(0)};x^3) - \Delta(y^3,x^3_{(0)};x^3_{(0)})\right)\right] - \int_{x^3_{(0)}}^{y^3} dy'^3\, r'''^{ab}_{J(10)}(\underset{\sim}{x};y^1,y^2,y'^3) \quad ,$$

$$\therefore \quad R^{ab}_{J,10}(\underset{\sim}{x};y) = R^{ab}_{J,10}(\underset{\sim}{x};y^1,y^2,x^3_{(0)}) + \delta^{ab}\delta^{J,4}\delta(x^1-y^1)\delta(x^2-y^2)\left(\Delta(y^3,x^3_{(0)};x^3) - \Delta(y^3,x^3_{(0)};x^3_{(0)})\right) - \int_{x^3_{(0)}}^{y^3} dy'^3\, r'''^{ab}_{J(10)}(\underset{\sim}{x};y^1,y^2,y'^3) \quad . \tag{C.46c}$$

Usando (C.46b) para expressar $R^{ab}_{J,10}(\underset{\sim}{x};x^1_{(0)},y^2,y^3)$ em (C.46a), encontramos

$$R^{ab}_{J,10}(\underset{\sim}{x};y) = R^{ab}_{J,10}(\underset{\sim}{x};x^1_{(0)},x^2_{(0)},y^3) + \delta^{J,4}\delta^{ab}\left[\delta(x^1-x^1_{(0)})\delta(x^2-x^2_{(0)}) - \delta(x^1-y^1)\delta(x^2-y^2)\right] . \Delta(x^3,x^3_{(0)};y^3) + \int_{x^2_{(0)}}^{y^2} dy'^2\, r'^{ab}_{J(10)}(\underset{\sim}{x};x^1_{(0)},y'^2,y^3) + \int_{x^1_{(0)}}^{y^1} dy'^1\, r''^{ab}_{J(10)}(\underset{\sim}{x};y'^1,y^2,y^3) \; . \tag{C.46d}$$

Por meio de (C.46c) e de (C.46d), chegamos a

$$\begin{aligned}
\underset{J,10}{R^{ab}}(\underset{\sim}{x};y) = {} & \underset{J,10}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \\
& + \delta^{J,4}\delta^{ab}\Big[\delta(x^1-x^1_{(0)})\delta(x^2-x^2_{(0)})\Big(\Delta(y^3,x^3_{(0)};x^3)-\Delta(y^3,x^3_{(0)};x^3_{(0)})+\Delta(x^3,x^3_{(0)};y^3)\Big)- \\
& - \delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\Big] + \int_{x^2_{(0)}}^{y^2} dy'^2\, \underset{J(10)}{r'^{ab}}(\underset{\sim}{x};x^1_{(0)},y'^2,y^3) + \\
& + \int_{x^1_{(0)}}^{y^1} dy'^1\, \underset{J(10)}{r''^{ab}}(\underset{\sim}{x};y'^1,y^2,y^3) - \int_{x^3_{(0)}}^{y^3} dy'^3\, \underset{J(10)}{r'''^{ab}}(\underset{\sim}{x};x^1_{(0)},x^2_{(0)},y^3) \qquad \text{(C.47)}
\end{aligned}$$

ou ainda, em forma mais compacta,

$$\boxed{\begin{aligned}
\underset{J,10}{R^{ab}}(\underset{\sim}{x};y) = {} & \delta^{ab}\delta^{J,4}\Big[\delta(x^1-x^1_{(0)})\delta(x^2-x^2_{(0)})\Big(\Delta(y^3,x^3_{(0)};x^3)-\Delta(y^3,x^3_{(0)};x^3_{(0)})+\Delta(x^3,x^3_{(0)};y^3)\Big)- \\
& - \delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\Big] + \underset{J(10)}{r^{ab}}(\underset{\sim}{x};y) \quad , \quad \text{(C.48)}
\end{aligned}}$$

onde

$$\boxed{\begin{aligned}
\underset{J(10)}{r^{ab}}(\underset{\sim}{x};y) \equiv {} & \underset{J,10}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \int_{x^1_{(0)}}^{y^1} dy'^1\, \underset{J(10)}{r''^{ab}}(\underset{\sim}{x};y'^1,y^2,y^3) + \\
& + \int_{x^2_{(0)}}^{y^2} dy'^2\, \underset{J(10)}{r'^{ab}}(\underset{\sim}{x};x^1_{(0)},y'^2,y^3) - \int_{x^3_{(0)}}^{y^3} dy'^3\, \underset{J(10)}{r'''^{ab}}(\underset{\sim}{x};x^1_{(0)},x^2_{(0)},y'^3). \quad \text{(C.49)}
\end{aligned}}$$

De posse do resultado (C.48), podemos concluir a determinação de todos os elementos da inversa R. Por exemplo, inserindo (C.48) em (C.25), chegamos a

$$\boxed{\begin{aligned}
&R^{ab}_{J,12}(\underset{\sim}{x};y) = \delta^{J,k}\, g f^{abc} F^{0k,c}(\underset{\sim}{x})\, \Delta(x^1,x^1_{(0)};y^1)\, \delta(x^2-y^2)\, \delta(x^3_{(0)}-y^3) + \\
&+ \delta^{J,4}\Big\{ D^{2,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) + g f^{abc} F^{23,c}(y) \Big[\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) - \delta(x^1-x^1_{(0)})\delta(x^2-x^2_{(0)}) \cdot \\
&\Big(\Delta(y^3,x^3_{(0)};x^3) - \Delta(y^3,x^3_{(0)};x^3_{(0)}) + \Delta(x^3,x^3_{(0)};y^3) \Big) \Big] \Big] \Big\} + \delta^{J,6}\delta^{ab}\partial^3_y\, \delta^{(3)}(\underset{\sim}{x}-y) + \\
&+ \delta^{J,7}\Big[\delta^{ab}\delta(x^1-y^1) + g f^{abc} A^{1,c}(\underset{\sim}{x})\, \Delta(x^1,x^1_{(0)};y^1) \Big] \delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \delta^{J,8}\Big[\delta^{ab}\partial^2_y\, \delta(x^2-y^2) + \\
&+ g f^{abc} A^{2,c}(\underset{\sim}{x})\, \delta(x^2-y^2) \Big] \Delta(x^1,x^1_{(0)};y^1)\delta(x^3_{(0)}-y^3) - \delta^{J,11}\delta^{ab}\Delta(x^1,x^1_{(0)};y^1)\,\delta(x^2-y^2)\,\delta(x^3_{(0)}-y^3) + \\
&+ g f^{bcd} F^{23,d}(y)\, r^{ac}_{J(10)}(\underset{\sim}{x};y) + r^{ab}_{J(12)}(\underset{\sim}{x};y) \quad . \qquad \text{(C.50)}
\end{aligned}}$$

Inserindo (C.48) em (C.34), obtemos

$$\begin{aligned}
\underset{J,13}{R}^{ab}(\underset{\sim}{x};y) = {} & \delta^{J,4}\Big\{ D^{1,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) + g f^{abc} F^{31,c}(y)\Big[\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) - \\
& - \delta(x^1-x^1_{(0)})\delta(x^2-x^2_{(0)})\Big(\Delta(y^3,x^3_{(0)};x^3) - \Delta(y^3,x^3_{(0)};x^3_{(0)}) + \Delta(x^3,x^3_{(0)};y^3)\Big)\Big]\Big\} + \\
& + \delta^{J,5}\delta^{ab}\partial^3_x\delta^{(3)}(\underset{\sim}{x}-y) + \delta^{J,8}\delta^{ab}\delta(x^1_{(0)}-y^1)\delta(x^2-y^2)\delta(x^3_{(0)}-y^3) + \\
& + \delta^{J,11}\delta^{ab}\delta(x^1_{(0)}-y^1)\,\Delta(y^2,x^2_{(0)};x^2)\delta(x^3_{(0)}-y^3) + g f^{bcd} F^{31,d}(y)\,\underset{J(10)}{r}^{ac}(\underset{\sim}{x};y) + \\
& + \underset{J(13)}{r}^{ab}(\underset{\sim}{x};y) \quad . \qquad (C.51)
\end{aligned}$$

Além disso, (C.48) em (C.20) fornece

$$\begin{aligned}
\underset{J,14}{R}^{ab}(\underset{\sim}{x};y) = {} & \delta^{J,k} g f^{abc} F^{0k,c}(\underset{\sim}{x})\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) + \\
& + \delta^{J,4} g f^{abc} F^{12,c}(y)\Big[\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) - \\
& - \delta(x^1-x^1_{(0)})\delta(x^2-x^2_{(0)})\Big(\Delta(y^3,x^3_{(0)};x^3) - \Delta(y^3,x^3_{(0)};x^3_{(0)}) + \Delta(x^3,x^3_{(0)};y^3)\Big)\Big] + \\
& + \delta^{J,5} D^{2,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) - \delta^{J,6} D^{1,ba}(y)\,\delta^{(3)}(\underset{\sim}{x}-y) + \\
& + \delta^{J,k+6} \underset{k}{D}^{ab}(\underset{\sim}{x})\Big(\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\Big) - \delta^{J,11}\delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3) + \\
& + g f^{bcd} F^{12,d}(y)\,\underset{J(10)}{r}^{ac}(\underset{\sim}{x};y) + \underset{J(14)}{r}^{ab}(\underset{\sim}{x};y) \quad . \qquad (C.52)
\end{aligned}$$

Os elementos $R^{ab}_{J,k+11}$ e $R^{ab}_{J,10}$ levados em (C.2e) conduzem a

$$\boxed{\begin{aligned}
R^{ab}_{J,1}(\underset{\sim}{x};y) = {} & \delta^{J,4} g f^{abc} F^{23,c}_{(y)} \Big[\delta(x^1-y^1)\delta(x^2-y^2)\,\Delta(x^3, x^3_{(0)}; y^3) - \\
& - \delta(x^1-x^1_{(0)})\,\delta(x^2-x^2_{(0)}) \Big(\Delta(y^3, x^3_{(0)}; x^3) - \Delta(y^3, x^3_{(0)}; x^3_{(0)}) + \Delta(x^3, x^3_{(0)}; y^3) \Big) \Big] + \\
& + \delta^{J,7} \delta^{ab} \delta^{(3)}(\underset{\sim}{x}-y) + g f^{cbd} F^{01,d}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) - g f^{cbd} F^{23,d}_{(y)} r^{ac}_{J(10)}(\underset{\sim}{x};y) - \\
& - R^{ab}_{J,12}(\underset{\sim}{x};y) \qquad , \qquad\qquad (C.53)
\end{aligned}}$$

$$\boxed{\begin{aligned}
R^{ab}_{J,2}(\underset{\sim}{x};y) = {} & \delta^{J,4} g f^{abc} F^{31,c}_{(y)} \Big[\delta(x^1-y^1)\delta(x^2-y^2)\,\Delta(x^3, x^3_{(0)}; y^3) - \\
& - \delta(x^1-x^1_{(0)})\,\delta(x^2-x^2_{(0)}) \Big(\Delta(y^3, x^3_{(0)}; x^3) - \Delta(y^3, x^3_{(0)}; x^3_{(0)}) + \Delta(x^3, x^3_{(0)}; y^3) \Big) \Big] + \\
& + \delta^{J,8} \delta^{ab} \delta^{(3)}(\underset{\sim}{x}-y) + g f^{cbd} F^{02,d}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) - g f^{cbd} F^{31,d}_{(y)} r^{ac}_{J(10)}(\underset{\sim}{x};y) - \\
& - R^{ab}_{J,13}(\underset{\sim}{x};y) \qquad , \qquad\qquad (C.54)
\end{aligned}}$$

$$\boxed{\begin{aligned}
R^{ab}_{J,3}(\underset{\sim}{x};y) = {} & \delta^{J,9} \delta^{ab} \delta^{(3)}(\underset{\sim}{x}-y) + g f^{cbd} F^{03,d}_{(y)} R^{ac}_{J,11}(\underset{\sim}{x};y) - \\
& - R^{ab}_{J,14}(\underset{\sim}{x};y) \qquad . \qquad\qquad (C.55)
\end{aligned}}$$

APÊNDICE D

RESTRIÇÕES DECORRENTES DA ANTISSIMETRIA DA MATRIZ R SOBRE AS FUNÇÕES $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$, $r^{ab}_{J(10)}(\underset{\sim}{x};\underset{\sim}{y})$ E $r^{ab}_{J(k+11)}(\underset{\sim}{x};\underset{\sim}{y})$

Os resultados obtidos para os elementos da matriz inversa R foram calculados a partir de (2.42a) (ver capítulo III e Apêndice C). Entretanto, dado que R é a inversa de uma matriz antissimétrica, ela também deve ser antissimétrica. Exigindo, então, antissimetria

$$R^{ab}_{JK}(\underset{\sim}{x};\underset{\sim}{y}) = -R^{ba}_{KJ}(\underset{\sim}{y};\underset{\sim}{x}) \qquad (D.1)$$

para todos os elementos de R, obtemos condições a serem satisfeitas pelas funções $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$, $r^{ab}_{J(10)}(\underset{\sim}{x};\underset{\sim}{y})$ e $r^{ab}_{J(k+11)}(\underset{\sim}{x};\underset{\sim}{y})$ que aparecem nos referidos resultados (ver (3.9), (3.10), (3.17), (C.3), (C.11), (C.48) e (C.50)-(C.55)), onde J = 1,...,14.

Consideremos inicialmente as funções $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$. para J = 14, (3.17) nos dá

$$R^{ab}_{14,11}(\underset{\sim}{x};\underset{\sim}{y}) = R^{ab}_{14,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) \;. \qquad (D.2a)$$

Por outro lado, (C.52) implica a igualdade

$$R^{ba}_{11,14}(\underset{\sim}{y};\underset{\sim}{x}) = -\delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) + gf^{acd}F^{12,d}(\underset{\sim}{x})\,r^{bc}_{11(10)}(\underset{\sim}{y};\underset{\sim}{x}) +$$
$$+ r^{ba}_{11(14)}(\underset{\sim}{y};\underset{\sim}{x}) \;. \qquad (D.2b)$$

A relação (D.1) nos leva então à seguinte relação

$$\boxed{R^{ab}_{14,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -r^{ba}_{11(14)}(y;\underset{\sim}{x}) - g f^{acd} F^{12,d}(\underset{\sim}{x})\, r^{bc}_{11(10)}(y;\underset{\sim}{x})} \quad . \qquad (D.3)$$

De forma similar, J = 13 em (3.17) conduz a

$$R^{ab}_{13,11}(\underset{\sim}{x};y) = R^{ab}_{13,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \delta^{ab}\delta(x^1-x^1_{(0)})\,\Delta(y^2,x^2_{(0)};x^2)\,\delta(x^3-x^3_{(0)})$$

enquanto (C.51) implica

$$\begin{aligned} R^{ba}_{11,13}(y;\underset{\sim}{x}) = {} & \delta^{ab}\delta(x^1-x^1_{(0)})\,\Delta(x^2,x^2_{(0)};y^2)\,\delta(x^3-x^3_{(0)}) + g f^{acd} F^{31,d}(\underset{\sim}{x})\, r^{bc}_{11(10)}(y;\underset{\sim}{x}) + \\ & + r^{ba}_{11(13)}(y;\underset{\sim}{x}) \end{aligned}$$

$\therefore$

$$\boxed{\begin{aligned} R^{ab}_{13,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = {} & -\delta^{ab}\delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\left(\Delta(y^2,x^2_{(0)};x^2)+\Delta(x^2,x^2_{(0)};y^2)\right) - \\ & - r^{ba}_{11(13)}(y;\underset{\sim}{x}) - g f^{acd} F^{31,d}(\underset{\sim}{x})\, r^{bc}_{11(10)}(y;\underset{\sim}{x}) \quad . \qquad (D.4) \end{aligned}}$$

Para J = 12, (3.17) fornece

$$R^{ab}_{12,11}(\underset{\sim}{x};y) = R^{ab}_{12,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \delta^{ab}\Delta(y^1,x^1_{(0)};x^1)\,\delta(x^2-y^2)\,\delta(x^3-x^3_{(0)})$$

enquanto (C.50) implica

$$\underset{11,12}{R}^{ba}(y;\underset{\sim}{x}) = -\delta^{ab}\Delta(y^1, x^1_{(0)}; x^1)\delta(x^2-y^2)\delta(x^3-x^3_{(0)}) + g f^{acd}\, \overset{23,d}{F}(\underset{\sim}{x})\, \underset{11(10)}{r}^{bc}(y;\underset{\sim}{x}) +$$

$$+ \underset{11(12)}{r}^{ba}(y;\underset{\sim}{x})$$

$$\therefore$$

$$\boxed{\underset{12,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -\underset{11(12)}{r}^{ba}(y;\underset{\sim}{x}) - g f^{acd}\, \overset{23,d}{F}(\underset{\sim}{x})\, \underset{11(10)}{r}^{bc}(y;\underset{\sim}{x})} \quad . \qquad (D.5)$$

Para J = 11, (3.17) dá por um lado

$$\underset{11,11}{R}^{ab}(\underset{\sim}{x};y) = \underset{11,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$$

e por outro lado

$$\underset{11,11}{R}^{ba}(y;\underset{\sim}{x}) = \underset{11,11}{R}^{ba}(y;\underset{\sim}{x}_{(0)})$$

$$\therefore \quad \underset{11,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -\underset{11,11}{R}^{ba}(y;\underset{\sim}{x}_{(0)}) = cte .$$

Consistentemente com a antissimetria de R (e com a própria expressão (3.17)), tomamos a constante acima igual a <u>zero</u>, i.e.,

$$\boxed{\underset{11,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0} \quad . \qquad (D.6)$$

Para J = 10, (3.17) $\Longrightarrow$

$$R^{ab}_{10,11}(\underset{\sim}{x};y) = R^{ab}_{10,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$$

enquanto (C.48) $\Longrightarrow$

$$R^{ba}_{11,10}(y;\underset{\sim}{x}) = r^{ba}_{11(10)}(y;\underset{\sim}{x})$$

$\therefore$

$$\boxed{R^{ab}_{10,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -r^{ba}_{11(10)}(y;\underset{\sim}{x})} \quad . \qquad (D.7)$$

De maneira análoga, levando em conta (D.6), obtemos

$$\boxed{R^{ab}_{k+6,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0} \qquad (D.8)$$

e

$$\boxed{R^{ab}_{k+3,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0} \quad . \qquad (D.9)$$

Para J = 3, (3.17) $\Longrightarrow$

$$R^{ab}_{3,11}(\underset{\sim}{x};y) = R^{ab}_{3,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) - \delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{10)};x^3)$$

enquanto (C.55) $\Longrightarrow$

$$R^{ba}_{11,3}(y;\underset{\sim}{x}) = \delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) - r^{ba}_{11(14)}(y;\underset{\sim}{x}) -$$

$$-gf^{acd}\,F^{12,d}(\underset{\sim}{x})\,r^{bc}_{11(10)}(y;\underset{\sim}{x})$$

$\therefore$ (ver D.3)

$$\boxed{R^{ab}_{3,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = r^{ba}_{11(14)}(y;\underset{\sim}{x}) + gf^{acd}F^{12,d}(\underset{\sim}{x})\, r^{bc}_{11(10)}(y;\underset{\sim}{x}) = -R^{ab}_{14,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})}. \quad (D.10)$$

Para J = 2, (3.17) $\Longrightarrow$

$$R^{ab}_{2,11}(\underset{\sim}{x};y) = R^{ab}_{2,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) - \delta^{ab}\delta(x^1 - x^1_{(0)})\,\Delta(y^2, x^2_{(0)}; x^2)\,\delta(x^3 - x^3_{(0)})$$

enquanto (C.54) $\Longrightarrow$

$$R^{ba}_{11,2}(y;\underset{\sim}{x}) = -\delta^{ab}\delta(x^1 - x^1_{(0)})\,\Delta(x^2, x^2_{(0)}; y^2)\,\delta(x^3 - x^3_{(0)}) - r^{ba}_{11(13)}(y;\underset{\sim}{x})$$

$\therefore$

$$\boxed{\begin{aligned} R^{ab}_{2,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = {} & \delta^{ab}\delta(x^1 - x^1_{(0)})\,\delta(x^3 - x^3_{(0)})\big(\Delta(y^2, x^2_{(0)}; x^2) + \Delta(x^2, x^2_{(0)}; y^2)\big) + \\ & + r^{ba}_{11(13)}(y;\underset{\sim}{x}) \quad . \qquad (D.11) \end{aligned}}$$

Para J = 1, (3.17) $\Longrightarrow$

$$R^{ab}_{1,11}(\underset{\sim}{x};y) = R^{ab}_{1,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) - \delta^{ab}\Delta(y^1, x^1_{(0)}; x^1)\,\delta(x^2 - y^2)\,\delta(x^3 - x^3_{(0)})$$

enquanto (C.53) $\Longrightarrow$

$$R^{ba}_{11,1}(y;\underset{\sim}{x}) = \delta^{ab}\Delta(y^1, x^1_{(0)}; x^1)\,\delta(x^2 - y^2)\,\delta(x^3 - x^3_{(0)}) - r^{ba}_{11(12)}(y;\underset{\sim}{x})$$

$$\therefore \quad \boxed{R^{ab}_{1,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = r^{ba}_{11(12)}(y;\underset{\sim}{x})} \quad . \qquad (D.12)$$

Tendo em vista (D.6), (D.8) e (D.9) cabe investigar se outras funções $R^{ab}_{J,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$ são também (ou poderão ser tomadas como) nulas. Isto, de fato, é o caso. Por exemplo, desde (C.2a) (ou (3.4a)), obtemos para $J = 10$

$$D^{k,bc}_{(y)} R^{ac}_{10,11}(\underset{\sim}{x};y) = 0 \quad . \qquad (D.13)$$

Usando (3.17), (D.13) se reduz a

$$D^{k,bc}_{(y)} R^{ac}_{10,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = gf^{bdc} A^{k,d}_{(y)} R^{ac}_{10,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0$$

o que fornece

$$\boxed{R^{ac}_{10,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0} \quad . \qquad (D.14)$$

A expressão (D.14) implica, por sua vez, desde (D.7)

$$\boxed{r^{ba}_{11(10)}(y;\underset{\sim}{x}) = 0} \quad . \qquad (D.15)$$

Por outro lado, a situação é diferente no que se refere às funções $R^{ab}_{k+11,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -R^{ab}_{k,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$ (ver (D.3)-(D.5) e (D.10)-(D.12)). Note-se que (D.4) e (D.5) junto com (C.35) e (C.26), respectivamente, implicam

$$\underset{13,11}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^1-x^1_{(0)})\delta(x^3-x^3_{(0)})\Big[-\delta^{ab}\big(\Delta(y^2,x^2_{(0)};x^2)+\Delta(x^2,x^2_{(0)};y^2)\big) - \underset{11(13)}{r'^{ba}}(y;x^2)\Big] \qquad (D.16)$$

e

$$\underset{12,11}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -\delta(x^3-x^3_{(0)})\,\underset{11(12)}{\tilde{r}^{ba}}(y;x^1,x^2)\ . \qquad (D.17)$$

Observação: Logo abaixo mostramos, usando as definições (C.32), (C.23) e (C.24), que a dependência em $\underset{\sim}{y}$ no lado direito de (D.16) e (D.17) desaparece realmente, conforme indicado no lado esquerdo dessas equações.

Particularizando a equação (C.16) para J = 11, teremos

$$D^{k,db}_{(y)}\,\underset{11,k+11}{R^{ab}}(\underset{\sim}{x};y) = -\Big(D^{k,db}_{(y)}\,\underset{k+11,11}{R^{ba}}(y;\underset{\sim}{x})\Big) = -\delta^{ad}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\ . \qquad (D.18)$$

O parêntese em (D.18) pode ser calculado desde (3.17) levando em conta os suportes das funções $R^{ab}_{13,11}$ e $R^{ab}_{12,11}$ expressos em (D.16) e (D.17), respectivamente, junto com as condições de gauge (2.1b) e (2.1c):

$$D^{k,db}_{(y)}\,\underset{k+11,11}{R^{ba}}(y;\underset{\sim}{x}) =$$

$$=\big(\delta^{db}\partial^1_y + g f^{dcb}A^{1,c}(y)\big)\Big[\underset{12,11}{R^{ba}}(y;\underset{\sim}{x}_{(0)}) + \delta^{ba}\Delta(x^1,x^1_{(0)};y^1)\delta(x^2-y^2)\delta(y^3-x^3_{(0)})\Big]+$$

$$+ \left(\delta^{db}\partial_y^2 + g f^{dcb} A^{2,c}(y)\right)\left[R^{ba}_{13,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ba}\delta(y^1 - x^1_{(0)})\Delta(x^2, x^2_{(0)}; y^2)\delta(y^3 - x^3_{(0)})\right] +$$

$$+ \partial_y^3 \left[R^{da}_{14,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ad}\delta(x^1 - y^1)\delta(x^2 - y^2)\Delta(x^3, x^3_{(0)}; y^3)\right] =$$

$$= \partial_y^k R^{da}_{k+11,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ad}\left[\delta(x^1 - y^1) - \delta(y^1 - x^1_{(0)})\right]\delta(x^2 - y^2)\delta(y^3 - x^3_{(0)}) +$$

$$+ \delta^{ad}\delta(y^1 - x^1_{(0)})\left[\delta(x^2 - y^2) - \delta(y^2 - x^2_{(0)})\right]\delta(y^3 - x^3_{(0)}) +$$

$$+ \delta^{ad}\delta(x^1 - y^1)\delta(x^2 - y^2)\left[\delta(x^3 - y^3) - \delta(y^3 - x^3_{(0)})\right] =$$

$$= \partial_y^k R^{da}_{k+11,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ad}\left(\delta^{(3)}(\underset{\sim}{x} - y) - \delta^{(3)}(y - \underset{\sim}{x}_{(0)})\right) . \qquad (D.19)$$

Substituindo (D.19) em (D.18), obtemos a equação

$$\boxed{\partial_y^k R^{da}_{k+11,11}(y;\underset{\sim}{x}_{(0)}) = \delta^{da}\delta^{(3)}(y - \underset{\sim}{x}_{(0)})} \qquad (D.20)$$

que implica na impossibilidade de tomarmos todas as funções $R^{ab}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x}_{(0)})$ (e $R^{ab}_{k,11}(\underset{\sim}{y};\underset{\sim}{x}_{(0)})$) iguais a zero. Entretanto, a partir das definições (C.21), (C.32), (C.23) e (C.24) podemos calcular explicitamente os lados direitos em (D.3), (D.16) e (D.17). Desde (D.3), (D.15), (C.21), (C.18) e (C.19)

$$\underset{14,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -\underset{11(14)}{r}^{ba}(y;\underset{\sim}{x}) = -\underset{11(14)}{r'}^{ba}(y;\underset{\sim}{x}) - \underset{11(14)}{r''}^{ba}(y;\underset{\sim}{x}) =$$

$$= -\underset{11,14}{R}^{ba}(y;x^1,x^2,\infty)\,\theta(x^3-x^3_{(0)}) - \underset{11,14}{R}^{ba}(y;x^1,x^2,-\infty)\,\theta(x^3_{(0)}-x^3) -$$

$$-\int_{-\infty}^{+\infty} dz^3\,\Delta(z^3,x^3_{(0)};x^3)\,g f^{acd}\Big[\big(A^{1,c}(\underset{\sim}{x}) - A^{1,c}(x^1,x^2,z^3)\big)\underset{11,12}{R}^{bd}(y;x^1,x^2,z^3) +$$

$$+\big(A^{2,c}(\underset{\sim}{x}) - A^{2,c}(x^1,x^2,z^3)\big)\underset{11,13}{R}^{bd}(y;x^1,x^2,z^3)\Big] =$$

$$= \underset{14,11}{R}^{ab}(x^1,x^2,\infty;y)\,\theta(x^3-x^3_{(0)}) + \underset{14,11}{R}^{ab}(x^1,x^2,-\infty;y)\,\theta(x^3_{(0)}-x^3) +$$

$$+ g f^{acd}\int_{-\infty}^{+\infty} dz^3\,\Delta(z^3,x^3_{(0)};x^3)\Big[\big(A^{1,c}(\underset{\sim}{x}) - A^{1,c}(x^1,x^2,z^3)\big)\underset{12,11}{R}^{db}(x^1,x^2,z^3;y) +$$

$$+\big(A^{2,c}(\underset{\sim}{x}) - A^{2,c}(x^1,x^2,z^3)\big)\underset{13,11}{R}^{db}(x^1,x^2,z^3;y)\Big] \Longrightarrow$$

$$\Longrightarrow$$

$$\boxed{\underset{14,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \underset{14,11}{R}^{ab}(x^1,x^2,\infty;\underset{\sim}{x}_{(0)})\,\theta(x^3-x^3_{(0)}) + \underset{14,11}{R}^{ab}(x^1,x^2,-\infty;\underset{\sim}{x}_{(0)})\,\theta(x^3_{(0)}-x^3)\,. \qquad (D.21)}$$

Na obtenção de (D.21) usamos a antissimetria de R, (3.17) e os

<u>suportes</u> (D.16) e (D.17). De modo similar, desde (D.16) e (C.32), segue que

$$R^{ab}_{13,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\Big[-\delta^{ab}\big(\Delta(y^2,x^2_{(0)};x^2)+\Delta(x^2,x^2_{(0)};y^2)\big)-$$

$$-\gamma'^{ba}_{11(13)}(y;x^2)\Big] =$$

$$= \delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\Big\{-\delta^{ab}\big[\theta(x^2-x^2_{(0)})-\theta(x^2-y^2)+\theta(y^2-x^2_{(0)})-\theta(y^2-x^2)\big]-$$

$$-\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\Big[R^{ba}_{11,13}(y;z^1,x^2_{(0)},z^3)+\int_{x^2_{(0)}}^{x^2}dy'^2\Big(\partial^1_z R^{ba}_{11,12}(y;z^1,y'^2,z^3)+$$

$$+\partial^3_z R^{ba}_{11,14}(y;z^1,y'^2,z^3)+gf^{acd}A^{1,c}(z^1,y'^2,z^3)\,R^{bd}_{11,12}(y;z^1,y'^2,z^3)+$$

$$+gf^{acd}A^{2,c}(z^1,y'^2,z^3)\,R^{bd}_{11,13}(y;z^1,y'^2,z^3)\Big)\Big]\Big\} =$$

$$= \delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\Big\{-\delta^{ab}\big[\theta(x^2-x^2_{(0)})+\theta(y^2-x^2_{(0)})-1\big]+$$

$$+\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\Big[R^{ab}_{13,11}(z^1,x^2_{(0)},z^3;\underset{\sim}{x}_{(0)})+\delta^{ab}\delta(z^1-x^1_{(0)})\,\Delta(y^2,x^2_{(0)};x^2_{(0)})\,\delta(z^3-x^3_{(0)})+$$

$$+\int_{x^2_{(0)}}^{x^2} dy'^2 \Bigg(\partial^1_z \Big(R^{ab}_{12,11}(z^1, y'^2, z^3; \underset{\sim}{x}_{(0)}) + \delta^{ab} \Delta(y^1, x^1_{(0)}; z^1)\, \delta(y'^2 - y^2)\, \delta(z^3 - x^3_{(0)}) \Big) +$$

$$+ \partial^3_z \Big(R^{ab}_{14,11}(z^1, y'^2, z^3; \underset{\sim}{x}_{(0)}) + \delta^{ab} \delta(z^1 - y^1)\, \delta(y'^2 - y^2)\, \Delta(y^3, x^3_{(0)}; z^3)\Big) +$$

$$+ g f^{acd} A^{1,c}(z^1, y'^2, z^3) \Big(R^{db}_{12,11}(z^1,y'^2,z^3;\underset{\sim}{x}_{(0)}) + \delta^{db}\Delta(y^1,x^1_{(0)};z^1)\,\delta(y'^2-y^2)\,\delta(z^3-x^3_{(0)})\Big) +$$

$$+ g f^{acd} A^{2,c}(z^1,y'^2,z^3)\Big(R^{db}_{13,11}(z^1,y'^2,z^3;\underset{\sim}{x}_{(0)}) + \delta^{db}\delta(z^1-x^1_{(0)})\,\Delta(y^2,x^2_{(0)};y'^2)\,\delta(z^3-x^3_{(0)})\Big)\Bigg)\Big]\Big\} =$$

$$= \delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\Big\{ -\delta^{ab}\big(\theta(x^2-x^2_{(0)}) - \theta(x^2_{(0)}-y^2)\big) \quad +$$

$$+\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\; R^{ab}_{13,11}(z^1,x^2_{(0)},z^3;\underset{\sim}{x}_{(0)}) \quad + \delta^{ab}\big(\theta(0)-\theta(x^2_{(0)}-y^2)\big) -$$

$$-\int_{-\infty}^{+\infty}dz^3\int_{x^2_{(0)}}^{x^2}dy'^2\Big[R^{ab}_{12,11}(\infty,y'^2,z^3;\underset{\sim}{x}_{(0)}) - R^{ab}_{12,11}(-\infty,y'^2,z^3;\underset{\sim}{x}_{(0)})\Big] \quad +$$

$$+\int_{-\infty}^{+\infty}dz^1\int_{-\infty}^{+\infty}dz^3\; \delta^{ab}\big[\delta(z^1-y^1)-\delta(z^1-x^1_{(0)})\big]\,\Delta(x^2,x^2_{(0)};y^2)\,\delta(z^3-x^3_{(0)}) \; -$$

$$-\int_{-\infty}^{+\infty}dz^1\int_{x^2_{(0)}}^{x^2}dy'^2\Big[R^{ab}_{14,11}(z^1,y'^2,\infty;\underset{\sim}{x}_{(0)}) - R^{ab}_{14,11}(z^1,y'^2,-\infty;\underset{\sim}{x}_{(0)})\Big] \quad +$$

$$+\int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3 \; \delta^{ab} \delta(z^1-y^1)\, \Delta(x^2, x^2_{(0)}; y^2) \left[\delta(z^3-y^3) - \delta(z^3-x^3_{(0)}) \right] \Big\} \quad \Longrightarrow$$

$$\Longrightarrow$$

$$\boxed{\underset{13,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = \delta(x^1-x^1_{(0)})\,\delta(x^3-x^3_{(0)})\; \underset{13,11}{\tilde{R}}^{ab}(x^2;\underset{\sim}{x}_{(0)})} \quad , \qquad (D.22)$$

onde

$$\underset{13,11}{\tilde{R}}^{ab}(x^2;\underset{\sim}{x}_{(0)}) = \delta^{ab}\left[\Theta(0) - \Theta(x^2-x^2_{(0)})\right] + \underset{13,11}{\tilde{R}}^{ab}(x^2_{(0)};\underset{\sim}{x}_{(0)}) -$$

$$-\int_{-\infty}^{+\infty} dz^3 \int_{x^2_{(0)}}^{x^2} dz^2 \left[\underset{12,11}{R}^{ab}(\infty, z^2, z^3; \underset{\sim}{x}_{(0)}) - \underset{12,11}{R}^{ab}(-\infty, z^2, z^3; \underset{\sim}{x}_{(0)}) \right] -$$

$$-\int_{-\infty}^{+\infty} dz^1 \int_{x^2_{(0)}}^{x^2} dz^2 \left[\underset{14,11}{R}^{ab}(z^1, z^2, \infty; \underset{\sim}{x}_{(0)}) - \underset{14,11}{R}^{ab}(z^1, z^2, -\infty; \underset{\sim}{x}_{(0)}) \right] \quad . \qquad (D.23)$$

Agora, desde (D.17), (C.26), (C.24) e (C.23)

$$\underset{12,11}{R}^{ab}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = -\delta(x^3-x^3_{(0)})\; \underset{11(12)}{\tilde{r}}^{ba}(y; x^1, x^2) =$$

$$= -\delta(x^3-x^3_{(0)}) \Big\{ \int_{-\infty}^{+\infty} dz^1\, \Delta(z^1, x^1_{(0)}; x^1) \left[\underset{11,14}{R}^{ba}(y; z^1, x^2, \infty) - \underset{11,14}{R}^{ba}(y; z^1, x^2, -\infty) \right] +$$

$$+\int_{-\infty}^{+\infty} dz^3 \left[\underset{11,12}{R}^{ba}(y; \infty, x^2, z^3)\, \Theta(x^1-x^1_{(0)}) + \underset{11,12}{R}^{ba}(y; -\infty, x^2, z^3)\, \Theta(x^1_{(0)}-x^1) \right] +$$

$$+\int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3\, \Delta(z^1, x^1_{(0)}; x^1)\, g f^{acd} \left[A^{2,c}(\underset{\sim}{x}) - A^{2,c}(z^1, x^2, z^3) \right] \underset{11,13}{R}^{bd}(y; z^1, x^2, z^3) \Big\} =$$

$$= \delta(x^3 - x^3_{(0)}) \left\{ \int_{-\infty}^{+\infty} dz^1 \, \Delta(z^1, x^1_{(0)}; x^1) \left[R^{ab}_{14,11}(z^1, x^2, \infty; \underset{\sim}{y}) - R^{ab}_{14,11}(z^1, x^2, -\infty; \underset{\sim}{y}) \right] + \right.$$

$$+ \int_{-\infty}^{+\infty} dz^3 \left[R^{ab}_{12,11}(\infty, x^2, z^3; \underset{\sim}{y}) \, \theta(x^1 - x^1_{(0)}) + R^{ab}_{12,11}(-\infty, x^2, z^3; \underset{\sim}{y}) \theta(x^1_{(0)} - x^1) + \right.$$

$$\left. + \int_{-\infty}^{+\infty} dz^1 \int_{-\infty}^{+\infty} dz^3 \, g f^{acd} \Delta(z^1, x^1_{(0)}; x^1) \left[A^{2,c}(\underset{\sim}{x}) - A^{2,c}(z^1, x^2, z^3) \right] R^{db}_{13,11}(z^1, x^2, z^3; \underset{\sim}{y}) \right\} \Rightarrow$$

$$\boxed{R^{ab}_{12,11}(\underset{\sim}{x}; \underset{\sim}{x}_{(0)}) = \delta(x^3 - x^3_{(0)}) \; \tilde{R}^{ab}_{12,11}(x^1, x^2; \underset{\sim}{x}_{(0)})} \; , \qquad (D.24)$$

onde

$$\boxed{\begin{aligned} \tilde{R}^{ab}_{12,11}(x^1, x^2; \underset{\sim}{x}_{(0)}) = {} & \tilde{R}^{ab}_{12,11}(\infty, x^2; \underset{\sim}{x}_{(0)}) \, \theta(x^1 - x^1_{(0)}) + \tilde{R}^{ab}_{12,11}(-\infty, x^2; \underset{\sim}{x}_{(0)}) \, \theta(x^1_{(0)} - x^1) + \\ & + \int_{-\infty}^{+\infty} dz^1 \, \Delta(z^1, x^1_{(0)}; x^1) \left[R^{ab}_{14,11}(z^1, x^2, \infty; \underset{\sim}{x}_{(0)}) - R^{ab}_{14,11}(z^1, x^2, -\infty; \underset{\sim}{x}_{(0)}) \right] \end{aligned}} \; . \qquad (D.25)$$

Na obtenção de (D.22) e (D.24), é claro, usamos novamente a antissimetria de R e os suportes das funções $R^{ab}_{12,11}(\underset{\sim}{x}; \underset{\sim}{x}_{(0)})$ e $R^{ab}_{13,11}(\underset{\sim}{x}; \underset{\sim}{x}_{(0)})$. Além disso, usando (D.24) em (D.23), obtemos

$$\boxed{\begin{aligned}\tilde{R}^{ab}_{13,11}(x^2;\underset{\sim}{x}_{(0)}) &= \delta^{ab}\left[\Theta(0)-\Theta(x^2-x^2_{(0)})\right] + \tilde{R}^{ab}_{13,11}(x^2_{(0)};\underset{\sim}{x}_{(0)}) - \\ &-\int_{x^2_{(0)}}^{x^2} dz^2 \left[\tilde{R}^{ab}_{12,11}(\infty,z^2;\underset{\sim}{x}_{(0)}) - \tilde{R}^{ab}_{12,11}(-\infty,z^2;\underset{\sim}{x}_{(0)})\right] - \\ &-\int_{-\infty}^{+\infty} dz^1 \int_{x^2_{(0)}}^{x^2} dz^2 \left[R^{ab}_{14,11}(z^1,z^2,\infty;\underset{\sim}{x}_{(0)}) - R^{ab}_{14,11}(z^1,z^2,-\infty;\underset{\sim}{x}_{(0)})\right] .\end{aligned}} \qquad (D.26)$$

