

A refinement of the rotation period of the roAp star HR 3831 and the discovery of a significant phase lag between the times of pulsation and magnetic extrema and the time of mean-light extremum

D. W. Kurtz,¹ A. Kanaan,² Peter Martinez¹ and P. Tripe¹

¹Department of Astronomy, University of Cape Town, Rondebosch 7700, South Africa

²Instituto de Física, Universidade Federal do Rio Grande do Sul, 90049 Porto Alegre, RS, Brazil

Accepted 1991 October 15. Received 1991 October 10; in original form 1991 June 27

SUMMARY

HR 3831 is a rapidly oscillating Ap star with a pulsational period of 11.67 min. The amplitude of this pulsation ranges from 0 to about 5 mmag as a function of the rotational aspect of the star. We use 268 hr of new high-speed photometric observations obtained in 1990 and 1991 plus 238 hr of published high-speed photometry from 1980, 1981, 1985 and 1986 to show that the period of pulsation amplitude modulation is equal to the period of mean-light variation. We use this to show that an interpretation of the pulsation-amplitude modulation as rotationally perturbed m -modes requires that $C_{nl} \leq 2 \times 10^{-5}$ at the 3σ confidence level, making that interpretation highly unlikely. We also show that the time of pulsation maximum precedes the time of mean-light extremum by 0.177 ± 0.007 d (0.062 ± 0.002 rotation periods), a 25σ difference. The time of magnetic extremum coincides with the time of pulsation-amplitude maximum but differs from mean-light extremum by 0.158 ± 0.032 d, a 5σ difference. This argues that HR 3831 is an oblique pulsator with the magnetic axis and pulsation axis aligned, rather than a spotted pulsator with amplitude modulation caused by surface inhomogeneities. It also indicates a deviation from cylindrical symmetry which makes the interpretation of the pulsation-amplitude modulation in terms of the oblique-pulsator model more complex than has yet been considered. By shifting the pulsation-amplitude variations into phase with the mean-light variations and normalizing both data sets, we refine the rotation frequency for HR 3831 to be $\nu_{\text{rot}} = 0.3506333 \pm 0.0000006 \text{ d}^{-1} = 4058.256 \pm 0.007 \text{ nHz}$, or $P_{\text{rot}} = 2.851982 \pm 0.000005 \text{ d}$.

1 INTRODUCTION

HR 3831 is a singly periodic rapidly oscillating Ap (roAp) star which pulsates with a frequency of $\nu = 1.42801 \text{ mHz}$ ($P = 11.67 \text{ min}$). Observed through a Johnson B filter, the semi-amplitude of the light variation associated with the pulsation ranges from about 5 mmag at the times of magnetic extrema to zero at magnetic quadrature [Kurtz 1982; Kurtz & Shibahashi 1986; Kurtz 1990; Kurtz, Shibahashi & Goode 1990; (hereafter KSG)]. The amplitude-modulation period is the same as the rotation period, $P_{\text{rot}} = 2.851962 \text{ d}$ (Kurtz & Marang 1988), determined from mean-light variations presumably associated with abundance-anomaly concentrations typical of the Ap stars (Renson & Manfroid 1978; Kurtz 1982; Renson *et al.* 1984; Kurtz & Marang 1988). Thompson (1983) measured the effective magnetic field nine times and Mathys (1991) measured it a further 12 times;

analysis of their data shows a reversing polarity field with a zero-point of $16 \pm 42 \text{ G}$ and an amplitude of $760 \pm 70 \text{ G}$ with the same rotation period. We show in this paper that the phase of the magnetic field variations coincides with the phase of the pulsation-amplitude variations and differs from the phase of the mean-light variations.

The amplitude modulation of the pulsational light variations shows the clear signature of an oblique dipole pulsation mode (KSG): The phase of the pulsation undergoes a 180° phase reversal as the star passes through quadrature. Fig. 1 shows this in a plot of pulsation phase and amplitude versus rotation phase for new data obtained in 1991. HR 3831 provides the best evidence in favour of the oblique-pulsator model which postulates that the pulsation axis and magnetic axis in the roAp stars coincide.

There are complications, however. It is clear that the pulsation mode in HR 3831 is not simply a dipole; details of the

phase of the pulsation mode as a function of rotational phase (which determines the observer's viewing aspect) remain unexplained. There are also clearly determined frequencies at the first and second harmonics of the principal frequency which have a pattern that has so far defied theoretical explanation (Kurtz 1990; KSG).