Em continuação, analisamos as funções $r^{ab}_{J(k+11)}(\underset{\sim}{x};\underset{\sim}{y})$ começando pelas $r^{ab}_{J(12)}(\underset{\sim}{x};\underset{\sim}{y})$. A partir de (C.50) encontramos as seguintes condições de consistência que garantem a antissimetria de R:

$$r^{ab}_{k(12)}(\underset{\sim}{x};\underset{\sim}{y}) = gf^{bdc}F^{23,d}_{(\underset{\sim}{y})}\,r^{ac}_{k(10)}(\underset{\sim}{x};\underset{\sim}{y}) + gf^{acd}F^{0k,d}_{(\underset{\sim}{x})}\,R^{bc}_{12,11}(\underset{\sim}{y};\underset{\sim}{x}_{(0)}) \;, \qquad (D.27a)$$

$$r^{ab}_{4(12)}(\underset{\sim}{x};\underset{\sim}{y}) = gf^{acd}F^{12,d}_{(\underset{\sim}{x})}\,R^{bc}_{12,11}(\underset{\sim}{y};\underset{\sim}{x}) + gf^{bdc}F^{23,d}_{(\underset{\sim}{y})}\,r^{ac}_{4(10)}(\underset{\sim}{x};\underset{\sim}{y}) +$$

$$+gf^{abc}F^{23f}_{(\underset{\sim}{y})}\,\delta(x^1-x^1_{(0)})\,\delta(x^2-x^2_{(0)})\left(\Delta(y^3,x^3_{(0)};x^3)-\Delta(y^3,x^3_{(0)};x^3_{(0)})+\Delta(x^3,x^3_{(0)};y^3)\right), \qquad (D.27b)$$

$$r^{ab}_{5(12)}(\underset{\sim}{x};\underset{\sim}{y}) = gf^{acd}F^{23,d}_{(\underset{\sim}{x})}\,R^{bc}_{12,11}(\underset{\sim}{y};\underset{\sim}{x}) + gf^{bdc}F^{23,d}_{(\underset{\sim}{y})}\,r^{ac}_{5(10)}(\underset{\sim}{x};\underset{\sim}{y}) \;, \qquad (D.27c)$$

$$r^{ab}_{6(12)}(\underset{\sim}{x};\underset{\sim}{y}) = gf^{acd}F^{31,d}_{(\underset{\sim}{x})}\,R^{bc}_{12,11}(\underset{\sim}{y};\underset{\sim}{x}) + gf^{bdc}F^{23,d}_{(\underset{\sim}{y})}\,r^{ac}_{6(10)}(\underset{\sim}{x};\underset{\sim}{y}) \;, \qquad (D.27d)$$

$$r^{ab}_{k+6(12)}(\underset{\sim}{x};\underset{\sim}{y}) = gf^{bdc}F^{23,d}_{(\underset{\sim}{y})}\,r^{ac}_{k+6(10)}(\underset{\sim}{x};\underset{\sim}{y}) + gf^{acd}A^{k,d}_{(\underset{\sim}{x})}\,R^{bc}_{12,11}(\underset{\sim}{y};\underset{\sim}{x}_{(0)}) \;, \qquad (D.27e)$$

$$r^{ab}_{10(12)}(\underset{\sim}{x};y) = -r^{ba}_{12(10)}(y;\underset{\sim}{x}) + gf^{bdc}F^{23,d}(y)\,r^{ac}_{10(10)}(\underset{\sim}{x};y) \quad , \qquad (D.27\,f)$$

$$r^{ab}_{11(12)}(\underset{\sim}{x};y) = gf^{bdc}F^{23,d}(y)\,r^{ac}_{11(10)}(\underset{\sim}{x};y) - R^{ba}_{12,11}(y;\underset{\sim}{x}_{(0)}) \quad , \qquad (D.27\,g)$$

$$r^{ab}_{k+11(12)}(\underset{\sim}{x};y) = gf^{bdc}F^{23,d}(y)\,r^{ac}_{k+11(10)}(\underset{\sim}{x};y) \quad . \qquad (D.27\,h)$$

De forma similar, analisamos as funções $r^{ab}_{J(13)}(\underset{\sim}{x};\underset{\sim}{y})$. Desde (C.51), encontramos:

$$r^{ab}_{1(13)}(\underset{\sim}{x};y) = r^{ba}_{13(12)}(y;\underset{\sim}{x}) + gf^{acd}F^{01,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{1(10)}(\underset{\sim}{x};y) \,, \quad (D.28a)$$

$$r^{ab}_{2(13)}(\underset{\sim}{x};y) = r^{ba}_{13(13)}(y;\underset{\sim}{x}) + gf^{acd}F^{02,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{2(10)}(\underset{\sim}{x};y) \,, \quad (D.28b)$$

$$r^{ab}_{3(13)}(\underset{\sim}{x};y) = gf^{acd}F^{03,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{3(10)}(\underset{\sim}{x};y) \,, \quad (D.28c)$$

$$r^{ab}_{4(13)}(\underset{\sim}{x};y) = gf^{acd}F^{12,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{4(10)}(\underset{\sim}{x};y) +$$

$$+ gf^{abc}F^{31,c}(y)\,\delta(x^1 - x^1_{(0)})\,\delta(x^2 - x^2_{(0)})\left(\Delta(y^3, x^3_{(0)}; x^3) - \Delta(y^3, x^3_{(0)}; x^3_{(0)}) + \Delta(x^3, x^3_{(0)}; y^3)\right), \quad (D.28d)$$

$$r^{ab}_{5(13)}(\underset{\sim}{x};y) = gf^{acd}F^{23,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{5(10)}(\underset{\sim}{x};y) \,, \quad (D.28\,e)$$

$$r^{ab}_{6(13)}(\underset{\sim}{x};y) = gf^{acd}F^{31,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{6(10)}(\underset{\sim}{x};y) \,, \quad (D.28\,f)$$

$$r^{ab}_{7(13)}(\underset{\sim}{x};y) = gf^{acd}A^{1,d}(\underset{\sim}{x})\,R^{bc}_{13,11}(y;\underset{\sim}{x}) + gf^{bdc}F^{31,d}(y)\,r^{ac}_{7(10)}(\underset{\sim}{x};y) \quad , \quad (D.28\,g)$$

$$r^{ab}_{8(13)}(\underset{\sim}{x};y) = g f^{acd} A^{2,d}_{(\underset{\sim}{x})} R^{bc}_{13,11}(y;\underset{\sim}{x}) + g f^{bdc} F^{31,d}_{(y)} r^{ac}_{8(10)}(\underset{\sim}{x};y) \,, \qquad (D.28\,h)$$

$$r^{ab}_{9(13)}(\underset{\sim}{x};y) = g f^{bdc} F^{31,d}_{(y)} r^{ac}_{9(10)}(\underset{\sim}{x};y) \,, \qquad (D.28\,i)$$

$$r^{ab}_{10(13)}(\underset{\sim}{x};y) = - r^{ba}_{13(10)}(y;\underset{\sim}{x}) + g f^{bdc} F^{31,d}_{(y)} r^{ac}_{10(10)}(\underset{\sim}{x};y) \,, \qquad (D.28\,j)$$

$$r^{ab}_{11(13)}(\underset{\sim}{x};y) = -\delta^{ab}\delta(y^1 - x^1_{(0)})\,\delta(y^3 - x^3_{(0)})\left(\Delta(x^2, x^2_{(0)}; y^2) + \Delta(y^2, x^2_{(0)}; x^2)\right) - R^{ba}_{13,11}(y;\underset{\sim}{x}_{(0)}) +$$

$$+ g f^{bdc} F^{31,d}_{(y)} r^{ac}_{11(10)}(\underset{\sim}{x};y) \,, \qquad (D.28\,k)$$

$$r^{ab}_{k+11(13)}(\underset{\sim}{x};y) = g f^{bdc} F^{31,d}_{(y)} r^{ac}_{k+11(10)}(\underset{\sim}{x};y) \,. \qquad (D.28\,\ell)$$

Por outro lado, desde (C.52), obtemos para as funções $r^{ab}_{J(14)}(\underset{\sim}{x};\underset{\sim}{y})$ as seguintes condições:

$$r^{ab}_{k(14)}(\underset{\sim}{x};y) = r^{ba}_{14(k+11)}(y;\underset{\sim}{x}) + g f^{bdc} F^{12,d}_{(y)} r^{ac}_{k(10)}(\underset{\sim}{x};y) + g f^{acd} F^{0k,d}_{(\underset{\sim}{x})} R^{bc}_{14,11}(y;\underset{\sim}{x}_{(0)}) \,, \qquad (D.29\,a)$$

$$r^{ab}_{4(14)}(\underset{\sim}{x};y) = g f^{bdc} F^{12,d}_{(y)} r^{ac}_{4(10)}(\underset{\sim}{x};y) + g f^{acd} F^{12,d}_{(\underset{\sim}{x})} R^{bc}_{14,11}(y;\underset{\sim}{x}_{(0)}) +$$

$$+ g f^{abc} F^{12,c}_{(y)} \delta(x^1 - x^1_{(0)})\delta(x^2 - x^2_{(0)})\Big(\Delta(y^3, x^3_{(0)}; x^3) - \Delta(y^3, x^3_{(0)}; x^3_{(0)}) + \Delta(x^3, x^3_{(0)}; y^3)\Big) \,, \qquad (D.29\,b)$$

$$r^{ab}_{5(14)}(\underset{\sim}{x};y) = g f^{acd} F^{23,d}_{(\underset{\sim}{x})} R^{bc}_{14,11}(y;\underset{\sim}{x}) + g f^{bdc} F^{12,d}_{(y)} r^{ac}_{5(10)}(\underset{\sim}{x};y) \,, \qquad (D.29\,c)$$

$$r^{ab}_{6(14)}(\underset{\sim}{x};y) = g f^{acd} F^{31,d}_{(\underset{\sim}{x})} R^{bc}_{14,11}(y;\underset{\sim}{x}) + g f^{bdc} F^{12,d}_{(y)} r^{ac}_{6(10)}(\underset{\sim}{x};y) \,, \qquad (D.29\,d)$$

$$r^{ab}_{k+6(14)}(\underset{\sim}{x};y) = gf^{bdc}F^{12,d}(y)\,r^{ac}_{k+6(10)}(\underset{\sim}{x};y) + gf^{acd}A^{k,d}(\underset{\sim}{x})R^{bc}_{14,11}(y;\underset{\sim}{x}_{(0)}) \;, \qquad (D.29\;e)$$

$$r^{ab}_{10(14)}(\underset{\sim}{x};y) = -r^{ba}_{14(10)}(y;\underset{\sim}{x}) + gf^{bdc}F^{12,d}(y)\,r^{ac}_{10(10)}(\underset{\sim}{x};y) \;, \qquad (D.29\;f)$$

$$r^{ab}_{11(14)}(\underset{\sim}{x};y) = gf^{bdc}F^{12,d}(y)\,r^{ac}_{11(10)}(\underset{\sim}{x};y) - R^{ba}_{14,11}(y;\underset{\sim}{x}_{(0)}) \;, \qquad (D.29\;g)$$

$$r^{ab}_{k+11(14)}(\underset{\sim}{x};y) = gf^{bdc}F^{12,d}(y)\,r^{ac}_{k+11(10)}(\underset{\sim}{x};y) \;. \qquad (D.29\;h)$$

Concluímos este Apêndice analisando em detalhe as funções restantes $r^{ab}_{J(10)}(\underset{\sim}{x};\underset{\sim}{y})$ desde (C.48). Veremos que praticamente todas estas funções são nulas (ou poderão ser tomadas consistentemente iguais a zero) à diferença do que acontece com as $R^{ab}_{k+11,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$ (ver (D.20)). Como conseqüência, as relações (D.27)-(D.29) tornam-se mais simples. Por conveniência, começamos fixando $J=10$ em (C.48). Por um lado, obtemos

$$R^{ab}_{10,10}(\underset{\sim}{x};y) = r^{ab}_{10(10)}(\underset{\sim}{x};y)$$

e por outro lado

$$R^{ba}_{10,10}(y;\underset{\sim}{x}) = r^{ba}_{10(10)}(y;\underset{\sim}{x}) \;.$$

Logo, $r^{ab}_{10(10)}(\underset{\sim}{x};\underset{\sim}{y}) = -r^{ba}_{10(10)}(\underset{\sim}{y};\underset{\sim}{x})$, ou seja,

$$R^{ab}_{10,10}(\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + \int_{x^1_{(0)}}^{y^1} dz^1\, r''^{ab}_{10(10)}(\underset{\sim}{x};z^1,y^2,y^3) + \int_{x^2_{(0)}}^{y^2} dz^2\, r'^{ab}(\underset{\sim}{x};x^1_{(0)},z^2,y^3) -$$

$$-\int_{x^3_{(0)}}^{y^3} dz^3 \, \underset{10(10)}{r'''^{ab}}(\underset{\sim}{x}; x^1_{(0)}, x^2_{(0)}, z^3) = -\underset{10,10}{R^{ba}}(\underset{\sim}{y}; \underset{\sim}{x}_{(0)}) - \int_{x^1_{(0)}}^{x^1} dz^1 \, \underset{10(10)}{r''^{ba}}(\underset{\sim}{y}; z^1, x^2, x^3) -$$

$$-\int_{x^2_{(0)}}^{x^2} dz^2 \, \underset{10(10)}{r'^{ba}}(\underset{\sim}{y}; x^1_{(0)}, z^2, x^3) + \int_{x^3_{(0)}}^{x^3} dz^3 \, \underset{10(10)}{r'''^{ba}}(\underset{\sim}{y}; x^1_{(0)}, x^2_{(0)}, z^3) \quad . \qquad (D.30)$$

A expressão (D.30) possibilita tomar (ver (C.37), (C.40), (C.43), (C.45) e (C.48))

$$\underset{10,10}{R^{ab}}(\underset{\sim}{x}; y^1, y^2, \pm\infty) = 0 \qquad (D.31)$$

o que significa

$$\underset{10(10)}{r'^{ab}}(\underset{\sim}{x}; \underset{\sim}{y}) = \underset{10(10)}{r''^{ab}}(\underset{\sim}{x}; \underset{\sim}{y}) = \underset{10(10)}{r'''^{ab}}(\underset{\sim}{x}; \underset{\sim}{y}) = 0 \quad e \quad \underset{10,10}{R^{ab}}(\underset{\sim}{x}; \underset{\sim}{x}_{(0)}) = 0 \quad . \qquad (D.32)$$

As relações (D.32), por seu turno, implicam

$$\boxed{\underset{10(10)}{r^{ab}}(\underset{\sim}{x}; \underset{\sim}{y}) = \underset{10,10}{R^{ab}}(\underset{\sim}{x}; \underset{\sim}{y}) = 0} \quad , \qquad (D.33)$$

o que é consistente com (D.31). Levando em conta (D.33), obtemos imediatamente desde (C.48) e (C.50)-(C.52)

$$\underset{k+11(10)}{r^{ab}}(\underset{\sim}{x}; \underset{\sim}{y}) = -\underset{10(k+11)}{r^{ba}}(\underset{\sim}{y}; \underset{\sim}{x}) \quad . \qquad (D.34)$$

Além disso, fazendo uso de (D.14), encontramos

$$\boxed{r^{ab}_{k+6(10)}(\underset{\sim}{x};y) = 0} \quad , \tag{D.35}$$

$$\boxed{r^{ab}_{6(10)}(\underset{\sim}{x};y) = 0} \quad , \tag{D.36}$$

$$\boxed{r^{ab}_{5(10)}(\underset{\sim}{x};y) = 0} \quad . \tag{D.37}$$

Entretanto, para J = 4, **(C.48)** $\Longrightarrow$

$$\begin{aligned} R^{ab}_{4,10}(\underset{\sim}{x};y) = -\delta^{ab}\Big[\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(10)};y^3) - \delta(x^1-x^1_{(10)})\delta(x^2-x^2_{(0)})\Big(\Delta(y^3,x^3_{(10)};x^3) - \\ -\Delta(y^3,x^3_{(10)};x^3_{(10)}) + \Delta(x^3,x^3_{(10)};y^3)\Big)\Big] + r^{ab}_{4(10)}(\underset{\sim}{x};y) \end{aligned}$$

enquanto (C.10) $\Longrightarrow$

$$R^{ba}_{10,4}(y;\underset{\sim}{x}) = \delta^{ab}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(10)};y^3)$$

$\therefore$

$$\boxed{r^{ab}_{4(10)}(\underset{\sim}{x};y) = -\delta^{ab}\delta(x^1-x^1_{(10)})\delta(x^2-x^2_{(0)})\Big(\Delta(y^3,x^3_{(10)};x^3) - \Delta(y^3,x^3_{(10)};x^3_{(0)}) + \Delta(x^3,x^3_{(10)};y^3)\Big).} \tag{D.38}$$

Por último, é direto obter desde (C.53)-(C.55)

$$r^{ab}_{k(10)}(\underset{\sim}{x};y) = r^{ba}_{10(k+11)}(y;\underset{\sim}{x}) \quad . \tag{D.39}$$

Consideremos agora a equação (C.16) em conexão com as funções $r^{ab}_{J(10)}(\underset{\sim}{x};\underset{\sim}{y})$. Para J = 10, levando em conta (D.14) e (D.28), obtemos

$$D^{k,cb}_{(y)} \, R^{ab}_{10,k+11}(\underset{\sim}{x};y) = 0 \quad . \qquad (D.40)$$

Usando (C.50)-(C.52), reescrevemos (D.40) como

$$D^{1,cb}_{(y)} \, r^{ab}_{10(12)}(\underset{\sim}{x};y) + D^{2,cb}_{(y)} \, r^{ab}_{10(13)}(\underset{\sim}{x};y) + \partial^{3}_{y} \, r^{ac}_{10(14)}(\underset{\sim}{x};y) = 0 \quad .$$

Tendo em vista os suportes das funções $r^{ab}_{10(12)}$ e $r^{ab}_{10(13)}$ (ver (C.26) e (C.35)) e as condições de gauge (2.1b,c), esta última equação se reduz a

$$\boxed{\partial^{k}_{y} \, r^{ac}_{10(k+11)}(\underset{\sim}{x};y) = 0} \quad . \qquad (D.41)$$

A equação (D.41), à diferença de (D.20) em relação às funções $R^{ab}_{k+11,11}(\underset{\sim}{x};\underset{\sim}{x}_{(0)})$, nos permite colocar

$$\boxed{r^{ab}_{10(k+11)}(\underset{\sim}{x};y) = 0} \qquad (D.42)$$

o que, por sua vez, implica (ver (D.39) e (D.34))

$$\boxed{r^{ab}_{k(10)}(\underset{\sim}{x};y) = r^{ba}_{10(k+11)}(y;\underset{\sim}{x}) = r^{ab}_{k+11(10)}(\underset{\sim}{x};y) = 0} \quad . \qquad (D.43)$$

Deste modo, a única função $r^{ab}_{J(10)}(\underset{\sim}{x};\underset{\sim}{y})$ <u>não</u> nula é $r^{ab}_{4(10)}(\underset{\sim}{x};\underset{\sim}{y})$ (ver (D.38)). Conforme antecipamos, este fato simplifica muito as funções $r^{ab}_{J(k+11)}(\underset{\sim}{x};\underset{\sim}{y})$ (ver (D.27)-(D.29)). Em particular, desde (D.27h), (D.28 1), (D.29h) e (D.43), obtemos

$$\boxed{r^{ab}_{j+11(k+11)}(\underset{\sim}{x};\underset{\sim}{y}) = 0} \quad . \qquad (D.44)$$

Além disso, uma vez conhecidas as funções $r^{ab}_{J(10)}(\underset{\sim}{x};\underset{\sim}{y})$, pode-se reescrever os elementos de matriz $R^{ab}_{J,k}$, $R^{ab}_{J,10}$ e $R^{ab}_{J,k+11}$ (ver (C.48) e (C.50)-(C.55)) o que fazemos no capítulo III (ver (3.18)-(3.24)).

APÊNDICE E

OBTENÇÃO DOS MULTIPLICADORES DE LAGRANGE E DOS PARÊNTESES DE DIRAC BÁSICOS DA CROMODINÂMICA NO GAUGE SUPERAXIAL

A determinação da inversa (R) da matriz de Faddeev-Popov possibilita, por sua vez, calcular todos os multiplicadores de Lagrange $u^a_J(\underset{\sim}{x})$, $J = 1,\ldots,14$, dados por (2.41)

$$u^a_J(\underset{\sim}{x}) = -\sum_{K=1}^{14}\int d^3y\; R^{ab}_{JK}(\underset{\sim}{x};\underset{\sim}{y})\left[\Phi^b_K(\underset{\sim}{y}), H\right]_{PP}\Big|_{\Sigma} \qquad (E.1)$$

com H definido em (2.32), i.e., com

$$H = \int d^3z\Big[\frac{1}{2}\Pi^c_j(\underset{\sim}{z})\Pi^c_j(\underset{\sim}{z}) + \frac{1}{4}F^{ij,c}(\underset{\sim}{z})F^{ij,c}(\underset{\sim}{z}) - \Pi_\psi(\underset{\sim}{z})\gamma^0(\gamma^k\partial^z_k + im)\psi(\underset{\sim}{z}) -$$

$$- ig\,\Pi_\psi(\underset{\sim}{z})\gamma^0\gamma^k\frac{\lambda^c}{2}\psi(\underset{\sim}{z})A^{k,c}(\underset{\sim}{z})\Big]\;. \qquad (E.2)$$

Por conveniência, calculamos em primeiro lugar o multiplicador $A^{0,a}(\underset{\sim}{x})$ da lei de Gauss, ou seja, o $u^a_{11}(\underset{\sim}{x})$ (ver (2.34k)). Desde (E.1), (3.17)

$$A^{0,a}(\underset{\sim}{x}) = -\sum_{K=1}^{14}\int d^3y\; R^{ab}_{11,K}(\underset{\sim}{x};\underset{\sim}{y})\left[\Phi^b_K(\underset{\sim}{y}), H\right]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14}\int d^3y\; R^{ba}_{K,11}(\underset{\sim}{y};\underset{\sim}{x})\left[\Phi^b_K(\underset{\sim}{y}), H\right]\Big|_{\Sigma} =$$

$$= \int d^3y\Big\{R^{ba}_{k,11}(\underset{\sim}{y};\underset{\sim}{x})\left[\Phi^b_k(\underset{\sim}{y}), H\right] + R^{ba}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x})\left[\Phi^b_{k+11}(\underset{\sim}{y}), H\right]\Big\}\Big|_{\Sigma} =$$

$$= \int d^3y \; R^{ba}_{k+11,11}(y;\underset{\sim}{x}) \, [\Phi^{b}_{k+11}(y), H]\Big|_{\Sigma} = \int d^3y \; R^{ba}_{k+11,11}(y;\underset{\sim}{x}) \, [A^{k,b}(y), H]\Big|_{\Sigma} =$$

$$= \int d^3y \; R^{ba}_{k+11,11}(y;\underset{\sim}{x}) \, F^{0k,b}(y) =$$

$$= \int d^3y \Big\{ \Big[R^{ba}_{12,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ab} \Delta(x^1, x^1_{(0)}; y^1) \delta(x^2 - y^2) \delta(y^3 - x^3_{(0)}) \Big] F^{01,b}(y) +$$

$$+ \Big[R^{ba}_{13,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ab} \delta(y^1 - x^1_{(0)}) \Delta(x^2, x^2_{(0)}; y^2) \delta(y^3 - x^3_{(0)}) \Big] F^{02,b}(y) +$$

$$+ \Big[R^{ba}_{14,11}(y;\underset{\sim}{x}_{(0)}) + \delta^{ab} \delta(x^1 - y^1) \delta(x^2 - y^2) \Delta(x^3, x^3_{(0)}; y^3) \Big] F^{03,b}(y) \Big\} =$$

$$= \int d^3y \; R^{ba}_{k+11,11}(y;\underset{\sim}{x}_{(0)}) \, F^{0k,b}(y) + \int dy^1 \, \Delta(x^1, x^1_{(0)}; y^1) \, F^{01,a}(y^1, x^2, x^3_{(0)}) +$$

$$+ \int dy^2 \, \Delta(x^2, x^2_{(0)}; y^2) \, F^{02,a}(x^1_{(0)}, y^2, x^3_{(0)}) + \int dy^3 \, \Delta(x^3, x^3_{(0)}; y^3) \, F^{03,a}(x^1, x^2, y^3) \; . \quad \text{(E.3)}$$

Mas, desde (3.2), segue que

$$\int dy^1 \, \Delta(x^1, x^1_{(0)}; y^1) \, F^{01,a}(y^1, x^2, x^3_{(0)}) = \int dy^1 \int_{x^1_{(0)}}^{x^1} dz^1 \, \delta(z^1 - y^1) \, F^{01,a}(y^1, x^2, x^3_{(0)}) =$$

$$= \int_{x^1_{(0)}}^{x^1} dz^1 \, F^{01,a}(z^1, x^2, x^3_{(0)}) \qquad , \qquad \text{(E.4a)}$$

$$\int dy^2 \Delta(x^2, x^2_{(0)}; y^2) F^{02,a}(x^1_{(0)}, y^2, x^3_{(0)}) = \int dy^2 \int_{x^2_{(0)}}^{x^2} dz^2 \delta(z^2 - y^2) F^{02,a}(x^1_{(0)}, y^2, x^3_{(0)}) =$$

$$= \int_{x^2_{(0)}}^{x^2} dz^2 F^{02,a}(x^1_{(0)}, z^2, x^3_{(0)}) \quad , \qquad (E.4b)$$

$$\int dy^3 \Delta(x^3, x^3_{(0)}; y^3) F^{03,a}(x^1, x^2, y^3) = \int dy^3 \int_{x^3_{(0)}}^{x^3} dz^3 \delta(z^3 - y^3) F^{03,a}(x^1, x^2, y^3) =$$

$$= \int_{x^3_{(0)}}^{x^3} dz^3 F^{03,a}(x^1, x^2, z^3) \quad . \qquad (E.4c)$$

Substituindo (E.4) em (E.3), obtemos

$$\boxed{\begin{aligned} A^{0,a}(\underset{\sim}{x}) = & \int_{x^1_{(0)}}^{x^1} dz^1 F^{01,a}(z^1, x^2, x^3_{(0)}) + \int_{x^2_{(0)}}^{x^2} dz^2 F^{02,a}(x^1_{(0)}, z^2, x^3_{(0)}) + \\ & + \int_{x^3_{(0)}}^{x^3} dz^3 F^{03,a}(x^1, x^2, z^3) + \int d^3z\, R^{ba}_{k+11,11}(\underset{\sim}{z}; \underset{\sim}{x}_{(0)}) F^{0k,b}(\underset{\sim}{z}) \end{aligned}} \quad . \qquad (E.5)$$

A seguir, calculamos os multiplicadores $\eta^{0k,a}(\underset{\sim}{x})$ (i. e., $u^a_k(\underset{\sim}{x})$). Desde (E.1)

$$\eta^{ok,a}(\underset{\sim}{x}) = -\sum_{K=1}^{14}\int d^3y \, R^{ab}_{k,K}(\underset{\sim}{x};y)\,[\Phi^{b}_{K}(y),H]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{k=1}^{14}\int d^3y \, R^{ba}_{K,k}(y,\underset{\sim}{x})\,[\Phi^{b}_{K}(y),H]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14}\int d^3y \Big\{\Big[\delta^{ab}\delta^{K,k+6}\delta^{(3)}(\underset{\sim}{x}-y) + gf^{bac}\big(F^{23,c}(\underset{\sim}{x})\delta^{k1} + F^{31,c}(\underset{\sim}{x})\delta^{k2}\big).$$

$$.\delta^{K,4}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3) + gf^{adc}F^{ok,d}(\underset{\sim}{x})\,R^{bc}_{K,11}(y;\underset{\sim}{x}) -$$

$$- R^{ba}_{K,k+11}(y;\underset{\sim}{x})\Big]\cdot[\Phi^{b}_{K}(y),H]_{PP}\Big\}\Big|_{\Sigma} =$$

$$= \Big\{[\Phi^{a}_{k+6}(\underset{\sim}{x}),H]_{PP} + gf^{acb}\big(F^{23,c}(\underset{\sim}{x})\delta^{k1} + F^{31,c}(\underset{\sim}{x})\delta^{k2}\big).$$

$$.\int d^3y\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3,x^3_{(0)};x^3)\,[\Phi^{b}_{4}(y),H]_{PP} +$$

$$+ gf^{adc}F^{ok,d}(\underset{\sim}{x})\int d^3y\,R^{bc}_{j+11,11}(y;\underset{\sim}{x})\,[\Phi^{b}_{j+11}(y),H]_{PP} -$$

$$- \sum_{K=1}^{14}\int d^3y\,R^{ba}_{K,k+11}(y;\underset{\sim}{x})\,[\Phi^{b}_{K}(y),H]_{PP}\Big\}\Big|_{\Sigma}\,. \qquad (E.6)$$

Mas

$$[\Phi^{a}_{k+6}(\underset{\sim}{x}), H]_{PP} = i g \, \pi_{\psi}(\underset{\sim}{x}) \gamma^0 \gamma^k \frac{\lambda^a}{2} \psi(\underset{\sim}{x}) \qquad ; \qquad (E.7)$$

$$\int d^3y \, \delta(x^1-y^1)\delta(x^2-y^2)\Delta(y^3, x^3_{(0)}; x^3)[\pi^{b}_{12}(y), H]_{PP} = -\int dy^3 \Delta(y^3, x^3_{(0)}; x^3) F^{12,b}(x^1, x^2, y^3) ;$$

$$(E.8)$$

$$\int d^3y \, R^{bc}_{j+11,11}(y; \underset{\sim}{x}) [\Phi^{b}_{j+11}(y), H]_{PP}\Big|_{\Sigma} = A^{0,c}(\underset{\sim}{x}) \qquad , \qquad (E.9)$$

de acordo com (E.3). Calculemos o último termo de (E.6).

$$-\sum_{K=1}^{14} \int d^3y \, R^{ba}_{K,k+11}(y; \underset{\sim}{x}) [\Phi^{b}_{K}(y), H]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14} \int d^3y \; R^{ab}_{k+11,K}(\underset{\sim}{x}; y) [\Phi^{b}_{K}(y), H]_{PP}\Big|_{\Sigma} =$$

$$= \int d^3y \Big\{ R^{ab}_{k+11,4}(\underset{\sim}{x}; y) [\pi^{b}_{12}(y), H]_{PP} + R^{ab}_{k+11,5}(\underset{\sim}{x}; y) [\pi^{b}_{23}(y), H]_{PP} + R^{ab}_{k+11,6}(\underset{\sim}{x}; y) [\pi^{b}_{31}(y), H]_{PP} +$$

$$+ R^{ab}_{k+11,j+6}(\underset{\sim}{x}; y) [\Phi^{b}_{j+6}(y), H]_{PP} + R^{ab}_{k+11,11}(\underset{\sim}{x}; y) [\Phi^{b}_{11}(y), H]_{PP} +$$

$$+ R^{ab}_{k+11,j+11}(\underset{\sim}{x}; y) [\Phi^{b}_{j+11}(y), H]_{PP} \Big\}\Big|_{\Sigma} . \qquad (E.10)$$

Desde (3.19)

$$R^{ab}_{k+11,4}(\underset{\sim}{x};y)\left[\pi^{b}_{12}(y),H\right]_{PP}=-F^{12,b}_{(y)}\Big\{-\delta^{k1}D^{2,ba}_{(y)}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+\delta^{k2}D^{1,ba}_{(y)}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+$$

$$+gf^{cbd}F^{12,d}_{(y)}R^{ac}_{k+11,11}(\underset{\sim}{x};y)+gf^{abc}\left(\delta^{k1}F^{23,c}(\underset{\sim}{x})+\delta^{k2}F^{31,c}(\underset{\sim}{x})\right)\delta(x^{1}-y^{1})\delta(x^{2}-y^{2})\Delta(y^{3},x^{3}_{(0)};x^{3})\Big\}\Big]$$

$$=F^{12,b}_{(y)}\Big[\delta^{k1}D^{2,ba}_{(y)}\delta^{(3)}(\underset{\sim}{x}-y)-\delta^{k2}D^{1,ba}_{(y)}\delta^{(3)}(\underset{\sim}{x}-y)+$$

$$-gf^{abc}\left(\delta^{k1}F^{23,c}(\underset{\sim}{x})+\delta^{k2}F^{31,c}(\underset{\sim}{x})\right)\delta(x^{1}-y^{1})\delta(x^{2}-y^{2})\Delta(y^{3},x^{3}_{(0)};x^{3})\Big].$$

(E.11)

Desde (3.9)

$$R^{ab}_{k+11,5}(\underset{\sim}{x};y)\left[\pi^{b}_{23}(y),H\right]_{PP}=-F^{23,b}_{(y)}\Big\{-\delta^{ab}\delta^{k2}\partial^{3}_{y}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+\delta^{k3}D^{2,ba}_{(y)}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+$$

$$+gf^{bdc}F^{23,d}_{(y)}R^{ac}_{k+11,11}(\underset{\sim}{x};y)\Big]=$$

$$=\delta^{k2}F^{23,a}_{(y)}\partial^{3}_{y}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})-\delta^{k3}F^{23,b}_{(y)}D^{2,bc}_{(y)}\delta^{(3)}(\underset{\sim}{x}-y).$$

(E.12)

Desde (3.10)

$$R^{ab}_{k+11,6}(\underset{\sim}{x};y)\left[\Pi^{b}_{31}(y),H\right]_{PP} = -F^{31,b}(y)\Big\{\delta^{ab}\delta^{k1}\partial^{3}_{y}\delta^{(3)}(\underset{\sim}{x}-y) - \delta^{k3}D^{1,bc}(y)\delta^{(3)}(\underset{\sim}{x}-y) +$$

$$+gf^{bdc}F^{31,d}(y)R^{ac}_{k+11,11}(\underset{\sim}{x};y)\Big] =$$

$$= -\delta^{k1}F^{31,a}(y)\partial^{3}_{y}\delta^{(3)}(\underset{\sim}{x}-y) + \delta^{k3}F^{31,b}(y)D^{1,ba}(y)\delta^{(3)}(\underset{\sim}{x}-y) . \qquad (E.13)$$

Desde (3.20) e (E.7)

$$R^{ab}_{k+11,j+6}(\underset{\sim}{x};y)\left[\Phi^{b}_{j+6}(y),H\right]_{PP} = \left(D^{j,bc}(y)R^{ac}_{k+11,11}(\underset{\sim}{x};y)\right)ig\pi_{\psi}(y)\gamma^{0}\gamma^{j}\frac{\lambda^{b}}{2}\psi(y) . \qquad (E.14)$$

Desde (2.33k) e (E.2)

$$\left[\Phi^{b}_{11}(y),H\right]_{PP} = \left[ig\pi_{\psi}(y)\frac{\lambda^{b}}{2}\psi(y),H\right]_{PP} =$$

$$= -D^{bc}_{j}(y)\left[ig\pi_{\psi}(y)\gamma^{0}\gamma^{j}\frac{\lambda^{c}}{2}\psi(y)\right] . \qquad (E.15)$$

Prova:

$$\left.\left[\Phi^{b}_{11}(y),H\right]_{PP}\right|_{\Sigma} = \left.\left[\partial^{y}_{j}F^{0j,b}(y) + gf^{bcd}A^{c}_{j}(y)F^{0j,d}(y) + ig\pi_{\psi}(y)\frac{\lambda^{b}}{2}\psi(y),H\right]_{PP}\right|_{\Sigma} =$$

$$= gf^{bcd}\int d^{3}x\left.\left[A^{c}_{j}(y),\Pi^{a}_{k}(\underset{\sim}{x})\right]_{PP}F^{0j,d}(y)\Pi^{a}_{k}(\underset{\sim}{x})\right|_{\Sigma} +$$

$$+ ig\left(\frac{\lambda^{b}}{2}\right)^{uv}\int d^{3}x\left.\left\{\frac{(\pi^{u}_{\psi}(y)\psi^{v}(y))\overleftarrow{\delta}}{\delta\psi^{w}(\underset{\sim}{x})}\frac{\overrightarrow{\delta}H}{\delta\pi^{w}_{\psi}(\underset{\sim}{x})} - \frac{H\overleftarrow{\delta}}{\delta\psi^{w}(\underset{\sim}{x})}\frac{\overrightarrow{\delta}(\pi^{u}_{\psi}(y)\psi^{v}(y))}{\delta\pi^{w}_{\psi}(\underset{\sim}{x})}\right\}\right|_{\Sigma} =$$

$$= -g f^{bcd} F^{0j,d}_{(\underset{\sim}{x})} F^{0j,c}_{(\underset{\sim}{x})} +$$

$$+ ig\left(\frac{\lambda^b}{2}\right)^{uv} \Big\{ \Pi^u_\psi(y)\left[-\gamma^0(\gamma^k \partial^y_k + im)\psi^v(y) - ig\,\gamma^0\gamma^k\left(\frac{\lambda^a}{2}\right)^{vw}\psi^w(y) A^{k,a}(y)\right] -$$

$$-\left[(\partial^y_k \Pi^u_\psi(y))\gamma^0\gamma^k - im\,\Pi^u_\psi(y)\gamma^0 - ig\,\Pi^w_\psi(y)\gamma^0\gamma^k\left(\frac{\lambda^a}{2}\right)^{wu} A^{k,a}(y)\right]\psi^v(y)\Big\}$$

$$= -ig\,\partial^y_k\left(\Pi_\psi(y)\gamma^0\gamma^k\frac{\lambda^b}{2}\psi(y)\right) + g^2\,\Pi^u_\psi(y)\gamma^0\gamma^k\left[\frac{\lambda^b}{2},\frac{\lambda^a}{2}\right]^{uw}\psi^w(y) A^{k,a}(y) =$$

$$= -\delta^{bc}\partial^y_k\left(ig\,\Pi_\psi(y)\gamma^0\gamma^k\frac{\lambda^c}{2}\psi(y)\right) - gf^{bac}A^a_k(y)\left(ig\,\Pi_\psi(y)\gamma^0\gamma^k\frac{\lambda^c}{2}\psi(y)\right) =$$

$$= -D^{bc}_k(y)\left(ig\,\Pi_\psi(y)\gamma^0\gamma^k\frac{\lambda^c}{2}\psi(y)\right) \cdot \quad (q.e.d.)$$

Desde (3.22)-(3.24) e (D.48)

$$R^{ab}_{k+11,j+11}(\underset{\sim}{x};y)\,[\Phi^b_{j+11}(y), H]_{PP} =$$

$$= r^{ab}_{k+11(12)}(\underset{\sim}{x};y)\,F^{01,b}(y) + r^{ab}_{k+11(13)}(\underset{\sim}{x};y)\,F^{02,b}(y) + r^{ab}_{k+11(14)}(\underset{\sim}{x};y)\,F^{03,b}(y) = 0\,. \quad (E.16)$$

Substituindo os resultados (E.11)-(E.16) em (E.10), obtemos

$$-\sum_{k=1}^{14}\int d^3y\, \underset{k,k+11}{R^{ba}}(\underset{\sim}{y};\underset{\sim}{x})\,[\Phi_k^b(\underset{\sim}{y}),H]_{PP}\Big|_{\Sigma} =$$

$$=\int d^3y\Big\{F^{12,b}(\underset{\sim}{y})\Big[\delta^{k1}D^{2,ba}(\underset{\sim}{y})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})-\delta^{k2}D^{1,ba}(\underset{\sim}{y})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\ -$$

$$-gf^{abc}\Big(\delta^{k1}F^{23,c}(\underset{\sim}{x})+\delta^{k2}F^{31,c}(\underset{\sim}{x})\Big)\delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(y^3,x^3_{(0)};x^3)\Big]\ +$$

$$+\,\delta^{k2}F^{23,a}(\underset{\sim}{y})\,\partial^3_y\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})-\delta^{k3}F^{23,b}(\underset{\sim}{y})\,D^{2,ba}(\underset{\sim}{y})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\ -$$

$$-\,\delta^{k1}F^{31,a}(\underset{\sim}{y})\,\partial^3_y\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+\delta^{k3}F^{31,b}(\underset{\sim}{y})\,D^{1,ba}(\underset{\sim}{y})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\ +$$

$$+\Big(D^{j,bc}(\underset{\sim}{y})\,\underset{k+11,11}{R^{ac}}(\underset{\sim}{x};\underset{\sim}{y})\Big)\,ig\,\pi_\psi(\underset{\sim}{y})\gamma^0\gamma^j\frac{\lambda^b}{2}\psi(\underset{\sim}{y})\ -$$

$$-\ \underset{k+11,11}{R^{ab}}(\underset{\sim}{x};\underset{\sim}{y})\,D^{bc}_j(\underset{\sim}{y})\Big(ig\,\pi_\psi(\underset{\sim}{y})\gamma^0\gamma^j\frac{\lambda^c}{2}\psi(\underset{\sim}{y})\Big)\Big\}\,. \qquad (E.17)$$

Notar que, se admitirmos $\pi_\psi(\underline{y}),\ \psi(\underline{y}) \xrightarrow[|\underline{y}|\to\infty]{} 0$ o que será justificado no capítulo IV, podemos integrar por partes o último termo de (E.17) e obter (b ↔ c)

$$\int d^3y\, \underset{k+11,11}{R^{ac}}(\underset{\sim}{x};\underset{\sim}{y})\,D^{j,cb}(\underset{\sim}{y})\Big(ig\,\pi_\psi(\underset{\sim}{y})\gamma^0\gamma^j\frac{\lambda^b}{2}\psi(\underset{\sim}{y})\Big)=$$

$$=-\int d^3y\Big(D^{j,bc}(\underset{\sim}{y})\,\underset{k+11,11}{R^{ac}}(\underset{\sim}{x};\underset{\sim}{y})\Big)\,ig\,\pi_\psi(\underset{\sim}{y})\gamma^0\gamma^j\frac{\lambda^b}{2}\psi(\underset{\sim}{y})\,. \qquad (E.18)$$

Levando (E.18) em (E.17) e o resultado obtido, junto com (E.7)-(E.9), na expressão (E.6), ficamos com

$$\eta^{ok,a}_{(\underset{\sim}{x})} = ig\,\pi_{\psi}(\underset{\sim}{x})\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\psi(\underset{\sim}{x}) +$$

$$+ gf^{abc}\left(F^{23,c}_{(\underset{\sim}{x})}\delta^{k1} + F^{31,c}_{(\underset{\sim}{x})}\delta^{k2}\right)\int dy^{3}\,\Delta(y^{3},x^{3}_{(0)};x^{3})F^{12,b}(x^{1},x^{2},y^{3}) +$$

$$+ gf^{abc}F^{ok,b}_{(\underset{\sim}{x})}A^{0,c}_{(\underset{\sim}{x})} + \delta^{k1}D^{ab}_{2}(\underset{\sim}{x})F^{12,b}_{(\underset{\sim}{x})} - \delta^{k2}D^{ab}_{1}(\underset{\sim}{x})F^{12,b}_{(\underset{\sim}{x})} -$$

$$- gf^{abc}\left(\delta^{k1}F^{23,c}_{(\underset{\sim}{x})} + \delta^{k2}F^{31,c}_{(\underset{\sim}{x})}\right)\int dy^{3}\,\Delta(y^{3},x^{3}_{(0)};x^{3})\,F^{12,b}(x^{1},x^{2},y^{3}) +$$

$$+ \delta^{k2}D^{ab}_{3}(\underset{\sim}{x})F^{23,b}_{(\underset{\sim}{x})} - \delta^{k3}D^{ab}_{2}(\underset{\sim}{x})F^{23,b}_{(\underset{\sim}{x})} - \delta^{k1}D^{ab}_{3}(\underset{\sim}{x})F^{31,a}_{(\underset{\sim}{x})} +$$

$$+ \delta^{k3}D^{ab}_{1}(\underset{\sim}{x})F^{31,b}_{(\underset{\sim}{x})} \implies$$

$$\boxed{\eta^{oka}_{(\underset{\sim}{x})} = -D^{ab}_{j}(\underset{\sim}{x})F^{jk,b}_{(\underset{\sim}{x})} - gf^{acb}A^{0,c}_{(\underset{\sim}{x})}F^{ok,b}_{(\underset{\sim}{x})} + ig\,\pi_{\psi}(\underset{\sim}{x})\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\psi(\underset{\sim}{x}) . \qquad (E.19)}$$

Na seqüência, calculamos os multiplicadores $\eta^{j,a}$ (ou u^{a}_{j+6}, por (2.34 g,h,i)):

$$\eta^{j,a}_{(\underset{\sim}{x})} = -\sum_{K=1}^{14}\int d\underset{\sim}{y}^{3}\; R^{ab}_{j+6,K}(\underset{\sim}{x};\underset{\sim}{y})\left[\Phi^{b}_{K}(\underset{\sim}{y}), H\right]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14}\int d\underset{\sim}{y}^{3}\; R^{ba}_{K,j+6}(\underset{\sim}{y};\underset{\sim}{x})\left[\Phi^{b}_{K}(\underset{\sim}{y}), H\right]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14} \int d^3y \left\{ \left[-\delta^{ab}\delta^{K,j}\delta^{(3)}(y-\underset{\sim}{x}) + D^{j,ac}_{(\underset{\sim}{x})} \underset{K,11}{R}^{bc}(y;\underset{\sim}{x}) \right] \left[\underset{K}{\Phi}^{b}(y), H \right]_{PP} \right\} \Big|_{\Sigma} =$$

$$= \Big\{ -\left[\underset{j}{\Phi}^{a}(\underset{\sim}{x}), H \right]_{PP} + D^{j,ac}_{(\underset{\sim}{x})} \int d^3y \; \underset{k,11}{R}^{bc}(y;\underset{\sim}{x}) \left[\underset{k}{\Phi}^{b}(y), H \right]_{PP} +$$

$$+ D^{j,ac}_{(\underset{\sim}{x})} \int d^3y \; \underset{k+11,11}{R}^{bc}(y;\underset{\sim}{x}) \left[\underset{k+11}{\Phi}^{b}(y), H \right]_{PP} \Big\} \Big|_{\Sigma} =$$

$$= D^{j,ac}_{(\underset{\sim}{x})} A^{0,c}_{(\underset{\sim}{x})}$$

onde usamos (3.20) e (E.9). Logo,

$$\boxed{\eta^{j,a}_{(\underset{\sim}{x})} = \partial^{j}_{x} A^{0,a}_{(\underset{\sim}{x})} - g f^{acb} A^{0,c}_{(\underset{\sim}{x})} A^{j,b}_{(\underset{\sim}{x})}} . \qquad (E.20)$$

Por outro lado, para os $\eta^{jk,a}$ (ou u^{a}_{j+3}) encontramos

$$\underset{j+3}{u}^{a}(\underset{\sim}{x}) = -\sum_{K=1}^{14} \int d^3y \; \underset{j+3,K}{R}^{ab}(\underset{\sim}{x};y) \left[\underset{K}{\Phi}^{b}(y), H \right]_{PP} \Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14} \int d^3y \; \underset{K,j+3}{R}^{ba}(y;\underset{\sim}{x}) \left[\underset{K}{\Phi}^{b}(y), H \right]_{PP} \Big|_{\Sigma} , \qquad (E.21)$$

a qual, para j = 1, fornece (ver (3.19))

$$\underset{4}{u}^{a}(\underset{\sim}{x}) = \eta^{12,a}_{(\underset{\sim}{x})} = \sum_{K=1}^{14} \int d^3y \; \underset{K,4}{R}^{ba}(y;\underset{\sim}{x}) \left[\underset{K}{\Phi}^{b}(y), H \right]_{PP} \Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14} \int d^3y \Big\{ \Big[-\delta^{K,12} D^{2,ab}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-y) + \delta^{K,13} D^{1,ab}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-y) +$$

$$+\delta^{ab}\delta^{K,10}\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)+gf^{cad}\overset{12,d}{F}(\underset{\sim}{x})\underset{K,11}{\overset{bc}{R}}(y;\underset{\sim}{x})+$$

$$+gf^{bac}(\delta^{K,12}\overset{23,c}{F}(y)+\delta^{K,13}\overset{31,c}{F}(y))\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\Big].$$

$$\cdot\Big[\underset{K}{\overset{b}{\Phi}}(y),H\Big]_{PP}\Big\}\Big|_{\Sigma}=$$

$$=\Big\{-D^{2,ab}_{(\underset{\sim}{x})}\Big[\underset{12}{\overset{b}{\Phi}}(\underset{\sim}{x}),H\Big]_{PP}+D^{1,ab}_{(\underset{\sim}{x})}\Big[\underset{13}{\overset{b}{\Phi}}(\underset{\sim}{x}),H\Big]_{PP}+$$

$$+\int d^3y\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\Big[\underset{10}{\overset{a}{\Phi}}(y),H\Big]_{PP}+$$

$$+gf^{adc}\overset{12,d}{F}(\underset{\sim}{x})\int d^3y\,\underset{k+11,11}{\overset{bc}{R}}(y;\underset{\sim}{x})\Big[\underset{k+11}{\overset{b}{\Phi}}(y),H\Big]_{PP}+$$

$$+gf^{acb}\int d^3y\,\delta(x^1-y^1)\delta(x^2-y^2)\Delta(x^3,x^3_{(0)};y^3)\Big(\overset{23,c}{F}(y)\Big[\underset{12}{\overset{b}{\Phi}}(y),H\Big]_{PP}+$$

$$+\overset{31,c}{F}(y)\Big[\underset{13}{\overset{b}{\Phi}}(y),H\Big]_{PP}\Big)\Big\}\Big|_{\Sigma}\Longrightarrow$$

$$\boxed{\overset{12,a}{\eta}(\underset{\sim}{x})=D^{1,ab}_{(\underset{\sim}{x})}\overset{02,b}{F}(\underset{\sim}{x})-D^{2,ab}_{(\underset{\sim}{x})}\overset{01,b}{F}(\underset{\sim}{x})+gf^{abc}\overset{12,b}{F}(\underset{\sim}{x})\overset{0,c}{A}(\underset{\sim}{x})},\quad(E.22a)$$

onde usamos novamente (E.9).

Para j = 2, (E.21) fornece (ver (3.9))

$$\underset{5}{\overset{a}{u}}(\underset{\sim}{x})=\overset{23,a}{\eta}(\underset{\sim}{x})=\sum_{K=1}^{14}\int d^3y\,\underset{K,5}{\overset{ba}{R}}(y;\underset{\sim}{x})\Big[\underset{K}{\overset{b}{\Phi}}(y),H\Big]_{PP}\Big|_{\Sigma}=$$

$$=\sum_{K=1}^{14}\int d^3y\Big\{\Big[-\delta^{K,13}D^{3,ab}_{(\underset{\sim}{x})}\overset{(3)}{\delta}(\underset{\sim}{x}-y)+\delta^{K,14}D^{2,ab}_{(\underset{\sim}{x})}\overset{(3)}{\delta}(\underset{\sim}{x}-y)+$$

$$+ g f^{adc} F^{23,d}_{(\underset{\sim}{x})} \underset{K,11}{R}^{bc}(y;\underset{\sim}{x}) \Big] . \Big[\underset{K}{\Phi}^{b}(y), H \Big]_{PP} \Big\} \Big|_{\Sigma} =$$

$$= \Big\{ - D^{3,ab}_{(\underset{\sim}{x})} \Big[\underset{13}{\Phi}^{b}(\underset{\sim}{x}), H \Big]_{PP} + D^{2,ab}_{(\underset{\sim}{x})} \Big[\underset{14}{\Phi}^{b}(\underset{\sim}{x}), H \Big]_{PP} +$$

$$+ g f^{adc} F^{23,d}_{(\underset{\sim}{x})} \int d^3y \underset{k+11,11}{R}^{bc}(y;\underset{\sim}{x}) \Big[\underset{k+11}{\Phi}^{b}(y), H \Big]_{PP} \Big\} \Big|_{\Sigma} \Rightarrow$$

$$\boxed{\eta^{23,a}_{(\underset{\sim}{x})} = D^{2,ab}_{(\underset{\sim}{x})} F^{03,b}_{(\underset{\sim}{x})} - D^{3,ab}_{(\underset{\sim}{x})} F^{02,b}_{(\underset{\sim}{x})} + g f^{abc} F^{23,b}_{(\underset{\sim}{x})} A^{0,c}_{(\underset{\sim}{x})}} \,, \quad (E.22b)$$

usando (E.9). Para j = 3, (E.21) e (3.10) implicam

$$u^{a}_{6}(\underset{\sim}{x}) = \eta^{31,a}_{(\underset{\sim}{x})} = \sum_{K=1}^{14} \int d^3y \underset{K,6}{R}^{ba}(y;\underset{\sim}{x}) \Big[\underset{K}{\Phi}^{b}(y), H \Big]_{PP} \Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14} \int d^3y \Big\{ \Big[\delta^{K,12} D^{3,ab}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-y) - \delta^{K,14} D^{1,ab}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-y) +$$

$$+ g f^{adc} F^{31,d}_{(\underset{\sim}{x})} \underset{K,11}{R}^{bc}(y;\underset{\sim}{x}) \Big] . \Big[\underset{K}{\Phi}^{b}(y), H \Big]_{PP} \Big\} \Big|_{\Sigma} \Rightarrow$$

$$\boxed{\eta^{31,a}_{(\underset{\sim}{x})} = D^{3,ab}_{(\underset{\sim}{x})} F^{01,b}_{(\underset{\sim}{x})} - D^{1,ab}_{(\underset{\sim}{x})} F^{03,b}_{(\underset{\sim}{x})} + g f^{abc} F^{31,b}_{(\underset{\sim}{x})} A^{0,c}_{(\underset{\sim}{x})}} \,. \quad (E.22c)$$

Claramente, as equações (E.22) podem ser compactadas em

$$\boxed{\eta^{jk,a}_{(\underset{\sim}{x})} = D^{j,ab}_{(\underset{\sim}{x})} F^{0k,b}_{(\underset{\sim}{x})} - D^{k,ab}_{(\underset{\sim}{x})} F^{0j,b}_{(\underset{\sim}{x})} + g f^{abc} F^{jk,b}_{(\underset{\sim}{x})} A^{0,c}_{(\underset{\sim}{x})}} \,. \quad (E.23)$$

O multiplicador de Lagrange associado à identidade de Bianchi temporal é obtido como segue

$$\underset{10}{u}^{a}(\underset{\sim}{x}) = -\sum_{K=1}^{14}\int d^3y \; \underset{10,K}{R}^{ab}(\underset{\sim}{x};y)\,[\underset{K}{\Phi}^{b}(y),H]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14}\int d^3y \; \underset{K,10}{R}^{ba}(y;\underset{\sim}{x})\,[\underset{K}{\Phi}^{b}(y),H]_{PP}\Big|_{\Sigma} =$$

$$= \int d^3y \; \underset{4,10}{R}^{ba}(y;\underset{\sim}{x})\,[\underset{4}{\Phi}^{b}(y),H]_{PP}\Big|_{\Sigma} =$$

$$= \int d^3y \; \delta(y^1-x^1)\,\delta(y^2-x^2)\,\Delta(y^3,x^3_{(0)};x^3)\,F^{12,a}(y) \quad\Longrightarrow$$

$$\boxed{\underset{10}{u}^{a}(\underset{\sim}{x}) = \int_{-\infty}^{+\infty} dz^3\,\Delta(z^3,x^3_{(0)};x^3)\,F^{12,a}(x^1,x^2,z^3)} \; . \qquad (E.24)$$

Por último, os multiplicadores $u^{a}_{k+11}(\underset{\sim}{x})$ são obtidos diretamente de (E.17) e (E.18), dado que

$$\underset{k+11}{u}^{a}(\underset{\sim}{x}) = -\sum_{K=1}^{14}\int d^3y \; \underset{k+11,K}{R}^{ab}(\underset{\sim}{x};y)\,[\underset{K}{\Phi}^{b}(y),H]_{PP}\Big|_{\Sigma} =$$

$$= \sum_{K=1}^{14}\int d^3y \; \underset{K,k+11}{R}^{ba}(y;\underset{\sim}{x})\,[\underset{K}{\Phi}^{b}(y),H]_{PP}\Big|_{\Sigma} = -(E.17)\,.$$

Desta forma, os u^{a}_{k+11} são dados por (ver (E.19) e (E.6))

$$\boxed{\begin{aligned} u^{a}_{k+11}(\underset{\sim}{x}) = {} & D^{ab}_{j}(\underset{\sim}{x})\, F^{jk,b}(\underset{\sim}{x}) + g f^{abc} \Big(\delta^{k1} F^{23,c}(\underset{\sim}{x}) + \\ & + \delta^{k2} F^{31,c}(\underset{\sim}{x}) \Big) . \int_{-\infty}^{+\infty} dz^3\, \Delta(z^3, x^3_{(0)}; x^3)\, F^{12,b}(x^1, x^2, z^3) \, . \end{aligned}} \quad (E.25)$$

Obtenção dos Parênteses de Dirac Básicos da Cromodinâmica no Gauge Superaxial

A partir da definição (2.44), i.e., de

$$[\Omega_1, \Omega_2]_{PD} = [\Omega_1, \Omega_2]_{PP} -$$

$$- \sum_{J,K=1}^{14} \int d^3z \int d^3z' \, [\Omega_1, \Phi^{a}_{J}(\underset{\sim}{z})]_{PP} \, R^{ab}_{JK}(\underset{\sim}{z}; \underset{\sim}{z}') \, [\Phi^{b}_{K}(\underset{\sim}{z}'), \Omega_2]_{PP}$$

onde o PP é definido por (2.26), calculamos em primeiro lugar o PD

$$[A^{j,a}(\underset{\sim}{x}), \pi^{b}_{k}(y)]_{PD} = [A^{j,a}(\underset{\sim}{x}), \pi^{b}_{k}(y)]_{PP} -$$

$$- \sum_{J,K=1}^{14} \int d^3z \int d^3z' \, [A^{j,a}(\underset{\sim}{x}), \Phi^{c}_{J}(\underset{\sim}{z})]_{PP} \, R^{cd}_{JK}(\underset{\sim}{z}; \underset{\sim}{z}') \, [\Phi^{d}_{K}(\underset{\sim}{z}'), \pi^{b}_{k}(y)]_{PP} =$$

$$= \delta^{ab} \delta^{jk} \delta^{(3)}(\underset{\sim}{x} - y) -$$

$$- \int d^3z \int d^3z' \, [A^{j,a}(\underset{\sim}{x}), \Phi^{c}_{\ell+6}(\underset{\sim}{z})]_{PP} \, R^{cd}_{\ell+6,\, \ell'+11}(\underset{\sim}{z}; \underset{\sim}{z}') \, [\Phi^{d}_{\ell'+11}(\underset{\sim}{z}'), \pi^{b}_{k}(y)] =$$

$$= \delta^{ab}\delta^{jk}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - \int d^3z\int d^3z'\, \delta^{ac}\delta^{jl}\delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x})\, R^{cd}_{l+6,l'+11}(\underset{\sim}{z};\underset{\sim}{z}')\, \delta^{db}\delta^{l'k}\delta^{(3)}(\underset{\sim}{z}'-\underset{\sim}{y})$$

$$= \delta^{ab}\delta^{jk}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + R^{ba}_{k+11,j+6}(\underset{\sim}{y};\underset{\sim}{x}) \implies \quad (\text{ver } (3.20))$$

$$\boxed{[A^{j,a}(\underset{\sim}{x}), \Pi^b_k(\underset{\sim}{y})]_{PD} = \delta^{ab}\delta^{jk}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + D^{j,ac}(\underset{\sim}{x})\, R^{bc}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x})} \,. \qquad (E.26)$$

De modo similar,

$$[F^{jl,a}(\underset{\sim}{x}), \Pi^b_k(\underset{\sim}{y})]_{PD} =$$

$$= -\sum_{J,K=1}^{14}\int d^3z\int d^3z'\,[F^{jl,a}(\underset{\sim}{x}), \Phi^c_J(\underset{\sim}{z})]_{PP}\, R^{cd}_{JK}(\underset{\sim}{z};\underset{\sim}{z}')\,[\Phi^d_K(\underset{\sim}{z}'), \Pi^b_k(\underset{\sim}{y})]_{PP} =$$

$$= -\int d^3z\int d^3z' \Big\{ [F^{jl,a}(\underset{\sim}{x}), \Phi^c_{l'+3}(\underset{\sim}{z})]_{PP}\, R^{cd}_{l'+3,10}(\underset{\sim}{z};\underset{\sim}{z}')\,[\Phi^d_{10}(\underset{\sim}{z}'), \Pi^b_k(\underset{\sim}{y})]_{PP} +$$

$$+ [F^{jl,a}(\underset{\sim}{x}), \Phi^c_{l'+3}(\underset{\sim}{z})]_{PP}\, R^{cd}_{l'+3,11}(\underset{\sim}{z};\underset{\sim}{z}')\,[\Phi^d_{10}(\underset{\sim}{z}'), \Pi^b_k(\underset{\sim}{y})]_{PP} +$$

$$+ [F^{jl,a}(\underset{\sim}{x}), \Phi^c_{l'+3}(\underset{\sim}{z})]_{PP}\, R^{cd}_{l'+3,k'+11}(\underset{\sim}{z};\underset{\sim}{z}')\,[\Phi^d_{k'+11}(\underset{\sim}{z}'), \Pi^b_k(\underset{\sim}{y})]_{PP} \Big\} =$$

$$= \int d^3z \Big\{ [F^{jl,a}(\underset{\sim}{x}), \Phi^c_{l'+3}(\underset{\sim}{z})]_{PP}\, R^{dc}_{10,l'+3}(\underset{\sim}{y};\underset{\sim}{z})\, g f^{dbe}\big(\delta^{1k}F^{23,e}(\underset{\sim}{y}) + \delta^{2k}F^{31,e}(\underset{\sim}{y})\big) +$$

$$+ [F^{jl,a}(\underset{\sim}{x}), \Phi^c_{l'+3}(\underset{\sim}{z})]_{PP}\, R^{bc}_{k+11,l'+3}(\underset{\sim}{y};\underset{\sim}{z}) \Big\} \,, \qquad (E.27)$$

de onde, para j = 1, l = 2, obtemos

$$[F^{12,a}(\underset{\sim}{x}), \pi^{b}_{k}(y)]_{PD} = g f^{dbe}(\delta^{1k} F^{23,e}(y) + \delta^{2k} F^{31,e}(y))\, R^{da}_{10,4}(y;\underset{\sim}{x}) +$$

$$+ R^{ba}_{k+11,4}(y;\underset{\sim}{x}) =$$

$$= g f^{abc}(\delta^{1k} F^{23,c}(y) + \delta^{2k} F^{31,c}(y))\, \delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3, x^3_{(0)}; y^3) -$$

$$- \delta^{k1} D^{2,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) + \delta^{k2} D^{1,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) + g f^{cad} F^{12,d}(\underset{\sim}{x})\, R^{bc}_{k+11,11}(y;\underset{\sim}{x}) +$$

$$+ g f^{bac}(\delta^{k1} F^{23,c}(y) + \delta^{k2} F^{31,c}(y))\, \delta(x^1-y^1)\,\delta(x^2-y^2)\,\Delta(x^3, x^3_{(10)}; y^3) \Longrightarrow$$

$$\boxed{\begin{aligned}[F^{12,a}(\underset{\sim}{x}), \pi^{b}_{k}(y)]_{PD} &= \delta^{2k} D^{1,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) - \delta^{1k} D^{2,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) + \\ &\quad + g f^{adc} F^{12,d}(\underset{\sim}{x})\, R^{bc}_{k+11,11}(y;\underset{\sim}{x}) \,, \qquad (E.28a)\end{aligned}}$$

usando (3.19). Para j = 2, l = 3, (E.27) $\Longrightarrow$

$$[F^{23,a}(\underset{\sim}{x}), \pi^{b}_{k}(y)]_{PD} = R^{ba}_{k+11,5}(y;\underset{\sim}{x}) \Longrightarrow \quad (\text{ver } (3.9))$$

$$\boxed{\begin{aligned}[F^{23,a}(\underset{\sim}{x}), \pi^{b}_{k}(y)]_{PD} &= \delta^{3k} D^{2,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) - \delta^{2k} D^{3,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-y) + \\ &\quad + g f^{adc} F^{23,d}(\underset{\sim}{x})\, R^{bc}_{k+11,11}(y;\underset{\sim}{x}) \,. \qquad (E.28b)\end{aligned}}$$

Para $j = 3$, $l = 1$, (E.27) $\Longrightarrow$

$$[F^{31,a}(\underset{\sim}{x}), \pi^{b}_{k}(\underset{\sim}{y})]_{PD} = R^{ba}_{k+11,6}(\underset{\sim}{y};\underset{\sim}{x}) \Longrightarrow \qquad (\text{ver } (3.10))$$

$$\boxed{\begin{aligned}[F^{31,a}(\underset{\sim}{x}), \pi^{b}_{k}(\underset{\sim}{y})]_{PD} &= \delta^{1k} D^{3,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - \delta^{3k} D^{1,ab}(\underset{\sim}{x})\, \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \\ &+ g f^{adc} F^{31,d}(\underset{\sim}{x})\, R^{bc}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x}) \ . \qquad (E.28c)\end{aligned}}$$

As equações (E.28) podem ser compactadas na forma

$$\boxed{\begin{aligned}[F^{jl,a}(\underset{\sim}{x}), \pi^{b}_{k}(\underset{\sim}{y})]_{PD} &= \left[\delta^{lk} D^{j,ab}(\underset{\sim}{x}) - \delta^{jk} D^{l,ab}(\underset{\sim}{x})\right] \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \\ &+ g f^{adc} F^{jl,d}(\underset{\sim}{x})\, R^{bc}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x}) \ . \qquad (E.29)\end{aligned}}$$

A seguir, calculamos

$$[\pi^{a}_{j}(\underset{\sim}{x}), \pi^{b}_{k}(\underset{\sim}{y})]_{PD} = -\sum_{J,K=1}^{14} \int d^3\underset{\sim}{z} \int d^3\underset{\sim}{z}' [\pi^{a}_{j}(\underset{\sim}{x}), \Phi^{c}_{J}(\underset{\sim}{z})]_{PP} R^{cd}_{JK}(\underset{\sim}{z};\underset{\sim}{z}') [\Phi^{d}_{K}(\underset{\sim}{z}'), \pi^{b}_{k}(\underset{\sim}{y})]_{PP} =$$

$$= -\sum_{K=1}^{14} \int d^3\underset{\sim}{z} \int d^3\underset{\sim}{z}' \Big\{ [\pi^{a}_{j}(\underset{\sim}{x}), \Phi^{c}_{10}(\underset{\sim}{z})]_{PP} R^{cd}_{10,K}(\underset{\sim}{z};\underset{\sim}{z}') [\Phi^{d}_{K}(\underset{\sim}{z}'), \pi^{b}_{k}(\underset{\sim}{y})]_{PP} +$$

$$+ [\pi^{a}_{j}(\underset{\sim}{x}), \Phi^{c}_{11}(\underset{\sim}{z})]_{PP} R^{cd}_{11,K}(\underset{\sim}{z};\underset{\sim}{z}') [\Phi^{d}_{K}(\underset{\sim}{z}'), \pi^{b}_{k}(\underset{\sim}{y})]_{PP} +$$

$$+ [\pi^{a}_{j}(\underset{\sim}{x}), \Phi^{c}_{l+11}(\underset{\sim}{z})]_{PP} R^{cd}_{l+11,K}(\underset{\sim}{z};\underset{\sim}{z}') [\Phi^{d}_{K}(\underset{\sim}{z}'), \pi^{b}_{k}(\underset{\sim}{y})]_{PP} \Big\} =$$

$$= -\int d^3\underset{\sim}{z} \int d^3\underset{\sim}{z}' \Big\{ [\pi^{a}_{j}(\underset{\sim}{x}), \Phi^{c}_{11}(\underset{\sim}{z})]_{PP} R^{cd}_{11,l+11}(\underset{\sim}{z};\underset{\sim}{z}') [\Phi^{d}_{l+11}(\underset{\sim}{z}'), \pi^{b}_{k}(\underset{\sim}{y})]_{PP} +$$

$$+ \left[\Pi^{a}_{j}(\underset{\sim}{x}), \Phi^{c}_{\ell+11}(\underset{\sim}{z}) \right]_{PP} \underset{\ell+11,\,11}{R^{cd}}(\underset{\sim}{z};\underset{\sim}{z}') \left[\Phi^{d}_{11}(\underset{\sim}{z}'), \Pi^{b}_{k}(y) \right]_{PP} \Big\} =$$

$$= \int d^3z \int d^3z' \Big\{ g f^{cae} F^{oj,e}(\underset{\sim}{z}) \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x}) \underset{\ell+11,11}{R^{dc}}(\underset{\sim}{z}';\underset{\sim}{z}) \delta^{db} \delta^{\ell k} \delta^{(3)}(\underset{\sim}{z}'-y) +$$

$$+ \delta^{ac} \delta^{j\ell} \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x}) \underset{\ell+11,11}{R^{cd}}(\underset{\sim}{z};\underset{\sim}{z}') g f^{deb} F^{ok,e}(\underset{\sim}{z}') \delta^{(3)}(\underset{\sim}{z}'-y) \Big\} \Rightarrow$$

$$\boxed{\left[\Pi^{a}_{j}(\underset{\sim}{x}), \Pi^{b}_{k}(y) \right]_{PD} = g\left[f^{cad} F^{oj,d}(\underset{\sim}{x}) \underset{k+11,11}{R^{bc}}(y;\underset{\sim}{x}) + f^{dcb} F^{ok,c}(y) \underset{j+11,11}{R^{ad}}(\underset{\sim}{x};y) \right]} .$$

(E.30)

Por outro lado,

$$\left[\Psi(\underset{\sim}{x}), \Pi_{\Psi}(y) \right]_{PD} = \left[\Psi(\underset{\sim}{x}), \Pi_{\Psi}(y) \right]_{PP} -$$

$$- \int d^3z \int d^3z' \left[\Psi(\underset{\sim}{x}), \Phi^{a}_{11}(\underset{\sim}{z}) \right] \underset{11,11}{R^{ab}}(\underset{\sim}{z};\underset{\sim}{z}') \left[\Phi^{b}_{11}(\underset{\sim}{z}'), \Pi_{\Psi}(y) \right] =$$

$$= \left[\Psi(\underset{\sim}{x}), \Pi_{\Psi}(y) \right]_{PP} \Longrightarrow$$

$$\boxed{\left[\Psi(\underset{\sim}{x}), \Pi_{\Psi}(y) \right]_{PD} = \delta^{(3)}(\underset{\sim}{x}-y)} \quad ; \qquad (E.31)$$

$$\left[\Psi^{u}(\underset{\sim}{x}), \Pi^{a}_{k}(y) \right]_{PD} =$$

$$- \int d^3z \int d^3z' \left[\Psi^{u}(\underset{\sim}{x}), \Phi^{b}_{11}(\underset{\sim}{z}) \right]_{PP} \underset{11,\,j+11}{R^{bc}}(\underset{\sim}{z};\underset{\sim}{z}') \left[\Phi^{c}_{j+11}(\underset{\sim}{z}'), \Pi^{a}_{k}(y) \right]_{PP} =$$

$$= + \int d^3z \int d^3z' \; i g \left(\frac{\lambda}{2}\right)^{b\,uv} \psi^{v}(\underset{\sim}{z})\, \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x})\, R^{cb}_{j+11,11}(\underset{\sim}{z}';\underset{\sim}{z})\, \delta^{ac} \delta^{jk} \delta^{(3)}(\underset{\sim}{z}'-\underset{\sim}{y}) \Longrightarrow$$

$$\boxed{\left[\psi^{u}(\underset{\sim}{x}), \pi^{a}_{k}(\underset{\sim}{y})\right]_{PD} = i g \left(\frac{\lambda}{2}\right)^{b\,uv} \psi^{v}(\underset{\sim}{x})\, R^{ab}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x})} \quad ; \qquad (E.32)$$

$$\left[\pi^{u}_{\psi}(\underset{\sim}{x}), \pi^{a}_{k}(\underset{\sim}{y})\right]_{PD} =$$

$$= - \int d^3z \int d^3z' \left[\pi^{u}_{\psi}(\underset{\sim}{x}), \Phi^{b}_{11}(\underset{\sim}{z})\right]_{PP} R^{bc}_{11,j+11}(\underset{\sim}{z};\underset{\sim}{z}') \left[\Phi^{c}_{j+11}(\underset{\sim}{z}'), \pi^{a}_{k}(\underset{\sim}{y})\right]_{PP} =$$

$$= \int d^3z \int d^3z' \,(-ig)\, \pi^{v}_{\psi}(\underset{\sim}{z}) \left(\frac{\lambda}{2}\right)^{b\,vu} \delta^{(3)}(\underset{\sim}{z}-\underset{\sim}{x})\, R^{cb}_{j+11,11}(\underset{\sim}{z}';\underset{\sim}{z})\, \delta^{ac} \delta^{jk} \delta^{(3)}(\underset{\sim}{z}'-\underset{\sim}{y}) \Longrightarrow$$

$$\boxed{\left[\pi^{u}_{\psi}(\underset{\sim}{x}), \pi^{a}_{k}(\underset{\sim}{y})\right]_{PD} = - i g\, \pi^{v}_{\psi}(\underset{\sim}{x}) \left(\frac{\lambda}{2}\right)^{b\,vu} R^{ab}_{k+11,11}(\underset{\sim}{y};\underset{\sim}{x})} \quad . \qquad (E.33)$$

Todos os outros PD's dos campos básicos ($A^{j,a}$, π^{a}_{j}, ψ, π_{ψ} e $F^{jk,a}$) entre si são nulos, o que é direto demonstrar.

Nota: Os PD's de $F^{0j,a}$ com todos os campos básicos coincidem com os PD's de π^{a}_{j} acima calculados. De fato, os PD's básicos são consistentes com todos os vínculos e condições de gauge, i.e., com $\{\Phi^{a}_{J}(\underset{\sim}{x}),\ J = 1,\ldots,14\}$.

APÊNDICE F

CARGA TOPOLÓGICA NO GAUGE AXIAL

A carga topológica é a quantidade invariante de gauge definida por [4,5,24]

$$q = -\frac{1}{16\pi^2}\int d^4x \; Tr \; F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad (F.1)$$

onde

$$F_{\mu\nu} = \frac{g}{i}\frac{\lambda^a}{2}F^a_{\mu\nu} \qquad (F.2)$$

e

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \quad . \qquad (F.3)$$

Levando (F.2) e (F.3) em (F.1), obtemos

$$q = \frac{g^2}{32\pi^2}\int d^4x \; F^a_{\mu\nu}\tilde{F}^{\mu\nu,a} \qquad (F.4)$$

usando $Tr(\lambda^a\lambda^b) = 2\delta^{ab}$. Como bem se sabe [5,22], o integrando em (F.1) (ou F.4) pode ser escrito como uma divergência total $\partial_\mu K^\mu$ onde

$$K^\mu = \epsilon^{\mu\nu\rho\sigma}\, Tr\left(A_\nu F_{\rho\sigma} - \frac{2}{3}A_\nu A_\rho A_\sigma\right) \quad . \qquad (F.5)$$

Prova

Desde (F.3) e (2.6) segue

$$Tr\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} Tr(F_{\rho\sigma} F_{\mu\nu}) =$$

$$= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} Tr\left\{\left[\partial_\rho A_\sigma - \partial_\sigma A_\rho + [A_\sigma, A_\rho]\right]\left[\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\right]\right\} =$$

$$= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} Tr\Big\{ (\partial_\rho A_\sigma)(\partial_\mu A_\nu) - (\partial_\rho A_\sigma)(\partial_\nu A_\mu) - (\partial_\sigma A_\rho)(\partial_\mu A_\nu) +$$

$$+ (\partial_\sigma A_\rho)(\partial_\nu A_\mu) + (\partial_\rho A_\sigma)[A_\mu, A_\nu] - \partial_\sigma A_\rho [A_\mu, A_\nu] +$$

$$+ [A_\sigma, A_\rho]\partial_\mu A_\nu - [A_\sigma, A_\rho]\partial_\nu A_\mu + [A_\sigma, A_\rho][A_\mu, A_\nu]\Big\} =$$

$$= \epsilon^{\mu\nu\rho\sigma} Tr\Big\{ (\partial_\rho A_\sigma)(\partial_\mu A_\nu) + (\partial_\sigma A_\rho)(\partial_\nu A_\mu) +$$

$$+ (\partial_\rho A_\sigma)[A_\mu, A_\nu] + [A_\sigma, A_\rho]\partial_\mu A_\nu + \underbrace{4 A_\sigma A_\rho A_\mu A_\nu}_{=0}\Big] =$$

$$= 2\epsilon^{\mu\nu\rho\sigma} Tr\Big[(\partial_\rho A_\sigma)(\partial_\mu A_\nu) + 2(\partial_\rho A_\sigma)A_\mu A_\nu\Big] . \qquad (F.6)$$

Por outro lado, é direto verificar que

$$(\partial_\rho A_\sigma)(\partial_\mu A_\nu) = \partial_\rho(A_\sigma \partial_\mu A_\nu) - A_\sigma(\partial_\rho\partial_\mu A_\nu) , \qquad (F.7)$$

$$\epsilon^{\mu\nu\rho\sigma} Tr\Big[(\partial_\rho A_\sigma)A_\mu A_\nu\Big] = \epsilon^{\mu\nu\rho\sigma} Tr\Big[\partial_\rho(A_\sigma A_\mu A_\nu) -$$

$$- (\partial_\rho A_\mu)A_\nu A_\sigma - (\partial_\rho A_\nu)A_\sigma A_\mu\Big] . \qquad (F.8)$$

Os dois últimos termos em (F.8) são iguais entre si e ambos iguais ao lado esquerdo, logo (F.8) implica

$$\epsilon^{\mu\nu\rho\sigma} Tr\left[(\partial_\rho A_\sigma) A_\mu A_\nu\right] = \frac{1}{3}\epsilon^{\mu\nu\rho\sigma} Tr\left[\partial_\rho (A_\sigma A_\mu A_\nu)\right]. \quad (F.9)$$

Substituindo (F.9) e (F.7) em (F.6), encontramos

$$Tr\, F_{\mu\nu} \tilde{F}^{\mu\nu} =$$

$$= 2\epsilon^{\mu\nu\rho\sigma} Tr\Big[\, \partial_\rho (A_\sigma \partial_\mu A_\nu) - A_\sigma (\partial_\rho \partial_\mu A_\nu) \; +$$

$$+ \frac{2}{3}\partial_\rho (A_\sigma A_\mu A_\nu) \Big] =$$

$$= 2\epsilon^{\mu\nu\rho\sigma} Tr\Big[\, \frac{2}{3}\partial_\mu (A_\nu A_\rho A_\sigma) + \partial_\mu (A_\nu \partial_\rho A_\sigma) \; +$$

$$+ \underbrace{A_\rho (\partial_\sigma \partial_\mu A_\nu)}_{=0} \Big] =$$

$$= 2\epsilon^{\mu\nu\rho\sigma} Tr\Big[\, \frac{2}{3}\partial_\mu (A_\nu A_\rho A_\sigma) + \partial_\mu (A_\nu \partial_\rho A_\sigma)\Big] .$$

$$(F.10)$$

Desde (F.10) segue que

$$Tr\, F_{\mu\nu} \tilde{F}^{\mu\nu} = \partial_\mu K^\mu \quad (F.11)$$

onde

$$K^\mu = 2\,\epsilon^{\mu\nu\rho\sigma}\, Tr\left[A_\nu \partial_\rho A_\sigma + \frac{2}{3} A_\nu A_\rho A_\sigma\right] . \quad (F.12)$$

Para completar a prova, mostramos que (F.12) coincide com (F.5). Pela antissimetria de $\varepsilon^{\mu\nu\rho\sigma}$, é evidente que

$$K^\mu = \epsilon^{\mu\nu\rho\sigma}\, Tr\left[2 A_\nu \partial_\rho A_\sigma + \frac{2}{3} A_\nu\, 2 A_\rho A_\sigma\right] =$$

$$= \epsilon^{\mu\nu\rho\sigma}\, Tr\left[A_\nu (\partial_\rho A_\sigma - \partial_\sigma A_\rho) + \frac{2}{3} A_\nu (A_\rho A_\sigma - A_\sigma A_\rho)\right] =$$

$$= \epsilon^{\mu\nu\rho\sigma}\, Tr\left\{\frac{1}{3} A_\nu (\partial_\rho A_\sigma - \partial_\sigma A_\rho) + \right.$$

$$\left. + \frac{2}{3} A_\nu \left[(\partial_\rho A_\sigma - \partial_\sigma A_\rho) + [A_\rho, A_\sigma]\right]\right\} =$$

$$= \epsilon^{\mu\nu\rho\sigma}\, Tr\left\{\frac{2}{3} A_\nu F_{\rho\sigma} + \underset{\checkmark}{\cancel{\frac{2}{3} A_\nu \partial_\rho A_\sigma}} + \right.$$

$$\left. + \frac{1}{3} A_\nu F_{\rho\sigma} - \frac{1}{3} A_\nu \left(\underbrace{\partial_\rho A_\sigma \cancel{-} \partial_\sigma}_{\checkmark} A_\rho + [A_\rho, A_\sigma]\right)\right\} \Rightarrow$$

$$K^\mu = \epsilon^{\mu\nu\rho\sigma}\, Tr\left(A_\nu F_{\rho\sigma} - \frac{2}{3} A_\nu A_\rho A_\sigma\right) . \quad q.e.d. \quad (F.13)$$