Since HR 3831 appears to be singly periodic, it also seems to be a good candidate to try to observe evolutionary changes in the pulsation frequency. Heller & Kawaler (1988) suggested that such changes may be detectable in a time-span of only about 10 years. Recent attempts to detect evolutionary frequency changes in four other roAp stars, HD 101065 (Martinez & Kurtz 1990), HD 137949 (Kurtz 1991), HD 134214 (Kurtz *et al.* 1991) and HD 217522 (Kreidl *et al.* 1991), have not given clear results. HD 101065 and 137949 appear to have frequencies changing at a rate respectively one and two orders of magnitude greater than expected; the data for HD 134214 cannot be phased with a single frequency, even though there is little or no indication of any secondary frequencies in any of the individual data sets; and HD 217522 appears to have changed pulsation modes.

Thus, there is substantial incentive for new observations of HR 3831. We have obtained 268 hr of new high-speed

photometric observations of HR 3831; 167 hr were obtained during a time-span of only 17 d from both Cerro Tololo Inter-American Observatory (CTIO) and the South African Astronomical Observatory (SAAO) for a duty cycle of 41 per cent, substantially reducing the 1 d^{-1} aliases inherent in single-site observations. Most of the new data have been obtained with a 1.0-m telescope which has reduced the noise compared to earlier observations and allowed higher accuracy frequency analysis. That frequency analysis is complex, however, and will be reported in detail in a future publication. In this paper we compare the rotation frequency and phase determined from the amplitude modulation of the pulsational light variations with those determined from the mean-light variations; both are compared with the magnetic phase determined from the observations of Thompson (1983) and Mathys (1991). The pulsation data span the years 1980 to 1991 and the mean-light data span the years 1975 to 1987.

The interesting result is that the pulsation-amplitude modulation is in phase with the magnetic variations and both are significantly out of phase with the mean-light variations. This discriminates between the oblique-pulsator model and an alternative spotted-pulsator model (Mathys 1985) in favour of the former. The magnetic poles coincide with the pulsation

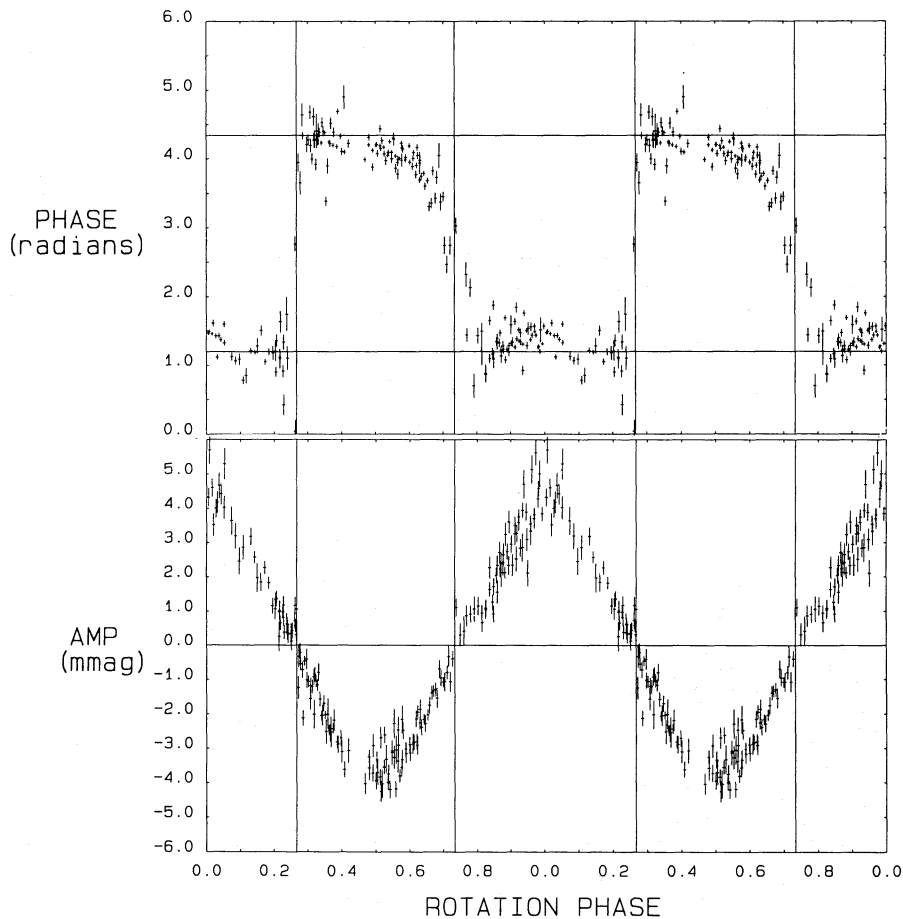


Figure 1. This plot shows pulsation phase and amplitude as a function of rotation phase for the 1991 data. The zero-point of the rotation phase has been taken to be pulsation maximum. This shows the 180° phase reversal that occurs at magnetic quadrature which is marked by the solid vertical lines. To emphasize this the amplitudes are given negative values between magnetic phases 0.267 and 0.733 where quadrature occurs.

poles, and the abundance-anomaly patches or rings which presumably give rise to the mean-light variations are asymmetrically offset from these poles.