Note-se que K^μ é dependente de gauge mas $\partial_\mu K^\mu$ não é. Desde (F.11) é evidente que q poderá ser escrita como uma integral de superfície

$$q = -\frac{1}{16\pi^2} \oint dS_\mu K^\mu \quad (F.14)$$

Se supusermos que $F_{\mu\nu} \to 0$ sobre a superfície suficientemente rápido, então no gauge axial $A^{3,a} = 0$ a equação (F.14) se reduzirá a

$$q = \frac{i g^3}{192\pi^2} \int_{-\infty}^{+\infty} dx^0 dx^1 dx^2 \left| \overset{0,a}{A}(x^0_{,}\underset{\sim}{x}) \overset{2,b}{A}(x^0_{,}\underset{\sim}{x}) \overset{1,c}{A}(x^0_{,}\underset{\sim}{x}) \right|_{x^3 = -\infty}^{x^3 = +\infty} Tr\left(\lambda^a [\lambda^b, \lambda^c]\right) . (F.15)$$

Prova de (F.15)

Tomamos, na forma usual, os elementos de hipersuperfície dS_μ dados por

$$\left.\begin{aligned} dS_0 &= dx^1 dx^2 dx^3 \\ dS_1 &= dx^0 dx^2 dx^3 \\ dS_2 &= dx^0 dx^3 dx^1 \\ dS_3 &= dx^0 dx^1 dx^2 \end{aligned}\right\} . \qquad (F.16)$$

Com isso, desde (F.13) e (F.14) teremos

$$q = \frac{-1}{16\pi^2} \left\{ \int dS_0 \left[\epsilon^{01\rho\sigma} Tr\left[A_1 F_{\rho\sigma} - \frac{2}{3} A_1 A_\rho A_\sigma \right] + \right.\right.$$

$$+ \epsilon^{02\rho\sigma} Tr\left[A_2 F_{\rho\sigma} - \frac{2}{3} A_2 A_\rho A_\sigma \right] +$$

$$+ \epsilon^{03\rho\sigma} Tr\left[A_3 \underset{\downarrow = 0}{F_{\rho\sigma}} - \frac{2}{3} A_3 \underset{\downarrow = 0}{A_\rho} A_\sigma \right] \Bigg]_{x^0 = -\infty}^{x^0 = +\infty} +$$

$$+ \int dS_1 \Big[\epsilon^{10\rho\sigma} \, Tr\big[A_0 F_{\rho\sigma} - \tfrac{2}{3} A_0 A_\rho A_\sigma \big] +$$

$$+ \epsilon^{12\rho\sigma} \, Tr\big[A_2 F_{\rho\sigma} - \tfrac{2}{3} A_2 A_\rho A_\sigma \big] +$$

$$+ \epsilon^{13\rho\sigma} \, Tr\big[\underset{\swarrow = 0}{\overset{/}{A_3}} F_{\rho\sigma} - \tfrac{2}{3} \underset{\swarrow = 0}{\overset{/}{A_3}} A_\rho A_\sigma \big] \Big]_{x^1 = -\infty}^{x^1 = +\infty} +$$

$$+ \int dS_2 \Big[\epsilon^{20\rho\sigma} \, Tr\big[A_0 F_{\rho\sigma} - \tfrac{2}{3} A_0 A_\rho A_\sigma \big] +$$

$$+ \epsilon^{21\rho\sigma} \, Tr\big[A_1 F_{\rho\sigma} - \tfrac{2}{3} A_1 A_\rho A_\sigma \big] \Big]_{x^2 = -\infty}^{x^2 = +\infty} +$$

$$+ \int dS_3 \Big[\epsilon^{30\rho\sigma} \, Tr\big[A_0 F_{\rho\sigma} - \tfrac{2}{3} A_0 A_\rho A_\sigma \big] +$$

$$+ \epsilon^{31\rho\sigma} \, Tr\big[A_1 F_{\rho\sigma} - \tfrac{2}{3} A_1 A_\rho A_\sigma \big] +$$

$$+ \epsilon^{32\rho\sigma} \, Tr\big[A_2 F_{\rho\sigma} - \tfrac{2}{3} A_2 A_\rho A_\sigma \big] \Big]_{x^3 = -\infty}^{x^3 = +\infty} \Bigg\} . \qquad \textbf{(F.17)}$$

É imediato que os únicos termos que sobrevivem na situação de gauge axial são os seguintes, procedentes da última integral,

$$q = \frac{-1}{16\pi^2} \int_{-\infty}^{+\infty} dx^0 dx^1 dx^2 \Big\{ \underbrace{\epsilon^{3012}}_{+1} Tr\left(-\tfrac{2}{3} A_0 A_1 A_2\right) + \underbrace{\epsilon^{3021}}_{-1} Tr\left(-\tfrac{2}{3} A_0 A_2 A_1\right) +$$

$$+ \underbrace{\epsilon^{3120}}_{-1} Tr\left(-\tfrac{2}{3} A_1 A_2 A_0\right) + \underbrace{\epsilon^{3102}}_{+1} Tr\left(-\tfrac{2}{3} A_1 A_0 A_2\right) +$$

$$+\underbrace{\epsilon^{3201}}_{-1} Tr\left(-\frac{2}{3} A_2 A_0 A_1\right) + \underbrace{\epsilon^{3210}}_{+1} Tr\left(-\frac{2}{3} A_2 A_1 A_0\right)\Bigg]_{x^3=-\infty}^{x^3=+\infty} =$$

$$= \frac{-1}{16\pi^2}\int_{-\infty}^{+\infty} dx^0 dx^1 dx^2 \Bigg\{ -\frac{2}{3}\, Tr\left(A_0[A_1,A_2]\right) -$$

$$-\frac{2}{3}\, Tr\left(A_1[A_0,A_2]\right) - \frac{2}{3}\, Tr\left(A_2[A_1,A_0]\right)\Bigg\}_{x^3=-\infty}^{x^3=+\infty} .$$

(F.18)

Introduzindo agora nossa parametrização (2.5), teremos

$$Tr\left(A_0[A_1,A_2]\right) + Tr\left(A_1[A_0,A_2]\right) + Tr\left(A_2[A_1,A_0]\right) =$$

$$= \left(\frac{g}{2i}\right)^3 A_0^a A_1^b A_2^c \Bigg\{ Tr\left(\lambda^a[\lambda^b,\lambda^c]\right) + Tr\left(\lambda^b[\lambda^a,\lambda^c]\right) +$$

$$+ Tr\left(\lambda^c[\lambda^b,\lambda^a]\right) \Bigg\} =$$

$$= \left(\frac{g}{2i}\right)^3 A_0^a A_1^b A_2^c \Bigg[Tr\Big(\underset{\swarrow}{\overset{/}{\lambda^a\lambda^b\lambda^c}} - \underset{\Downarrow}{\overset{\|}{\lambda^a\lambda^c\lambda^b}} + \underset{\Downarrow}{\overset{\|}{\lambda^b\lambda^a\lambda^c}} - \underset{\swarrow}{\overset{/}{\lambda^b\lambda^c\lambda^a}} +$$

$$+ \lambda^c\lambda^b\lambda^a - \lambda^c\lambda^a\lambda^b \Big)\Bigg] = \left(\frac{g}{2i}\right)^3 A_0^a A_1^b A_2^c\, Tr\left(\lambda^c[\lambda^b,\lambda^a]\right), \quad (F.19)$$

onde usamos a ciclicidade do traço. Assim, levando (F.19) em (F.18), obtemos (F.15), i.e.,

$$q = \frac{i g^3}{192\pi^2}\int_{-\infty}^{+\infty} dx^0 dx^1 dx^2 \left| A^{0,a}_{(x^0,\underline{x})} A^{2,b}_{(x^0,\underline{x})} A^{1,c}_{(x^0,\underline{x})}\right|_{x^3=-\infty}^{x^3=+\infty} Tr\left[\lambda^a[\lambda^b,\lambda^c]\right] . \quad (F.20)$$

q.e.d.

APÊNDICE G

AÇÃO DOS GERADORES DE POINCARÉ SOBRE OS CAMPOS BÁSICOS

A partir de (5.1)-(5.3), fazendo uso de (4.2), calculamos em primeiro lugar a ação do operador Hamiltoniano $\hat{H}$ sobre os campos básicos. É direto obter

$$\left[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{A}^{j,a}(\underset{\sim}{y})\right] = \left[\frac{1}{2}\underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x})\,\underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x}), \hat{A}^{j,a}(\underset{\sim}{y})\right] = \underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x}).\left[\underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x}), \hat{A}^{j,a}(\underset{\sim}{y})\right] =$$

$$= -i\,\underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x}).\left(\delta^{ab}\delta^{jk}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + \hat{D}^{j,ab}(\underset{\sim}{y})\,\underset{k}{R}(\underset{\sim}{x};\underset{\sim}{y})\right) \Rightarrow$$

$$\boxed{\left[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{A}^{j,a}(\underset{\sim}{y})\right] = -i\,\underset{j}{\hat{\pi}}^{a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - i\left(\hat{D}^{j,ab}(\underset{\sim}{y})\,\underset{k}{R}(\underset{\sim}{x};\underset{\sim}{y})\right).\underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x})}. \qquad (G.1)$$

Integrando (G.1) sobre $\underset{\sim}{x}$ em todo o espaço, obtemos

$$\left[\hat{H}, \hat{A}^{j,a}(\underset{\sim}{y})\right] = -i\,\underset{j}{\hat{\pi}}^{a}(\underset{\sim}{y}) - i\,\hat{D}^{j,ab}(\underset{\sim}{y}).\int d^3x\,\underset{k}{R}(\underset{\sim}{x};\underset{\sim}{y})\,\underset{k}{\hat{\pi}}^{b}(\underset{\sim}{x}) \Rightarrow$$

$$\boxed{\left[\hat{H}, \hat{A}^{j,a}(\underset{\sim}{y})\right] = -i\,\underset{j}{\hat{\pi}}^{a}(\underset{\sim}{y}) - i\,\hat{D}^{j,ab}(\underset{\sim}{y}).\,\hat{A}^{0,b}(\underset{\sim}{y})} \qquad (G.2)$$

uma vez que (ver (E.3) e (3.50))

$$\int d^3x\,\underset{k}{R}(\underset{\sim}{x};\underset{\sim}{y})\,\underset{k}{\hat{\pi}}^{a}(\underset{\sim}{x}) = \hat{A}^{0,a}(\underset{\sim}{y}) \quad . \qquad (G.3)$$

De forma similar,

$$\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\, \hat{F}^{jk,a}(\underset{\sim}{y})\right] = \hat{\pi}^{b}_{\ell}(\underset{\sim}{x}).\left[\hat{\pi}^{b}_{\ell}(\underset{\sim}{x}),\, \hat{F}^{jk,a}(\underset{\sim}{y})\right] =$$

$$= -i\hat{\pi}^{b}_{\ell}(\underset{\sim}{x}).\left\{\left[\delta^{\ell k}\hat{D}^{j,ab}(\underset{\sim}{y}) - \delta^{j\ell}\hat{D}^{k,ab}(\underset{\sim}{y})\right]\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + g f^{acb}\hat{F}^{jk,c}(\underset{\sim}{y}) R_{\ell}(\underset{\sim}{x};\underset{\sim}{y})\right] \Rightarrow$$

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\, \hat{F}^{jk,a}(\underset{\sim}{y})\right] = & -i\hat{\pi}^{b}_{k}(\underset{\sim}{x}).\hat{D}^{j,ab}(\underset{\sim}{y})\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) + i\,\hat{\pi}^{b}_{j}(\underset{\sim}{x}).\hat{D}^{k,ab}(\underset{\sim}{y})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) - \\ & - i g f^{acb}\hat{F}^{jk,c}(\underset{\sim}{y}).\hat{\pi}^{b}_{\ell}(\underset{\sim}{x}) R_{\ell}(\underset{\sim}{x};\underset{\sim}{y}) \quad , \quad (G.4)\end{aligned}}$$

de onde, por integração sobre $\underset{\sim}{x}$, encontra-se

$$\boxed{\begin{aligned}\left[\hat{H},\, \hat{F}^{jk,a}(\underset{\sim}{y})\right] = & -i\,\hat{D}^{j,ab}(\underset{\sim}{y}).\hat{\pi}^{b}_{k}(\underset{\sim}{y}) + i\,\hat{D}^{k,ab}(\underset{\sim}{y}).\hat{\pi}^{b}_{j}(\underset{\sim}{y}) - \\ & - i g f^{acb}\hat{F}^{jk,c}(\underset{\sim}{y}).\hat{A}^{0,b}(\underset{\sim}{y})\end{aligned}} \qquad (G.5)$$

usando (G.3);

$$\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right] = \hat{\pi}^{a}_{j}(\underset{\sim}{x}).\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right] + \frac{1}{2}\hat{F}^{j\ell,a}(\underset{\sim}{x}).\left[\hat{F}^{j\ell,a}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right] -$$

$$-\frac{1}{2}(\gamma^{0}\gamma^{\ell})_{rs}\left\{\left[\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right].\partial^{x}_{\ell}\hat{\psi}^{u}_{s}(\underset{\sim}{x}) + \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}).\partial^{x}_{\ell}\left[\hat{\psi}^{u}_{s}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right]\right\} +$$

$$+\frac{1}{2}(\gamma^{0}\gamma^{\ell})_{rs}\left\{\left(\partial^{x}_{\ell}\left[\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right]\right).\hat{\psi}^{u}_{s}(\underset{\sim}{x}) + \left(\partial^{x}_{\ell}\hat{\pi}^{u}_{\psi}(\underset{\sim}{x})\right).\left[\hat{\psi}^{u}_{s}(\underset{\sim}{x}),\, \hat{\pi}^{b}_{k}(\underset{\sim}{y})\right]\right\} -$$

$$-ig(\gamma^0\gamma^l)_{rs}\left(\frac{\lambda^a}{2}\right)^{uv}\Big\{[\hat{\pi}^u_{\psi}(\underset{\sim}{x}),\hat{\pi}^b_k(y)]\cdot\hat{\psi}^v_s(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}) + \hat{\pi}^u_{\psi_r}(\underset{\sim}{x})\cdot[\hat{\psi}^v_s(\underset{\sim}{x}),\hat{\pi}^b_k(y)]\hat{A}^{l,a}(\underset{\sim}{x})+$$

$$+\hat{\pi}^u_{\psi_r}(\underset{\sim}{x})\cdot\hat{\psi}^v_s(\underset{\sim}{x})[\hat{A}^{l,a}(\underset{\sim}{x}),\hat{\pi}^b_k(y)]\Big\} - im(\gamma^0)_{rs}\Big\{[\hat{\pi}^u_{\psi}(\underset{\sim}{x}),\hat{\pi}^b_k(y)]\cdot\hat{\psi}^u_s(\underset{\sim}{x})+$$

$$+\hat{\pi}^u_{\psi_r}(\underset{\sim}{x})\cdot[\hat{\psi}^u_s(\underset{\sim}{x}),\hat{\pi}^b_k(y)]\Big\} =$$

$$= igf^{acb}\hat{\pi}^a_j(\underset{\sim}{x})\cdot[\hat{\pi}^c_j(\underset{\sim}{x})R_k(y;\underset{\sim}{x}) + \hat{\pi}^c_k(y)R_j(\underset{\sim}{x};y)] +$$

$$+\frac{i}{2}\hat{F}^{jl,a}(\underset{\sim}{x})\cdot(\delta^{lk}\hat{D}^{j,ab}(\underset{\sim}{x}) - \delta^{jk}\hat{D}^{l,ab}(\underset{\sim}{x}))\overset{(3)}{\delta}(\underset{\sim}{x-y}) + \frac{i}{2}\hat{F}^{jl,a}(\underset{\sim}{x})\cdot gf^{acb}\hat{F}^{jl,c}(\underset{\sim}{x})R_k(y;\underset{\sim}{x})-$$

$$-\frac{g}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}(\partial^x_l\hat{\psi}(\underset{\sim}{x}))R_k(y;\underset{\sim}{x}) + \frac{g}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}(\partial^x_l\hat{\psi}(\underset{\sim}{x}))R_k(y;\underset{\sim}{x})+$$

$$+\frac{g}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})\partial^x_lR_k(y;\underset{\sim}{x}) + \frac{g}{2}(\partial^x_l\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})R_k(y;\underset{\sim}{x})+$$

$$+\frac{g}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})\partial^x_lR_k(y;\underset{\sim}{x}) - \frac{g}{2}(\partial^x_l\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})R_k(y;\underset{\sim}{x})-$$

$$-im\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})R_k(y;\underset{\sim}{x}) + im\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})R_k(y;\underset{\sim}{x}) -$$

$$-ig^2\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^b}{2}\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})R_k(y;\underset{\sim}{x}) + ig^2\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^a}{2}\frac{\lambda^b}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})R_k(y;\underset{\sim}{x})+$$

$$+g\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\left(\delta^{ab}\delta^{lk}\overset{(3)}{\delta}(\underset{\sim}{x-y}) + \delta^{ab}\partial^l_xR_k(y;\underset{\sim}{x}) + gf^{acb}\hat{A}^{l,c}(\underset{\sim}{x})R_k(y;\underset{\sim}{x})\right)$$

$$\Longrightarrow$$

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\,\hat{\pi}^{b}_{k}(y)\right] &= igf^{acb}\,\hat{\pi}^{c}_{k}(y).\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\,R_{j}(\underset{\sim}{x};y) - i\,\hat{F}^{jk,a}(\underset{\sim}{x})\,\hat{D}^{j,ba}(y)\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})+\\ &+ g\,\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^{0}\gamma^{k}\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) \quad , \qquad (G.6)\end{aligned}}$$

∴

$$\boxed{\begin{aligned}\left[\hat{H},\,\hat{\pi}^{b}_{k}(y)\right] &= igf^{acb}\,\hat{\pi}^{c}_{k}(y).\,\hat{A}^{0,a}(y) - i\,\hat{D}^{j,ba}(y)\,\hat{F}^{jk,a}(y) +\\ &+ g\,\hat{\pi}_{\psi}(y).\gamma^{0}\gamma^{k}\frac{\lambda^{b}}{2}\,\hat{\psi}(y) \qquad ;\end{aligned}} \qquad (G.7)$$

$$\begin{aligned}\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right] &= \hat{\pi}^{a}_{j}(\underset{\sim}{x}).\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right] + \frac{1}{2}(\gamma^{0}\gamma^{k})_{r's}\left\{\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right\}.\partial^{x}_{k}\hat{\psi}^{v}_{s}(\underset{\sim}{x})-\\ &-\frac{1}{2}(\gamma^{0}\gamma^{k})_{r's}\left(\partial^{x}_{k}\left\{\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right]\right).\hat{\psi}^{v}_{s}(\underset{\sim}{x}) + ig(\gamma^{0}\gamma^{k})_{r's}\left(\frac{\lambda}{2}\right)^{a\,vw}\left\{\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right\}.\hat{\psi}^{w}_{s}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})+\\ &+im(\gamma^{0})_{r's}\left\{\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right\}.\hat{\psi}^{v}_{s}(\underset{\sim}{x}) \quad \Longrightarrow\end{aligned}$$

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\,\hat{\psi}^{u}_{r}(y)\right] &= g\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\,R_{j}(\underset{\sim}{x};y) + \frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})(\gamma^{0}\gamma^{k})_{rs}\,\partial^{x}_{k}\hat{\psi}^{u}_{s}(\underset{\sim}{x})-\\ &-\frac{i}{2}\left(\partial^{x}_{k}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})\right)(\gamma^{0}\gamma^{k})_{rs}\,\hat{\psi}^{u}_{s}(\underset{\sim}{x}) - g\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})(\gamma^{0}\gamma^{k})_{rs}\left(\frac{\lambda}{2}\right)^{a\,uv}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}) - m\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y})(\gamma^{0})_{rs}\,\hat{\psi}^{u}_{s}(\underset{\sim}{x}),\end{aligned}}$$

(G.8)

∴

$$\boxed{\begin{aligned}\left[\hat{H},\hat{\psi}^{u}(y)\right] &= g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}(y)\cdot\hat{A}^{0,a}(y) + i\gamma^{0}\gamma^{k}\partial^{y}_{k}\hat{\psi}^{u}(y) - \\ &\quad - g\,\gamma^{0}\gamma^{k}\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}(y)\hat{A}^{k,a}(y) - m\gamma^{0}\hat{\psi}^{u}(y)\ ;\end{aligned}} \tag{G.9}$$

$$\begin{aligned}\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right] &= \hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right] - \frac{1}{2}(\gamma^{0}\gamma^{k})_{sr'}\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x})\cdot\partial^{x}_{k}\left\{\hat{\psi}^{v}_{r'}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right\} + \\ &+ \frac{1}{2}(\gamma^{0}\gamma^{k})_{sr'}\left(\partial^{x}_{k}\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x})\right)\cdot\left\{\hat{\psi}^{v}_{r'}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right\} - ig(\gamma^{0}\gamma^{k})_{sr'}\left(\frac{\lambda^{a}}{2}\right)^{wv}\hat{\pi}^{w}_{\psi_s}(\underset{\sim}{x})\cdot\left\{\hat{\psi}^{v}_{r'}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right\}\hat{A}^{k,a}(\underset{\sim}{x}) - \\ &- im(\gamma^{0})_{sr'}\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x})\cdot\left\{\hat{\psi}^{v}_{r'}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right\} \qquad \Longrightarrow\end{aligned}$$

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{00}(\underset{\sim}{x}),\hat{\pi}^{u}_{\psi_r}(y)\right] &= -g\,\hat{\pi}^{v}_{\psi}(y)\left(\frac{\lambda^{a}}{2}\right)^{vu}\cdot\hat{\pi}^{a}_{j}(\underset{\sim}{x})R_{j}(\underset{\sim}{x};y) - \frac{i}{2}\hat{\pi}^{u}_{\psi_s}(\underset{\sim}{x})(\gamma^{0}\gamma^{k})_{sr}\partial^{x}_{k}\delta^{(3)}(x-y) + \\ &+ \frac{i}{2}\left(\partial^{x}_{k}\hat{\pi}^{u}_{\psi_s}(\underset{\sim}{x})\right)\cdot(\gamma^{0}\gamma^{k})_{sr}\delta^{(3)}(\underset{\sim}{x}-y) + g\,\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x})\cdot(\gamma^{0}\gamma^{k})_{sr}\left(\frac{\lambda^{a}}{2}\right)^{vu}\hat{A}^{k,a}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-y) + \\ &+ m\,\hat{\pi}^{u}_{\psi_s}(\underset{\sim}{x})\cdot(\gamma^{0})_{sr}\delta^{(3)}(\underset{\sim}{x}-y) \qquad ,\end{aligned}} \tag{G.10}$$

$\therefore$

$$\boxed{\begin{aligned}\left[\hat{H},\hat{\pi}^{u}_{\psi}(y)\right] &= -g\,\hat{\pi}^{v}_{\psi}(y)\left(\frac{\lambda^{a}}{2}\right)^{vu}\cdot\hat{A}^{0,a}(y) + i\left(\partial^{y}_{k}\hat{\pi}^{u}_{\psi}(y)\right)\gamma^{0}\gamma^{k} + \\ &+ g\,\hat{\pi}^{v}_{\psi}(y)\cdot\gamma^{0}\gamma^{k}\left(\frac{\lambda^{a}}{2}\right)^{vu}\hat{A}^{k,a}(y) + m\,\hat{\pi}^{u}_{\psi}(y)\gamma^{0}\end{aligned}}\ . \tag{G.11}$$

Calculemos o efeito de $\hat{P}^k$ sobre os campos básicos. Desde (5.4)-(5.6) e (4.2), encontramos

$$\left[\hat{\Theta}^{0k}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(\underset{\sim}{y})}\right] = \left[\hat{\pi}^{b}_{\ell}{}_{(\underset{\sim}{x})} . \hat{F}^{k\ell,b}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(\underset{\sim}{y})}\right] = \left[\hat{\pi}^{b}_{\ell}{}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(\underset{\sim}{y})}\right] . \hat{F}^{k\ell,b}_{(\underset{\sim}{x})} =$$

$$= -i\left(\delta^{ab}\delta^{j\ell}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} + \hat{D}^{j,ab}_{(\underset{\sim}{y})} R_{\ell}(\underset{\sim}{x};\underset{\sim}{y})\right) . \hat{F}^{k\ell,b}_{(\underset{\sim}{x})} \qquad \Longrightarrow$$

$$\boxed{\left[\hat{\Theta}^{0k}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(\underset{\sim}{y})}\right] = i\, \hat{F}^{jk,a}_{(\underset{\sim}{x})} \delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} + i\, \hat{F}^{\ell k,b}_{(\underset{\sim}{x})} \hat{D}^{j,ab}_{(\underset{\sim}{y})} R_{\ell}(\underset{\sim}{x};\underset{\sim}{y})} \, , \qquad (G.12)$$

de onde, por integração sobre $\underset{\sim}{x}$, obtemos

$$\left[\hat{P}^{k}, \hat{A}^{j,a}_{(\underset{\sim}{y})}\right] = i\, \hat{F}^{jk,a}_{(\underset{\sim}{y})} +$$

$$+ i\, \hat{D}^{j,ab}_{(\underset{\sim}{y})} \int d^3x \left[\left(\partial^{\ell}_{x} \hat{A}^{k,b}_{(\underset{\sim}{x})} - \partial^{k}_{x} \hat{A}^{\ell,b}_{(\underset{\sim}{x})} + g f^{bcd} \hat{A}^{\ell,c}_{(\underset{\sim}{x})} \hat{A}^{k,d}_{(\underset{\sim}{x})}\right) R_{\ell}(\underset{\sim}{x};\underset{\sim}{y})\right]$$

$$= i\, \hat{F}^{jk,a}_{(\underset{\sim}{y})} - i\, \hat{D}^{j,ab}_{(\underset{\sim}{y})} \int d^3x \, R_{\ell}(\underset{\sim}{x};\underset{\sim}{y})\, \partial^{k}_{x} \hat{A}^{\ell,b}_{(\underset{\sim}{x})} -$$

$$- i\, \hat{D}^{j,ab}_{(\underset{\sim}{y})} \int d^3x \, \hat{A}^{k,b}_{(\underset{\sim}{x})} \delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{y})} =$$

$$= i\left(\partial^{j}_{y} \hat{A}^{k,a}_{(\underset{\sim}{y})} - \partial^{k}_{y} \hat{A}^{j,a}_{(\underset{\sim}{y})} + g f^{acb} \hat{A}^{j,c}_{(\underset{\sim}{y})} \hat{A}^{k,b}_{(\underset{\sim}{y})}\right) - i\, \partial^{j}_{y} \hat{A}^{k,a}_{(\underset{\sim}{y})} -$$

$$- i g f^{acb} \hat{A}^{j,c}_{(\underset{\sim}{y})} \hat{A}^{k,b}_{(\underset{\sim}{y})} - i\, \hat{D}^{j,ab}_{(\underset{\sim}{y})} \hat{B}^{k,b}_{(\underset{\sim}{y})} \quad , \qquad (G.13)$$

onde definimos

$$\hat{B}^{k,a}_{(y)} \equiv \int d^3x \; R_\ell(x;y) \, \partial^k_x \hat{A}^{\ell,a}_{(x)} \qquad (G.14)$$

e onde usamos o fato que (ver (3.52) e (2.1a,c))

$$\hat{A}^{\ell,a}_{(x)} R_\ell(x;y) = 0 \quad . \qquad (G.15)$$

Além disso, foi realizada uma integração por partes levando em conta (3.51) e (3.49). Claramente, (G.13) implica

$$\boxed{\left[\hat{P}^k, \hat{A}^{j,a}_{(x)}\right] = -i\,\partial^k_x \hat{A}^{j,a}_{(x)} - i\,\hat{D}^{j,ab}_{(x)} \hat{B}^{k,b}_{(x)}} \quad . \qquad (G.16)$$

Semelhantemente, calculamos

$$\left[\hat{\Theta}^{0k}_{(x)}, \hat{F}^{m\ell,b}_{(y)}\right] = \left[\hat{\pi}^a_j(x) . \hat{F}^{kj,a}_{(x)}, \hat{F}^{m\ell,b}_{(y)}\right] =$$

$$= \left[\hat{\pi}^a_j(x), \hat{F}^{m\ell,b}_{(y)}\right] . \hat{F}^{kj,a}_{(x)} =$$

$$= -i\,\hat{F}^{kj,a}_{(x)} \Big\{ \left[\delta^{j\ell} \hat{D}^{m,ba}_{(y)} - \delta^{mj} \hat{D}^{\ell,ba}_{(y)}\right] \delta^{(3)}(x-y) +$$

$$+ g f^{bca} \hat{F}^{m\ell,c}_{(y)} R_j(x;y) \Big\} \Longrightarrow$$

$$\boxed{\begin{aligned} \left[\hat{\Theta}^{0k}_{(x)}, \hat{F}^{m\ell,b}_{(y)}\right] = {} & i\left(\hat{F}^{km,a}_{(x)} \hat{D}^{\ell,ba}_{(y)} - \hat{F}^{k\ell,a}_{(x)} \hat{D}^{m,ba}_{(y)}\right) \delta^{(3)}(x-y) + \\ & + i g f^{bac} \hat{F}^{m\ell,c}_{(y)} \hat{F}^{kj,a}_{(x)} R_j(x;y) \quad . \qquad (G.17) \end{aligned}}$$

Desde (G.17), usando o fato que (ver (G.13))

$$\int d^3_x \, \hat{F}^{kj,a}_{(\underset{\sim}{x})} R_j(\underset{\sim}{x};y) = \hat{A}^{k,a}_{(y)} + \hat{B}^{k,a}_{(y)} \qquad (G.18)$$

junto com a identidade de Bianchi

$$\hat{D}^{l,ba}_{(y)} \hat{F}^{km,a}_{(y)} - \hat{D}^{m,ba}_{(y)} \hat{F}^{kl,a}_{(y)} = -\hat{D}^{k,ba}_{(y)} \hat{F}^{ml,a}_{(y)} \quad , \qquad (G.19)$$

obtemos

$$[\hat{P}^k, \hat{F}^{ml,b}_{(y)}] = -i\hat{D}^{k,ba}_{(y)} \hat{F}^{ml,a}_{(y)} - igf^{bca} \hat{F}^{ml,c}_{(y)} (\hat{A}^{k,a}_{(y)} + \hat{B}^{k,a}_{(y)})$$

$$= -i\partial^k_y \hat{F}^{ml,b}_{(y)} - igf^{bca} \hat{F}^{ml,c}_{(y)} \hat{B}^{k,a}_{(y)} \qquad \Longrightarrow$$

$$\boxed{[\hat{P}^k, \hat{F}^{jl,a}_{(\underset{\sim}{x})}] = -i\partial^k_x \hat{F}^{jl,a}_{(\underset{\sim}{x})} - igf^{acb} \hat{F}^{jl,c}_{(\underset{\sim}{x})} \hat{B}^{k,b}_{(\underset{\sim}{x})}} \quad . \qquad \textbf{(G.20)}$$

Por outro lado,

$$\begin{aligned}
[\hat{\Theta}^{ok}(\underset{\sim}{x}), \hat{\pi}^b_l(y)] &= \hat{\pi}^a_j(\underset{\sim}{x}).[\hat{F}^{kj,a}(\underset{\sim}{x}), \hat{\pi}^b_l(y)] + [\hat{\pi}^a_j(\underset{\sim}{x}), \hat{\pi}^b_l(y)].\hat{F}^{kj,a}(\underset{\sim}{x}) + \\
&+ \frac{1}{2}\hat{\pi}^u_\psi(\underset{\sim}{x}).\partial^k_x[\hat{\psi}^u(\underset{\sim}{x}), \hat{\pi}^b_l(y)] + \frac{1}{2}[\hat{\pi}^u_\psi(\underset{\sim}{x}), \hat{\pi}^b_l(y)].\partial^k_x\hat{\psi}^u(\underset{\sim}{x}) - \frac{1}{2}(\partial^k_x\hat{\pi}^u_\psi(\underset{\sim}{x})).[\hat{\psi}^u(\underset{\sim}{x}), \hat{\pi}^b_l(y)] - \\
&- \frac{1}{2}\left(\partial^k_x[\hat{\pi}^u_\psi(\underset{\sim}{x}), \hat{\pi}^b_l(y)]\right).\hat{\psi}^u(\underset{\sim}{x}) - ig\left(\frac{\lambda^a}{2}\right)^{uv}\Big\{[\hat{\pi}^u_\psi(\underset{\sim}{x}), \hat{\pi}^b_l(y)].\hat{\psi}^v(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}) + \hat{\pi}^u_\psi(\underset{\sim}{x}).[\hat{\psi}^v(\underset{\sim}{x}), \hat{\pi}^b_l(y)]\hat{A}^{ka}(\underset{\sim}{x}) + \\
&+ \hat{\pi}^u_\psi(\underset{\sim}{x}).\hat{\psi}^v(\underset{\sim}{x})[\hat{A}^{ka}(\underset{\sim}{x}), \hat{\pi}^b_l(y)]\Big\} - \frac{i}{4}(\sigma^{jk})_{rs}\partial^x_j\Big\{[\hat{\pi}^u_{\psi_r}(\underset{\sim}{x}), \hat{\pi}^b_l(y)].\hat{\psi}^u_s(\underset{\sim}{x}) + \hat{\pi}^u_{\psi_r}(\underset{\sim}{x}).[\hat{\psi}^u_s(\underset{\sim}{x}), \hat{\pi}^b_l(y)]\Big\} =
\end{aligned}$$

$$= i\,\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\Big[\big(\delta^{jl}\hat{D}^{k,ab}(\underset{\sim}{x}) - \delta^{kl}\hat{D}^{j,ab}(\underset{\sim}{x})\big)\overset{(3)}{\delta}(\underset{-}{x}-y) + igf^{acb}\hat{F}^{kj,c}(\underset{\sim}{x})\,R_{l}(y;\underset{\sim}{x})\Big] +$$

$$+igf^{acb}\big(\hat{\pi}^{c}_{j}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x}) + \hat{\pi}^{c}_{l}(y)R_{j}(\underset{\sim}{x};y)\big).\hat{F}^{kj,a}(\underset{\sim}{x}) +$$

$$+\Big\{\frac{1}{2}\hat{\pi}^{u}_{\psi}(\underset{\sim}{x}).\partial^{k}_{x}\Big[-g\Big(\frac{\lambda^{b}}{2}\Big)^{uv}\psi^{v}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x})\Big] + \frac{1}{2}\Big[g\hat{\pi}^{v}_{\psi}(\underset{\sim}{x})\Big(\frac{\lambda^{b}}{2}\Big)^{vu}R_{l}(y;\underset{\sim}{x})\Big].\partial^{k}_{x}\hat{\psi}^{u}(\underset{\sim}{x}) -$$

$$-\frac{1}{2}(\partial^{k}_{x}\hat{\pi}^{u}_{\psi}(\underset{\sim}{x})).\Big[-g\Big(\frac{\lambda^{b}}{2}\Big)^{uv}\hat{\psi}^{v}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x})\Big] - \frac{1}{2}\Big(\partial^{k}_{x}\Big[g\hat{\pi}^{v}_{\psi}(\underset{\sim}{x})\Big(\frac{\lambda^{b}}{2}\Big)^{vu}R_{l}(y;\underset{\sim}{x})\Big]\Big).\hat{\psi}^{u}(\underset{\sim}{x})\Big\} -$$

$$-\Big\{-ig\Big(\frac{\lambda^{a}}{2}\Big)^{uv}\Big[g\hat{\pi}^{w}_{\psi}(\underset{\sim}{x})\Big(\frac{\lambda^{b}}{2}\Big)^{wu}R_{l}(y;\underset{\sim}{x}).\hat{\psi}^{v}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}) +$$

$$+\hat{\pi}^{u}_{\psi}(\underset{\sim}{x}).\Big(-g\Big(\frac{\lambda^{b}}{2}\Big)^{vw}\hat{\psi}^{w}(\underset{\sim}{x}).R_{l}(y;\underset{\sim}{x})\Big)\hat{A}^{k,a}(\underset{\sim}{x}) + \hat{\pi}^{u}_{\psi}(\underset{\sim}{x}).\hat{\psi}^{v}(\underset{\sim}{x})i\Big(\delta^{ab}\delta^{kl}\overset{(3)}{\delta}(\underset{\sim}{x}-y) + \hat{D}^{k,ab}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x})\Big)\Big]\Big\} -$$

$$-\frac{i}{4}(\sigma^{jk})_{rs}\,\partial^{x}_{j}\Big[g\,\hat{\pi}^{v}_{\psi_{r}}(\underset{\sim}{x})\Big(\frac{\lambda^{b}}{2}\Big)^{vu}R_{l}(y;\underset{\sim}{x}).\hat{\psi}^{u}_{s}(\underset{\sim}{x}) - \hat{\pi}^{u}_{\psi_{r}}(\underset{\sim}{x}).g\Big(\frac{\lambda^{b}}{2}\Big)^{uv}\hat{\psi}^{v}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x})\Big]. \quad \textbf{(G.21)}$$

A primeira chave que aparece no lado direito de (G.21) é igual a

$$-\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}).\Big[\frac{\lambda^{b}}{2}(\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}))R_{l}(y;\underset{\sim}{x}) + \frac{\lambda^{b}}{2}\psi(\underset{\sim}{x})(\partial^{k}_{x}R_{l}(y;\underset{\sim}{x}))\Big] +$$

$$+\frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}).\frac{\lambda^{b}}{2}(\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}))R_{l}(y;\underset{\sim}{x}) + \frac{g}{2}(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x}) -$$

$$-\frac{g^{2}}{2}(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\frac{\lambda^{6}}{2}\hat{\psi}(\underset{\sim}{x})R_{l}(y;\underset{\sim}{x}) - \frac{g}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}).\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})(\partial^{k}_{x}R_{l}(y;\underset{\sim}{x})) =$$

$$= -g\,\hat{\pi}_{\psi}(\underset{\sim}{x}).\frac{\lambda^{6}}{2}\hat{\psi}(\underset{\sim}{x})\,\partial^{k}_{x}R_{l}(y;\underset{\sim}{x}) \quad . \qquad (G.22)$$

A segunda chave no lado direito de (G.21) é

$$-ig^{2}\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\left[\frac{\lambda^{b}}{2},\frac{\lambda^{a}}{2}\right]\hat{\psi}(\underset{\sim}{x})\,\hat{A}^{k,a}(\underset{\sim}{x})\,R_{\ell}(\underset{\sim}{y};\underset{\sim}{x})\;+$$

$$+g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\left(\delta^{ab}\delta^{k\ell}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+\hat{D}^{k,ab}(\underset{\sim}{x})\,R_{\ell}(\underset{\sim}{y};\underset{\sim}{x})\right)\;=$$

$$=g^{2}f^{bac}\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{c}}{2}\hat{\psi}(\underset{\sim}{x})\,\hat{A}^{k,a}(\underset{\sim}{x})\,R_{\ell}(\underset{\sim}{y};\underset{\sim}{x})+g\delta^{k\ell}\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+$$

$$+g^{2}f^{acb}\,\hat{A}^{k,c}(\underset{\sim}{x})\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\,R_{\ell}(\underset{\sim}{y};\underset{\sim}{x})+g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\,\partial^{k}_{x}R_{\ell}(\underset{\sim}{y};\underset{\sim}{x})\;=$$

$$=\delta^{k\ell}g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\,\partial^{k}_{x}R_{\ell}(\underset{\sim}{y};\underset{\sim}{x})\quad.\qquad(G.23)$$

Levando (G.23) e (G.22) em (G.21), encontramos

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\pi}^{b}_{\ell}(\underset{\sim}{y})\right]&=i\left(\hat{\pi}^{a}_{\ell}(\underset{\sim}{x})\cdot\hat{D}^{k,ab}(\underset{\sim}{x})-\delta^{k\ell}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\cdot\hat{D}^{j,ab}(\underset{\sim}{x})\right)\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})+\\&+igf^{acb}\,\hat{\pi}^{c}_{\ell}(\underset{\sim}{y})\cdot\hat{F}^{kj,a}(\underset{\sim}{x})\,R_{j}(\underset{\sim}{x};\underset{\sim}{y})+\delta^{k\ell}g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^{b}}{2}\hat{\psi}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})\quad.\qquad(G.24)\end{aligned}}$$