By judiciously combining the mean-light data and the amplitude-modulation data we improve the accuracy of the rotational frequency to $\nu_{\text{rot}} = 4058.256 \pm 0.007$ nHz ($P_{\text{rot}} = 2.851982 \pm 0.000005$ d).

2 NEW PHOTOMETRIC OBSERVATIONS

New high-speed photometric observations of HR 3831 were obtained in 1990 February, March and May and in 1991 February. The 1991 observations were obtained using the 0.6-m telescope of CTIO and the 1.0-m telescope of the SAAO contemporaneously. The observations were obtained through Johnson *B* filters using continuous 10-s integrations with occasional interruptions for sky-brightness measurements. The observations were corrected for coincidence losses, sky background and extinction while the times were corrected to Heliocentric Julian Date (HJD) to an accuracy of 10^{-5} d. Some low-frequency filtering was performed and the observations were averaged to produce 40-s integrations.

Table 1 gives a journal of the new observations. Listed are the civil data of the observations, the HJD of the first 40-s integration, the number of 40-s integrations obtained, the time-span of each observing run in hours, the standard deviation of the data with respect to the mean for the run, the telescope used and the observer. The standard deviations listed include all sources of noise plus the actual variation of HR 3831.

3 DETERMINATION OF THE ROTATION PERIOD AND PHASE

HR 3831 pulsates with a frequency of $\nu = 1.42801$ mHz and with an amplitude that varies with rotational aspect (KSG). We have fitted by linear least squares the frequency $\nu = 1.42801$ mHz to sections of the HR 3831 *B* data four cycles (46.69 min) long. The data analysed are: 135 hr from 1980/1981 (Kurtz 1982), 43 hr from 1985 (Kurtz & Shibahashi 1986), 60 hr from 1986 (KSG) and the 268 hr listed in Table 1. This gave us 615 data points consisting of the central HJD of the four cycles of data and the amplitude of the 1.42801-mHz sinusoid which best fits the four cycles of data.

A Fourier analysis of these data followed by a non-linear least-squares fit showed that the frequency $\nu = 0.7012672 \pm 0.0000018$ d⁻¹ best fits the pulsation-amplitude data with no significant alias ambiguities. We do not show the amplitude spectrum here since it is similar to that shown for the mean-light data by Kurtz & Marang (1988). HR 3831 is well known to have a double-wave pulsation-amplitude variation with a 180° reversing phase (Fig. 1), so the true rotation frequency is half that derived in the Fourier analysis: $\nu_{\text{rot}} = 0.3506336 \pm 0.0000009$ d⁻¹; and $P = 2.851980 \pm 0.000007$ d. The time of pulsation-amplitude maximum is $t_0(\text{pul}) = \text{HJD } 2444576.150 \pm 0.004$.

Fig. 2 shows the pulsation-amplitude data plotted with this period and epoch. The amplitudes plotted here are actually the absolute value of the pulsation amplitude. Since HR 3831 reverses phase at magnetic quadrature, the ampli-

Table 1. Journal of new observations of HR 3831.