Integrando (G.24) sobre $\underset{\sim}{x}$, com o uso de (G.18) e da lei de Gauss (4.3), obtemos

$$\left[\hat{P}^{k},\hat{\pi}^{b}_{\ell}(\underset{\sim}{y})\right]=-i\left(\hat{D}^{k,ba}(\underset{\sim}{y})\cdot\hat{\pi}^{a}_{\ell}(\underset{\sim}{y})\right)+igf^{acb}\,\hat{\pi}^{c}_{\ell}(\underset{\sim}{y})\cdot\left(\hat{A}^{k,a}(\underset{\sim}{y})+\hat{B}^{k,a}(\underset{\sim}{y})\right)=$$

$$=-i\,\partial^{k}_{y}\,\hat{\pi}^{b}_{\ell}(\underset{\sim}{y})-igf^{bca}\,\hat{\pi}^{c}_{\ell}(\underset{\sim}{y})\cdot\hat{B}^{k,a}(\underset{\sim}{y})\qquad\Longrightarrow$$

$$\boxed{\left[\hat{P}^{k},\hat{\pi}^{a}_{j}(\underset{\sim}{x})\right]=-i\,\partial^{k}_{x}\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})-i\,g\,f^{acb}\,\hat{\pi}^{c}_{j}(\underset{\sim}{x}).\,\hat{B}^{k,b}(\underset{\sim}{x})}\quad . \qquad (G.25)$$

Agora,

$$\left[\hat{\Theta}^{ok}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right]=\left[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right]\cdot\hat{F}^{kj,a}(\underset{\sim}{x})-\frac{1}{2}\left\{\hat{\pi}^{v}_{\psi_{s}}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right\}.\partial^{k}_{x}\hat{\psi}^{v}_{s}(\underset{\sim}{x})+$$

$$+\frac{1}{2}\left(\partial^{k}_{x}\left\{\hat{\pi}^{v}_{\psi_{s}}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right\}\right).\hat{\psi}^{v}_{s}(\underset{\sim}{x})+i\,g\left(\frac{\lambda^{a}}{2}\right)^{vw}\left\{\hat{\pi}^{v}_{\psi_{s}}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right\}.\hat{\psi}^{w}_{s}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})+$$

$$+\frac{i}{4}\left(\partial^{x}_{j}\left\{\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right\}\right).(\sigma^{jk})_{r's}\hat{\psi}^{v}_{s}(\underset{\sim}{x})+\frac{i}{4}\left\{\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right\}.(\sigma^{jk})_{r's}\,\partial^{x}_{j}\hat{\psi}^{v}_{s}(\underset{\sim}{x})\Longrightarrow$$

$$\boxed{\begin{aligned}\left[\hat{\Theta}^{ok}(\underset{\sim}{x}),\hat{\psi}^{u}_{r}(y)\right]&=g\left(\frac{\lambda}{2}\right)^{a\,uv}\hat{\psi}^{v}_{r}(y)\,R_{j}(\underset{\sim}{x};y)\,\hat{F}^{kj,a}(\underset{\sim}{x})-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-y)\,\partial^{k}_{x}\hat{\psi}^{u}_{r}(\underset{\sim}{x})+\\&+\frac{i}{2}\left(\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-y)\right)\hat{\psi}^{u}_{r}(\underset{\sim}{x})-g\left(\frac{\lambda^{a}}{2}\right)^{uw}\hat{\psi}^{w}_{r}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-y)\;-\\&-\frac{1}{4}\left(\partial^{x}_{j}\delta^{(3)}(\underset{\sim}{x}-y)\right)(\sigma^{jk})_{rs}\,\hat{\psi}^{u}_{s}(\underset{\sim}{x})-\frac{1}{4}\,\delta^{(3)}(\underset{\sim}{x}-y)(\sigma^{jk})_{rs}\,\partial^{x}_{j}\hat{\psi}^{u}_{s}(\underset{\sim}{x})\end{aligned}}\quad . \qquad (G.26)$$

Integrando (G.26) sobre $\underset{\sim}{x}$ e usando novamente (G.18), encontramos

$$\left[\hat{P}^{k},\hat{\psi}^{u}_{r}(y)\right]=g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\left(\hat{A}^{k,a}(y)+\hat{B}^{k,a}(y)\right)-i\,\partial^{k}_{y}\hat{\psi}^{u}_{r}(y)\;-$$

$$-g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\,\hat{A}^{k,a}(y)\qquad\Longrightarrow$$

$$\boxed{\left[\hat{P}^{k},\hat{\psi}^{u}(\underset{\sim}{x})\right]=-i\,\partial^{k}_{x}\hat{\psi}^{u}(\underset{\sim}{x})+g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}(\underset{\sim}{x})\,\hat{B}^{k,a}(\underset{\sim}{x})}\quad . \qquad (G.27)$$

Por último,

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right] = \left[\hat{\pi}^{a}_{\cdot j}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right].\hat{F}^{kj,a}(\underset{\sim}{x}) + \frac{1}{2}\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x}).\partial^{k}_{x}\left\{\hat{\psi}^{v}_{s}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right\} -$$

$$-\frac{1}{2}\left(\partial^{k}_{x}\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x})\right).\left\{\hat{\psi}^{v}_{s}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right\} - ig\left(\frac{\lambda^{a}}{2}\right)^{vw}\hat{\pi}^{v}_{\psi_s}(\underset{\sim}{x}).\left\{\hat{\psi}^{w}_{s}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right\}\hat{A}^{k,a}(\underset{\sim}{x}) -$$

$$-\frac{i}{4}\left(\partial^{x}_{\cdot j}\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x})\right).\left(\sigma^{jk}\right)_{r's}\left\{\hat{\psi}^{v}_{s}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right\} - \frac{i}{4}\hat{\pi}^{v}_{\psi_{r'}}(\underset{\sim}{x}).\left(\sigma^{jk}\right)_{r's}\partial^{x}_{\cdot j}\left\{\hat{\psi}^{v}_{s}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right\} \Longrightarrow$$

$$\boxed{\begin{aligned}&\left[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right] = -g\,\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{y})\left(\frac{\lambda^{a}}{2}\right)^{vu}R_{\cdot j}(\underset{\sim}{x};\underset{\sim}{y})\hat{F}^{kj,a}(\underset{\sim}{x}) + \frac{i}{2}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\,\partial^{k}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) -\\ &-\frac{i}{2}\left(\partial^{k}_{x}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\right)\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + g\,\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x})\left(\frac{\lambda^{a}}{2}\right)^{vu}\hat{A}^{k,a}(\underset{\sim}{x})\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) +\\ &+\frac{1}{4}\left(\partial^{x}_{\cdot j}\hat{\pi}^{u}_{\psi_s}(\underset{\sim}{x})\right)\left(\sigma^{jk}\right)_{sr}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}) + \frac{1}{4}\hat{\pi}^{u}_{\psi_s}(\underset{\sim}{x})\left(\sigma^{jk}\right)_{sr}\partial^{x}_{\cdot j}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{y}).\ (G.28)\end{aligned}}$$

Integrando (G.28) sobre $\underset{\sim}{x}$, ficamos com

$$\left[\hat{P}^{k}, \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y})\right] = -g\,\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{y})\left(\frac{\lambda^{a}}{2}\right)^{vu}\left(\hat{A}^{k,a}(\underset{\sim}{y}) + \hat{B}^{k,a}(\underset{\sim}{y})\right) - i\,\partial^{k}_{y}\hat{\pi}^{u}_{\psi_r}(\underset{\sim}{y}) +$$

$$+g\,\hat{\pi}^{v}_{\psi_r}(\underset{\sim}{y})\left(\frac{\lambda^{a}}{2}\right)^{vu}\hat{A}^{k,a}(\underset{\sim}{y}) \Longrightarrow$$

$$\boxed{\left[\hat{P}^{k}, \hat{\pi}^{u}_{\psi}(\underset{\sim}{x})\right] = -i\,\partial^{k}_{x}\hat{\pi}^{u}_{\psi}(\underset{\sim}{x}) - g\,\hat{\pi}^{v}_{\psi}(\underset{\sim}{x})\left(\frac{\lambda^{a}}{2}\right)^{vu}\hat{B}^{k,a}(\underset{\sim}{x})}\ . \qquad (G.29)$$

Passemos ao cálculo do efeito do gerador de rotações espaciais infinitesimais $\hat{J}^{k\ell}$ sobre cada campo básico. Desde (5.46),

$$[\hat{J}^{k\ell}, \hat{A}^{j,a}_{(y)}] = \int d^3_x \left\{ x^k [\hat{\Theta}^{0\ell}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(y)}] - x^\ell [\hat{\Theta}^{0k}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(y)}] \right\} =$$

$$= \int d^3_x \, x^k [\hat{\Theta}^{0\ell}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(y)}] - (k \leftrightarrow \ell) \quad , \qquad (G.30)$$

onde a notação ($k \leftrightarrow \ell$) significa o mesmo termo anterior (i.e., à esquerda) com a troca de k por ℓ e vice-versa. Fazendo uso de (G.12) e de (G.15)

$$\int d^3_x \, x^k [\hat{\Theta}^{0\ell}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(y)}] = \int d^3_x \, x^k \left[i \hat{F}^{j\ell,a}_{(\underset{\sim}{x})} \delta^{(3)}(\underset{\sim}{x}-y) + i \hat{F}^{m\ell,b}_{(\underset{\sim}{x})} \hat{D}^{j,ab}_{(y)} R_m(\underset{\sim}{x};y) \right] =$$

$$= i y^k \hat{F}^{j\ell,a}_{(y)} + i \hat{D}^{j,ab}_{(y)} \int d^3_x \, x^k \left[\partial^m_x \hat{A}^{\ell,b}_{(\underset{\sim}{x})} - \partial^\ell_x \hat{A}^{m,b}_{(\underset{\sim}{x})} \right] R_m(\underset{\sim}{x};y) \, . \qquad (G.31)$$

Usando (3.49) e (3.51), a integral acima pode ser escrita como

$$\int d^3_x \, x^k [\partial^m_x \hat{A}^{\ell,b}_{(\underset{\sim}{x})} - \partial^\ell_x \hat{A}^{m,b}_{(\underset{\sim}{x})}] R_m(\underset{\sim}{x};y) =$$

$$= \int d^3_x \, x^k \hat{A}^{\ell,b}_{(\underset{\sim}{x})} \partial^x_m R_m(\underset{\sim}{x};y) + \delta^{mk} \int d^3_x \, R_m(\underset{\sim}{x};y) \hat{A}^{\ell,b}_{(\underset{\sim}{x})} -$$

$$- \int d^3_x \, R_m(\underset{\sim}{x};y) \, x^k \partial^\ell_x \hat{A}^{m,b}_{(\underset{\sim}{x})} \quad =$$

$$= -y^k \hat{A}^{\ell,b}_{(y)} + \int d^3_x \, R_j(\underset{\sim}{x};y) [\delta^{jk} \delta^{\ell m} - \delta^{mj} x^k \partial^\ell_x] \hat{A}^{m,b}_{(\underset{\sim}{x})} \, . \qquad (G.32)$$

Levando (G.32) em (G.31)

$$\int d^3_x\, x^k [\hat{\Theta}^{0l}_{(\underset{\sim}{x})}, \hat{A}^{j,a}_{(y)}] = i y^k (\partial^j_y \hat{A}^{l,a}_{(y)} - \partial^l_y \hat{A}^{j,a}_{(y)} + g f^{abc} \hat{A}^{j,b}_{(y)} \hat{A}^{l,c}_{(y)}) +$$

$$+ i \partial^j_y (- y^k \hat{A}^{l,a}_{(y)} + \int d^3_x\, R_{j'}(\underset{\sim}{x};y) [\delta^{j'k}\delta^{lm} - \delta^{mj'} x^k \partial^l_x] \hat{A}^{m,a}_{(\underset{\sim}{x})}) +$$

$$+ i g f^{acb} \hat{A}^{j,c}_{(y)} (- y^k \hat{A}^{l,b}_{(y)} + \int d^3_x\, R_{j'}(\underset{\sim}{x};y) [\delta^{j'k}\delta^{lm} - \delta^{mj'} x^k \partial^l_x] \hat{A}^{m,b}_{(\underset{\sim}{x})}) =$$

$$= i \delta^{jk} \hat{A}^{l,a}_{(y)} - i y^k \partial^l_y \hat{A}^{j,a}_{(y)} +$$

$$+ i \hat{D}^{j,ab}_{(y)} (\int d^3_x\, R_{j'}(\underset{\sim}{x};y) [\delta^{j'k}\delta^{lm} - \delta^{mj'} x^k \partial^l_x] \hat{A}^{m,b}_{(\underset{\sim}{x})}) \quad \Longrightarrow$$

$\Longrightarrow$ (ver (G.30))

$$[\hat{J}^{kl}, \hat{A}^{j,a}_{(y)}] = i (y^l \partial^k_y - y^k \partial^l_y) \hat{A}^{j,a}_{(y)} + i \delta^{jk} \hat{A}^{l,a}_{(y)} - i \delta^{jl} \hat{A}^{k,a}_{(y)} +$$

$$+ i \hat{D}^{j,ab}_{(y)} \Big\{ \int d^3_x\, R_{j'}(\underset{\sim}{x};y) [\delta^{j'm} (x^l \partial^k_x - x^k \partial^l_x) + \delta^{j'k}\delta^{lm} -$$

$$- \delta^{j'l}\delta^{km}] \hat{A}^{m,b}_{(\underset{\sim}{x})} \Big\} \quad \Longrightarrow$$

$$\boxed{\begin{aligned} [\hat{J}^{kl}, \hat{A}^{j,a}_{(\underset{\sim}{x})}] = {} & i (x^l \partial^k - x^k \partial^l) \hat{A}^{j,a}_{(\underset{\sim}{x})} + i \delta^{jk} \hat{A}^{l,a}_{(\underset{\sim}{x})} - i \delta^{jl} \hat{A}^{k,a}_{(\underset{\sim}{x})} + \\ & + i \hat{D}^{j,ab}_{(\underset{\sim}{x})} \hat{C}^{kl,b}_{(\underset{\sim}{x})} \quad , \qquad (G.33) \end{aligned}}$$

onde introduzimos

$$\hat{C}^{kl,a} \equiv \int d^3_x\, R_j(y;\underset{\sim}{x}) [\delta^{jm} (y^l \partial^k_y - y^k \partial^l_y) + \delta^{jk}\delta^{lm} - \delta^{jl}\delta^{km}] \hat{A}^{m,a}_{(y)} . \quad (G.34)$$

De forma semelhante, computamos

$$\left[\hat{J}^{kl}, \hat{F}^{jm,a}_{(y)}\right] = \int d^3x \left\{ x^k \left[\hat{\Theta}^{ol}_{(\underset{\sim}{x})}, \hat{F}^{jm,a}_{(y)}\right] - x^l \left[\hat{\Theta}^{ok}_{(\underset{\sim}{x})}, \hat{F}^{jm,a}_{(y)}\right]\right\} =$$

$$= \int d^3x\, x^k \left[\hat{\Theta}^{ol}_{(\underset{\sim}{x})}, \hat{F}^{jm,a}_{(y)}\right] - (k \leftrightarrow l) \quad . \qquad (G.35)$$

Fazendo uso de (G.17) e (G.15)

$$\int d^3x\, x^k \left[\hat{\Theta}^{ol}_{(\underset{\sim}{x})}, \hat{F}^{jm,a}_{(y)}\right] =$$

$$= \int d^3x\, x^k \Big[i\left(\hat{F}^{lj,b}_{(\underset{\sim}{x})} \hat{D}^{m,ab}_{(y)} - \hat{F}^{lm,b}_{(\underset{\sim}{x})} \hat{D}^{j,ab}_{(y)}\right) \delta^{(3)}(\underset{\sim}{x} - y) +$$

$$+ i g f^{abc} \hat{F}^{jm,c}_{(y)} \hat{F}^{ln,b}_{(\underset{\sim}{x})} R_n(\underset{\sim}{x}; y) \Big] =$$

$$= i \hat{D}^{m,ab}_{(y)} \left(y^k \hat{F}^{lj,b}_{(y)}\right) - i \hat{D}^{j,ab}_{(y)} \left(y^k \hat{F}^{lm,b}_{(y)}\right) +$$

$$+ i g f^{abc} \hat{F}^{jm,c}_{(y)} \int d^3x\, x^k \left[\partial^l_x \hat{A}^{n,b}_{(\underset{\sim}{x})} - \partial^n_x \hat{A}^{l,b}_{(\underset{\sim}{x})}\right] R_n(\underset{\sim}{x}; y) =$$

$$= -i \delta^{mk} \hat{F}^{lj,a}_{(y)} + i \delta^{jk} \hat{F}^{lm,a}_{(y)} + i y^k \left(\hat{D}^{m,ab}_{(y)} \hat{F}^{lj,b}_{(y)} - \hat{D}^{j,ab}_{(y)} \hat{F}^{ln,b}_{(y)}\right)$$

$$+ i g f^{abc} \hat{F}^{jm,c}_{(y)} \left[y^k \hat{A}^{l,b}_{(y)} - \delta^{nk} \int d^3x\, \hat{A}^{l,b}_{(\underset{\sim}{x})} R_n(\underset{\sim}{x}; y)\right] +$$

$$+ i g f^{abc} \hat{F}^{jm,c}_{(y)} \int d^3x\, x^k \left(\partial^l_x \hat{A}^{n,b}_{(\underset{\sim}{x})}\right) R_n(\underset{\sim}{x}; y) =$$

$$= -i \delta^{mk} \hat{F}^{lj,a}_{(y)} + i \delta^{jk} \hat{F}^{lm,a}_{(y)} + i y^k \hat{D}^{l,ab}_{(y)} \hat{F}^{mj,b}_{(y)} +$$

$$+ igf^{abc}\hat{F}^{jm,c}_{(y)}\, y^k \hat{A}^{l,b}_{(y)} \;+$$

$$+ igf^{acb}\hat{F}^{jm,c}_{(y)} \int d^3x\, R_n(\underset{\sim}{x};y)\left[\delta^{nk}\delta^{lj'} - \delta^{nj'}x^k\partial^l_x\right]\hat{A}^{j',b}_{(\underset{\sim}{x})} =$$

$$= -i\delta^{mk}\hat{F}^{lj,a}_{(y)} + i\delta^{jk}\hat{F}^{lm,a}_{(y)} + iy^k\partial^l_y\hat{F}^{mj,a}_{(y)} \;+$$

$$+ igf^{acb}\hat{F}^{jm,c}_{(y)} \int d^3x\, R_n(\underset{\sim}{x};y)\left[\delta^{nk}\delta^{lj'} - \delta^{nj'}x^k\partial^l_x\right]\hat{A}^{j',b}_{(\underset{\sim}{x})} \quad , (G.36)$$

onde realizamos uma integração por partes usando (3.49) e (3.51) e também voltamos a utilizar a identidade de Bianchi (G.19). Desde (G.36) e (G.35), encontramos

$$[\hat{J}^{kl}, \hat{F}^{jm,a}_{(y)}] = i(y^l\partial^k_y - y^k\partial^l_y)\hat{F}^{jm,a}_{(y)} \;+$$

$$+ i(\delta^{mk}\hat{F}^{jl,a}_{(y)} - \delta^{ml}\hat{F}^{jk,a}_{(y)} - \delta^{jk}\hat{F}^{ml,a}_{(y)} + \delta^{jl}\hat{F}^{mk,a}_{(y)}) \;+$$

$$+ igf^{acb}\hat{F}^{jm,c}_{(y)} \int d^3x\, R_n(\underset{\sim}{x};y)\left[\delta^{nj'}(x^l\partial^k_x - x^k\partial^l_x) + \delta^{nk}\delta^{lj'} - \delta^{nl}\delta^{kj'}\right]\hat{A}^{j',b}_{(\underset{\sim}{x})} \quad \Longrightarrow$$

$$\boxed{\begin{aligned} [\hat{J}^{kl}, \hat{F}^{jm,a}_{(\underset{\sim}{x})}] &= i(x^l\partial^k - x^k\partial^l)\hat{F}^{jm,a}_{(\underset{\sim}{x})} \;+ \\ &+ i(\delta^{mk}\hat{F}^{jl,a}_{(\underset{\sim}{x})} - \delta^{ml}\hat{F}^{jk,a}_{(\underset{\sim}{x})} - \delta^{jk}\hat{F}^{ml,a}_{(\underset{\sim}{x})} + \delta^{jl}\hat{F}^{mk,a}_{(\underset{\sim}{x})}) \;+ \\ &+ igf^{acb}\hat{F}^{jm,c}_{(\underset{\sim}{x})}\hat{C}^{kl,b}_{(\underset{\sim}{x})} \quad . \end{aligned}} \qquad (G.37)$$

Por outro lado,

$$[\hat{J}^{kl}, \hat{\pi}^{a}_{j}(y)] = \int d^{3}x \left\{ x^{k}[\hat{\Theta}^{0l}(\underset{\sim}{x}), \hat{\pi}^{a}_{j}(y)] - x^{l}[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\pi}^{a}_{j}(y)] \right\} =$$

$$= \int d^{3}x \; x^{k}[\hat{\Theta}^{0l}(\underset{\sim}{x}), \hat{\pi}^{a}_{j}(y)] - (k \leftrightarrow l) \qquad (G.38)$$

Desde (G.24), (G.32) e da lei de Gauss (4.3), obtemos

$$\int d^{3}x \; x^{k}[\hat{\Theta}^{0l}(\underset{\sim}{x}), \hat{\pi}^{a}_{j}(y)] = \int d^{3}x \; x^{k}\Big[i \big(\hat{\pi}^{b}_{j}(\underset{\sim}{x}) \cdot \hat{D}^{l,ba}(\underset{\sim}{x}) - \delta^{lj} \hat{\pi}^{b}_{m}(\underset{\sim}{x}) \cdot \hat{D}^{m,ba}(\underset{\sim}{x})\big) \delta^{(3)}(\underset{\sim}{x}-y) +$$

$$+ i g f^{abc} \hat{F}^{lm,b}(\underset{\sim}{x}) \cdot \hat{\pi}^{c}_{j}(y) R_{m}(\underset{\sim}{x};y) + g \delta^{lj} \hat{\pi}_{\psi}(\underset{\sim}{x}) \cdot \frac{\lambda^{a}}{2} \hat{\psi}(\underset{\sim}{x}) \delta^{(3)}(\underset{\sim}{x}-y) \Big] =$$

$$= -i \delta^{lj} \hat{D}^{m,ab}(y) \cdot (y^{k} \hat{\pi}^{b}_{m}(y)) - i \hat{D}^{l,ab}(y) \cdot (y^{k} \hat{\pi}^{b}_{j}(y)) +$$

$$+ i g f^{abc} \hat{\pi}^{c}_{j}(y) \cdot \int d^{3}x \; x^{k} \left[\partial^{l}_{x} \hat{A}^{m,b}(\underset{\sim}{x}) - \partial^{m}_{x} \hat{A}^{l,b}(\underset{\sim}{x}) \right] R_{m}(\underset{\sim}{x};y) +$$

$$+ g \delta^{lj} y^{k} \hat{\pi}_{\psi}(y) \cdot \frac{\lambda^{a}}{2} \hat{\psi}(y) =$$

$$= -i \delta^{lj} \hat{\pi}^{a}_{k}(y) + i \delta^{lk} \hat{\pi}^{a}_{j}(y) - i y^{k} \hat{D}^{l,ab}(y) \cdot \hat{\pi}^{b}_{j}(y) +$$

$$+ i g f^{abc} \hat{\pi}^{c}_{j}(y) \cdot \left(y^{k} \hat{A}^{l,b}(y) - \int d^{3}x \; R_{n}(\underset{\sim}{x};y) \left[\delta^{nk} \delta^{lm} - \delta^{mn} x^{k} \partial^{l}_{x} \right] \hat{A}^{m,b}(\underset{\sim}{x}) \right) =$$

$$= -i \delta^{lj} \hat{\pi}^{a}_{k}(y) + i \delta^{lk} \hat{\pi}^{a}_{j}(y) - i y^{k} \partial^{l}_{y} \hat{\pi}^{a}_{j}(y) +$$

$$+ i g f^{acb} \hat{\pi}^{c}_{j}(y) \cdot \int d^{3}x \; R_{n}(\underset{\sim}{x};y) \left[\delta^{nk} \delta^{lm} - \delta^{mn} x^{k} \partial^{l}_{x} \right] \hat{A}^{m,b}(\underset{\sim}{x}) \Longrightarrow$$

$\Longrightarrow$ (ver (G.38))

$$[\hat{J}^{kl}, \hat{\pi}^a_j(y)] = -i\delta^{lj}\hat{\pi}^a_k(y) + i\delta^{kj}\hat{\pi}^a_l(y) + i(y^l\partial^k_y - y^k\partial^l_y)\hat{\pi}^a_j(y) +$$

$$+ igf^{acb}\hat{\pi}^c_j(y).\int d^3_x\, R_n(\underset{\sim}{x};y)\Big[\delta^{mn}(x^l\partial^k_x - x^k\partial^l_x) + \delta^{nk}\delta^{lm} -$$

$$-\delta^{nl}\delta^{km}\Big]\hat{A}^{m,b}(\underset{\sim}{x}) \qquad \Longrightarrow$$

$$\boxed{\begin{aligned}[\hat{J}^{kl}, \hat{\pi}^a_j(\underset{\sim}{x})] = i(x^l\partial^k - x^k\partial^l)\hat{\pi}^a_j(\underset{\sim}{x}) + i\delta^{jk}\hat{\pi}^a_l(\underset{\sim}{x}) - i\delta^{jl}\hat{\pi}^a_k(\underset{\sim}{x}) +\\ + igf^{acb}\hat{\pi}^c_j(\underset{\sim}{x}).\hat{C}^{kl,b}(\underset{\sim}{x}) \quad . \qquad (G.39)\end{aligned}}$$

Agora,

$$[\hat{J}^{kl}, \hat{\psi}^u_r(y)] = \int d^3_x\, x^k[\hat{\Theta}^{0l}(\underset{\sim}{x}), \hat{\psi}^u_r(y)] - (k\leftrightarrow l) \quad . (G.40)$$

Desde (G.26) e (G.32)

$$\int d^3_x\, x^k[\hat{\Theta}^{0l}(\underset{\sim}{x}), \hat{\psi}^u_r(y)] = \int d^3_x\, x^k\Big[g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_r(y)\, R_j(\underset{\sim}{x};y)\hat{F}^{lj,a}(\underset{\sim}{x}) -$$

$$-\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-y)\,\partial^l_x\hat{\psi}^u_r(\underset{\sim}{x}) + \frac{i}{2}\left(\partial^l_x\delta^{(3)}(\underset{\sim}{x}-y)\right)\hat{\psi}^u_r(\underset{\sim}{x}) - g\left(\frac{\lambda^a}{2}\right)^{uw}\hat{\psi}^w_r(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})\,\delta^{(3)}(x-y) -$$

$$-\frac{1}{4}\left(\partial^x_j\delta^{(3)}(\underset{\sim}{x}-y)\right)(\sigma^{jl})_{rs}\hat{\psi}^u_s(\underset{\sim}{x}) - \frac{1}{4}\delta^{(3)}(\underset{\sim}{x}-y)(\sigma^{jl})_{rs}\,\partial^x_j\hat{\psi}^u_s(\underset{\sim}{x})\Big] =$$

$$= \frac{i}{2}\delta^{lk}\hat{\psi}^u_r(y) - iy^k\partial^l_y\hat{\psi}^u_r(y) - g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_r(y)\hat{A}^{l,a}(y)\,y^k +$$

$$+\frac{\delta^{jk}}{4}(\sigma^{jl})_{rs}\hat{\psi}^{u}_{s}(y)+\frac{1}{4}y^{k}(\sigma^{jl})_{rs}\left(\partial^{y}_{j}\hat{\psi}^{u}_{s}(\underset{\sim}{x})\right)-\frac{1}{4}(\sigma^{jl})_{rs}\left(\partial^{y}_{j}\hat{\psi}^{u}_{s}(y)\right)y^{k}+$$

$$+g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\int d^{3}_{x}\;x^{k}R_{j}(\underset{\sim}{x};y)\left(\partial^{l}_{x}\hat{A}^{j,a}(\underset{\sim}{x})-\partial^{j}_{x}\hat{A}^{l,a}(\underset{\sim}{x})\right)=$$

$$=\frac{i}{2}\delta^{lk}\hat{\psi}^{u}_{r}(y)-iy^{k}\partial^{l}_{y}\hat{\psi}^{u}_{r}(y)-g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\hat{A}^{l,a}(y)y^{k}+$$

$$+\frac{1}{4}(\sigma^{kl})_{rs}\hat{\psi}^{u}_{s}(y)+g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\Big\{y^{k}\hat{A}^{l,a}(y)-\int d^{3}_{x}R_{j}(\underset{\sim}{x};y)\Big[\delta^{jk}\delta^{lm}-$$

$$-\delta^{mj}x^{k}\partial^{l}_{x}\Big]\hat{A}^{m,a}(\underset{\sim}{x})\Big\}=$$

$$=\frac{i}{2}\delta^{lk}\hat{\psi}_{r}(y)-iy^{k}\partial^{l}_{y}\hat{\psi}^{u}_{r}(y)+\frac{1}{4}(\sigma^{kl})_{rs}\hat{\psi}^{u}_{s}(y)-$$

$$-g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\int d^{3}_{x}R_{j}(\underset{\sim}{x};y)\left[\delta^{jk}\delta^{lm}-\delta^{mj}x^{k}\partial^{l}_{x}\right]\hat{A}^{m,a}(\underset{\sim}{x})\quad .(G.41)$$

Levando (G.41) em (G.40), chegamos a

$$[\hat{J}^{kl},\hat{\psi}^{u}_{r}(y)]=i\left(y^{l}\partial^{k}_{y}-y^{k}\partial^{l}_{y}\right)\hat{\psi}^{u}_{r}(y)+\frac{1}{4}(\sigma^{kl})_{rs}\hat{\psi}^{u}_{s}(y)-\frac{1}{4}(\sigma^{lk})_{rs}\hat{\psi}^{u}_{s}(y)-$$

$$-g\left(\frac{\lambda^{a}}{2}\right)^{uv}\hat{\psi}^{v}_{r}(y)\int d^{3}_{x}R_{j}(\underset{\sim}{x};y)\left[\delta^{mj}\left(x^{l}\partial^{k}_{x}-x^{k}\partial^{l}_{x}\right)+\delta^{jk}\delta^{lm}-\delta^{jl}\delta^{km}\right]\hat{A}^{m,a}(\underset{\sim}{x})\Longrightarrow$$

$\Longrightarrow$

$$\boxed{\begin{aligned}[\hat{J}^{kl},\hat{\psi}(\underset{\sim}{x})]&=i\left(x^{l}\partial^{k}-x^{k}\partial^{l}\right)\hat{\psi}(\underset{\sim}{x})+\frac{1}{2}\sigma^{kl}\hat{\psi}(\underset{\sim}{x})-\\&-g\,\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\,\hat{C}^{kl,a}(\underset{\sim}{x})\quad .\end{aligned}}\qquad \textbf{(G.42)}$$

Por último,

$$[\hat{J}^{k\ell}, \hat{\pi}^{u}_{\psi_r}(y)] = \int d^3x \, x^k [\hat{\Theta}^{0\ell}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(y)] - (k \leftrightarrow \ell) \quad . \quad (G.43)$$

Desde (G.28) e (G.32)

$$\int d^3x \, x^k [\hat{\Theta}^{0\ell}(\underset{\sim}{x}), \hat{\pi}^{u}_{\psi_r}(y)] = \int d^3x \, x^k \Big[-g \, \hat{\pi}^{v}_{\psi}(y) \left(\frac{\lambda^a}{2}\right)^{vu} R_j(\underset{\sim}{x};y) \, \hat{F}^{\ell j,a}(\underset{\sim}{x}) \; +$$

$$+ \frac{i}{2} \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x}) \partial^\ell_x \delta^{(3)}(\underset{\sim}{x}-y) - \frac{i}{2}\left(\partial^\ell_x \hat{\pi}^{u}_{\psi_r}(\underset{\sim}{x})\right) \delta^{(3)}(\underset{\sim}{x}-y) + g \, \hat{\pi}^{v}_{\psi_r}(\underset{\sim}{x}) \left(\frac{\lambda^a}{2}\right)^{vu} \hat{A}^{\ell,a}(\underset{\sim}{x}) \, \delta^{(3)}(\underset{\sim}{x}-y) +$$

$$+ \frac{1}{4}\left(\partial^r_j \hat{\pi}^{u}_{\psi_s}(\underset{\sim}{x})\right)(\sigma^{j\ell})_{sr} \, \delta^{(3)}(\underset{\sim}{x}-y) + \frac{1}{4} \hat{\pi}^{k}_{\psi_s}(\underset{\sim}{x}) (\sigma^{j\ell})_{sr} \, \partial^r_j \delta^{(3)}(\underset{\sim}{x}-y) \Big] =$$

$$= \frac{i}{2} \delta^{\ell k} \hat{\pi}^{u}_{\psi_r}(y) - i \, y^k \partial^\ell_y \hat{\pi}^{u}_{\psi_r}(y) \; + g \, \hat{\pi}^{v}_{\psi_r}(y) \left(\frac{\lambda^a}{2}\right)^{vu} \hat{A}^{\ell,a}(y) y^k -$$

$$- \frac{1}{4} \hat{\pi}^{u}_{\psi_s}(y) (\sigma^{k\ell})_{sr} \; - g \, \hat{\pi}^{v}_{\psi}(y) \left(\frac{\lambda^a}{2}\right)^{vu} \int d^3x \, x^k \left[\partial^\ell_x \hat{A}^{j,a}(\underset{\sim}{x}) - \partial^j_x \hat{A}^{\ell,a}(\underset{\sim}{x}) \right] R_j(\underset{\sim}{x};y) =$$

$$= \frac{i}{2} \delta^{\ell k} \hat{\pi}^{u}_{\psi_r}(y) - i y^k \partial^\ell_y \hat{\pi}^{u}_{\psi_r}(y) + g \, \hat{\pi}^{v}_{\psi_r}(y) \left(\frac{\lambda^a}{2}\right)^{vu} \hat{A}^{\ell,a}(y) \, y^k -$$

$$- \frac{1}{4} \hat{\pi}^{u}_{\psi_s}(y) (\sigma^{k\ell})_{sr} + g \, \hat{\pi}^{v}_{\psi}(y) \left(\frac{\lambda^a}{2}\right)^{vu} \Big[-y^k \hat{A}^{\ell,a}(y) \; +$$

$$+ \int d^3x \, R_j(\underset{\sim}{x};y) \left[\delta^{jk} \delta^{\ell m} - \delta^{mj} x^k \partial^\ell_x \right] \hat{A}^{m,a}(\underset{\sim}{x}) \Big] \; =$$

$$= \frac{i}{2} \delta^{\ell k} \hat{\pi}^{u}_{\psi_r}(y) - i y^k \partial^\ell_y \hat{\pi}^{u}_{\psi_r}(y) - \frac{1}{4} \hat{\pi}^{u}_{\psi_s}(y) (\sigma^{k\ell})_{sr} \; +$$

$$+ g \, \hat{\pi}^{v}_{\psi}(y) \left(\frac{\lambda^a}{2}\right)^{vu} \int d^3x \, R_j(\underset{\sim}{x};y) \left[\delta^{jk} \delta^{\ell m} - \delta^{mj} x^k \partial^\ell_x \right] \hat{A}^{m,a}(\underset{\sim}{x}) \quad . \quad (G.44)$$

Substituindo (G.44) em (G.43), ficamos com

$$[\hat{J}^{kl}, \hat{\pi}^{u}_{\psi_r}(y)] = i(y^l \partial^k_y - y^k \partial^l_y)\hat{\pi}^{u}_{\psi_r}(y) - \frac{1}{4}\hat{\pi}^{u}_{\psi_s}(y)(\sigma^{kl})_{sr} +$$

$$+ \frac{1}{4}\hat{\pi}^{u}_{\psi_s}(y)(\sigma^{lk})_{sr} + g\hat{\pi}^{v}_{\psi}(y)\left(\frac{\lambda^a}{2}\right)^{vu}\int d^3_x R_j(x;y)[\delta^{nj}(x^l\partial^k_x - x^k\partial^l_x) +$$

$$+ \delta^{jk}\delta^{ln} - \delta^{jl}\delta^{kn}]\hat{A}^{n,a}(x) \Longrightarrow$$

$$\boxed{\begin{aligned}[\hat{J}^{kl}, \hat{\pi}_{\psi}(x)] = {} & i(x^l\partial^k - x^k\partial^l)\hat{\pi}_{\psi}(x) - \frac{1}{2}\hat{\pi}_{\psi}(x)\sigma^{kl} + \\ & + g\,\hat{\pi}_{\psi}(x)\frac{\lambda^a}{2}\hat{C}^{kl,a}(x) \; .\end{aligned}} \quad \text{(G.45)}$$

A seguir, computamos o efeito do gerador de transformações de Lorentz puras $\hat{J}^{0k} = x^0\hat{P}^k - \int d^3x\, x^k\hat{\Theta}^{00}(x)$ sobre cada campo básico. Desde (G.16) e (G.1), segue que $(y^0 = x^0)$

$$[\hat{J}^{0k}, \hat{A}^{j,a}(y)] = x^0[\hat{P}^k, \hat{A}^{j,a}(y)] - \int d^3_x\, x^k[\hat{\Theta}^{00}(x), \hat{A}^{j,a}(y)] =$$

$$= x^0\left(-i\partial^k_y \hat{A}^{j,a}(y) - i\hat{D}^{j,ab}(y)\hat{B}^{k,b}(y)\right) - \int d^3_x\, x^k\Big[-i\hat{\pi}^{a}_{j}(x)\delta^{(3)}(x-y) -$$

$$- i\left(\hat{D}^{j,ab}(y) R_l(x;y)\right)\cdot\hat{\pi}^{b}_{l}(x)\Big] \Longrightarrow$$

$$\boxed{\begin{aligned}[\hat{J}^{0k}, \hat{A}^{j,a}(x)] = {} & -ix^0(\partial^k_x\hat{A}^{j,a}(x) + \hat{D}^{j,ab}(x)\hat{B}^{k,b}(x)) + i\,x^k\hat{\pi}^{a}_{j}(x) + \\ & + i\,\hat{D}^{j,ab}(x)\cdot\hat{E}^{k,b}(x) \quad ,\end{aligned}} \quad \text{(G.46)}$$

onde introduzimos

$$\hat{E}^{k,a}_{(\underset{\sim}{x})} \equiv \int d^3_y\, y^k\, R_\ell(y;\underset{\sim}{x})\, \hat{\pi}^a_\ell(y) \quad . \qquad (G.47)$$

Similarmente, desde (G.20) e (G.4), segue

$$[\hat{J}^{0k}, \hat{F}^{j\ell,a}_{(y)}] = x^0[\hat{P}^k, \hat{F}^{j\ell,a}_{(y)}] - \int d^3_x\, x^k [\hat{\Theta}^{00}_{(\underset{\sim}{x})}, \hat{F}^{j\ell,a}_{(y)}] =$$

$$= x^0\left(-i\partial^k_y \hat{F}^{j\ell,a}_{(y)} - igf^{acb}\hat{F}^{j\ell,c}_{(y)}\hat{B}^{k,b}_{(y)}\right) -$$

$$-\int d^3_x\, x^k \Big[-i\hat{\pi}^b_\ell(\underset{\sim}{x}).\hat{D}^{j,ab}_{(y)}\delta^{(3)}(\underset{\sim}{x}-y) + i\hat{\pi}^b_j(\underset{\sim}{x}).\hat{D}^{\ell,ab}_{(y)}\delta^{(3)}(\underset{\sim}{x}-y) -$$

$$-igf^{acb}\hat{F}^{j\ell,c}_{(y)}.\hat{\pi}^b_m(\underset{\sim}{x})\, R_m(\underset{\sim}{x};y)\Big] =$$

$$= -ix^0\left(\partial^k_y \hat{F}^{j\ell,a}_{(y)} + gf^{acb}\hat{F}^{j\ell,c}_{(y)}\hat{B}^{k,b}_{(y)}\right) +$$

$$+ iy^k\left(\hat{D}^{j,ab}_{(y)}.\hat{\pi}^b_\ell(y) - \hat{D}^{\ell,ab}_{(y)}.\hat{\pi}^b_j(y)\right) +$$

$$+ igf^{acb}\hat{F}^{j\ell,c}_{(y)}.\int d^3_x\, x^k R_m(\underset{\sim}{x};y)\,\hat{\pi}^b_m(\underset{\sim}{x}) \quad \Longrightarrow$$

$$\boxed{\begin{aligned}&[\hat{J}^{0k}, \hat{F}^{j\ell,a}_{(\underset{\sim}{x})}] = -ix^0\left(\partial^k_x \hat{F}^{j\ell,a}_{(\underset{\sim}{x})} + gf^{acb}\hat{F}^{j\ell,c}_{(\underset{\sim}{x})}\hat{B}^{k,b}_{(\underset{\sim}{x})}\right) +\\ &+ ix^k\left(\hat{D}^{j,ab}_{(\underset{\sim}{x})}.\hat{\pi}^b_\ell(\underset{\sim}{x}) - \hat{D}^{\ell,ab}_{(\underset{\sim}{x})}.\hat{\pi}^b_j(\underset{\sim}{x})\right) +\\ &+ igf^{acb}\hat{F}^{j\ell,c}_{(\underset{\sim}{x})}.\,\hat{E}^{k,b}_{(\underset{\sim}{x})} \quad .\end{aligned}} \qquad (G.48)$$