| Date | HJD(1) 2440000.+ | n | t hr | σ mmag | tel | obs |
|-----------|---------------------|-------|---------|------------------|-------------|-----|
| 1990 | | | | | | |
| Feb 06/07 | 7929.48704 | 282 | 3.21 | 1.78 | SAAO 1.0-m | PM |
| Feb 08/09 | 7931.35161 | 588 | 6.75 | 3.10 | SAAO 1.0-m | PM |
| Feb 10/11 | 7933.27887 | 652 | 8.49 | 1.63 | SAAO 1.0-m | PM |
| Feb 11/12 | 7934.29758 | 714 | 8.06 | 3.18 | SAAO 1.0-m | PM |
| Feb 16/17 | 7939.27405 | 332 | 7.96 | 1.41 | SAAO 1.0-m | DWK |
| Mar 06/07 | 7957.26676 | 245 | 2.94 | 3.27 | SAAO 1.0-m | DWK |
| Mar 07/08 | 7958.25538 | 659 | 7.40 | 2.98 | SAAO 1.0-m | DWK |
| Mar 08/09 | 7959.25735 | 670 | 7.48 | 1.24 | SAAO 1.0-m | DWK |
| Mar 09/10 | 7960.25354 | 597 | 7.51 | 2.41 | SAAO 1.0-m | DWK |
| Mar 10/11 | 7961.25401 | 712 | 7.97 | 3.10 | SAAO 1.0-m | DWK |
| Mar 11/12 | 7962.25494 | 642 | 7.18 | 1.70 | SAAO 1.0-m | DWK |
| Mar 12/13 | 7963.25112 | 683 | 7.66 | 1.51 | SAAO 1.0-m | DWK |
| May 09/10 | 8021.20886 | 114 | 1.35 | 2.97 | SAAO 1.0-m | DWK |
| May 04/05 | 8016.24234 | 275 | 3.60 | 1.21 | SAAO 0.75-m | PT |
| May 05/06 | 8017.25278 | 242 | 2.92 | 2.66 | SAAO 0.75-m | PT |
| May 06/07 | 8018.23342 | 358 | 4.08 | 3.17 | SAAO 0.75-m | PT |
| May 10/11 | 8022.20210 | 383 | 4.36 | 1.57 | SAAO 1.0-m | DWK |
| May 11/12 | 8023.20137 | 76 | 0.86 | 1.74 | SAAO 1.0-m | DWK |
| May 12/13 | 8024.19670 | 136 | 1.55 | 2.83 | SAAO 1.0-m | DWK |
| Σ | | 8360 | 101.33 | | | |
| 1991 | | | | | | |
| Feb 15/16 | 8303.57449 | 574 | 6.93 | 2.88 | CTIO 0.6-m | AK |
| Feb 16/17 | 8304.56144 | 553 | 6.33 | 2.41 | CTIO 0.6-m | AK |
| Feb 17/18 | 8305.67489 | 450 | 5.19 | 1.50 | CTIO 0.6-m | AK |
| Feb 18/19 | 8306.52016 | 526 | 6.06 | 2.87 | CTIO 0.6-m | AK |
| Feb 19/20 | 8307.28164 | 728 | 8.33 | 2.05 | SAAO 1.0-m | PM |
| | 8307.53234 | 505 | 5.77 | 2.47 | CTIO 0.6-m | AK |
| Feb 20/21 | 8308.31627 | 345 | 3.92 | 2.29 | SAAO 1.0-m | PM |
| | 8308.54462 | 739 | 8.40 | 2.00 | SAAO 1.0-m | PM |
| Feb 21/22 | 8309.31339 | 241 | 3.02 | 3.17 | SAAO 1.0-m | PM |
| | 8309.52231 | 398 | 4.60 | 2.88 | CTIO 0.6-m | AK |
| Feb 22/23 | 8310.35520 | 228 | 2.72 | 1.93 | SAAO 1.0-m | PM |
| | 8310.54258 | 485 | 5.59 | 3.05 | CTIO 0.6-m | AK |
| Feb 23/24 | 8311.25429 | 754 | 8.68 | 1.60 | SAAO 1.0-m | PM |
| Feb 24/25 | 8312.27410 | 711 | 8.16 | 2.98 | SAAO 1.0-m | PM |
| Feb 25/26 | 8313.26636 | 637 | 7.26 | 2.85 | SAAO 1.0-m | PM |
| | 8313.52886 | 572 | 7.06 | 3.42 | CTIO 0.6-m | AK |
| Feb 26/27 | 8314.26447 | 678 | 7.65 | 1.65 | SAAO 1.0-m | DWK |
| | 8314.54184 | 610 | 6.94 | 2.35 | CTIO 0.6-m | AK |
| Feb 27/28 | 8315.26044 | 687 | 7.73 | 2.37 | SAAO 1.0-m | DWK |
| | 8315.52102 | 738 | 8.42 | 2.08 | SAAO 1.0-m | DWK |
| Feb 28/29 | 8316.26050 | 654 | 7.63 | 4.14 | SAAO 1.0-m | DWK |
| | 8316.54812 | 574 | 7.43 | 3.31 | CTIO 0.6-m | AK |
| Mar 01/02 | 8317.25902 | 676 | 7.61 | 1.83 | SAAO 1.0-m | DWK |
| Mar 02/03 | 8318.26292 | 673 | 7.60 | 1.65 | SAAO 1.0-m | DWK |
| Mar 04/05 | 8320.25489 | 696 | 7.84 | 2.13 | SAAO 1.0-m | DWK |
| | | 14432 | 166.87 | | | |

tude would more correctly be plotted as negative between magnetic phases 0.267 and 0.733 as it is in Fig. 1. However, because the mean-light variations show a double wave each rotation (Kurtz & Marang 1988), we have chosen to analyse the absolute value of the pulsation-amplitude variations so that they also undergo a double wave each rotation. In this way the comparison of the pulsation-amplitude variations and the mean-light variations is made under similar conditions.

We have reanalysed Kurtz & Marang's (1988) normalized mean-light data and find essentially the same frequency they found: $\nu = 0.7012712 \pm 0.0000035$ d⁻¹. Of course, the rotation frequency is half this. The important point here is that the frequency derived from the pulsation-amplitude variations, $\nu = 0.7012672 \pm 0.0000018$ d⁻¹, and from the mean-light variations, $\nu = 0.7012712 \pm 0.0000035$ d⁻¹, differ by only 0.0000040 ± 0.0000037 d⁻¹, which is a little over 1σ difference. That is, these two independently derived frequencies are the same.