Por outro lado, a partir de (G.25) e (G.6), encontramos

$$[\hat{J}^{0k}, \hat{\pi}^{a}_{\cdot j}(y)] = x^0[\hat{P}^k, \hat{\pi}^{a}_{\cdot j}(y)] - \int d^3_x\, x^k[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{\pi}^{a}_{\cdot j}(y)] =$$

$$= x^0\left(-i\partial^k_y \hat{\pi}^{a}_{\cdot j}(y) - igf^{acb}\hat{\pi}^{c}_{\cdot j}(y).\hat{B}^{k,b}(y)\right) - \int d^3_x\, x^k\Big[igf^{abc}\hat{\pi}^{c}_{\cdot j}(y).\hat{\pi}^{b}_{m}(\underset{\sim}{x})R_m(\underset{\sim}{x};y) -$$

$$- i\hat{F}^{lj,b}(\underset{\sim}{x})\hat{D}^{l,ab}(y)\delta^{(3)}(\underset{\sim}{x}-y) + g\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\delta^{(3)}(\underset{\sim}{x}-y)\Big] =$$

$$= -ix^0\left(\partial^k_y\hat{\pi}^{a}_{\cdot j}(y) + gf^{acb}\hat{\pi}^{c}_{\cdot j}(y).\hat{B}^{k,b}(y)\right) - iy^k\hat{D}^{l,ab}(y)\hat{F}^{jl,b}(y) -$$

$$- gy^k\hat{\pi}_{\psi}(y).\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(y) + igf^{acb}\hat{\pi}^{c}_{\cdot j}(y).\int d^3_x\, x^k R_m(\underset{\sim}{x};y)\hat{\pi}^{b}_{m}(\underset{\sim}{x}) \Rightarrow$$

$$\boxed{\begin{aligned}[\hat{J}^{0k}, \hat{\pi}^{a}_{\cdot j}(\underset{\sim}{x})] = {} & -ix^0\left(\partial^k_x\hat{\pi}^{a}_{\cdot j}(\underset{\sim}{x}) + gf^{acb}\hat{\pi}^{c}_{\cdot j}(\underset{\sim}{x}).\hat{B}^{k,b}(\underset{\sim}{x})\right) - \\ & - ix^k\hat{D}^{l,ab}(\underset{\sim}{x})\hat{F}^{jl,b}(\underset{\sim}{x}) - gx^k\hat{\pi}_{\psi}(\underset{\sim}{x}).\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}) + \\ & + igf^{acb}\hat{\pi}^{c}_{\cdot j}(\underset{\sim}{x}).\hat{E}^{k,b}(\underset{\sim}{x}) \quad .\end{aligned}} \qquad (G.49)$$

Agora, desde (G.27) e (G.8), segue

$$[\hat{J}^{0k}, \hat{\psi}^{u}(y)] = x^0[\hat{P}^k, \hat{\psi}^{u}(y)] - \int d^3_x\, x^k[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{\psi}^{u}(y)] =$$

$$= x^0\left(-i\partial^k_y\hat{\psi}^{u}(y) + g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}(y)\hat{B}^{k,a}(y)\right) - \int d^3_x\, x^k\Big[g\hat{\pi}^{a}_{j}(\underset{\sim}{x}).\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}(y)R_{\cdot j}(\underset{\sim}{x};y) +$$

$$+ \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-y)\gamma^0\gamma^l\partial^r_l\hat{\psi}^{u}(\underset{\sim}{x}) - \frac{i}{2}\left(\partial^r_l\delta^{(3)}(\underset{\sim}{x}-y)\right)\gamma^0\gamma^l\hat{\psi}^{u}(\underset{\sim}{x}) - g\delta^{(3)}(\underset{\sim}{x}-y)\gamma^0\gamma^l\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^{v}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}) -$$

$$- m\delta^{(3)}(\underset{\sim}{x}-y)\gamma^0\hat{\psi}^{u}(\underset{\sim}{x})\Big] =$$

$$= -ix^0\left(\partial^k_y\hat{\psi}^u_{(y)} + ig\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_{(y)}\hat{B}^{k,a}_{(y)}\right) - iy^k\gamma^0\gamma^l\partial^y_l\hat{\psi}_{(y)} -$$

$$-\frac{i}{2}\gamma^0\gamma^k\hat{\psi}^u_{(y)} + g\,y^k\gamma^0\gamma^l\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_{(y)}\hat{A}^{l,a}_{(y)} + my^k\gamma^0\hat{\psi}^u_{(y)} -$$

$$-g\left(\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_{(y)}\cdot\int d^3x\,x^k R_j(\underset{\sim}{x};y)\,\hat{\pi}^a_j(\underset{\sim}{x}) \qquad \Longrightarrow$$

$$\boxed{\begin{aligned}[\hat{J}^{0k},\hat{\psi}(\underset{\sim}{x})] &= -ix^0\left(\partial^k_x\hat{\psi}(\underset{\sim}{x}) + ig\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{B}^{k,a}(\underset{\sim}{x})\right) - ix^k\gamma^0\gamma^l\partial^x_l\hat{\psi}(\underset{\sim}{x}) - \\ &-\frac{i}{2}\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}) + g\,x^k\gamma^0\gamma^l\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}) + mx^k\gamma^0\hat{\psi}(\underset{\sim}{x}) - g\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\cdot\hat{E}^{ka}(\underset{\sim}{x})\end{aligned}}.$$

(G.50)

Por último, (G.29) e (G.10) implicam

$$[\hat{J}^{0k},\hat{\pi}^u_\psi(y)] = x^0[\hat{P}^k,\hat{\pi}^u_\psi(y)] - \int d^3x\,x^k[\hat{\Theta}^{00}(\underset{\sim}{x}),\hat{\pi}^u_\psi(y)] =$$

$$= x^0\left(-i\partial^k_y\hat{\pi}^u_\psi(y) - g\,\hat{\pi}^v_\psi(y)\left(\frac{\lambda^a}{2}\right)^{vu}\hat{B}^{k,a}(y)\right) - \int d^3x\,x^k\Big[-g\,\hat{\pi}^v_\psi(y)\left(\frac{\lambda^a}{2}\right)^{vu}\cdot\hat{\pi}^a_j(\underset{\sim}{x})R_j(\underset{\sim}{x};y) -$$

$$-\frac{i}{2}\hat{\pi}^u_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\partial^x_l\delta^{(3)}(\underset{\sim}{x}-y) + \frac{i}{2}\left(\partial^x_l\hat{\pi}^u_\psi(\underset{\sim}{x})\right)\gamma^0\gamma^l\delta^{(3)}(\underset{\sim}{x}-y) +$$

$$+g\,\hat{\pi}^v_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^l\left(\frac{\lambda^a}{2}\right)^{vu}\hat{A}^{l,a}(\underset{\sim}{x})\,\delta^{(3)}(\underset{\sim}{x}-y) + n\,\hat{\pi}^u_\psi(\underset{\sim}{x})\gamma^0\delta^{(3)}(\underset{\sim}{x}-y)\Big] =$$

$$= -ix^0\left(\partial^k_y\hat{\pi}^u_\psi(y) - ig\,\hat{\pi}^v_\psi(y)\left(\frac{\lambda^a}{2}\right)^{vu}\hat{B}^{k,a}(y)\right) - iy^k\left(\partial^y_l\hat{\pi}^u_\psi(y)\right)\gamma^0\gamma^l -$$

$$-\frac{i}{2}\hat{\pi}^u_\psi(y)\gamma^0\gamma^k - g\,y^k\hat{\pi}^v_\psi(y)\gamma^0\gamma^l\left(\frac{\lambda^a}{2}\right)^{vu}\hat{A}^{l,a}(y) - my^k\hat{\pi}^u_\psi(y)\gamma^0 +$$

$$+ g\, \hat{\pi}_{\psi}^{v}(y)\left(\frac{\lambda^a}{2}\right)^{vu} \cdot \int d^3x\, x^k R_{j}(\underset{\sim}{x};y)\, \hat{\pi}_{j}^{a}(y) \qquad \Longrightarrow$$

$$\boxed{\begin{aligned} \left[\hat{J}^{0k}, \hat{\pi}_{\psi}(\underset{\sim}{x})\right] = & -ix^0\left(\partial_x^k \hat{\pi}_{\psi}(\underset{\sim}{x}) - ig\,\hat{\pi}_{\psi}(\underset{\sim}{x})\frac{\lambda^a}{2}\hat{B}^{k,a}(\underset{\sim}{x})\right) - ix^k\left(\partial_l^x \hat{\pi}_{\psi}(\underset{\sim}{x})\right)\gamma^0\gamma^l - \\ & -\frac{i}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\gamma^0\gamma^k - g x^k \hat{\pi}_{\psi}(\underset{\sim}{x})\gamma^0\gamma^l\frac{\lambda^a}{2}\hat{A}^{l,a}(\underset{\sim}{x}) - m x^k \hat{\pi}_{\psi}(\underset{\sim}{x})\gamma^0 + g\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{E}^{k,a}(\underset{\sim}{x}) \; . \end{aligned}}$$

(G.51)

APÊNDICE H

PROVA DAS EXPRESSÕES (5.69) E (5.74)

Provaremos (5.69) da seguinte forma. Mostraremos que o lado direito coincide com a soma de nossos resultados (5.68), (5.59) e (5.58). Desde (5.7)-(5.9) é claro que

$$\frac{i}{2}\Big(\partial_x^j \overset{(3)}{\delta}(x-x')\Big)\Big[\hat{\Theta}^{kj}(\underset{\sim}{x})+\hat{\Theta}^{kj}(\underset{\sim}{x}')\Big]=\frac{i}{2}\partial_x^j\overset{(3)}{\delta}(x-x')\Big\{\hat{F}^{km,a}(\underset{\sim}{x})\hat{F}^{jm,a}(\underset{\sim}{x})-\hat{\pi}_k^a(\underset{\sim}{x})\hat{\pi}_j^a(\underset{\sim}{x})+$$

$$+\delta^{jk}\Big(\frac{1}{2}\hat{\pi}_l^a(\underset{\sim}{x})\hat{\pi}_l^a(\underset{\sim}{x})-\frac{1}{4}\hat{F}^{lm,a}(\underset{\sim}{x})\hat{F}^{lm,a}(\underset{\sim}{x})\Big)+\frac{1}{4}\Big[\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\partial_x^k\hat{\psi}(\underset{\sim}{x})-(\partial_x^k\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x})+$$

$$+\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\partial_x^j\hat{\psi}(\underset{\sim}{x})-(\partial_x^j\hat{\pi}_\psi(\underset{\sim}{x}))\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x})\Big]-\frac{ig}{2}\Big[\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})+$$

$$+\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\Big]+\hat{F}^{km,a}(\underset{\sim}{x}')\hat{F}^{jm,a}(\underset{\sim}{x}')-\hat{\pi}_k^a(\underset{\sim}{x}')\hat{\pi}_j^a(\underset{\sim}{x}')+$$

$$+\delta^{jk}\Big(\frac{1}{2}\hat{\pi}_l^a(\underset{\sim}{x}')\hat{\pi}_l^a(\underset{\sim}{x}')-\frac{1}{4}\hat{F}^{lm,a}(\underset{\sim}{x}')\hat{F}^{lm,a}(\underset{\sim}{x}')\Big)+\frac{1}{4}\Big[\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^j\partial_{x'}^k\hat{\psi}(\underset{\sim}{x}')-(\partial_{x'}^k\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}')+$$

$$+\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^k\partial_{x'}^j\hat{\psi}(\underset{\sim}{x}')-(\partial_{x'}^j\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}')\Big]-\frac{ig}{2}\Big[\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x}')+$$

$$+\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x}')\Big]\Big\}=$$

$$=\frac{i}{2}\partial_x^j\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\hat{F}^{km,a}(\underset{\sim}{x})\hat{F}^{jm,a}(\underset{\sim}{x}')+\hat{F}^{km,a}(\underset{\sim}{x}')\hat{F}^{jm,a}(\underset{\sim}{x})-\pi_k^a(\underset{\sim}{x})\hat{\pi}_j^a(\underset{\sim}{x}')-\hat{\pi}_k^a(\underset{\sim}{x}')\hat{\pi}_j^a(\underset{\sim}{x})+$$

$$+\delta^{jk}\Big(\hat{\pi}_l^a(\underset{\sim}{x})\hat{\pi}_l^a(\underset{\sim}{x}')-\frac{1}{2}\hat{F}^{lm,a}(\underset{\sim}{x})\hat{F}^{lm,a}(\underset{\sim}{x}')\Big)\quad+$$

$$+\frac{1}{4}\Big[\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\partial^{k}_{x'}\hat{\psi}(\underset{\sim}{x}')+\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})-(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x}')-(\partial^{k}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x})+$$

$$+\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\partial^{j}_{x'}\hat{\psi}(\underset{\sim}{x}')+\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{k}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x})-(\partial^{j}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x}')-(\partial^{j}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x})\Big]-$$

$$-\frac{ig}{2}\Big[\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}')+\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})+\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x}')+$$

$$+\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x})\Big]\Bigg\} \quad , \qquad (H.1)$$

onde usamos a propriedade (5.17). Desde (5.1)-(5.3) e (5.17), segue também

$$\frac{i}{2}\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{\Theta}^{00}(\underset{\sim}{x})+\hat{\Theta}^{00}(\underset{\sim}{x}')\Big]=\frac{i}{2}\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\frac{1}{2}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x})+\frac{1}{4}\hat{F}^{jl,a}(\underset{\sim}{x})\hat{F}^{jl,a}(\underset{\sim}{x})-$$

$$-\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})+\frac{1}{2}(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x})-ig\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})-$$

$$-im\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x})+\frac{1}{2}\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\hat{\pi}^{a}_{j}(\underset{\sim}{x}')+\frac{1}{4}\hat{F}^{jl,a}(\underset{\sim}{x}')\hat{F}^{jl,a}(\underset{\sim}{x}')-\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}')+$$

$$+\frac{1}{2}(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x}')-ig\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x}')-im\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')\Big\}=$$

$$=\frac{i}{2}\partial^{k}_{x}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\hat{\pi}^{a}_{j}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x}')+\frac{1}{2}\hat{F}^{jl,a}(\underset{\sim}{x})\hat{F}^{jl,a}(\underset{\sim}{x}')-\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}')-$$

$$-\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})+\frac{1}{2}(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x}')+\frac{1}{2}(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}'))\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x})-$$

$$-ig\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x}')-ig\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x})-im\,\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x}')-im\,\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x})\Big\}. \quad (H.2)$$

Além disso, (5.1)-(5.3) e (5.7)-(5.9) implicam

$$-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\partial^k_x\hat{\Theta}^{00}(\underset{\sim}{x})+\partial^x_j\hat{\Theta}^{jk}(\underset{\sim}{x})\right]=-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\hat{\pi}^a_j(\underset{\sim}{x})\partial^k_x\hat{\pi}^a_j(\underset{\sim}{x})+\frac{1}{2}\hat{F}^{j\ell,a}(\underset{\sim}{x})\partial^k_x\hat{F}^{j\ell,a}(\underset{\sim}{x})-$$

$$-\frac{1}{2}(\partial^k_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^j(\partial^x_j\hat{\psi}(\underset{\sim}{x}))-\frac{1}{2}\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^j(\partial^k_x\partial^x_j\hat{\psi}(\underset{\sim}{x}))+\frac{1}{2}(\partial^k_x\partial^x_j\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x})+$$

$$+\frac{1}{2}(\partial^x_j\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^j(\partial^k_x\hat{\psi}(\underset{\sim}{x}))-ig(\partial^k_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})-ig\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}(\partial^k_x\hat{\psi}(\underset{\sim}{x}))\hat{A}^{j,a}(\underset{\sim}{x})-$$

$$-ig\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})(\partial^k_x\hat{A}^{j,a}(\underset{\sim}{x}))-im(\partial^k_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\hat{\psi}(\underset{\sim}{x})-im\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0(\partial^k_x\hat{\psi}(\underset{\sim}{x}))+$$

$$+(\partial^x_j\hat{F}^{j\ell,a}(\underset{\sim}{x}))\hat{F}^{k\ell,a}(\underset{\sim}{x})+\hat{F}^{j\ell,a}(\underset{\sim}{x})\partial^x_j\hat{F}^{k\ell,a}(\underset{\sim}{x})-(\partial^x_j\hat{\pi}^a_j(\underset{\sim}{x}))\hat{\pi}^a_k(\underset{\sim}{x})-\hat{\pi}^a_j(\underset{\sim}{x})\partial^x_j\hat{\pi}^a_k(\underset{\sim}{x})+$$

$$+\hat{\pi}^a_\ell(\underset{\sim}{x})\partial^x_k\hat{\pi}^a_\ell(\underset{\sim}{x})-\frac{1}{2}\hat{F}^{\ell m,a}(\underset{\sim}{x})\partial^x_k\hat{F}^{\ell m,a}(\underset{\sim}{x})+\frac{1}{4}\Big[(\partial^x_j\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^k\partial^j_x\hat{\psi}(\underset{\sim}{x})+$$

$$+\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^k\partial^x_j\partial^j_x\hat{\psi}(\underset{\sim}{x})-(\partial^x_j\partial^j_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x})-(\partial^j_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^k\partial^x_j\hat{\psi}(\underset{\sim}{x})+$$

$$+(\partial^x_j\hat{\pi}_+(\underset{\sim}{x}))\gamma^0\gamma^j\partial^k_x\hat{\psi}(\underset{\sim}{x})+\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^j\partial^x_j\partial^k_x\hat{\psi}(\underset{\sim}{x})-(\partial^x_j\partial^k_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x})-$$

$$-(\partial^k_x\hat{\pi}_+(\underset{\sim}{x}))\cdot\gamma^0\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x})\Big]-\frac{ig}{2}\Big[\partial^x_j\Big(\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\Big)+$$

$$+\partial^x_j\Big(\hat{\pi}_+(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\Big)\Big]\Big\}\qquad\therefore$$

$$-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\left[\partial^k_x\hat{\Theta}^{00}(\underset{\sim}{x})+\partial^x_j\hat{\Theta}^{jk}(\underset{\sim}{x})\right]=-\frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\hat{F}^{j\ell,a}(\underset{\sim}{x})\partial^k_x\hat{F}^{j\ell,a}(\underset{\sim}{x})+$$

$$+(\partial^x_j\hat{F}^{j\ell,a}(\underset{\sim}{x}))\hat{F}^{k\ell,a}(\underset{\sim}{x})+\hat{F}^{j\ell,a}(\underset{\sim}{x})\partial^x_j\hat{F}^{k\ell,a}(\underset{\sim}{x})+(\partial^j_x\hat{\pi}^a_j(\underset{\sim}{x}))\hat{\pi}^a_k(\underset{\sim}{x})+\hat{\pi}^a_j(\underset{\sim}{x})\partial^j_x\hat{\pi}^a_k(\underset{\sim}{x})-$$

$$-ig\,\partial^{k}_{x}\Big(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\Big)-i\frac{g}{2}\,\partial^{x}_{j}\Big(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\Big)-$$

$$-i\frac{g}{2}\,\partial^{x}_{j}\Big(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\Big)-\frac{3}{4}\big(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\big)\cdot\gamma^{0}\gamma^{j}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})\;-$$

$$-\frac{1}{4}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\big(\partial^{x}_{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})\big)+\frac{1}{4}\big(\partial^{x}_{j}\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\big)\cdot\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x})+\frac{3}{4}\big(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})\big)\cdot\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x})+$$

$$+\frac{1}{4}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\partial^{x}_{j}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x})-\frac{1}{4}\big(\partial^{x}_{j}\partial^{j}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})\big)\cdot\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x})-im\,\partial^{k}_{x}\big(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\hat{\psi}(\underset{\sim}{x})\big)\Big\}.\;(H.3)$$

O lado direito de (5.69) é igual à soma dos resultados (H.1), (H.2) e (H.3). Vejamos inicialmente se os termos puramente bosônicos nesta soma coincidem com o resultado (5.68). Desde (H.1)-(H.3) segue que

$$\big[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')\big]_{t.p.b.}=\frac{i}{2}\partial^{j}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{F}^{km,a}(\underset{\sim}{x})\hat{F}^{jm,a}(\underset{\sim}{x}')+\hat{F}^{km,a}(\underset{\sim}{x}')\hat{F}^{jm,a}(\underset{\sim}{x})-$$

$$-\hat{\pi}^{a}_{k}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x}')-\hat{\pi}^{a}_{k}(\underset{\sim}{x}')\hat{\pi}^{a}_{j}(\underset{\sim}{x})+\delta^{jk}\Big(\hat{\pi}^{a}_{l}(\underset{\sim}{x})\hat{\pi}^{a}_{l}(\underset{\sim}{x}')-\frac{1}{2}\hat{F}^{lm,a}(\underset{\sim}{x})F^{lm,a}(\underset{\sim}{x}')\Big)\Big]+$$

$$+\frac{i}{2}\partial^{k}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{\pi}^{a}_{j}(\underset{\sim}{x})\hat{\pi}^{a}_{j}(\underset{\sim}{x}')+\frac{1}{2}\hat{F}^{jl,a}(\underset{\sim}{x})\hat{F}^{jl,a}(\underset{\sim}{x}')\Big]-$$

$$-\frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{F}^{jl,a}(\underset{\sim}{x})\partial^{k}_{x}\hat{F}^{jl,a}(\underset{\sim}{x})+\big(\partial^{x}_{j}\hat{F}^{jl,a}(\underset{\sim}{x})\big)\hat{F}^{kl,a}(\underset{\sim}{x})+\hat{F}^{jl,a}(\underset{\sim}{x})\partial^{x}_{j}\hat{F}^{kl,a}(\underset{\sim}{x})+$$

$$+\big(\partial^{j}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\big)\hat{\pi}^{a}_{k}(\underset{\sim}{x})+\hat{\pi}^{a}_{j}(\underset{\sim}{x})\partial^{j}_{x}\hat{\pi}^{a}_{k}(\underset{\sim}{x})\Big]=$$

$$=\frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\hat{F}^{jm,a}(\underset{\sim}{x}')\partial^{x}_{j}F^{km,a}(\underset{\sim}{x})+\big(\partial^{x}_{j}\hat{F}^{jm,a}(\underset{\sim}{x})\big)F^{km,a}(\underset{\sim}{x}')+\hat{\pi}^{a}_{j}(\underset{\sim}{x}')\partial^{j}_{x}\hat{\pi}^{a}_{k}(\underset{\sim}{x})+$$

$$+\left(\partial^{j}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\right)\hat{\pi}^{a}_{k}(\underset{\sim}{x}') - \hat{\pi}^{a}_{\ell}(\underset{\sim}{x}')\,\partial^{k}_{x}\hat{\pi}^{a}_{\ell}(\underset{\sim}{x}) - \left(\partial^{k}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\right)\hat{\pi}^{a}_{j}(\underset{\sim}{x}') -$$

$$- \hat{F}^{j\ell,a}(\underset{\sim}{x})\,\partial^{k}_{x}\hat{F}^{j\ell,a}(\underset{\sim}{x}) - \left(\partial^{x}_{j}\hat{F}^{j\ell,a}(\underset{\sim}{x})\right)\hat{F}^{k\ell,a}(\underset{\sim}{x}) - \hat{F}^{j\ell,a}(\underset{\sim}{x})\,\partial^{x}_{j}\hat{F}^{k\ell,a}(\underset{\sim}{x}) -$$

$$-\left(\partial^{j}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x})\right)\hat{\pi}^{a}_{k}(\underset{\sim}{x}) - \hat{\pi}^{a}_{j}(\underset{\sim}{x})\,\partial^{j}_{x}\hat{\pi}^{a}_{k}(\underset{\sim}{x})\Big] =$$

$$= -i\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{\pi}^{a}_{j}(\underset{\sim}{x})\,\partial^{k}_{x}\hat{\pi}^{a}_{j}(\underset{\sim}{x}) - \frac{i}{2}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\hat{F}^{j\ell,a}(\underset{\sim}{x})\,\partial^{k}_{x}\hat{F}^{j\ell,a}(\underset{\sim}{x})\,, \quad (H.4)$$

que coincide com (5.68). Nesta prova, é claro, usamos o fato que $[\hat{\pi}^{a}_{j}(\underset{\sim}{x}),\hat{\pi}^{a}_{k}(\underset{\sim}{x}')] = 0$ (ver (4.2c)). A seguir, olhemos para os termos fermiônicos com acoplamento. Desde (H.1)-(H.3), obtemos

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{00}(\underset{\sim}{x}')\right]_{t.f.c.a.} = \frac{g}{2}\,\partial^{j}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}') +$$

$$+\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x}) + \frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x}') +$$

$$+\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x})\Big] + \frac{g}{2}\,\partial^{k}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x}') +$$

$$+\hat{\pi}_{\psi}(\underset{\sim}{x}')\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{j,a}(\underset{\sim}{x})\Big] - \frac{g}{2}\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[+\partial^{k}_{x}\left(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\right) +$$

$$-\frac{1}{2}\partial^{j}_{x}\left(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{k}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\right) - \frac{1}{2}\partial^{j}_{x}\left(\hat{\pi}_{\psi}(\underset{\sim}{x})\cdot\gamma^{0}\gamma^{j}\frac{\lambda^{a}}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\right)\Big] =$$

$$= \frac{g}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{-\frac{1}{2}\partial^j_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\hat{A}^{k,a}(\underset{\sim}{x}') - \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\partial^j_x\hat{A}^{k,a}(\underset{\sim}{x}) -$$

$$-\frac{1}{2}\partial^j_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\hat{A}^{j,a}(\underset{\sim}{x}') - \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\partial^j_x\hat{A}^{j,a}(\underset{\sim}{x}) -$$

$$-\partial^k_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\hat{A}^{j,a}(\underset{\sim}{x}') - \hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\partial^k_x\hat{A}^{j,a}(\underset{\sim}{x}) -$$

$$-\partial^k_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\hat{A}^{j,a}(\underset{\sim}{x}) - \Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\partial^k_x\hat{A}^{j,a}(\underset{\sim}{x}) +$$

$$+\frac{1}{2}\partial^j_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\hat{A}^{j,a}(\underset{\sim}{x}) + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\partial^j_x\hat{A}^{j,a}(\underset{\sim}{x}) +$$

$$+\frac{1}{2}\partial^j_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\Big)\hat{A}^{k,a}(\underset{\sim}{x}) + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\partial^j_x\hat{A}^{k,a}(\underset{\sim}{x})\Big\} =$$

$$= \frac{g}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[-2\partial^k_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\Big)\Big] =$$

$$= -g\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^k_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{j,a}(\underset{\sim}{x})\Big) \quad , \qquad (H.5)$$

que coincide com (5.59). Finalmente, desde (H.1)-(H.3), os termos puramente fermiônicos no lado direito de (5.69) são os seguintes

$$[\hat{\Theta}^{0k}(\underset{\sim}{x}), \hat{\Theta}^{00}(\underset{\sim}{x}')]_{t.p.f.} = \frac{i}{2}\partial^j_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\frac{1}{4}\Big[\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^j\partial^k_{x'}\hat{\psi}(\underset{\sim}{x}') +$$

$$+\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^j\partial^k_x\hat{\psi}(\underset{\sim}{x}) - (\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}') - (\partial^k_{x'}\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}) +$$

$$+\hat{\pi}_\psi(\underset{\sim}{x})\cdot\gamma^0\gamma^k\partial^j_{x'}\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_\psi(\underset{\sim}{x}')\cdot\gamma^0\gamma^k\partial^j_x\hat{\psi}(\underset{\sim}{x}) - (\partial^j_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}') -$$

$$-\left(\partial^j_{x'}\hat{\pi}_\psi(\underset{\sim}{x}')\right).\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x})\Big]\Big\} + \frac{i}{2}\partial^k_x\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[-\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^0\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}').\gamma^0\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x}) + \frac{1}{2}(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}') + \frac{1}{2}(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}) -$$

$$-im\,\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^0\hat{\psi}(\underset{\sim}{x}') - im\,\hat{\pi}_\psi(\underset{\sim}{x}').\gamma^0\hat{\psi}(\underset{\sim}{x})\Big] -$$

$$-\frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[-\frac{3}{4}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\gamma^0\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x}) - \frac{1}{4}\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^0\gamma^j(\partial^x_j\partial^k_x\hat{\psi}(\underset{\sim}{x})) +$$

$$+\frac{1}{4}(\partial^x_j\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}) + \frac{3}{4}(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\partial^k_x\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{1}{4}\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^0\gamma^k\partial^x_j\partial^j_x\hat{\psi}(\underset{\sim}{x}) - \frac{1}{4}(\partial^x_j\partial^j_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}) - im\,\partial^k_x(\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^0\hat{\psi}(\underset{\sim}{x}))\Big] =$$

$$= \frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{-\frac{1}{4}(\partial^j_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\partial^k_{x'}\hat{\psi}(\underset{\sim}{x}') - \frac{1}{4}\hat{\pi}_\psi(\underset{\sim}{x}').\gamma^0\gamma^j\partial^j_x\partial^k_x\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{1}{4}(\partial^j_x\partial^k_x\hat{\pi}_4(\underset{\sim}{x})).\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}') + \frac{1}{4}(\partial^k_{x'}\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^0\gamma^j\partial^j_x\hat{\psi}(\underset{\sim}{x}) - \frac{1}{4}(\partial^j_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^k\partial^j_{x'}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{4}\hat{\pi}_\psi(\underset{\sim}{x}').\gamma^0\gamma^k\partial^j_x\partial^j_x\hat{\psi}(\underset{\sim}{x}) + \frac{1}{4}(\partial^j_x\partial^j_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^k\hat{\psi}(\underset{\sim}{x}') + \frac{1}{4}(\partial^j_{x'}\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^0\gamma^k\partial^j_x\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{1}{2}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}') + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}').\gamma^0\gamma^j\partial^k_x\partial^x_j\hat{\psi}(\underset{\sim}{x}) - \frac{1}{2}(\partial^k_x\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{2}(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}')).\gamma^0\gamma^j\partial^k_x\hat{\psi}(\underset{\sim}{x}) + im(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\hat{\psi}(\underset{\sim}{x}') + im\,\hat{\pi}_\psi(\underset{\sim}{x}').\gamma^0(\partial^k_x\hat{\psi}(\underset{\sim}{x})) +$$

$$+\frac{3}{4}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\partial^x_j\hat{\psi}(\underset{\sim}{x}) + \frac{1}{4}\hat{\pi}_\psi(\underset{\sim}{x}).\gamma^0\gamma^j(\partial^x_j\partial^k_x\hat{\psi}(\underset{\sim}{x})) -$$

$$-\frac{1}{4}(\partial^x_j\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\hat{\psi}(\underset{\sim}{x}) - \frac{3}{4}(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x})).\gamma^0\gamma^j\partial^k_x\hat{\psi}(\underset{\sim}{x}) -$$

$$-\frac{1}{4}\hat{\pi}_{+}(\underset{\sim}{x}).\gamma^{0}\gamma^{k}\partial^{x}_{j}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x}) + \frac{1}{4}(\partial^{x}_{j}\partial^{j}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{k}\hat{\psi}(\underset{\sim}{x}) + i m\,\partial^{k}_{x}(\hat{\pi}_{+}(\underset{\sim}{x})\gamma^{0}\hat{\psi}(\underset{\sim}{x}))\Big\} =$$

$$= \frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{(\partial^{j}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - \hat{\pi}_{+}(\underset{\sim}{x}').\gamma^{0}\gamma^{j}\partial^{j}_{x}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+(\partial^{j}_{x}\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x}') - (\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+ 2 i m\,\partial^{k}_{x}(\hat{\pi}_{+}(\underset{\sim}{x}).\gamma^{0}\hat{\psi}(\underset{\sim}{x}))\Big\} =$$

$$= \frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial^{j}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x})\Big] +$$

$$+\frac{i}{2}\partial^{j}_{x}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{\pi}_{+}(\underset{\sim}{x}').\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\hat{\psi}(\underset{\sim}{x})\Big] -$$

$$- m\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}(\hat{\pi}_{+}(\underset{\sim}{x}).\gamma^{0}\hat{\psi}(\underset{\sim}{x})) =$$

$$= \frac{i}{2}\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial^{j}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+(\partial^{j}_{x'}\hat{\pi}_{+}(\underset{\sim}{x}')).\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{j}_{x'}\hat{\psi}(\underset{\sim}{x}')\Big] -$$

$$- m\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}(\hat{\pi}_{+}(\underset{\sim}{x}).\gamma^{0}\hat{\psi}(\underset{\sim}{x})) =$$

$$= i\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial^{j}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{+}(\underset{\sim}{x})).\gamma^{0}\gamma^{j}\partial^{j}_{x}\hat{\psi}(\underset{\sim}{x})\Big] -$$

$$- m\,\overset{(3)}{\delta}(\underset{\sim}{x}-\underset{\sim}{x}')\,\partial^{k}_{x}(\hat{\pi}_{+}(\underset{\sim}{x}).\gamma^{0}\hat{\psi}(\underset{\sim}{x})) \quad , \tag{H.6}$$

que coincide com (5.58). Isto completa à prova de (5.69). Q.E.D.

Para provar (5.74) procedemos de forma similar ao que fizemos na demonstração de (5.69). Mostramos que o lado direito de (5.74), com $\hat{\Theta}^{0k}$ dada por (5.4)-(5.6), coincide com a soma de nossos resultados (5.71)-(5.73). É evidente que o lado direito de (5.74) pode ser escrito como

$$\frac{i}{2}\Big(\hat{\Theta}^{0\ell}(\underset{\sim}{x}) + \hat{\Theta}^{0\ell}(\underset{\sim}{x}')\Big)\partial^k_x \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') + \frac{i}{2}\Big(\hat{\Theta}^{0k}(\underset{\sim}{x}) + \hat{\Theta}^{0k}(\underset{\sim}{x}')\Big)\partial^\ell_x \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}') +$$

$$+ \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big(\partial^\ell_x \hat{\Theta}^{0k}(\underset{\sim}{x}) - \partial^k_x \hat{\Theta}^{0\ell}(\underset{\sim}{x})\Big) =$$

$$= \frac{i}{2}\partial^k_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{\pi}^a_j(\underset{\sim}{x})\cdot\hat{F}^{\ell j,a}(\underset{\sim}{x}) + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\partial^\ell_x\hat{\psi}(\underset{\sim}{x}) - \frac{1}{2}(\partial^\ell_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\hat{\psi}(\underset{\sim}{x}) -$$

$$- ig\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{\ell,a}(\underset{\sim}{x}) - \frac{i}{4}\partial^x_j(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{j\ell}\hat{\psi}(\underset{\sim}{x})) + \hat{\pi}^a_j(\underset{\sim}{x}')\cdot\hat{F}^{\ell j,a}(\underset{\sim}{x}') + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\partial^\ell_{x'}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{2}(\partial^\ell_{x'}\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\hat{\psi}(\underset{\sim}{x}') - ig\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{\ell,a}(\underset{\sim}{x}') - \frac{i}{4}\partial^{x'}_j(\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\sigma^{j\ell}\hat{\psi}(\underset{\sim}{x}'))\Big] +$$

$$+ \frac{i}{2}\partial^\ell_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\hat{\pi}^a_j(\underset{\sim}{x})\cdot\hat{F}^{kj,a}(\underset{\sim}{x}) + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\partial^k_x\hat{\psi}(\underset{\sim}{x}) - \frac{1}{2}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\hat{\psi}(\underset{\sim}{x}) -$$

$$- ig\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}) - \frac{i}{4}\partial^x_j(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jk}\hat{\psi}(\underset{\sim}{x})) + \hat{\pi}^a_j(\underset{\sim}{x}')\cdot\hat{F}^{kj,a}(\underset{\sim}{x}') + \frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\partial^k_{x'}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{2}(\partial^k_{x'}\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\hat{\psi}(\underset{\sim}{x}') - ig\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x}') - \frac{i}{4}\partial^{x'}_j(\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\sigma^{jk}\hat{\psi}(\underset{\sim}{x}'))\Big] +$$

$$+ \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial^\ell_x\hat{\pi}^a_j(\underset{\sim}{x}))\cdot\hat{F}^{kj,a}(\underset{\sim}{x}) + \hat{\pi}^a_j(\underset{\sim}{x})\cdot\partial^\ell_x\hat{F}^{kj,a}(\underset{\sim}{x}) + \frac{1}{2}(\partial^\ell_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\partial^k_x\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\partial^\ell_x\partial^k_x\hat{\psi}(\underset{\sim}{x}) - \frac{1}{2}(\partial^\ell_x\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\hat{\psi}(\underset{\sim}{x}) - \frac{1}{2}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\partial^\ell_x\hat{\psi}(\underset{\sim}{x}) -$$

$$-ig\,\partial^l_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\Big)-\frac{i}{4}\,\partial^l_x\partial^x_j\big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jk}\hat{\psi}(\underset{\sim}{x})\big)\;-$$

$$-(\partial^k_x\hat{\pi}^a_j(\underset{\sim}{x}))\cdot\hat{F}^{lj,a}(\underset{\sim}{x})-\hat{\pi}^a_j(\underset{\sim}{x})\cdot\partial^k_x\hat{F}^{lj,a}(\underset{\sim}{x})-\frac{1}{2}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\partial^l_x\hat{\psi}(\underset{\sim}{x})-\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\partial^k_x\partial^l_x\hat{\psi}(\underset{\sim}{x})+$$

$$+\frac{1}{2}(\partial^k_x\partial^l_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\hat{\psi}(\underset{\sim}{x})+\frac{1}{2}(\partial^l_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\partial^k_x\hat{\psi}(\underset{\sim}{x})+ig\,\partial^k_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})\Big)+$$

$$+\frac{i}{4}\,\partial^k_x\partial^x_j\big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jl}\hat{\psi}(\underset{\sim}{x})\big)\Big]\;=$$

$$=\frac{i}{2}\partial^k_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\Big[\hat{\pi}^a_j(\underset{\sim}{x})\cdot\hat{F}^{lj,a}(\underset{\sim}{x}')+\hat{\pi}^a_j(\underset{\sim}{x}')\cdot\hat{F}^{lj,a}(\underset{\sim}{x})+\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\partial^l_{x'}\hat{\psi}(\underset{\sim}{x}')+\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\partial^l_x\hat{\psi}(\underset{\sim}{x})-$$

$$-\frac{1}{2}(\partial^l_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\hat{\psi}(\underset{\sim}{x}')-\frac{1}{2}(\partial^l_{x'}\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\hat{\psi}(\underset{\sim}{x})-ig\,\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x}')-ig\,\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{l,a}(\underset{\sim}{x})$$

$$-\frac{i}{4}\Big[(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\sigma^{jl}\hat{\psi}(\underset{\sim}{x}')+(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\sigma^{jl}\hat{\psi}(\underset{\sim}{x})+\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jl}\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}')+\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\sigma^{jl}\partial^x_j\hat{\psi}(\underset{\sim}{x})\Big]\Big]\Big\}$$

$$+\frac{i}{2}\partial^l_x\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{\hat{\pi}^a_j(\underset{\sim}{x})\cdot\hat{F}^{kj,a}(\underset{\sim}{x}')+\hat{\pi}^a_j(\underset{\sim}{x}')\cdot\hat{F}^{kj,a}(\underset{\sim}{x})+\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x})\cdot\partial^k_{x'}\hat{\psi}(\underset{\sim}{x}')+\frac{1}{2}\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\partial^k_x\hat{\psi}(\underset{\sim}{x})-$$

$$-\frac{1}{2}(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\hat{\psi}(\underset{\sim}{x}')-\frac{1}{2}(\partial^k_{x'}\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\hat{\psi}(\underset{\sim}{x})-ig\,\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x}')-ig\,\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x}')\hat{A}^{k,a}(\underset{\sim}{x})-$$