If we should wish to try to explain the pulsation-amplitude modulation in terms of the excitation of a rotationally perturbed triplet of ($\ell=1$, $m=-1, 0, 1$), then the pulsation-amplitude modulation frequency would be $(1-C_{nl})\Omega$ (Ledoux 1951) and the mean-light modulation frequency would be Ω , where Ω is the rotation frequency, presuming that the abundance-anomaly concentrations and magnetic field are frozen in. We can put an upper limit on $C_{nl} \leq 2 \times 10^{-5}$ at the 3σ confidence level from the above derived frequencies, strongly arguing that rotationally perturbed m modes are not a viable explanation for the amplitude modulation of the pulsation in HR 3831; C_{nl} is theoretically expected to be in the range 0.001 to 0.01.

In Fig. 3 we have normalized the pulsation-amplitude variations to the same range as the mean-light variations normal-

ized by Kurtz & Marang (1988) and plotted the two with the same epoch. It is obvious that the time of mean-light extremum lags behind the pulsation-amplitude maximum. We can quantify this. From our least-squares fits the time of pulsation-amplitude maximum is $t_0(\text{pul}) = \text{HJD } 2444576.150 \pm 0.004$; the time of mean-light extremum is $t_0(\text{mean}) = \text{HJD } 2444576.327 \pm 0.006$. Hence $t_0(\text{pul}) - t_0(\text{mean}) = -0.177 \pm 0.007 \text{ d} = -0.062 \pm 0.002$ rotation periods, a 25σ difference, which is what we see in Fig. 3.

Since the frequency of variation of both the mean light and pulsation amplitude are the same, we have shifted the latter in phase to match the mean-light variations and then used the combined normalized data set to refine the rotation frequency. Fig. 4 is a plot of the mean-light variations and the pulsation-amplitude variations as a function of time

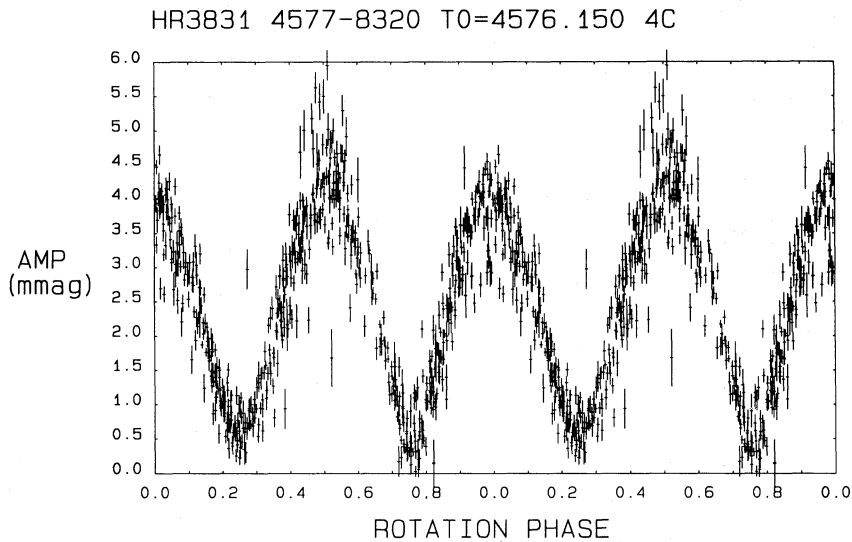


Figure 2. Pulsation amplitude determined by fitting $\nu = 1.42801$ mHz to sections of the B data four cycles (46.69 min) long are plotted against rotational phase where phase zero has been taken to be the time of pulsation-amplitude maximum.

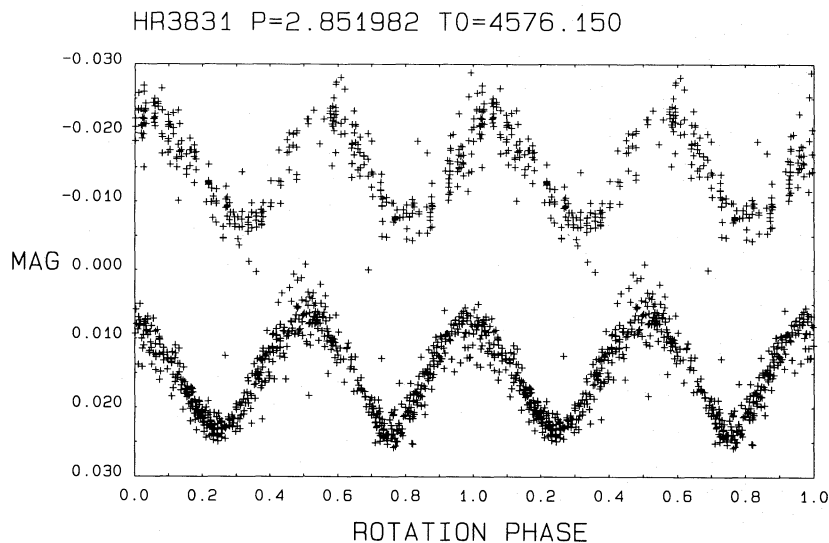


Figure 3. The top curve shows the normalized mean-light data and the bottom curve shows the normalized pulsation-amplitude data plotted against rotation phase, where phase zero is the time of pulsation-amplitude maximum. The time of mean-light extremum obviously lags behind the pulsation maximum by 0.062 ± 0.002 rotation periods. The two curves have been arbitrarily given a 0.03-mag vertical shift with respect to each other and have been scaled to the same amplitude for display purposes.

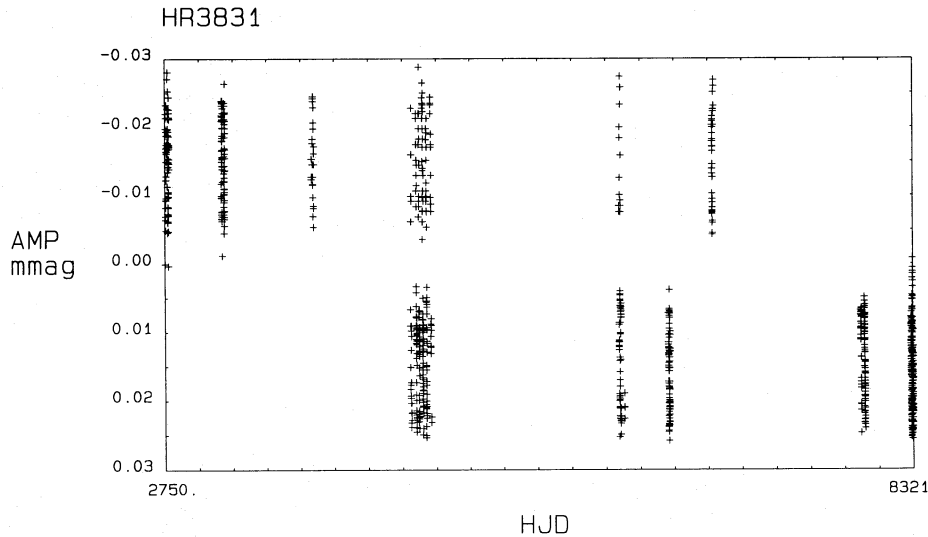


Figure 4. This shows the mean-light data on top and the pulsation-amplitude data on the bottom just as in Fig. 3, except that they are plotted linearly against time instead of being phased. This shows the relative time distribution of the two data sets. The apparent vertical scatter is mostly true variability as can be seen in Fig. 3.

from 1975 to 1991 showing the time distribution of the data sets. From our Fourier analysis and non-linear least-squares fit of the derived frequency and its subharmonic (which is at the true rotation frequency), we find $\nu = 0.7012666 \pm 0.0000012 \text{ d}^{-1}$. The subharmonic was fitted because the primary and secondary maxima and minima are not exactly equal for the pulsation-amplitude variations, as can be seen in Fig. 3. Neglect of the subharmonic only changes the derived frequency by 1σ . Since the rotation frequency is half the frequency derived in the Fourier analysis, the refined rotation frequency for HR 3831 is $\nu_{\text{rot}} = 0.3506333 \pm 0.0000006 \text{ d}^{-1} = 4058.256 \pm 0.007 \text{ nHz}$, or $P_{\text{rot}} = 2.851982 \pm 0.000005 \text{ d}$.

We have reanalysed Thompson's (1983) nine magnetic measurements and eight of Mathys' (1991) magnetic measurements, excluding four 1986 measurements which he considered unreliable. The period determined from these data, $P = 2.8519 \pm 0.0011 \text{ d}$, is of low accuracy, but is consistent with the much more accurate period determinations made from the pulsation-amplitude variations and the mean-light variations above. We therefore assume that the magnetic period is the same ($P = 2.851982 \text{ d}$) as the rotation period we determined in the last paragraph. Fitting that period by least squares to the magnetic data gives a time of positive magnetic extremum of $t_0(+\text{ext}) = \text{HJD } 2444574.743 \pm 0.031$ and a time of negative magnetic extremum of $t_0(-\text{ext}) = \text{HJD } 2444576.169 \pm 0.031$. Mathys (1991) gives a maximum $t_0 = \text{HJD } 2440001.66 \pm 0.04$, but it can be seen from his table 4 that magnetic phase zero refers to negative extremum, not positive extremum. Using our period $P = 2.851982 \text{ d}$ and $t_0(-\text{ext}) = \text{HJD } 2444576.169 \pm 0.031$ we find $t_0(-\text{ext}) = \text{HJD } 2440001.59$; using the period Mathys used of $P = 2.851962 \text{ d}$ we get $t_0(-\text{ext}) = \text{HJD } 2440001.62$, both of which are consistent with Mathys' (1991) determination. Mathys correctly points out that the time of positive magnetic extremum given by Kurtz & Marang (1988) is incorrect. It can be seen from the phase of