$$-\frac{i}{4}\Big[(\partial^x_j\hat{\pi}_\psi(\underset{\sim}{x}))\cdot\sigma^{jk}\hat{\psi}(\underset{\sim}{x}')+(\partial^{x'}_j\hat{\pi}_\psi(\underset{\sim}{x}'))\cdot\sigma^{jk}\hat{\psi}(\underset{\sim}{x})+\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jk}\partial^{x'}_j\hat{\psi}(\underset{\sim}{x}')+\hat{\pi}_\psi(\underset{\sim}{x}')\cdot\sigma^{jk}\partial^x_j\hat{\psi}(\underset{\sim}{x})\Big]\Big\}+$$

$$+\frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{(\partial^l_x\hat{\pi}^a_j(\underset{\sim}{x}))\cdot\hat{F}^{kj,a}(\underset{\sim}{x})+\hat{\pi}^a_j(\underset{\sim}{x})\cdot\partial^l_x\hat{F}^{kj,a}(\underset{\sim}{x})-(\partial^k_x\hat{\pi}^a_j(\underset{\sim}{x}))\cdot\hat{F}^{lj,a}(\underset{\sim}{x})-\hat{\pi}^a_j(\underset{\sim}{x})\cdot\partial^k_x\hat{F}^{lj,a}(\underset{\sim}{x})+$$

$$+(\partial^l_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot(\partial^k_x\hat{\psi}(\underset{\sim}{x}))-(\partial^k_x\hat{\pi}_\psi(\underset{\sim}{x}))\cdot(\partial^l_x\hat{\psi}(\underset{\sim}{x}))-ig\,\partial^l_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{k,a}(\underset{\sim}{x})\Big)+$$

$$+ig\,\partial^k_x\Big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\frac{\lambda^a}{2}\hat{\psi}(\underset{\sim}{x})\hat{A}^{l,a}(\underset{\sim}{x})\Big)-\frac{i}{4}\partial^l_x\partial^x_j\big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jk}\hat{\psi}(\underset{\sim}{x})\big)+\frac{i}{4}\partial^k_x\partial^x_j\big(\hat{\pi}_\psi(\underset{\sim}{x})\cdot\sigma^{jl}\hat{\psi}(\underset{\sim}{x})\big)\Big\},\quad\text{(H.7)}$$

onde, no último passo, usamos a propriedade (5.17). Desde (H.7), obtemos

$$\left[\hat{\Theta}^{0k}(\underset{\sim}{x}),\hat{\Theta}^{0\ell}(\underset{\sim}{x}')\right]_{t.p.f.} = \frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{-\frac{1}{2}(\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\partial^{\ell}_{x'}\hat{\psi}(\underset{\sim}{x}') -$$

$$-\frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}').\partial^{k}_{x}\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) + \frac{1}{2}(\partial^{k}_{x}\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\hat{\psi}(\underset{\sim}{x}') + \frac{1}{2}(\partial^{\ell}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{i}{4}\Big[(\partial^{k}_{x}\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{j\ell}\hat{\psi}(\underset{\sim}{x}') + (\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\sigma^{j\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) + (\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{j\ell}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}') +$$

$$+\hat{\pi}_{\psi}(\underset{\sim}{x}').\sigma^{j\ell}\partial^{k}_{x}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})\Big] - \frac{1}{2}(\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\partial^{k}_{x'}\hat{\psi}(\underset{\sim}{x}') - \frac{1}{2}\hat{\pi}_{\psi}(\underset{\sim}{x}').\partial^{\ell}_{x}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{1}{2}(\partial^{\ell}_{x}\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\hat{\psi}(\underset{\sim}{x}') + \frac{1}{2}(\partial^{k}_{x'}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) + \frac{i}{4}\Big[(\partial^{\ell}_{x}\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\hat{\psi}(\underset{\sim}{x}') +$$

$$+(\partial^{x'}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x}')).\sigma^{jk}\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) + (\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\partial^{x'}_{j}\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_{\psi}(\underset{\sim}{x}').\sigma^{jk}\partial^{\ell}_{x}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})\Big] +$$

$$+(\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) - \frac{i}{4}\Big[(\partial^{\ell}_{x}\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\hat{\psi}(\underset{\sim}{x}) +$$

$$+(\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) + (\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{jk}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}) + \hat{\pi}_{\psi}(\underset{\sim}{x}).\sigma^{jk}\partial^{\ell}_{x}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x}) -$$

$$-(\partial^{k}_{x}\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{j\ell}\hat{\psi}(\underset{\sim}{x}) - (\partial^{x}_{j}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{j\ell}\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\sigma^{j\ell}(\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})) -$$

$$-\hat{\pi}_{\psi}(\underset{\sim}{x}).\sigma^{j\ell}\partial^{k}_{x}\partial^{x}_{j}\hat{\psi}(\underset{\sim}{x})\Big]\Big\} =$$

$$= \frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{-\hat{\pi}_{\psi}(\underset{\sim}{x}').\partial^{k}_{x}\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) + (\partial^{k}_{x}\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\hat{\psi}(\underset{\sim}{x}') +$$

$$+(\partial^{\ell}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\partial^{k}_{x}\hat{\psi}(\underset{\sim}{x}) - (\partial^{k}_{x}\hat{\pi}_{\psi}(\underset{\sim}{x})).\partial^{\ell}_{x}\hat{\psi}(\underset{\sim}{x}) +$$

$$+\frac{i}{2}\Big[(\partial_x^k\partial_j^x\hat{\pi}_\psi(\underset{\sim}{x})).\sigma^{jl}\hat{\psi}(\underset{\sim}{x}') + (\partial_j^{x'}\hat{\pi}_\psi(\underset{\sim}{x}')).\sigma^{jl}\partial_x^k\hat{\psi}(\underset{\sim}{x}) +$$

$$+(\partial_x^k\hat{\pi}_\psi(\underset{\sim}{x})).\sigma^{jl}\partial_j^{x'}\hat{\psi}(\underset{\sim}{x}') + \hat{\pi}_\psi(\underset{\sim}{x}').\sigma^{jl}\partial_x^k\partial_j^x\hat{\psi}(\underset{\sim}{x})\Big] =$$

$$= \frac{i}{2}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[\,2(\partial_x^l\hat{\pi}_\psi(\underset{\sim}{x})).\partial_x^k\hat{\psi}(\underset{\sim}{x}) - 2(\partial_x^k\hat{\pi}_\psi(\underset{\sim}{x})).\partial_x^l\hat{\psi}(\underset{\sim}{x})\Big] =$$

$$= i\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial_x^l\hat{\pi}_\psi(\underset{\sim}{x})).\partial_x^k\hat{\psi}(\underset{\sim}{x}) - (\partial_x^k\hat{\pi}_\psi(\underset{\sim}{x})).\partial_x^l\hat{\psi}(\underset{\sim}{x})\Big]\,, \qquad (H8)$$

dado que, por exemplo,

$$\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[-\hat{\pi}_\psi(\underset{\sim}{x}').\partial_x^k\partial_x^l\hat{\psi}(\underset{\sim}{x})\Big] = \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[-\hat{\pi}_\psi(\underset{\sim}{x}).\partial_{x'}^k\partial_{x'}^l\hat{\psi}(\underset{\sim}{x}')\Big] =$$

$$= +\quad \partial_{x'}^l\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\,\hat{\pi}_\psi(\underset{\sim}{x}).\partial_{x'}^k\hat{\psi}(\underset{\sim}{x}')) = -\partial_x^l\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\hat{\pi}_\psi(\underset{\sim}{x}).\partial_{x'}^k\hat{\psi}(\underset{\sim}{x}')) =$$

$$= \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial_x^l\hat{\pi}_\psi(\underset{\sim}{x})).(\partial_x^k\hat{\psi}(\underset{\sim}{x}))\Big] \quad ;$$

$$\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial_x^k\partial_j^x\hat{\pi}_\psi(\underset{\sim}{x})).\sigma^{jl}\hat{\psi}(\underset{\sim}{x}')\Big] = \delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial_{x'}^k\,\partial_j^{x'}\hat{\pi}_\psi(\underset{\sim}{x}'))\sigma^{jl}\hat{\psi}(\underset{\sim}{x})\Big] =$$

$$= -\partial_{x'}^k\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big[(\partial_j^{x'}\hat{\pi}_\psi(\underset{\sim}{x}')).\sigma^{jl}\hat{\psi}(\underset{\sim}{x})\Big] = \partial_x^k\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')(\partial_j^{x'}\hat{\pi}_\psi(\underset{\sim}{x}').\sigma^{jl}\hat{\psi}(\underset{\sim}{x})) =$$

$$= -\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\,(\partial_j^{x'}\hat{\pi}_\psi(\underset{\sim}{x}')).\sigma^{jl}\partial_x^k\hat{\psi}(\underset{\sim}{x}) \quad .$$

Claramente, o resultado (H.8) coincide com (5.71). Por outro lado, desde (H.7), também segue que

$$[\hat{\Theta}^{0k}_{(\underset{\sim}{x})},\hat{\Theta}^{0l}_{(\underset{\sim}{x}')}]_{t.f.c.a.} =$$

$$= \frac{i}{2}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\Big\{ -ig\,\partial^k_x(\hat{\pi}_{\psi(\underset{\sim}{x})}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x})})\hat{A}^{l,a}_{(\underset{\sim}{x}')} + ig\,\hat{\pi}_{\psi(\underset{\sim}{x}')}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x}')}\partial^k_x\hat{A}^{l,a}_{(\underset{\sim}{x})} +$$

$$+ ig\,\partial^l_x(\hat{\pi}_{\psi(\underset{\sim}{x})}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x})})\hat{A}^{k,a}_{(\underset{\sim}{x}')} + ig\,\hat{\pi}_{\psi(\underset{\sim}{x}')}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x}')}\partial^l_x\hat{A}^{k,a}_{(\underset{\sim}{x})} -$$

$$- ig\,\partial^l_x(\hat{\pi}_{\psi(\underset{\sim}{x})}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x})}\hat{A}^{k,a}_{(\underset{\sim}{x})}) + ig\,\partial^k_x(\hat{\pi}_{\psi(\underset{\sim}{x})}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x})}\hat{A}^{l,a}_{(\underset{\sim}{x})})\Big\} =$$

$$= -g\,\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\,\partial^k_x(\hat{\pi}_{\psi(\underset{\sim}{x})}.\frac{\lambda^a}{2}\hat{\psi}_{(\underset{\sim}{x})}\hat{A}^{l,a}_{(\underset{\sim}{x})}) \quad , \qquad (H.9)$$

que coincide com (5.72). Por último, desde (H.7)

$$[\hat{\Theta}^{0k}_{(\underset{\sim}{x})},\hat{\Theta}^{0l}_{(\underset{\sim}{x}')}]_{t.p.b.} = \frac{i}{2}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{x}')\Big\{ -(\partial^k_x\hat{\pi}^a_{j}(\underset{\sim}{x})).\hat{F}^{lj,a}_{(\underset{\sim}{x}')} -$$

$$-\hat{\pi}^a_{j}(\underset{\sim}{x}').\partial^k_x\hat{F}^{lj,a}_{(\underset{\sim}{x})} - (\partial^l_x\hat{\pi}^a_{j}(\underset{\sim}{x})).\hat{F}^{kj,a}_{(\underset{\sim}{x}')} - \hat{\pi}^a_{j}(\underset{\sim}{x}').\partial^l_x\hat{F}^{kj,a}_{(\underset{\sim}{x})} +$$

$$+(\partial^l_x\hat{\pi}^a_{j}(\underset{\sim}{x})).\hat{F}^{kj,a}_{(\underset{\sim}{x})} + \hat{\pi}^a_{j}(\underset{\sim}{x}).\partial^l\hat{F}^{kj,a}_{(\underset{\sim}{x})} - (\partial^k_x\hat{\pi}^a_{j}(\underset{\sim}{x})).\hat{F}^{lj,a}_{(\underset{\sim}{x})} -$$

$$-\hat{\pi}^a_{j}(\underset{\sim}{x}).\partial^k_x\hat{F}^{lj,a}_{(\underset{\sim}{x})}\Big\} = \frac{i}{2}\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\Big[-2\hat{\pi}^a_{j}(\underset{\sim}{x}).\partial^k_x\hat{F}^{lj,a}_{(\underset{\sim}{x})} -$$

$$-2(\partial^k_x\hat{\pi}^a_{j}(\underset{\sim}{x})).\hat{F}^{lj,a}_{(\underset{\sim}{x})}\Big] = -i\,\delta^{(3)}_{(\underset{\sim}{x}-\underset{\sim}{x}')}\,\partial^k_x(\hat{\pi}^a_{j}(\underset{\sim}{x}).\hat{F}^{lj,a}_{(\underset{\sim}{x})}) \; , \qquad (H.10)$$

que coincide com (5.73). Isto completa a prova de (5.74). Q.E.D.

APÊNDICE I

OBTENÇÃO DA ÁLGEBRA DE POINCARÉ A PARTIR DA ÁLGEBRA DAS DENSIDADES DE MOMENTUM

Neste Apêndice provamos as expressões (5.75) partindo de (5.43), (5.69) e (5.74) e levando em conta (5.49). Desde (5.43), por integração sobre $\underset{\sim}{x}$ em todo o espaço*,

$$[\hat{P}^0, \hat{\Theta}^{00}(\underset{\sim}{x}')] = i\,\partial_k^{x'}\,\hat{\Theta}^{0k}(\underset{\sim}{x}') \qquad \therefore \quad ,$$

integrando sobre $\underset{\sim}{x}'$, segue

$$[\hat{P}^0, \hat{P}^0] = i\int d^3x'\,\partial_k^{x'}\,\hat{\Theta}^{0k}(\underset{\sim}{x}') = i\left[\hat{\Theta}^{0k}(+\infty) - \hat{\Theta}^{0k}(-\infty)\right] = 0 \;,$$

de acordo com (5.49). Logo,

$$[\hat{P}^0, \hat{P}^0] = 0 \quad . \qquad (I.1)$$

Por outro lado, integrando (5.69) sobre $\underset{\sim}{x}$ encontramos

$$[\hat{P}^k, \hat{\Theta}^{00}(\underset{\sim}{x}')] = \underset{\downarrow}{\overset{|}{-\frac{i}{2}\,\partial^j_{x'}\,\hat{\Theta}^{kj}(\underset{\sim}{x}')}} - \frac{i}{2}\,\partial^k_{x'}\,\hat{\Theta}^{00}(\underset{\sim}{x}') - \frac{i}{2}\,\partial^k_{x'}\,\hat{\Theta}^{00}(\underset{\sim}{x}') +$$

$$+ \underset{\downarrow}{\overset{|}{\frac{i}{2}\,\partial^j_{x'}\,\hat{\Theta}^{jk}(\underset{\sim}{x}')}} = -i\,\partial^k_{x'}\,\hat{\Theta}^{00}(\underset{\sim}{x}') \qquad ,$$

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[*] Neste Apêndice, todas as integrações espaciais serão de $-\infty$ a $+\infty$.

devido a simetria do tensor $\hat{\Theta}^{ik}$ ($\hat{\Theta}^{ik} = \hat{\Theta}^{ki}$, ver (5.7)-(5.9)). Uma integração adicional sobre $\underset{\sim}{x}'$ conduz a

$$[\hat{P}^k, \hat{P}^0] = -i\,[\hat{\Theta}^{00}(+\infty) - \hat{\Theta}^{00}(-\infty)] = 0 \,,$$

devido à (5.49). Logo,

$$[\hat{P}^k, \hat{P}^0] = 0 \,, \qquad (I.2)$$

que corresponde à conservação do momentum linear total. Agora, desde (5.74), por integração sobre $\underset{\sim}{x}$ segue

$$[\hat{P}^k, \hat{\Theta}^{0\ell}(\underset{\sim}{x}')] = -\frac{i}{2}\,\partial^k_{x'}\hat{\Theta}^{0\ell}(\underset{\sim}{x}') \cancel{-\frac{i}{2}\,\partial^\ell_{x'}\hat{\Theta}^{0k}(\underset{\sim}{x}')} \cancel{+\frac{i}{2}\,\partial^\ell_{x'}\hat{\Theta}^{0k}(\underset{\sim}{x}')} -$$

$$-\frac{i}{2}\,\partial^k_{x'}\hat{\Theta}^{0\ell}(\underset{\sim}{x}') = -i\,\partial^k_{x'}\hat{\Theta}^{0\ell}(\underset{\sim}{x}')$$

$\therefore$, após integrar sobre $\underset{\sim}{x}'$,

$$[\hat{P}^k, \hat{P}^\ell] = +i\,[\hat{\Theta}^{0\ell}(+\infty) - \hat{\Theta}^{0\ell}(-\infty)] = 0 \,,$$

levando em conta (5.49). Logo,

$$[\hat{P}^k, \hat{P}^\ell] = 0 \,. \qquad (I.3)$$

Desta forma, combinando os resultados (I.1), (I.2) e (I.3), acabamos de provar (5.75a), i.e.,

$$[\hat{P}^\mu, \hat{P}^\nu] = 0 \,. \qquad (I.4)$$

Passemos à prova de (5.75b). Desde (5.43), integrando sobre x'

$$[\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{P}^{0}] = -i\,\partial^{x}_{\ell}\,\hat{\Theta}^{0\ell}(\underset{\sim}{x}) \tag{I.5}$$

∴ multiplicando ambos lados por x^k e, após, integrando sobre $\underset{\sim}{x}$, segue

$$\int d^3_x\,[x^k\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{P}^{0}] = -i\int d^3_x\, x^k\,\partial^{x}_{\ell}\,\hat{\Theta}^{0\ell}(\underset{\sim}{x}) = i\,\delta^{k\ell}\int d^3x\,\hat{\Theta}^{0\ell}(\underset{\sim}{x}) =$$

$$= i\,\hat{P}^{k} \quad . \tag{I.6}$$

Mas, de acordo com (5.48), o lado esquerdo de (I.6) é igual ao comutador $[\hat{K}^k,\hat{P}^0]$. Logo, (I.6)

$$[\hat{K}^{k}, \hat{P}^{0}] = i\,\hat{P}^{k} \quad . \tag{I.7}$$

Combinando (I.7) com (I.2), obtemos (ver (5.47))

$$x^0[\hat{P}^{0}, \hat{P}^{k}] - [\hat{P}^{0}, \hat{K}^{k}] = i\,\hat{P}^{k} \quad \Rightarrow$$

$$[\hat{P}^{0}, \hat{J}^{0k}] = i\,\hat{P}^{k} \quad . \tag{I.8}$$

Note-se que o lado esquerdo de (I.5) pode também ser escrito na forma $[\hat{\Theta}^{00}(\underset{\sim}{x}),\hat{H}] = +i\partial_0\hat{\Theta}^{00}(\underset{\sim}{x})$ o que, desde (I.5), significa

$$\partial_0\hat{\Theta}^{00}(\underset{\sim}{x}) + \partial^{x}_{\ell}\,\hat{\Theta}^{0\ell}(\underset{\sim}{x}) = \partial_{\mu}\hat{\Theta}^{0\mu}(\underset{\sim}{x}) = 0 \quad , \tag{I.9}$$

que é a equação da conservação da energia local. Agora, desde (5.46), (5.44) e (5.69), temos

$$[\hat{J}^{k\ell},\hat{P}^{o}]=\Big[\int d\overset{3}{x}\big(x^{k}\hat{\Theta}^{o\ell}(\underset{\sim}{x})-x^{\ell}\hat{\Theta}^{ok}(\underset{\sim}{x})\big),\int d\overset{3}{x}{}'\,\hat{\Theta}^{oo}(\underset{\sim}{x}')\Big]=$$

$$=\int d\overset{3}{x}\int d\overset{3}{x}{}'\;x^{k}[\hat{\Theta}^{o\ell}_{(\underset{\sim}{x})},\hat{\Theta}^{oo}_{(\underset{\sim}{x}')}]-\int d\overset{3}{x}\int d\overset{3}{x}{}'\;x^{\ell}[\hat{\Theta}^{ok}_{(\underset{\sim}{x})},\hat{\Theta}^{oo}_{(\underset{\sim}{x}')}]=$$

$$=\int d\overset{3}{x}\Big\{x^{k}\Big[\frac{i}{2}\underset{x}{\partial^{j}}\hat{\Theta}^{\ell j}_{(\underset{\sim}{x})}+\cancel{\frac{i}{2}\underset{x}{\partial^{\ell}}\hat{\Theta}^{oo}_{(\underset{\sim}{x})}}-\cancel{\frac{i}{2}\underset{x}{\partial^{\ell}}\hat{\Theta}^{oo}_{(\underset{\sim}{x})}}+\frac{i}{2}\underset{x}{\partial^{j}}\hat{\Theta}^{j\ell}_{(\underset{\sim}{x})}\Big]-$$

$$-x^{\ell}\Big[\frac{i}{2}\underset{x}{\partial^{j}}\hat{\Theta}^{kj}_{(\underset{\sim}{x})}+\cancel{\frac{i}{2}\underset{x}{\partial^{k}}\hat{\Theta}^{oo}_{(\underset{\sim}{x})}}-\cancel{\frac{i}{2}\underset{x}{\partial^{k}}\hat{\Theta}^{oo}_{(\underset{\sim}{x})}}+\frac{i}{2}\underset{x}{\partial^{j}}\hat{\Theta}^{jk}_{(\underset{\sim}{x})}\Big]\Big\}=$$

$$=i\int d\overset{3}{x}\Big[x^{k}\,\underset{x}{\partial^{j}}\hat{\Theta}^{\ell j}_{(\underset{\sim}{x})}-x^{\ell}\,\underset{x}{\partial^{j}}\hat{\Theta}^{kj}_{(\underset{\sim}{x})}\Big]=$$

$$=i\int d\overset{3}{x}\Big[\delta^{jk}\hat{\Theta}^{\ell j}{}_{(\underset{\sim}{x})}-\delta^{j\ell}\hat{\Theta}^{kj}{}_{(\underset{\sim}{x})}\Big]=i\int d\overset{3}{x}\Big[\hat{\Theta}^{\ell k}_{(\underset{\sim}{x})}-\hat{\Theta}^{k\ell}_{(\underset{\sim}{x})}\Big]=$$

$$=0\quad,$$

o que associa a simetria da densidade $\hat{\Theta}^{k\ell}$ à conservação do momentum angular total $\hat{J}^{k\ell}$, i.e.,

$$[\hat{P}^{o},\hat{J}^{k\ell}]=0\quad.\qquad\qquad(I.10)$$

Agora, lançando mão de (I.3) e (5.69)

$$[\hat{P}^{k},\hat{J}^{o\ell}]=[\hat{P}^{k},x^{o}\hat{P}^{\ell}-\hat{K}^{\ell}]=-[\hat{P}^{k},\hat{K}^{\ell}]=$$

$$= -\int d^3x \int d^3x' \left[\hat{\Theta}^{0k}(\underset{\sim}{x}), x'^{l} \hat{\Theta}^{00}(\underset{\sim}{x}') \right] = -\int d^3x'\, x'^{l} \left[\cancel{-\frac{i}{2} \partial^{j}_{x'} \hat{\Theta}^{kj}(\underset{\sim}{x}')} - \right.$$

$$\left. -\frac{i}{2} \partial^{k}_{x'} \hat{\Theta}^{00}(\underset{\sim}{x}') - \frac{i}{2} \partial^{k}_{x'} \hat{\Theta}^{00}(\underset{\sim}{x}') \cancel{+ \frac{i}{2} \partial^{j}_{x'} \hat{\Theta}^{jk}(\underset{\sim}{x}')} \right] =$$

$$= -i \int d^3x'\, x'^{l} \partial^{x'}_{k} \hat{\Theta}^{00}(\underset{\sim}{x}') = i \int d^3x'\, \delta^{kl} \hat{\Theta}^{00}(\underset{\sim}{x}') = i \delta^{kl} \hat{P}^{0}$$

∴ seguem as relações

$$[\hat{P}^{k}, \hat{J}^{0l}] = i \delta^{kl} \hat{P}^{0} \quad , \qquad (I.11)$$

$$[\hat{P}, \hat{K}^{l}] = -i \delta^{kl} \hat{H} \quad . \qquad (I.12)$$

Por outro lado, desde (5.74) calculamos

$$[\hat{P}^{k}, \hat{J}^{lm}] = \int d^3x \int d^3x' \left[\hat{\Theta}^{0k}(\underset{\sim}{x}), x'^{l} \hat{\Theta}^{0m}(\underset{\sim}{x}') - x'^{m} \hat{\Theta}^{0l}(\underset{\sim}{x}') \right] =$$

$$= \int d^3x' \left\{ x'^{l} \left[-\frac{i}{2} \partial^{k}_{x'} \hat{\Theta}^{0m}(\underset{\sim}{x}') \cancel{- \frac{i}{2} \partial^{m}_{x'} \hat{\Theta}^{0k}(\underset{\sim}{x}')} \cancel{+ \frac{i}{2} \partial^{m}_{x'} \hat{\Theta}^{0k}(\underset{\sim}{x}')} - \frac{i}{2} \partial^{k}_{x'} \hat{\Theta}^{0m}(\underset{\sim}{x}') \right] - \right.$$

$$\left. - x'^{m} \left[-\frac{i}{2} \partial^{k}_{x'} \hat{\Theta}^{0l}(\underset{\sim}{x}') \cancel{- \frac{i}{2} \partial^{l}_{x'} \hat{\Theta}^{0k}(\underset{\sim}{x}')} \cancel{+ \frac{i}{2} \partial^{l}_{x'} \hat{\Theta}^{0k}(\underset{\sim}{x}')} - \frac{i}{2} \partial^{k}_{x'} \hat{\Theta}^{0l}(\underset{\sim}{x}') \right] \right\} =$$

$$= i \int d^3x' \left[x'^{l} \partial^{x'}_{k} \hat{\Theta}^{0m}(\underset{\sim}{x}') - x'^{m} \partial^{x'}_{k} \hat{\Theta}^{0l}(\underset{\sim}{x}') \right] =$$

$$= i \int d^3x' \left[\delta^{km} \hat{\Theta}^{0l}(\underset{\sim}{x}') - \delta^{kl} \hat{\Theta}^{0m}(\underset{\sim}{x}') \right] = i \delta^{km} \hat{P}^{l} - i \delta^{kl} \hat{P}^{m} ,$$

ou seja,

$$[\hat{P}^k, \hat{J}^{\ell m}] = i\delta^{km}\hat{P}^{\ell} - i\delta^{k\ell}\hat{P}^m \quad . \qquad (I.13)$$

De acordo com nossa métrica definida na Introdução, p.1, é direto ver que os resultados (I.13) e (I.11) podem ser agrupados em

$$[\hat{P}^k, \hat{J}^{\rho\sigma}] = i(g^{k\rho}\hat{P}^{\sigma} - g^{k\sigma}\hat{P}^{\rho}) \qquad (I.14)$$

e os resultados (I.10) e (I.8) em

$$[\hat{P}^0, \hat{J}^{\rho\sigma}] = i(g^{0\rho}\hat{P}^{\sigma} - g^{0\sigma}\hat{P}^{\rho}) \quad . \qquad (I.15)$$

Por seu turno, (I.15) e (I.14) podem ser englobados em

$$[\hat{P}^{\mu}, \hat{J}^{\rho\sigma}] = i(g^{\mu\rho}\hat{P}^{\sigma} - g^{\mu\sigma}\hat{P}^{\rho}) \qquad (I.16)$$

que é a expressão (5.75b). A seguir, provaremos (5.75c). Usando (5.43), calculamos

$$[\hat{J}^{0k}, \hat{J}^{0\ell}] = [x^0\hat{P}^k - \hat{K}^k, x^0\hat{P}^{\ell} - \hat{K}^{\ell}] =$$

$$= -x^0\underset{\swarrow}{\overset{/}{[\hat{P}^k, \hat{K}^{\ell}]}} - x^0\underset{\swarrow}{\overset{/}{[\hat{K}^k, \hat{P}^{\ell}]}} + [\hat{K}^k, \hat{K}^{\ell}] =$$

$$= \int d^3x \int d^3x'\, x^k x'^{\ell} [\hat{\Theta}^{00}(\underset{\sim}{x}), \hat{\Theta}^{00}(\underset{\sim}{x}')] = i\int d^3x'\, x'^{\ell} [\partial^{x'}_m (x'^k \hat{\Theta}^{0m}(\underset{\sim}{x}')) +$$

$$+ \hat{\Theta}^{om}(\underset{\sim}{x}')\partial^{x'}_{m}(x'^{k})\Big] = i\int d^3x'\, x'^{l}\Big[\delta^{km}\hat{\Theta}^{om}(\underset{\sim}{x}') + x'^{k}\partial^{x'}_{m}\hat{\Theta}^{om}(\underset{\sim}{x}') +$$

$$+ \hat{\Theta}^{om}(\underset{\sim}{x}')\delta^{km}\Big] = i\int d^3x'\Big[x'^{l}\hat{\Theta}^{ok}(\underset{\sim}{x}') + (x'^{l}x'^{k})\partial^{x'}_{m}\hat{\Theta}^{om}(\underset{\sim}{x}') + x'^{l}\hat{\Theta}^{ok}(\underset{\sim}{x}')\Big] =$$

$$= i\int d^3x'\Big[x'^{l}\hat{\Theta}^{ok}(\underset{\sim}{x}') - (\underset{\swarrow}{\cancel{x'^{l}\delta^{km}}} + x'^{k}\delta^{ml})\hat{\Theta}^{om}(\underset{\sim}{x}') + \underset{\swarrow}{\cancel{x'^{l}\hat{\Theta}^{ok}(\underset{\sim}{x}')}}\Big] =$$

$$= -i\int d^3x'\Big[x'^{k}\hat{\Theta}^{ol}(\underset{\sim}{x}') - x'^{l}\hat{\Theta}^{ok}(\underset{\sim}{x}')\Big] = -i\hat{J}^{kl}$$

$\therefore$

$$[\hat{J}^{ok}, \hat{J}^{ol}] = [\hat{K}^{k}, \hat{K}^{l}] = -i\hat{J}^{kl} \quad . \qquad (I.17)$$

Desde (5.69), pode-se calcular também

$$[\hat{J}^{ok}, \hat{J}^{lm}] = [x^{o}\hat{P}^{k} - \hat{K}^{k}, \hat{J}^{lm}] = x^{o}[\hat{P}^{k}, \hat{J}^{lm}] -$$

$$-[\hat{K}^{k}, \hat{J}^{lm}] = i\delta^{km}x^{o}\hat{P}^{l} - i\delta^{kl}x^{o}\hat{P}^{m} -$$

$$-\int d^3x\int d^3x'\Big[x'^{k}\hat{\Theta}^{oo}(\underset{\sim}{x}'), x^{l}\hat{\Theta}^{om}(\underset{\sim}{x}) - x^{m}\hat{\Theta}^{ol}(\underset{\sim}{x})\Big] =$$

$$= i\delta^{km}x^{o}\hat{P}^{l} - i\delta^{kl}x^{o}\hat{P}^{m} -$$

$$-\int d^3x\int d^3x'\Big\{x'^{k}x^{l}[\hat{\Theta}^{oo}(\underset{\sim}{x}'), \hat{\Theta}^{om}(\underset{\sim}{x})] - x'^{k}x^{m}[\hat{\Theta}^{oo}(\underset{\sim}{x}'), \hat{\Theta}^{ol}(\underset{\sim}{x})]\Big\} =$$

$$= i\delta^{km}x^{o}\hat{P}^{l} - i\delta^{kl}x^{o}\hat{P}^{m} + \int d^3x'\, x'^{k}\Big\{\int d^3x\, x^{l}[\hat{\Theta}^{om}(\underset{\sim}{x}), \hat{\Theta}^{oo}(\underset{\sim}{x}')] -$$

$$-\int d^3x\, x^m \left[\hat{\Theta}^{0\ell}(\underset{\sim}{x}), \hat{\Theta}^{00}(\underset{\sim}{x}')\right]\Big\} = i\delta^{km} x^0 \hat{P}^\ell - i\delta^{k\ell} x^0 \hat{P}^m +$$

$$+\int d^3x'\, x'^k \Big\{ \frac{i}{2}\left[\underset{\swarrow}{\overset{/}{\delta^{j\ell} \hat{\Theta}^{mj}(\underset{\sim}{x}')}} + x'^\ell \underset{\downarrow}{\overset{/}{\partial^{x'}_j \hat{\Theta}^{mj}(\underset{\sim}{x}')}}\right] + \frac{i}{2}\hat{\Theta}^{m\ell}(\underset{\sim}{x}') +$$

$$+\frac{i}{2}\partial^{x'}_m\left(x'^\ell \hat{\Theta}^{00}(\underset{\sim}{x}')\right) + \frac{i}{2}\underset{\searrow}{\overset{\wr}{\delta^{m\ell} \hat{\Theta}^{00}(\underset{\sim}{x}')}} - \frac{i}{2} x'^\ell \partial^m_{x'} \hat{\Theta}^{00}(\underset{\sim}{x}') +$$

$$+\frac{i}{2} x'^\ell \underset{\swarrow}{\overset{/}{\partial^j_{x'} \hat{\Theta}^{jm}(\underset{\sim}{x}')}} - \frac{i}{2}\left[\underset{\swarrow}{\overset{/}{\delta^{jm} \hat{\Theta}^{\ell j}(\underset{\sim}{x}')}} + x'^m \underset{\Downarrow}{\overset{//}{\partial^{x'}_j \hat{\Theta}^{\ell j}(\underset{\sim}{x}')}}\right] -$$

$$-\frac{i}{2}\hat{\Theta}^{\ell m}(\underset{\sim}{x}') - \frac{i}{2}\partial^{x'}_\ell\left(x'^m \hat{\Theta}^{00}(\underset{\sim}{x}')\right) - \frac{i}{2}\underset{\searrow}{\overset{\wr}{\delta^{\ell m} \hat{\Theta}^{00}(\underset{\sim}{x}')}} + \frac{i}{2} x'^m \partial^\ell_{x'} \hat{\Theta}^{00}(\underset{\sim}{x}') -$$

$$-\frac{i}{2} x'^m \underset{\Downarrow}{\overset{//}{\partial^j_{x'} \hat{\Theta}^{j\ell}(\underset{\sim}{x}')}} \Big\} =$$

$$= i\delta^{km} x^0 \hat{P}^\ell - i\delta^{k\ell} x^0 \hat{P}^m - i\int d^3x' \Big\{ \hat{\Theta}^{00}(\underset{\sim}{x}')\Big[\partial^{x'}_m(x'^k x'^\ell) -$$

$$- \partial^{x'}_\ell(x'^k x'^m)\Big]\Big\} =$$

$$= i\delta^{km} x^0 \hat{P}^\ell - i\delta^{k\ell} x^0 \hat{P}^m - i\int d^3x'\, \hat{\Theta}(\underset{\sim}{x}')\left[\delta^{km} x'^\ell - \delta^{k\ell} x'^m\right] =$$

$$= i\delta^{km}(x^0 \hat{P}^\ell - \hat{K}^\ell) - i\delta^{k\ell}(x^0 \hat{P}^m - \hat{K}^m) =$$

$$= i\delta^{km} \hat{J}^{0\ell} - i\delta^{k\ell} \hat{J}^{0m} \quad \Longrightarrow$$

$$[\hat{J}^{0k}, \hat{J}^{\ell m}] = i\delta^{km}\hat{J}^{0\ell} - i\delta^{k\ell}\hat{J}^{0m} \quad . \qquad (I.18)$$

Além disso, do mesmo cálculo anterior concluímos que

$$[\hat{K}^k, \hat{J}^{lm}] = i(\delta^{km}\hat{K}^l - \delta^{kl}\hat{K}^m) \quad . \quad (I.19)$$

Finalmente, recorrendo a (5.74) podemos obter

$$[\hat{J}^{kl}, \hat{J}^{mn}] = \int d^3_x \int d^3_{x'} \left[x^k \hat{\Theta}^{0l}_{(\underset{\sim}{x})} - x^l \hat{\Theta}^{0k}_{(\underset{\sim}{x})}, \; x'^m \hat{\Theta}^{0n}_{(\underset{\sim}{x}')} - x'^n \hat{\Theta}^{0m}_{(\underset{\sim}{x}')} \right] = \int d^3_x \int d^3_{x'} \left\{ x^k x'^m [\hat{\Theta}^{0l}_{(\underset{\sim}{x})}, \hat{\Theta}^{0n}_{(\underset{\sim}{x}')}] - \right.$$

$$\left. - x^k x'^n [\hat{\Theta}^{0l}_{(\underset{\sim}{x})}, \hat{\Theta}^{0m}_{(\underset{\sim}{x}')}] \right\} - \int d^3_x \int d^3_{x'} \left\{ \begin{matrix} \text{chave anterior} \\ \text{com} \\ (k \leftrightarrow l) \end{matrix} \right\} =$$

$$= \int d^3_x \left\{ x^k \left[-\frac{i}{2} \hat{\Theta}^{0n}_{(\underset{\sim}{x})} \delta^{lm} - \frac{i}{2} \partial^x_l (x^m \hat{\Theta}^{0n}_{(\underset{\sim}{x})}) - \frac{i}{2} \hat{\Theta}^{0l}_{(\underset{\sim}{x})} \delta^{nm} - \right.\right.$$

$$-\frac{i}{2} \partial^x_n (x^m \hat{\Theta}^{0l}_{(\underset{\sim}{x})}) + \frac{i}{2} x^m \partial^x_l \hat{\Theta}^{0n}_{(\underset{\sim}{x})} - \frac{i}{2} x^m \partial^x_n \hat{\Theta}^{0l}_{(\underset{\sim}{x})} +$$

$$+ \frac{i}{2} \hat{\Theta}^{0m}_{(\underset{\sim}{x})} \delta^{ln} + \frac{i}{2} \partial^x_l (x^n \hat{\Theta}^{0m}_{(\underset{\sim}{x})}) + \frac{i}{2} \hat{\Theta}^{0l}_{(\underset{\sim}{x})} \delta^{mn} +$$

$$\left.\left. + \frac{i}{2} \partial^x_m (x^n \hat{\Theta}^{0l}_{(\underset{\sim}{x})}) - \frac{i}{2} x^n \partial^x_l \hat{\Theta}^{0m}_{(\underset{\sim}{x})} + \frac{i}{2} x^n \partial^x_m \hat{\Theta}^{0l}_{(\underset{\sim}{x})} \right] \right\}$$

$$- \left(\begin{matrix} \text{o mesmo} \\ \text{c/ } k \leftrightarrow l \end{matrix} \right) =$$

$$= \left\{ i \int d^3_x \left[x^k \hat{\Theta}^{0m}_{(\underset{\sim}{x})} \delta^{ln} - x^k \hat{\Theta}^{0n}_{(\underset{\sim}{x})} \delta^{lm} \right] + \right.$$

$$\left. + i \int d^3_x \left[x^k x^n \partial^x_m \hat{\Theta}^{0l}_{(\underset{\sim}{x})} - x^k x^m \partial^x_n \hat{\Theta}^{0l}_{(\underset{\sim}{x})} \right] \right\} - \{ k \leftrightarrow l \} =$$

$$= \left\{ i \int d^3x \left[x^k \hat{\Theta}^{0m}(\underset{\sim}{x}) \delta^{ln} - x^k \hat{\Theta}^{0n}(\underset{\sim}{x}) \delta^{lm} - \right.\right.$$

$$\left.\left. - \hat{\Theta}^{0l}(\underset{\sim}{x}) \left(\delta^{km} x^n + \cancel{x^k \delta^{mn}} - \delta^{nk} x^m - \cancel{x^k \delta^{mn}} \right) \right] \right\} -$$

$$- \{ k \leftrightarrow l \} =$$

$$= i \int d^3x \left[x^k \hat{\Theta}^{0m}(\underset{\sim}{x}) \delta^{ln} - x^k \hat{\Theta}^{0n}(\underset{\sim}{x}) \delta^{lm} - x^n \hat{\Theta}^{0l}(\underset{\sim}{x}) \delta^{km} + x^m \hat{\Theta}^{0l}(\underset{\sim}{x}) \delta^{nk} - \right.$$

$$\left. - x^l \hat{\Theta}^{0m}(\underset{\sim}{x}) \delta^{kn} + x^l \hat{\Theta}^{0n}(\underset{\sim}{x}) \delta^{km} + x^n \hat{\Theta}^{0k}(\underset{\sim}{x}) \delta^{lm} - x^m \hat{\Theta}^{0k}(\underset{\sim}{x}) \delta^{nl} \right] =$$

$$= i \left(\delta^{ln} \hat{J}^{km} + \delta^{lm} \hat{J}^{nk} + \delta^{km} \hat{J}^{ln} + \delta^{nk} \hat{J}^{ml} \right) \Rightarrow$$

$$[\hat{J}^{kl}, \hat{J}^{mn}] = i \left(\delta^{km} \hat{J}^{ln} + \delta^{lm} \hat{J}^{nk} + \delta^{kn} \hat{J}^{ml} + \delta^{ln} \hat{J}^{km} \right). \quad (I.20)$$

Os resultados (I.17) e (I.18) podem ser englobados no seguinte

$$[\hat{J}^{0k}, \hat{J}^{\rho\lambda}] = -i \left(g^{\rho k} \hat{J}^{\lambda 0} + g^{k\lambda} \hat{J}^{0\rho} + g^{\lambda 0} \hat{J}^{\rho k} + g^{0\rho} \hat{J}^{k\lambda} \right) \qquad (I.21)$$

o qual junto com (I.20) implica ($g^{k\ell} = -\delta^{k\ell}$)

$$[\hat{J}^{\nu\mu}, \hat{J}^{\rho\lambda}] = -i \left(g^{\rho\mu} \hat{J}^{\lambda\nu} + g^{\mu\lambda} \hat{J}^{\nu\rho} + g^{\lambda\nu} \hat{J}^{\rho\mu} + g^{\nu\rho} \hat{J}^{\mu\lambda} \right)$$

$$(I.22)$$

que coincide com (5.75c). Q.E.D.