the least-squares fit of the rotation period to Thompson's magnetic data given by Kurtz & Marang that the correct time of positive magnetic extremum from their analysis is $t_0(+\text{ext}) = \text{HJD } 2440000.21 \pm 0.06$ and $t_0(-\text{ext}) = \text{HJD } 2440001.64 \pm 0.06$ which are consistent with the above determinations.

The time of negative magnetic extremum, $t_0(-\text{ext}) = \text{HJD } 2444576.169 \pm 0.031$, coincides with the time of pulsation-amplitude maximum, $t_0(\text{pul}) = \text{HJD } 2444576.150 \pm 0.004$; the difference between them, $t_0(\text{pul}) - t_0(-\text{ext}) = -0.019 \pm 0.031 \text{ d}$, is insignificant. The time of mean-light extremum, $t_0(\text{mean}) = \text{HJD } 2444576.327 \pm 0.006$, differs from negative magnetic extremum by $t_0(\text{mean}) - t_0(-\text{ext}) = 0.158 \pm 0.032 \text{ d}$, a 5σ difference.

4 DISCUSSION

We interpret the roAp stars to be oblique pulsators, i.e. oblique rotators in which the pulsation axis and the magnetic axis coincide (see Kurtz 1990; Matthews 1991). An alternative model is the spotted-pulsator model (Mathys 1985) in which the pulsation axis coincides with the rotation axis, but inhomogeneous distributions of surface flux, the ratio of surface flux to radius variations, and phase lag between the flux and radius variations give rise to the observed amplitude modulation with the rotation period.

If we suppose that these proposed inhomogeneous distributions follow the mean-light variations, then our determination that the pulsation-amplitude modulation is in phase with the magnetic variations, and the mean-light variations are not, suggests that oblique pulsation, rather than spotted pulsation, is the better model. However, if the relevant inhomogeneities of the spotted-pulsator model are aligned with the magnetic field, and are separate from the cause of the mean-light variations, then our observations do not distinguish between oblique pulsation and spotted pulsation.

A similar situation to that in HR 3831 also occurs in the roAp star HR 1217 where the pulsation-amplitude maximum differs from the mean-light extremum by 3σ (Kurtz *et al.* 1989). For HR 1217, as for HR 3831, the mean-light extremum lags behind the time of pulsation-amplitude maximum; in this case by 0.031 ± 0.010 rotation periods. This may be a consequence of the rare-earth abundance-anomaly concentrations lagging behind the magnetic pole in rotational longitude (Bonsack 1979). The time of magnetic maximum is too poorly determined in HR 1217 to distinguish whether the magnetic axis coincides with the pulsation axis, or the abundance-anomaly distribution, or neither. More accurate magnetic phases are highly desirable for these stars. Mathys (1991) gives two new measurements of the magnetic field in HR 1217, but these are insufficient to improve the phase determination sufficiently for our purposes. We find from Preston's (1972) TiCrFe-line magnetic measurements, Landstreet's (unpublished) H β measurements and Mathys' (1991) Fe-line measurements that $t_0(+\text{magnetic maximum}) = \text{HJD } 2446744.31 \pm 0.28$, which is intermediate between (and indistinguishable from) Kurtz *et al.*'s (1989) time of pulsation amplitude $t_0(\text{pul}) = \text{HJD } 2446744.15 \pm 0.10$ and Kurtz & Marang's (1987) time of mean-light minimum $t_0(\text{mean}) = \text{HJD } 2446744.54 \pm 0.09$.

Mathys (1991) finds a magnetic period for HR 1217 of $P = 12.4610 \pm 0.0025$ d; the rotation period determined from mean-light observations and Eu II line strengths by Kurtz & Marang (1987) is $P = 12.4572 \pm 0.0003$ d. Mathys suggests that the latter period is inconsistent with the magnetic measurements, an important result if it is correct. However, we do not consider the 1.5σ formal difference between the two period determinations to be convincing, and we do not see any convincing differences in the magnetic curves plotted with the two different periods [see figs 38 and 39 in Mathys (1991)]. We suggest that one rotation period is sufficient to explain the mean-light variations, line-strength variations, pulsation-amplitude variations and magnetic variations in HR 1217 and that the best determination of that period is the $P = 12.4572 \pm 0.0003$ d given by Kurtz & Marang (1987).

In general, the magnetic Ap stars may be modelled using an axisymmetric oblique-rotator model with cylindrical symmetry. Differences in the field strengths at opposite poles can equivalently be modelled either by an offset dipole, or by a linear sum of spherical harmonics. See Landstreet's (1988) derivation of the abundance distribution and magnetic-field configuration of 53 Cam for the best modelling of this sort that has been done to date.

It is clear, however, that cylindrical symmetry is not adequate to explain all Ap stars. HR 3831 and 1217 are probably two such cases. This problem has been known for some time (see Stift 1974, 1975; Michaud 1980; Landstreet 1980), but there are no published, high-accuracy determinations of

non-cylindrically symmetric magnetic-field geometry and abundance distributions (i.e. including spherical-harmonic contributions from $m \neq 0$ components). In Michaud's (1980) view the abundance concentrations in the magnetic Ap stars occur in rings about the magnetic poles where axisymmetric ($m = 0$) dipole-plus-quadrupole field components give rise to rings of closed field lines which trap diffusing ions. The lag between the pulsation maximum and mean-light extremum in HR 3831 and 1217 indicates a deviation from cylindrical symmetry which makes the interpretation of the pulsation-amplitude modulation in terms of the oblique-pulsator model more complex than has yet been considered.

ACKNOWLEDGMENTS

We thank Dr Gautier Mathys for useful conversations and for providing his new magnetic data prior to publication. DWK and PM acknowledge financial support from the Foundation for Research Development.

REFERENCES

- Bonsack, W. K., 1979. *Publs astr. Soc. Pacif.*, **91**, 648.
 Heller, C. H. & Kawaler, S. D., 1988. *Astrophys. J. Lett.*, **329**, L43.
 Kriedl, T. J., Kurtz, D. W., Kuschnig, R., Bus, S. J., Birch, P. B., Candy, M. P. & Weiss, W. W., 1991. *Mon. Not. R. astr. Soc.*, **250**, 477.
 Kurtz, D. W., 1982. *Mon. Not. R. astr. Soc.*, **200**, 807.
 Kurtz, D. W., 1990. *Ann. Rev. Astr. Astrophys.*, **28**, 607.
 Kurtz, D. W., 1991. *Mon. Not. R. astr. Soc.*, **249**, 468.
 Kurtz, D. W. & Shibahashi, H., 1986. *Mon. Not. R. astr. Soc.*, **223**, 557.
 Kurtz, D. W. & Marang, F., 1987. *Mon. Not. R. astr. Soc.*, **229**, 285.
 Kurtz, D. W. & Marang, F., 1988. *Mon. Not. R. astr. Soc.*, **231**, 1039.
 Kurtz, D. W., Shibahashi, H. & Goode, P. R., 1990. *Mon. Not. R. astr. Soc.*, **247**, 558 (KSG).
 Kurtz, D. W., Kriedl, T. J., O'Donoghue, D., Osip, D. J. & Tripe, P., 1991. *Mon. Not. R. astr. Soc.*, **251**, 152.
 Kurtz, D. W. *et al.*, 1989. *Mon. Not. R. astr. Soc.*, **240**, 881.
 Landstreet, J. D., 1980. *Astr. J.*, **85**, 611.
 Landstreet, J. D., 1988. *Astrophys. J.*, **326**, 967.
 Ledoux, P., 1951. *Astrophys. J.*, **114**, 373.
 Martinez, P. & Kurtz, D. W., 1990. *Mon. Not. R. astr. Soc.*, **242**, 636.
 Matthews, J., 1991. *Publs astr. Soc. Pacif.*, **103**, 5.
 Mathys, G., 1985. *Astr. Astrophys.*, **151**, 315.
 Mathys, G., 1991. *Astr. Astrophys. Suppl.*, **89**, 121.
 Michaud, G., 1980. *Astr. J.*, **85**, 589.
 Preston, G. W., 1972. *Astrophys. J.*, **175**, 465.
 Renson, P. & Manfroid, J., 1978. *Astr. Astrophys. Suppl.*, **34**, 445.
 Renson, P., Manfroid, J., Heck, A. & Mathys, G., 1984. *Astr. Astrophys.*, **131**, 63.
 Stift, M. J., 1974. *Mon. Not. R. astr. Soc.*, **169**, 471.
 Stift, M. J., 1975. *Mon. Not. R. astr. Soc.*, **172**, 133.
 Thompson, I. B., 1983. *Mon. Not. R. astr. Soc.*, **205**, 43p.