APÊNDICE J

PROVA DE $\partial_\mu \hat{j}^{\mu,a} = 0$

Tendo definido as densidades $\hat{j}^{0,a}$ e $\hat{j}^{k,a}$ por (5.76) e (5.77), respectivamente, mostraremos aqui que

$$\partial_0 \hat{j}^{0,a} + \partial_k \hat{j}^{k,a} = 0 \quad , \qquad (J.1)$$

fazendo uso essencialmente das equações de movimento quânticas (5.50), das equações de vínculo (2.33) e dos CTI's (4.2) e (4.6). É claro que

$$\partial_0 \hat{j}^{0,a} = \partial_0 \Big\{ \frac{f^{acb}}{2} \big[\hat{A}^{k,c} \hat{\pi}_k^b + \hat{\pi}_k^b \hat{A}^{k,c} \big] - \frac{i}{2} \Big(\frac{\lambda^a}{2}\Big)^{uv} \big[\hat{\pi}_\psi^u \hat{\psi}^v -$$

$$- \hat{\psi}^v \hat{\pi}_\psi^u \big] \Big\} =$$

$$= \frac{f^{acb}}{2} \big[\dot{\hat{A}}^{k,c} \hat{\pi}_k^b + \hat{A}^{k,c} \dot{\hat{\pi}}_k^b + \dot{\hat{\pi}}_k^b \hat{A}^{k,c} + \hat{\pi}_k^b \dot{\hat{A}}^{k,c} \big] -$$

$$- \frac{i}{2} \Big(\frac{\lambda^a}{2}\Big)^{uv} \big[\dot{\hat{\pi}}_{\psi_r}^u \hat{\psi}_r^v + \hat{\pi}_{\psi_r}^u \dot{\hat{\psi}}_r^v - \dot{\hat{\psi}}_r^v \hat{\pi}_{\psi_r}^u - \hat{\psi}_r^v \dot{\hat{\pi}}_{\psi_r}^u \big] =$$

$$= \frac{f^{acb}}{2} \Big[\big(\hat{\pi}_k^c + \hat{D}^{k,cd} \cdot \hat{A}^{0,d} \big) \hat{\pi}_k^b + \hat{\pi}_k^b \big(\hat{\pi}_k^c + \hat{D}^{k,cd} \cdot \hat{A}^{0,d} \big) +$$

$$+ \hat{A}^{k,c} \Big(-g f^{bed} \hat{A}^{0,e} \cdot \hat{\pi}_k^d + \hat{D}^{j,bd} \hat{F}^{jk,d} + i g \hat{\pi}_\psi \cdot \gamma^0 \gamma^k \frac{\lambda^b}{2} \hat{\psi} \Big) +$$

$$+ \Big(-g f^{bed} \hat{A}^{0,e} \cdot \hat{\pi}_k^d + \hat{D}^{j,bd} \hat{F}^{jk,d} + i g \hat{\pi}_\psi \cdot \gamma^0 \gamma^k \frac{\lambda^b}{2} \hat{\psi} \Big) \hat{A}^{k,c} \Big] -$$

$$-\frac{i}{2}\left(\frac{\lambda}{2}\right)^{auv}\left\{\left[(\partial^k\hat{\pi}_{\psi_s}^{u})(\gamma^0\gamma^k)_{sr}+ig\,\hat{\pi}_{\psi_s}^{w}(\gamma^0\gamma^k)_{sr}\left(\frac{\lambda^b}{2}\right)^{wu}\hat{A}^{k,b}-ig\,\hat{\pi}_{\psi_r}^{w}\cdot\left(\frac{\lambda}{2}\right)^{bwu}\hat{A}^{0,b}+\right.\right.$$

$$\left.+\,im\,\hat{\pi}_{\psi_s}^{u}(\gamma^0)_{sr}\right]\hat{\psi}_r^{v}\quad-$$

$$-\hat{\psi}_r^{v}\left[(\partial^k\hat{\pi}_{\psi_s}^{u})(\gamma^0\gamma^k)_{sr}+ig\,\hat{\pi}_{\psi_s}^{w}(\gamma^0\gamma^k)_{sr}\left(\frac{\lambda^b}{2}\right)^{wu}\hat{A}^{k,b}-ig\,\hat{\pi}_{\psi_r}^{w}\cdot\left(\frac{\lambda^b}{2}\right)^{wu}\hat{A}^{0,b}+\right.$$

$$\left.+\,im\,\hat{\pi}_{\psi_s}^{u}(\gamma^0)_{sr}\right]\quad+$$

$$+\,\hat{\pi}_{\psi_r}^{u}\left[(\gamma^0\gamma^k)_{rs}(\partial^k\hat{\psi}_s^{v})-ig(\gamma^0\gamma^k)_{rs}\left(\frac{\lambda^b}{2}\right)^{vw}\hat{A}^{k,b}\hat{\psi}_s^{w}+ig\left(\frac{\lambda^b}{2}\right)^{vw}\hat{A}^{0,b}\cdot\hat{\psi}_r^{w}-\right.$$

$$\left.-\,im(\gamma^0)_{rs}\hat{\psi}_s^{v}\right]\quad-$$

$$-\left[(\gamma^0\gamma^k)_{rs}(\partial^k\hat{\psi}_s^{v})-ig(\gamma^0\gamma^k)_{rs}\left(\frac{\lambda^b}{2}\right)^{vw}\hat{A}^{k,b}\hat{\psi}_s^{w}+ig\left(\frac{\lambda^b}{2}\right)^{vw}\hat{A}^{0,b}\cdot\hat{\psi}_r^{w}-\right.$$

$$\left.\left.-\,im(\gamma^0)_{rs}\hat{\psi}_s^{v}\right]\hat{\pi}_{\psi_r}^{u}\right\}\,. \qquad (\mathbf{J}.2)$$

Por outro lado:

$$\partial^k\hat{j}^{k,a}=\partial^k\left[\frac{f^{acb}}{2}\left(\hat{A}^{0,c}\hat{\pi}_k^{b}+\hat{\pi}_k^{b}\hat{A}^{0,c}\right)+f^{acb}\hat{A}^{j,c}\hat{F}^{kj,b}-\right.$$

$$\left.-\frac{i}{2}(\gamma^0\gamma^k)_{rs}\left(\frac{\lambda^a}{2}\right)^{uv}\left(\hat{\pi}_{\psi_r}^{u}\hat{\psi}_s^{v}-\hat{\psi}_s^{v}\hat{\pi}_{\psi_r}^{u}\right)\right]=$$

$$=\frac{f^{acb}}{2}\left[(\partial^k\hat{A}^{0,c})\hat{\pi}_k^{b}+\hat{A}^{0,c}(\partial^k\hat{\pi}_k^{b})+(\partial^k\hat{\pi}_k^{b})\hat{A}^{0,c}+\hat{\pi}_k^{b}(\partial^k\hat{A}^{0,c})\right]+$$

$$+ f^{acb}\partial^k(\hat{A}^{j,c}\hat{F}^{kj,b}) - \frac{i}{2}(\gamma^0\gamma^k)_{rs}\left(\frac{\lambda^a}{2}\right)^{uv}\left[(\partial^k\hat{\pi}^u_{\psi_r})\hat{\psi}^v_s + \right.$$

$$\left. + \hat{\pi}^u_{\psi_r}(\partial^k\hat{\psi}^v_s) - (\partial^k\hat{\psi}^v_s)\hat{\pi}^u_{\psi_r} - \hat{\psi}^v_s(\partial^k\hat{\pi}^u_{\psi_r})\right] . \qquad (J.3)$$

A diferença (J.2)-(J.3), de acordo com (J.1), deverá ser zero. Antes de tomar essa diferença, fazemos notar que a antissimetria de $\hat{F}^{jk}$ implica

$$\hat{D}^{k,ab}_{\cdot}(\hat{D}^{j,bd}_{\cdot}\hat{F}^{jk,d}) = 0$$

$$\therefore$$

$$\delta^{ab}\partial^k(\hat{D}^{j,bd}\hat{F}^{jk,d}) = -gf^{acb}\hat{A}^{k,c}(\hat{D}^{j,bd}\hat{F}^{jk,d}) .$$

O lado esquerdo desta equação é igual a

$$\delta^{ab}\partial^k(gf^{bed}\hat{A}^{j,e}\hat{F}^{jk,d}) = gf^{acb}\partial^k(\hat{A}^{j,c}\hat{F}^{jk,b}) .$$

Logo,

$$f^{acb}\hat{A}^{k,c}(\hat{D}^{j,bd}\hat{F}^{jk,d}) = +f^{acb}\partial^k(\hat{A}^{j,c}\hat{F}^{kj,b}). \qquad (J.4)$$

Agora, tomamos (J.2)-(J.3), i.e.,

$$\partial_0\hat{j}^{0,a} - \partial^k\hat{j}^{k,a} =$$

$$= \frac{f^{acb}}{2}\left\{(\hat{D}^{k,cd}_{\cdot}\hat{A}^{0,d})\hat{\pi}^b_k + \hat{\pi}^b_k(\hat{D}^{k,cd}_{\cdot}\hat{A}^{0,d}) - \right.$$

$$-gf^{bed}\left[\hat{A}^{k,c}(\hat{A}^{0,e}_{\cdot}\hat{\pi}^d_k) + (\hat{A}^{0,e}_{\cdot}\hat{\pi}^d_k)\hat{A}^{k,c}\right] -$$

$$-(\partial^k\hat{A}^{0,c})\hat{\pi}^b_k - \hat{A}^{0,c}(\partial^k\hat{\pi}^b_k) - (\partial^k\hat{\pi}^b_k)\hat{A}^{0,c} - \hat{\pi}^b_k(\partial^k\hat{A}^{0,c})\Big]\Big]\Big\}+$$

$$+f^{acb}ig\,\hat{\pi}_\psi\cdot\gamma^0\gamma^k\frac{\lambda^b}{2}\hat{\psi}\hat{A}^{k,c} -$$

$$-\frac{i}{2}\Big\{(\partial^k\hat{\pi}_\psi)\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi} + ig\,\hat{\pi}_\psi\gamma^0\gamma^k\frac{\lambda^b}{2}\frac{\lambda^a}{2}\hat{\psi}\hat{A}^{k,b} - ig\Big(\hat{\pi}^u_{\psi_r}\cdot\Big(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\Big)^{uv}\hat{A}^{0,b}\Big)\hat{\psi}^v_r+$$

$$+im\,\hat{\pi}_\psi\gamma^0\frac{\lambda^a}{2}\hat{\psi} - ig\,\hat{\psi}^v_r\Big(\hat{\pi}^w_{\psi_s}(\gamma^0\gamma^k)_{sr}\Big(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\Big)^{wv}\hat{A}^{k,b}\Big) - \hat{\psi}^v_r\Big(\partial^k\hat{\pi}^u_{\psi_s}(\gamma^0\gamma^k)_{sr}\Big(\frac{\lambda^a}{2}\Big)^{uv}\Big)+$$

$$+ig\,\hat{\psi}^v_r\Big(\hat{\pi}^w_{\psi_r}\cdot\Big(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\Big)^{wv}\hat{A}^{0,b}\Big) - im\,\hat{\psi}^v_r\hat{\pi}^u_{\psi_s}(\gamma^0)_{sr}\Big(\frac{\lambda^a}{2}\Big)^{uv}+$$

$$+\hat{\pi}_\psi\gamma^0\gamma^k\frac{\lambda^a}{2}\partial^k\hat{\psi} - ig\,\hat{\pi}_\psi\gamma^0\gamma^k\frac{\lambda^a}{2}\frac{\lambda^b}{2}\hat{A}^{k,b}\hat{\psi} + ig\,\hat{\pi}^u_{\psi_r}\Big(\Big(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\Big)^{uv}\hat{A}^{0,b}\cdot\hat{\psi}^v_r\Big)-$$

$$-im\,\hat{\pi}_\psi\gamma^0\frac{\lambda^a}{2}\hat{\psi} - (\gamma^0\gamma^k)_{rs}\Big(\frac{\lambda^a}{2}\Big)^{uv}(\partial^k\hat{\psi}^v_s)\hat{\pi}^u_{\psi_r} + ig(\gamma^0\gamma^k)_{rs}\Big(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\Big)^{uw}\hat{A}^{k,b}\hat{\psi}^w_s\hat{\pi}^u_{\psi_r}-$$

$$-ig\Big(\Big(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\Big)^{uw}\hat{A}^{0,b}\cdot\hat{\psi}^w_r\Big)\hat{\pi}^u_{\psi_r} + im(\gamma^0)_{rs}\Big(\frac{\lambda^a}{2}\Big)^{uv}\hat{\psi}^v_s\hat{\pi}^u_{\psi_r} -$$

$$-(\partial^k\hat{\pi}_\psi)\gamma^0\gamma^k\frac{\lambda^a}{2}\hat{\psi} - \hat{\pi}_\psi\gamma^0\gamma^k\frac{\lambda^a}{2}\partial^k\hat{\psi} + \Big((\gamma^0\gamma^k)_{rs}\Big(\frac{\lambda^a}{2}\Big)^{uv}\partial^k\hat{\psi}^v_s\Big)\hat{\pi}^u_{\psi_r}+$$

$$+\Big((\gamma^0\gamma^k)_{rs}\Big(\frac{\lambda^a}{2}\Big)^{uv}\hat{\psi}^v_s\Big)\partial^k\hat{\pi}^u_{\psi_r}\Big\} \qquad \Longrightarrow$$

$$\partial_0\hat{j}^{0,a} - \partial^k\hat{j}^{k,a} = igf^{acb}\hat{\pi}_\psi\cdot\gamma^0\gamma^k\frac{\lambda^b}{2}\hat{\psi}\hat{A}^{k,c} +$$

$$+\frac{f^{acb}}{2}\Big\{(\partial^k\hat{A}^{0,c})\hat{\pi}^b_k + gf^{ced}(\hat{A}^{k,e}\cdot\hat{A}^{0,d})\hat{\pi}^b_k + \hat{\pi}^b_k(\partial^k\hat{A}^{0,c}) +$$

$$+ g f^{ced}\, \hat{\pi}^{b}_{k} (\hat{A}^{k,e} \cdot \hat{A}^{0,d}) - g f^{bed}\, \hat{A}^{k,c} (\hat{A}^{0,e} \cdot \hat{\pi}^{d}_{k}) - g f^{bed} (\hat{A}^{0,e} \cdot \hat{\pi}^{d}_{k}) \hat{A}^{k,c} -$$

$$- (\partial^{k} \hat{A}^{0,c}) \hat{\pi}^{b}_{k} - \hat{A}^{0,c} (\partial^{k} \hat{\pi}^{b}_{k}) - (\partial^{k} \hat{\pi}^{b}_{k}) \hat{A}^{0,c} - \hat{\pi}^{b}_{k} (\partial^{k} \hat{A}^{0,c}) \Big\} +$$

$$+ \frac{g}{2}\, \hat{\pi}_{\psi} \gamma^{0}\gamma^{k} \left[\frac{\lambda^{b}}{2}, \frac{\lambda^{a}}{2}\right] \hat{\psi} \hat{A}^{k,b} + \frac{g}{2}\, \hat{\psi}^{v}_{s}\, \hat{\pi}^{u}_{\psi_r} (\gamma^{0}\gamma^{k})_{rs} \left[\frac{\lambda^{a}}{2}, \frac{\lambda^{b}}{2}\right]^{uv} \hat{A}^{k,b} -$$

$$- \frac{g}{2} \left(\hat{\pi}^{u}_{\psi_r} \cdot \hat{A}^{0,b}\right) \left(\frac{\lambda^{b}}{2}\frac{\lambda^{a}}{2}\right)^{uv} \hat{\psi}^{v}_{r} + \frac{g}{2}\, \hat{\psi}^{v}_{r} \left(\hat{\pi}^{u}_{\psi_r} \cdot \hat{A}^{0,b}\right) \left(\frac{\lambda^{b}}{2}\frac{\lambda^{a}}{2}\right)^{uv} +$$

$$+ \frac{g}{2}\, \hat{\pi}^{u}_{\psi_r} \left(\hat{A}^{0,b} \cdot \hat{\psi}^{v}_{r}\right) \left(\frac{\lambda^{a}}{2}\frac{\lambda^{b}}{2}\right)^{uv} - \frac{g}{2} \left(\hat{A}^{0,b} \cdot \hat{\psi}^{v}_{r}\right) \hat{\pi}^{u}_{\psi_r} \left(\frac{\lambda^{a}}{2}\frac{\lambda^{b}}{2}\right)^{uv} . \qquad (J.5)$$

Consideremos, na expressão (J.5), inicialmente os termos fermiônicos acoplados ao campo $\hat{A}^{k,a}$:

$$i g f^{acb}\, \hat{\pi}_{\psi} \cdot \gamma^{0}\gamma^{k} \frac{\lambda^{b}}{2} \hat{\psi} \hat{A}^{k,c} +$$

$$+ \frac{ig}{2} f^{bac}\, \hat{\pi}^{u}_{\psi_r} (\gamma^{0}\gamma^{k})_{rs} \left(\frac{\lambda^{c}}{2}\right)^{uv} \hat{\psi}^{v}_{s} \hat{A}^{k,b} - \frac{ig}{2} f^{bac}\, \hat{\psi}^{v}_{s}\, \hat{\pi}^{u}_{\psi_r} (\gamma^{0}\gamma^{k})_{rs} \left(\frac{\lambda^{c}}{2}\right)^{uv} \hat{A}^{k,b} =$$

$$= i g f^{acb}\, \hat{\pi}_{\psi} \cdot \gamma^{0}\gamma^{k} \frac{\lambda^{b}}{2} \hat{\psi} \hat{A}^{k,c} +$$

$$+ i g f^{bac} (\gamma^{0}\gamma^{k})_{rs} \left(\frac{\lambda^{c}}{2}\right)^{uv} \frac{1}{2} \left[\hat{\pi}^{u}_{\psi_r} \hat{\psi}^{v}_{s} - \hat{\psi}^{v}_{s} \hat{\pi}^{u}_{\psi_r}\right] \hat{A}^{k,b} =$$

$$= i g f^{acb}\, \hat{\pi}_{\psi} \cdot \gamma^{0}\gamma^{k} \frac{\lambda^{b}}{2} \hat{\psi} \hat{A}^{k,c} + i g f^{acb} \left(\hat{\pi}_{\psi} \cdot \gamma^{0}\gamma^{k} \frac{\lambda^{c}}{2} \hat{\psi}\right) \hat{A}^{k,b} =$$

$$= 0 \quad . \qquad (J.6)$$

A seguir, tomemos em (J.5) os termos fermiônicos restantes (acoplados ao campo $\hat{A}^{0,a}$)*:

$$\frac{g}{4}\left\{\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv}\left[-\hat{\pi}^{u}_{\psi_r}\hat{A}^{0,b}\hat{\psi}^{v}_{r}-\hat{A}^{0,b}\hat{\pi}^{u}_{\psi_r}\hat{\psi}^{v}_{r}+\hat{\psi}^{v}_{r}\hat{\pi}^{u}_{\psi_r}\hat{A}^{0,b}+\right.\right.$$

$$\left.+\hat{\psi}^{v}_{r}\hat{A}^{0,b}\hat{\pi}^{u}_{\psi_r}\right]+$$

$$+\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv}\left[\hat{\pi}^{u}_{\psi_r}\hat{A}^{0,b}\hat{\psi}^{v}_{r}+\hat{\pi}^{u}_{\psi_r}\hat{\psi}^{v}_{r}\hat{A}^{0,b}-\hat{A}^{0,b}\hat{\psi}^{v}_{r}\hat{\pi}^{u}_{\psi_r}-\right.$$

$$\left.\left.-\psi^{v}_{r}\hat{A}^{0,b}\hat{\pi}^{u}_{\psi_r}\right]\right\}-$$

$$-\frac{ig}{4}f^{acb}\left(\frac{\lambda}{2}\right)^{buv}\left[\hat{A}^{0,c}\hat{\pi}^{u}_{\psi_r}\hat{\psi}^{v}_{r}-\hat{A}^{0,c}\hat{\psi}^{v}_{r}\hat{\pi}^{u}_{\psi_r}+\hat{\pi}^{u}_{\psi_r}\hat{\psi}^{v}_{r}\hat{A}^{0,c}-\hat{\psi}^{v}_{r}\hat{\pi}^{u}_{\psi_r}\hat{A}^{0,c}\right].$$

$$(J.7)$$

A chave em (J.7) pode ser reescrita lançando mão de (4.2d), de $\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right] = i\,f^{abc}\frac{\lambda^c}{2}$ e do fato que $\mathrm{Tr}\left(\frac{\lambda^a}{2}\right) = 0$. Realmente,

$$-\hat{\pi}^{u}_{\psi_r}\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv}\hat{A}^{0,b}\hat{\psi}^{v}_{r}+\hat{\pi}^{u}_{\psi_r}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv}\hat{A}^{0,b}\hat{\psi}^{v}_{r}=\hat{\pi}^{u}_{\psi_r}\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]^{uv}\hat{A}^{0,b}\hat{\psi}^{v}_{r}=$$

$$=if^{abc}\,\hat{\pi}^{u}_{\psi_r}\left(\frac{\lambda^c}{2}\right)^{uv}\hat{A}^{0,b}\hat{\psi}^{v}_{r}=if^{abc}\left(\frac{\lambda^c}{2}\right)^{uv}\hat{A}^{0,b}\hat{\pi}^{u}_{\psi_r}\hat{\psi}^{v}_{r}+$$

$$+if^{abc}\left(\frac{\lambda^c}{2}\right)^{uv}\left[\hat{\pi}^{u}_{\psi_r},\hat{A}^{0,b}\right]\hat{\psi}^{v}_{r}=$$

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*Note-se que, usando a equação de vínculo (4.3) (a lei de Gauss), podemos escrever $\partial^k\bar{\pi}^b_k = -g\,f^{bde}\,\bar{A}^{k,d}\cdot\bar{\pi}^e_k + i\,g\,\hat{\pi}_\psi\cdot\frac{\lambda^b}{2}\,\hat{\psi}$.

$$= -if^{abc}\left(\frac{\lambda^c}{2}\right)^{uv}\hat{A}^{0,b}\,\hat{\psi}^v_r\,\hat{\pi}^u_{\psi_r} - f^{abc}\,Tr\left(\frac{\lambda^c}{2}\right)\hat{A}^{0,b}\,\delta^{(3)}(0) +$$

$$+ if^{abc}\left(\frac{\lambda^c}{2}\right)^{uv}\left[\hat{\pi}^u_{\psi_r},\hat{A}^{0,b}\right]\hat{\psi}^v_r \qquad ; \qquad (J.8)$$

$$\hat{\psi}^v_r\,\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv} - \hat{\psi}^v_r\,\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv} = \hat{\psi}^v_r\,\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left[\frac{\lambda^b}{2},\frac{\lambda^a}{2}\right]^{uv}$$

$$= if^{acb}\,\hat{\psi}^v_r\,\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^c}{2}\right)^{uv} =$$

$$= if^{acb}\left(\frac{\lambda^c}{2}\right)^{uv}\hat{\psi}^v_r\,\hat{\pi}^u_{\psi_r}\,\hat{A}^{0,b} + if^{acb}\left(\frac{\lambda^c}{2}\right)^{uv}\hat{\psi}^v_r\left[\hat{A}^{0,b},\hat{\pi}^u_{\psi_r}\right] \; ; \quad (J.9)$$

$$-\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_r - \hat{A}^{0,b}\,\hat{\psi}^v_r\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv} =$$

$$= -\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv}\hat{\psi}^v_r + \hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\,\hat{\psi}^v_r\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv} - 4i\,\hat{A}^{0,b}\,\delta^{(3)}(0)\,Tr\left[\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right] =$$

$$= \hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]^{uv}\hat{\psi}^v_r - 4i\,\hat{A}^{0,b}\,\delta^{(3)}(0)\,Tr\left[\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right] =$$

$$= if^{abc}\,\hat{A}^{0,b}\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^c}{2}\right)^{uv}\hat{\psi}^v_r - i\,\hat{A}^{0,b}\,\delta^{(3)}(0)\,Tr\left[\lambda^a\lambda^b\right] \; ; \quad (J.10)$$

$$\hat{\psi}^v_r\,\hat{\pi}^u_{\psi_r}\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv}\hat{A}^{0,b} + \hat{\pi}^u_{\psi_r}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv}\hat{\psi}^v_r\,\hat{A}^{0,b} =$$

$$= -\hat{\pi}^u_{\psi_r}\,\hat{\psi}^v_r\left(\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)^{uv}\hat{A}^{0,b} + 4i\,\delta^{(3)}(0)\,Tr\left[\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right]\hat{A}^{0,b} + \hat{\pi}^u_{\psi_r}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right)^{uv}\hat{\psi}^v_r\,\hat{A}^{0,b} =$$

$$= \hat{\pi}_{\psi_r}^{u} \left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right]^{uv} \hat{\psi}_r^{v} \hat{A}^{0,b} + i\,\delta^{(3)}(0)\, Tr\left[\lambda^b \lambda^a\right] \hat{A}^{0,b} =$$

$$= i f^{abc}\, \hat{\pi}_{\psi_r}^{u} \left(\frac{\lambda^c}{2}\right)^{uv} \hat{\psi}_r^{v} \hat{A}^{0,b} + i\,\delta^{(3)}(0)\, \hat{A}^{0,b}\, Tr\left[\lambda^a, \lambda^b\right] . \qquad (J.11)$$

A chave em (J.7) ē igual ā soma das expressões (J.8)-(J.11). Realizando tal soma, obtemos

$$\left\{ \begin{matrix} \text{chave em} \\ (J.7) \end{matrix} \right] = i f^{acb} \left(\frac{\lambda^b}{2}\right)^{uv} \Big[-\hat{A}^{0,c} \hat{\psi}_r^{v} \hat{\pi}_{\psi_r}^{u} - \hat{\psi}_r^{v} \hat{\pi}_{\psi_r}^{u} \hat{A}^{0,c} +$$

$$+ \hat{A}^{0,c} \hat{\pi}_{\psi_r}^{u} \hat{\psi}_r^{v} + \hat{\pi}_{\psi}^{u} \hat{\psi}_r^{v} \hat{A}^{0,c} \Big] +$$

$$+ i f^{abc} \left(\frac{\lambda^c}{2}\right)^{uv} \left\{ \left[\hat{\pi}_{\psi_r}^{u}, \hat{A}^{0,b} \right] \hat{\psi}_r^{v} - \hat{\psi}_r^{v} \left[\hat{A}^{0,b}, \hat{\pi}_{\psi_r}^{u} \right] \right\} .$$

$$(J.12)$$

Agora, o termo da chave no lado direito de (J.12) ē, de fato, nulo, conforme segue de (4.6d):

$$i f^{abc} \left(\frac{\lambda^c}{2}\right)^{uv} \left\{ \left[\hat{\pi}_{\psi_r}^{u}(\underset{\sim}{x}), \hat{A}^{0,b}(\underset{\sim}{x}) \right] \hat{\psi}_r^{v}(\underset{\sim}{x}) - \hat{\psi}_r^{v}(\underset{\sim}{x}) \left[\hat{A}^{0,b}(\underset{\sim}{x}), \hat{\pi}_{\psi_r}^{u}(\underset{\sim}{x}) \right] \right\} =$$

$$= i f^{abc} \left(\frac{\lambda^c}{2}\right)^{uv} \Big\{ g\, \hat{\pi}_{\psi_r}^{w}(\underset{\sim}{x}) \left(\frac{\lambda^b}{2}\right)^{wu} \Omega(\underset{\sim}{x}, \underset{\sim}{x}; \underset{\sim}{x}_{(0)})\, \hat{\psi}_r^{v}(\underset{\sim}{x}) +$$

$$+ g\, \hat{\psi}_r^{v}(\underset{\sim}{x})\, \hat{\pi}_{\psi}^{w}(\underset{\sim}{x}) \left(\frac{\lambda^b}{2}\right)^{wu} \Omega(\underset{\sim}{x}, \underset{\sim}{x}; \underset{\sim}{x}_{(0)}) \Big] =$$

$$= i g f^{abc} \Big\{ \hat{\pi}_{\psi_r}^{w}(\underset{\sim}{x}) \left(\frac{\lambda^b}{2}\frac{\lambda^c}{2}\right)^{wv} \hat{\psi}_r^{v}(\underset{\sim}{x})\, \Omega(\underset{\sim}{x}, \underset{\sim}{x}; \underset{\sim}{x}_{(0)}) -$$

$$- \hat{\pi}^{w}_{\psi_r}(\underset{\sim}{x}) \left(\frac{\lambda^b}{2} \frac{\lambda^c}{2} \right)^{wv} \hat{\psi}^{v}_{r}(\underset{\sim}{x})\, \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)}) + i\, \delta^{(3)}_{(0)}\, Tr[\lambda^b \lambda^c]\, \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)})$$

$$= - g\, \delta^{(3)}_{(0)}\, f^{abc}\, Tr[\lambda^b \lambda^c]\, \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0 \quad , \quad (J.13)$$

uma vez que $Tr[\lambda^b\lambda^c] \propto \delta^{bc}$. Substituindo, então, (J.12) em (J.7) vemos que os termos fermiônicos acoplados ao campo $\hat{A}^{0,a}$ também se cancelam em (J.5). Por último, tomamos os termos puramente bosônicos em (J.5):

$$\frac{g}{4} f^{acb} \Big\{ f^{ced} \left[\hat{A}^{k,e} \hat{A}^{0,d} + \hat{A}^{0,d} \hat{A}^{k,e} \right] \hat{\pi}^{b}_{k} +$$

$$+ f^{ced}\, \hat{\pi}^{b}_{k} \left[\hat{A}^{k,e} \hat{A}^{0,d} + \hat{A}^{0,d} \hat{A}^{k,e} \right] -$$

$$- f^{bed}\, \hat{A}^{k,c} \left[\hat{A}^{0,e} \hat{\pi}^{d}_{k} + \hat{\pi}^{d}_{k} \hat{A}^{0,e} \right] -$$

$$- f^{bed} \left[\hat{A}^{0,e} \hat{\pi}^{d}_{k} + \hat{\pi}^{d}_{k} \hat{A}^{0,e} \right] \hat{A}^{k,c} +$$

$$+ f^{bde}\, \hat{A}^{0,c} \left[\hat{A}^{k,d} \hat{\pi}^{e}_{k} + \hat{\pi}^{e}_{k} \hat{A}^{k,d} \right] +$$

$$+ f^{bde} \left[\hat{A}^{k,d} \hat{\pi}^{e}_{k} + \hat{\pi}^{e}_{k} \hat{A}^{k,d} \right] \hat{A}^{0,c} \Big\} =$$

$$= \frac{g}{4} \Big\{ f^{acb} f^{ced} \left[\hat{A}^{k,e} \hat{A}^{0,d} \hat{\pi}^{b}_{k} + \hat{A}^{0,d} \hat{A}^{k,e} \hat{\pi}^{b}_{k} + \hat{\pi}^{b}_{k} \hat{A}^{k,e} \hat{A}^{0,d} + \hat{\pi}^{b}_{k} \hat{A}^{0,d} \hat{A}^{k,e} \right] -$$

$$- f^{aec} f^{cdb} \left[\hat{A}^{k,e} \hat{A}^{0,d} \hat{\pi}^{b}_{k} + \hat{A}^{k,e} \hat{\pi}^{b}_{k} \hat{A}^{0,d} + \hat{A}^{0,d} \hat{\pi}^{b}_{k} \hat{A}^{k,e} + \hat{\pi}^{b}_{k} \hat{A}^{0,d} \hat{A}^{k,e} \right] +$$

$$+f^{adc}f^{ceb}\left[\hat{A}^{0,d}\left(\hat{A}^{k,e}\hat{\pi}_k^{b}+\hat{\pi}_k^{b}\hat{A}^{k,e}\right)+\left(\hat{A}^{k,e}\hat{\pi}_k^{b}+\hat{\pi}_k^{b}\hat{A}^{k,e}\right)\hat{A}^{0,d}\right]\Bigg\}.$$

(J.14)

No que segue, mostramos que os dois primeiros colchetes em (J.14) cancelam exatamente o terceiro. Para provar este enunciado começamos reescrevendo os dois primeiros colchetes com o auxílio da identidade de Jacobi $f^{acb}f^{ced} - f^{aec}f^{cdb} = -f^{adc}f^{ceb}$:

$$-f^{adc}f^{ceb}\left(\hat{A}^{k,e}\hat{A}^{0,d}\hat{\pi}_k^{b}+\hat{\pi}_k^{b}\hat{A}^{0,d}\hat{A}^{k,e}\right)\ +$$

$$+f^{acb}f^{ced}\left(\hat{A}^{0,d}\hat{A}^{k,e}\hat{\pi}_k^{b}+\hat{\pi}_k^{b}\hat{A}^{k,e}\hat{A}^{0,d}\right)\ -$$

$$-f^{aec}f^{cdb}\left(\hat{A}^{k,e}\hat{\pi}_k^{b}\hat{A}^{0,d}+\hat{A}^{0,d}\hat{\pi}_k^{b}\hat{A}^{k,e}\right)\ =$$

$$=-f^{adc}f^{ceb}\Big\{\hat{A}^{0,d}\hat{A}^{k,e}\hat{\pi}_k^{b}+[\hat{A}^{k,e},\hat{A}^{0,d}]\hat{\pi}_k^{b}+\hat{\pi}_k^{b}\hat{A}^{k,e}\hat{A}^{0,d}+$$

$$+\hat{\pi}_k^{b}[\hat{A}^{0,d},\hat{A}^{k,e}]\Big\}\ +$$

$$+f^{acb}f^{ced}\Big\{\hat{A}^{0,d}\hat{\pi}_k^{b}\hat{A}^{k,e}+\hat{A}^{0,d}[\hat{A}^{k,e},\hat{\pi}_k^{b}]+\hat{\pi}_k^{b}\hat{A}^{k,e}\hat{A}^{0,d}\Big\}-$$

$$-f^{aec}f^{cdb}\Big\{\hat{\pi}_k^{b}\hat{A}^{k,e}\hat{A}^{0,d}+[\hat{A}^{k,e},\hat{\pi}_k^{b}]\hat{A}^{0,d}+\hat{A}^{0,d}\hat{\pi}_k^{b}\hat{A}^{k,e}\Big\}=$$

$$=-f^{adc}f^{ceb}\Big\{\hat{A}^{0,d}\hat{A}^{k,e}\hat{\pi}_k^{b}+[\hat{A}^{k,e},\hat{A}^{0,d}]\hat{\pi}_k^{b}+\hat{A}^{k,e}\hat{\pi}_k^{b}\hat{A}^{0,d}+$$

$$+ [\hat{\pi}^{b}_{k}, \hat{A}^{k,e}]\hat{A}^{0,d} + \hat{\pi}^{b}_{k}[\hat{A}^{0,d}, \hat{A}^{k,e}]\Big\} -$$

$$-f^{adc}f^{ceb}\Big[\hat{A}^{0,d}\hat{\pi}^{b}_{k}\hat{A}^{k,e} + \hat{\pi}^{b}_{k}\hat{A}^{k,e}\hat{A}^{0,d}\Big] +$$

$$+f^{acb}f^{ced}\hat{A}^{0,d}[\hat{A}^{k,e}, \hat{\pi}^{b}_{k}] - f^{aec}f^{cdb}[\hat{A}^{k,e}, \hat{\pi}^{b}_{k}]\hat{A}^{0,d} =$$

$$= -f^{adc}f^{ceb}\Big[\hat{A}^{0,d}(\hat{A}^{k,e}\hat{\pi}^{b}_{k} + \hat{\pi}^{b}_{k}\hat{A}^{k,e}) + (\hat{A}^{k,e}\hat{\pi}^{b}_{k} + \hat{\pi}^{b}_{k}\hat{A}^{k,e})\hat{A}^{0,d}\Big] +$$

$$+\Big\{(+f^{adc}f^{ceb} - f^{aec}f^{cdb})[\hat{A}^{k,e}, \hat{\pi}^{b}_{k}]\hat{A}^{0,d} +$$

$$+ f^{acb}f^{ced}\hat{A}^{0,d}[\hat{A}^{k,e}, \hat{\pi}^{b}_{k}] -$$

$$- f^{adc}f^{ceb}\Big[[\hat{A}^{k,e}, \hat{A}^{0,d}]\hat{\pi}^{b}_{k} - \hat{\pi}^{b}_{k}[\hat{A}^{k,e}, \hat{A}^{0,d}]\Big]\Big\} . \quad (J.15)$$

O cancelamento exato dos termos de (J.14) decorre do fato que a chave em (J.15) é identicamente nula. De fato,

$$\left\{\begin{matrix}\text{chave em}\\ (J.15)\end{matrix}\right\} = -f^{acb}f^{ced}[\hat{A}^{k,e}, \hat{\pi}^{b}_{k}]\hat{A}^{0,d} +$$

$$+f^{acb}f^{ced}\hat{A}^{0,d}[\hat{A}^{k,e}, \hat{\pi}^{b}_{k}] - f^{adc}f^{ceb}\Big[[\hat{A}^{k,e}, \hat{A}^{0,d}], \hat{\pi}^{b}_{k}\Big] =$$

$$= f^{acb}f^{ced}\Big[\hat{A}^{0,d}(\underset{\sim}{x}), [\hat{A}^{k,e}(\underset{\sim}{x}), \hat{\pi}^{b}_{k}(\underset{\sim}{x})]\Big] - f^{adc}f^{ceb}\Big[[\hat{A}^{k,e}(\underset{\sim}{x}), \hat{A}^{0,d}(\underset{\sim}{x})], \hat{\pi}^{b}_{k}(\underset{\sim}{x})\Big] =$$

$$= f^{acb}f^{ced}\Big[\hat{A}^{0,d}(\underset{\sim}{x}), igf^{emb}\hat{A}^{k,m}(\underset{\sim}{x})R_{k}(\underset{\sim}{x};\underset{\sim}{x})\Big] -$$

$$-f^{adc} f^{ceb} \left[-ig f^{dme} \hat{A}^{k,m}(\underset{\sim}{x})\, \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)}),\ \hat{\pi}^{b}_{k}(\underset{\sim}{x}) \right] =$$

$$= ig f^{adc} f^{ceb} f^{dme} \left[\hat{A}^{k,m}(\underset{\sim}{x}),\ \hat{\pi}^{b}_{k}(\underset{\sim}{x}) \right] \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)}) =$$

$$= ig f^{adc} f^{ceb} f^{dme} \left[i\delta^{mb} \left(\delta^{(3)}(0) + \partial^{k}_{x} R_{k}(\underset{\sim}{x};\underset{\sim}{x}) \right) \right] \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)}) =$$

$$= -g f^{adc} f^{bce} f^{edb} \left(\delta^{(3)}(0) + \partial^{k}_{x} R_{k}(\underset{\sim}{x};\underset{\sim}{x}) \right) \Omega(\underset{\sim}{x},\underset{\sim}{x};\underset{\sim}{x}_{(0)}) = 0, \quad (J.16)$$

pela antissimetria das constantes de estrutura. Com isso, todos os termos no lado direito de (J.5) se cancelam deixando-nos com

$$\partial_0 \hat{j}^{0,a} - \partial^{k} \hat{j}^{k,a} = 0 \quad \Rightarrow \quad \partial_\mu \hat{j}^{\mu,a} = 0 \ .$$

Q.E.D.

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